NVESTIGATING SCHOOL MATHEMATICS

TEACHERS' EDITION

VESTIGATION DISCUSSION UTILIZATION



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	Kindergarten Units P and R	Book One Units a, b, c, and d	Book Two Units e, f, g, and h
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Teachers' Edition to accompany

Investigating School Mathematics

ROBERT E. EICHOLZ

PHARES G. O'DAFFER

CHARLES R. FLEENOR

Collaborator, Teachers' Edition
THERESA BURKE

Collaborators, Reference Material and Metrication
JOHN BATES
DONALD IRWIN
J. NORMAN C. SHARP
JAMES SHERRILL



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Foreword

The *Investigating School Mathematics* series co-ordinates the precise concepts of modern mathematics with an approach that stimulates the child to actively participate in his own learning experiences. The series provides for the necessary mastery of basic number skills, and presents the material in a way that emphasizes the exciting, creative nature of mathematics. As the child becomes involved in exciting explorations and investigations, the structure and beauty of mathematics unfolds. The children are encouraged to investigate and discover ideas for themselves, to look for interesting patterns and relationships, and to develop their own generalizations. New and fascinating topics are explored not solely for their mathematical value, but also because they stimulate interest and motivate children to put forth their best efforts.

In our view, the development of a sound mathematical structure need not be hindered by an exciting, activity-oriented approach. Rather, the activity approach can and should reinforce the child's experiences as he investigates mathematical topics in an orderly, structured fashion. The same, sound mathematical structure that was called "modern" in the 1960's is present in *Investigating School Mathematics*. The important difference in this new series lies in its approach. The child learns through continual active participation in activities and investigations that lead to the discovery of each new idea.

As each new concept unfolds, the child is given an opportunity to investigate the ideas by using a wide variety of manipulative materials and activities. Then, through guided discussion, he is led to a deeper understanding of the ideas and their relation to the overall structure of mathematics. Following the investigation and discussion, he is provided with sufficient problem-solving practice to develop speed and accuracy.

The *Investigating School Mathematics* series is unprecedented in its careful provision for individual differences. Throughout each text, the child is challenged to do what he *can* do, not what someone else *thinks* he can do. Each child has the opportunity to experience individual success in an environment that stresses co-operation and communication rather than competition. This careful provision for individual differences makes the *Investigating School Mathematics* series unusually adaptable to such diverse teaching situations as ungraded schools, individual or small-group instruction, or whole-class instruction.

The essence of the *Investigating School Mathematics* series is reflected in the beliefs to which we are committed: that there are fundamental mathematical concepts which can be isolated and set forth with sharpness and clarity; that these concepts, when truly understood, provide powerful tools for extending knowledge; that children of every level should be encouraged to actively participate, to think, to question, and to seek understanding; that, although a certain body of knowledge must be passed on to each generation from preceding generations, the individual creativity of each new generation must not be stifled by pedagogy which forces upon its pupils patterns of thought which have served us well in the past but which may be inadequate for the future.

Mathematics can be successfully taught in this spirit. At every stage in the learning of mathematics the discovery of new relationships can be a delight. It is in this spirit that *Investigating School Mathematics* has been written.

The authors wish to express their appreciation to Ball State University and to the Educational Research Council of Greater Cleveland, where many of the ideas were generated and tested for the *Elementary School Mathematics* series, which served as a forerunner of *Investigating School Mathematics;* to Edith Biggs and the Nuffield Project in England, for their leadership in bringing the activity-oriented laboratory approach into prominence; to Mrs. Nancy Hildebrand, whose contributions to the teachers' manuals for *Elementary School Mathematics* are still reflected in this manual; to Theresa Burke, who assisted in the preparation of this manual by bringing, from a wealth of classroom experience, many of the activities and teaching suggestions found in each lesson; and finally, to the many teachers and children who have proved that studying mathematics can be an exciting and stimulating experience in the elementary school.

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Mathematics of the Book 6 Program

Before using Book 6 of this series in the classroom, you are urged to take the time to study *A Text for Teachers*, which comprises the last section of this Teachers' Edition. A careful study of this material will help you gain a better understanding and appreciation of the philosophy and the teaching strategy that underlie the mathematics of the *Investigating School Mathematics* program.

All of the important mathematical concepts that were covered in Book 5 are reviewed, expanded, and reinforced in the Book 6 program.

In structuring Book 6, we have assumed some minimal understanding and skill on the part of the children. We expect that children know addition and subtraction facts through 18 and multiplication and related division facts through 81. The children should also be able to find sums and differences that involve regrouping and should have some skill in finding products involving two-digit factors and in working long division with two-digit divisors. They should also have an understanding of the basic ideas concerning fractional numbers and should be able to build and test sets of equivalent fractions.

All of the preceding ideas are reviewed thoroughly but briefly in Book 6. The following ideas are introduced or expanded in the Book 6 program.

Sets and logic: identification of sets; quantifiers (all, none, some); attribute pieces; similarities and differences; number and geometric patterns

Whole-number concepts: place value through 24 places; exponents and scientific notation; estimation; multiplication using three- and four-digit factors; extension of the division algorithm using two- and three-digit divisors; arithmetic mean (average); numeration systems other than base ten

Fractional - number concepts: equivalent fractions; lowest-terms fractions; inequalities; basic principles for addition and multiplication of fractional numbers; addition and subtraction; mixed numerals and improper fractions; multiplication and division

Number theory: factors and factor trees; prime numbers; union and intersection of sets; greatest common factor; least common multiple; modular (clock) arithmetic

Decimals: decimal names for fractional numbers; addition, subtraction, multiplication, and division using decimals; mixed decimal numerals

Ratio and Percent: equal ratios; scale drawings; meaning and notation for percent; discount; sales price; interest; problem solving

Geometry and Measurement: plane and space figures; congruent figures; symmetry; cross sections; tessellations; length; area; surface area; volume; circumference and area of circles; Pythagorean Theorem; graphing geometric figures; rotations; translations; similar figures; magnifications

Probability: experiments; equally likely outcomes; meaning of probability

Integers: negative numbers; basic principles for addition; adding integers; graphing integers and functions

A wide variety of story problems, optional exercises, flow charts, and special challenge problems (called *Think* problems) are integrated throughout the text to help the children improve their skills in problem analysis and problem solving.

Teachers' Bibliography

The books in the following list will provide a broader understanding of the overall mathematical development in the *Investigating School Mathematics* series.

Forbes, Jack, and Robert Eicholz, *Mathematics for Elementary Teachers* (Menlo Park, Calif.: Addison-Wesley, 1971).

Biggs, Edith, and James MacLean, *Freedom to Learn: An Active Learning Approach to Mathematics* (Don Mills, Ont.: Addison-Wesley, 1969).

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Dienes, Z. P., *Building Up Mathematics* (London: Hutchinson Educational Ltd., 1960; available from J. M. Dent & Sons (Canada) Ltd., Don Mills, Ont.).

Copeland, Richard, *How Children Learn Mathematics: Teaching Implications of Piaget's Research* (New York: The Macmillan Co., 1970).

The Schools Council, *Mathematics in Primary Schools* (Curriculum Bulletin No. 1, 1964; available from Selective Educational Equipment, Newton, Mass.).

Boyer, Carl B., *A History of Mathematics,* (New York: John Wiley & Sons, Inc., 1968).

Elliott, H. A., James MacLean, and Janet Jorden, *Geometry in the Classroom: New Concepts and Methods* (Toronto, Ontario: Holt, Rinehart and Winston of Canada, Ltd., 1968).

National Council of Teachers of Mathematics, *Insights into Modern Mathematics* (23rd Yearbook, 1957); *The Growth of Mathematical Ideas, Grades K-12* (24th Yearbook, 1959); *Enrichment Mathematics for the Grades* (27th Yearbook, 1963); *Topics in Mathematics for Elementary School Teachers* (29th Yearbook, 1964); *More Topics in Mathematics for Elementary School Teachers* (30th Yearbook, 1969), (Washington, D.C.:

National Council of Teachers of Mathematics).

Newman, J. R., *The World of Mathematics* (New York: Simon and Schuster, 1956; available from Musson Book Company, Don Mills, Ont.).

School Mathematics Study Group, *Studies in Mathematics, Volume IX, A Brief Course in Mathematics for Elementary School Teachers,* Revised Edition (Stanford University, 1963).

Members of the Association of Teachers of Mathematics, *Notes on Mathematics in Primary Schools* (New York: Cambridge University Press, 1967; available from the Macmillan Company of Canada Ltd., Toronto).

Williams, E. M., and Hilary Shuard, *Elementary Mathematics Today, Grades 1–8* (Menlo Park, Calif.: Addison-Wesley, 1971).

Teaching the Book 6 Program

Design Features of Book 6

Each lesson is titled with either a question or a provocative phrase inviting the child to explore a given idea. The core lessons of Book 6 are designed in one of two ways: those lessons which include an investigation activity begin on the left-hand page with the investigation, followed by a set of discussion exercises and then by a set of exercises on the right-hand page for the children to work independently. (The titles of these sections are *Investigating the Ideas, Discussing the Ideas,* and *Using the Ideas,* respectively.) Other lessons are designed around a set of discussion exercises, titled *Discussing the Ideas,* and a set of independent exercises, titled *Using the Ideas.*

Many lessons throughout the book contain starred exercises for enrichment and a *Think* problem for the more able or interested children. Each chapter contains a chapter review (*Reviewing the Chapter*) and every chapter, except the first two, contains at least one cumulative review (*Keeping in Touch*).

Though each lesson in Book 6 is presented on facing pages, this format may be treated with considerable flexibility. You may find that for some lessons you will want to spend your entire allotment of time for mathematics on the Investigation and Discussion, saving the exercises for another day. In other lessons, however, you may find that you can cover as many as two facing-page lessons in one day. Keep in mind, though, that, in general, the core lessons are designed to be used for a single mathematics lesson in one day.

Color is used functionally throughout the text whenever it is felt that color-coding of numerals and symbols will facilitate learning and understanding of key concepts.

Teaching Strategies for Book 6

While specific teaching strategies will be made clear through your teachers' edition notes, there is a broad general plan for the teaching strategy throughout the book. The organization of the teachers' manual, as well as the material in the child's book, continually suggests this strategy. It is intended that each day's lesson in which the child is presented with a new concept be divided into these parts: Preparation, Investigation, Discussion, Using the Exercises, and when appropriate, a follow-up. The preparation usually should be kept fairly short, and care should be taken to see that this work does not preempt either the Investigation or the Discussion. Generally, the Preparation should do nothing more than provide the children with that readiness which they need before they begin the Investigation. The Investigation presents the rudiment of the concept treated in the lesson and should be the "main event" in terms of pupil activity and involvement in the unfolding of the concept.

In general, the Investigation should be done by the children either independently or in small groups. Think of the Investigation as a student-centred activity. It is fully anticipated that the students will grope, question, search, and explore. Most Investigations are designed to provide for individual differences; that is, the child is frequently asked to perform a certain task as many ways as *he* can, or to find how many ways he can do a certain thing. By presenting the child with this type of challenge, at least some degree of success is assured. That is, your slowest student will find that he can do something more than one way, while your more able children will find many ways to do a given task. Thus, as you guide the children through an investigation, it is important for you to recognize that they will achieve in widely differing ways, and that you should give recognition for all levels of achievement. Perhaps the most important thing to remember in working with the children during the Investigation is to encourage them to do the thinking and exploring. It is their section. Do not help them too much.

Following the Investigation, the children are given an opportunity in the Discussion section to talk about what they did and to summarize the mathematical ideas of the lesson in preparation for working independently in the Using the Ideas section. Generally, the beginning discussion exercises are designed to stimulate the children to talk about what they did in the Investigation. You should encourage them to discuss the various methods that they used to investigate and explore the concepts. Also, you should follow your teachers' guide carefully to be sure that whatever mathematical ideas are to be developed in the section are actually summarized and understood by the children.

The section titled *Using the Ideas* (called *Using the Exercises* in the teachers' edition) does exactly what the title implies. The children, having come through the first three parts of the day's lesson—the Preparation, the Investigation, and the Discussion—should be ready to work on their own to use the ideas of the lesson. Again, you should

provide for individual differences in assigning work in this section; in other words, base your assignments on the needs and abilities of the children. The exercise sets in this section are generally graded, beginning with the easier exercises and ending with the more challenging (starred) problems. Often, there will be a challenging puzzle-type problem for a given lesson. From time to time, a fairly easy challenge problem is provided to encourage less able children to attempt it. Also, quite often, all of the children will benefit from a discussion of the challenge problem.

At the top of each left-hand page of the teachers' edition, under the heading *Objective(s)*, the goal of the lesson is stated in terms of what the child should be able to do as a result of the lesson. This objective summarizes the key idea of the lesson in terms of the child's performance. Throughout the teaching suggestions for the Investigation and Discussion, the most important ideas of each lesson are reemphasized and clarified. It is important that you carefully consider the objective of each lesson so that you know how to direct the children through the development of ideas to the desired goal.

Following the statement of the objective for each lesson, the teaching suggestions follow the format of the general teaching strategy mentioned above. The Preparation lists any materials the child will need for the Investigation, and, occasionally, materials recommended for use by the teacher during the Discussion. The preparation section is designed to review ideas, motivate children, or give them necessary background information regarding the Investigation. Depending on your classroom organization, you may choose to use only those Preparations which are essential to the Investigation. At times it is recommended that you have children begin immediately with the Investigation.

The next three sections of the teaching suggestions correspond to the sections of the child's text:

Investigating the Ideas – Investigation

Discussing the Ideas – Discussion

Using the Ideas – Using the Exercises

Each section contains teaching suggestions related specifically to the corresponding section in the child's text. When lessons in the child's text deviate from the standard format, the teaching suggestions fall under the heading of Discussion or Using the Exercises or both.

A section titled *Mathematics* is included in certain lessons to provide background or to clarify for the teacher the principal mathematical concepts treated in the lesson. This section is strictly for the teacher; terms used here need not be used with the children other than as indicated in the teaching suggestions or in the child's text.

The *Follow-up* suggests various activities, games, or worksheets to help reinforce the concepts developed in the lesson. These are included only as suggestions to be used as your schedule allows. However, some of these activities and many of the materials recommended in the *Resources for Active Learning* may be used by the children independently at various activity or free periods throughout the day.

Since effective use of the investigative approach requires knowledge of the materials available for activity-oriented classrooms, suggestions are provided in the section titled *Resources for Active Learning*. The lists of resources in the chapter introductions and in most of the lessons offer an ample start. Provided some of these resources are available, you should be able to use or adapt the ideas contained in them to various situations.

The "Manipulative Devices" can be used to support the lesson. The "Commercial Games" and "General Activities" in the Chapter Introduction should be useful as continuous activities throughout the chapter, for review or practice of basic skills and concepts. The Resources listed for specific lessons are directly related to those lessons' objectives. Choose one or two, and try them out in a variety of situations.

At the time of this writing, the authors cited those resources which are of high quality and which directly complement the active-learning approach. Familiarize yourself with those materials which have been marketed subsequently. Check with your principal or supervisor for more recent materials to support this type of learning approach. Such materials can often be obtained from your local supplier. Additional sources are listed on pages xi and xii.

Provision for Individual Differences

Minimum, average, and maximum assignments are provided for each lesson other than review lessons. These assignments are given to assist you in providing for the individual needs of the children. It is *not* intended that you give the minimum assignment to the slower children, the average assignment to the average children, and the maximum assignment to the more able children. Rather, these designations are given to assist you in making individual assignments according to needs, abilities, and time available for each individual child. For example, if time is short and you need to move rapidly through a particular lesson, you may choose to use the minimum assignment for all children. The minimum assignment will, in general, provide the children with sufficient practice and mastery of skills to move ahead to the next lessons. On the other hand, you may sometimes choose to use the maximum assignment with slower children over a period of two or three days. Also, it is highly likely that you will not want to assign the maximum assignment to the more able children, since quite often they need less practice than some average and below average children. For example, when your more able children demonstrate the ability to perform a particular skill with great efficiency,

LONG-RANGE PLANNING CHART						
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5	84-85, 88-93, 100-107, 110-113, 120-123	4 weeks	84-113, 120-123 114-117	4 weeks	84-117, 120-123 118-119	4 weeks
6	124-135, 138-140 136-137	2 weeks	124-135, 138-141 136-137	2 weeks	124-141	2 weeks
7	142-151, 154-155	2½ weeks	142-151, 154-155 152-153	2 weeks	142-155	2 weeks
8	156-169, 178-181 170-173	3½ weeks	156-171, 176-181 172-175	3 weeks	156-171, 176-181 172-175	3 weeks
9	182-191, 202-206, 212-216 192-197	4 weeks	182-197, 202-207, 212-215, 220-223 198-201, 208-211	3½ weeks	182-215 216-219	3½ weeks
10	224-239, 242-245, 254-257 246-247, 252-253	4 weeks	224-239, 242-245, 254-257 250-253	4 weeks	224-251, 254-257 252-253	4 weeks
11	258-267, 272-275, 278, 280-281 268-271	2½ weeks	258-267, 272-275, 278-281 268-271	2½ weeks	258-275, 278-281 276-277	2½ weeks
12	282-289, 300, 302-303 290-295	2 weeks	282-292, 300-303 293-297	2 weeks	282-297, 300-303 298-299	2 weeks
13	304-307, 312-315, 326, 328-329 316-319	2½ weeks	304-319, 326-329 320-323	2½ weeks	304-323, 326-329 324-325	2½ weeks
14	330-336		330-335, 339 336-337	1½ weeks	330-339	1½ weeks
15	340-343 344-347	1 week	340-345 346-347	2 weeks	340-347, 352-354 348-351	2 weeks

they should not be made to drill excessively in that skill. In some cases, an asterisk is placed beside an assignment to indicate that the lesson could be omitted without loss of continuity in the flow of ideas.

Long-Range Planning Chart

The long-range planning chart is designed to provide the teacher with some guidelines for planning basic, average, and maximum course coverage of *Investigating School Mathematics*.

The basic course outline covers all the essential parts of the program, but provides little in-depth or extension material. The average course covers all the material of the basic course plus considerable extension material. The maximum course provides for nearly total coverage of all topics presented in the text. Optional material for each of the three courses is shown by page numbers shaded gray.

The suggested time schedule, covering 36 weeks, should be viewed only as an *aid* in helping you plan a time schedule that allows for the individual differences within your own class. You should not view it as a rigid schedule. For

example, the basic course may be used with children who are achieving below the average grade level. This same course might be used in other ways, such as with a brighter child who may move rapidly through these pages to make up for time lost through an absence.

General Suggestions

We offer two general suggestions regarding use of the chapter and page lesson notes:

(1) Read and consider each point as it applies to the immediate objectives for the lesson and the overall objectives of the unit.

(2) Do not allow the teachers' manual notes to deter you from using your own effective teaching methods or to stifle creative efforts.

Your manual does not attempt to dictate all of the activities in the day-to-day handling of your class and the individuals in it. You should use your manual as a guide to be co-ordinated with those methods which you have found to be most effective in teaching mathematics in the past. One of the key techniques in presenting a structured system of mathematics consists of developing a topic and pursuing it until you have reached a desired level of understanding with regard to that topic. This is one of the guiding philosophies behind the development of *Investigating School Mathematics*. You will notice this philosophy particularly in the development of some of the more intense sections of work. That is, when the topic is being introduced, it is explored in great detail; however, we have tried to provide interesting activities to relieve the intensity of these longer sections. Treat the relief materials with a light touch and in the spirit of having fun and playing games with mathematics.

Classroom Organization

The Book 6 program can be used in situations where the entire class works together on the same lesson, where small groups work together on the same lesson, or where individual students are allowed to proceed at their own rate of speed. You will want to employ the type of classroom organization that best suits the physical facilities of your particular situation. Teachers often find it stimulating to vary group sizes for different lessons and units of work. Smaller-size groups can often work more effectively together and allow greater opportunity to participate in the Investigation and the Discussion. Whatever class organization you choose, keep in mind that the key to the *Investigating School Mathematics* program is active student participation.

Evaluation of Progress

A child's attitude toward mathematics is often influenced by the methods used for evaluating his progress. All too often, evaluation procedures focus attention on what the child *did not* understand or master, rather than on what the child *did* accomplish. In evaluating a child's progress, try to maintain a positive view, one which capitalizes on successes and develops confidence.

Achievement and diagnostic tests for Book 6 may be obtained from the publisher. Chapter reviews as well as cumulative reviews are provided in the text, to aid in evaluation of the child's progress. However, you may find that a day-to-day evaluation of the children, often involving personal interviews, will help you determine how well a child grasps the concepts and how well he is able to apply them.

Children's Bibliography

Books listed in the Books to Explore section of the pupils' text (pages *A25-A26*) were selected to appeal to the natural interests of children as well as to provide historical background and reference material. Additional books which may be of interest to you and the children are listed below.

More Books to Explore

Adler, Irving, *Magic House of Numbers* (New York: The John Day Company, 1957. Also available in paperback from the New American Library of World Literature, 1962). An interesting book about whole numbers, number systems with bases other than ten, and number curiosities. There are also many games, card tricks, and puzzles with and without numbers. (Answers are included.)

Cutler, Ann, *Instant Math* (New York: Doubleday & Co., 1962). This book presents shortcuts for calculating based on the Trachtenberg system. Learn how to do "speedy elevens."

Dudeney, Henry, *536 Puzzles & Curious Problems* (New York: Charles Scribner's Sons, 1967). This book contains problems about arithmetic and geometry. There are magic squares, dominoes and other games, and match puzzles.

Feravolo, Rocco, *Wonders of Mathematics* (New York: Dodd-Mead, 1963). The author offers activities and problems for developing numeration systems other than base ten.

Heath, Royal Vale, *Mathemagic: Magic, Puzzles and Games with Numbers* (New York: Dover Publications, 1953). In addition to the many puzzles and magic tricks involving numbers, this book contains pointers for you and your opponents in games of strategy. The explanations of why the tricks work make this book fascinating. Especially recommended are "The Robber and the Sheep" (page 9) and "The Lost Digit" (page 45).

Hogben, Lancelot, *The Wonderful World of Mathematics* (Garden City, New York: Doubleday & Co., 1968). This book shows how the civilization of man parallels the development of mathematics as a science. It starts with the cave man and tells the story of man's progress as he learned to measure and to count, to build and to navigate, to design computers and to calculate with them.

Jonas, Arthur, *New Ways in Math* (Englewood Cliffs, N.J.: Prentice-Hall, 1962). This book uses cartoons to tell the story of mathematics—including sets, probability, and algebra. "Men in

Math" (page 62) summarizes the lives of modern mathematicians such as Einstein and VonNeumann as well as those of antiquity, such as Pythagoras and Archimedes. You might also want to read *More New Ways in Math* (1964) by the same author.

Kadesch, Robert, *Math Menagerie* (New York: Harper & Row, 1970). The author demonstrates the fun, usefulness, and beauty of the world of mathematics. Building flexagons, breaking codes, and blowing bubbles are some of the activities that reveal how mathematics works. Readers can do the suggested experiments by themselves using simple, inexpensive materials.

Ruchlis, Hy, and Jack Englehardt, *The Story of Mathematics* (Irvington-Hudson, New York: Harvey House, 1958). The illustrations depict mathematical shapes and forms as they appear in art, buildings, shells, and microscopic plants. Explanations for problems in space and experiments with drawing and cutouts add interest.

Smith, David Eugene, *Number Stories of Long Ago* (Washington, D.C.: National Council of Teachers of Mathematics, 1969). A storyteller gives an account of the numbers used by boys and girls of long ago, including their ancient methods of counting, writing numerals, and computing. Sample pages from schoolbooks hundreds of years old, number puzzles, and curious problems are also included.

Thurber, James, *The Great Quillow* (New York: Harcourt Brace Jovanovich, 1944); *Many Moons* (New York: Harcourt Brace Jovanovich, 1943). Both of these fairy tales make use of ideas of number, measurement, and comparison.

Zarchy, Harry, *Wheel of Time* (New York: Crowell, 1957). This book develops various concepts of time from years to parts of seconds and emphasizes the dependence of everything in our lives on time, including science, navigation, and motion.

Other Books of Interest

Bell, Thelma and Corydon, *The Riddle of Time* (New York: Viking Press, 1963). A history of time

Bendick, Jeanne, *Names, Sets, and Numbers* (New York: Franklin Watts, 1971). Sets, numbers, and classifying (science)

Cutler, Ann, and Rudolph McShane, *The Trachtenberg Speed System of Basic Mathematics* (Garden City, New York: Doubleday & Co, 1960). "Speedy" arithmetic

Hertzberg, Hendrik, *One Million* (New York: Simon & Schuster, 1970). Number

Hunter, James, *Fun with Figures* (New York: Dover Publications, 1956); *More Fun with Figures* (New York: Dover Publications, 1966). Puzzles

Jones, Madeline, *The Mysterious Flexagons* (New York: Crown Publishers, 1965)

Krahn, Maria and Fernando, *The Life of Numbers* (New York: Simon & Schuster, 1970). Number

Linn, Charles, *Puzzles, Patterns, Pastimes* (New York: Doubleday & Co., 1969). Puzzles

Neely, Henry, *Triangles: Getting Ready for Triogonometry* (New York: Crowell, 1962). History of trigonometry and applications

Newell, Homer, *Space Book for Young People* (New York: McGraw-Hill Book Co., 1968). Space-oriented mathematics

Razzel, Arthur, and K. G. O. Watts, Exploring Mathematics Series (Garden City, New York: Doubleday & Co.); *Circles and Curves* (1969), Geometry; *Probability—The Science of Chance* (1967), Statistics; *A Question of Accuracy* (1969), Weights and measures; *Symmetry* (1968), Mathematics; *This Is 4: The Idea of a Number* (1967), Number; *Three and the Shape of Three* (1969), Number

Stonaker, Frances, *Famous Mathematicians* (Philadelphia: J. B. Lippincott, 1966). Biographical

Bibliography of Resources for Active Learning

Abbott, Janet, et al., Franklin Mathematics Series: *From Fingers to Computers; Making and Using Graphs and Nomographs; Mathematics Around the Clock; Patterns and Puzzles in Mathematics; Probability: The Science of Chance* (Chicago: Lyons and Carnahan, 1970; available from McGraw-Hill Ryerson, Scarboro, Ont.).

Bates, John, Donald Irwin, and Garry Hamilton, *Developmental Math Cards* (Don Mills, Ont.: Addison-Wesley, 1970).

Biggs, Edith, and James MacLean, *Freedom to Learn* (Don Mills, Ont.: Addison-Wesley, 1969).

Buckeye, Donald, William Ewbank, and John Ginther, *A Cloudburst of Math Experiments*, Volumes 1, 2, and 3 (Birmingham, Mich.: Midwest Publications, 1971).

Clarkson, Dave, *Math Activity Cards* (New York: Macmillan Co., 1969).

Cohen, Donald, *Inquiry in Mathematics via the Geo-board* (New York: Walker, 1967; available from Fitzhenry & Whiteside, Don Mills, Ont.).

Cohen, Donald, *Mathematics Using String* (Danbury, Conn.: Math Media, Inc., 1968).

Cohen, Donald, *Maths Mini-lab* (Newton, Mass.: Selective Educational Equipment, Inc., 1971).

Davis, Robert, Madison Project: *Discovery in Mathematics* and *Explorations in Mathematics:* A Text for Teachers and Student Discussion Guide (Menlo Park, Calif.: Addison-Wesley, 1964).

Elementary Science Study: *Attribute Games and Problems; Geo Blocks; Mirror Cards; Pattern Blocks; Peas and Particles; Pendulums; Senior Balancing; Tangrams* (St. Louis: Webster Division, McGraw-Hill Book Co., 1968–1970).

Elliott, H. A., James MacLean, and Janet Jorden, *Geometry in the Classroom: New Concepts and Methods* (Toronto, Canada: Holt, Rinehart and Winston of Canada, Ltd., 1968).

Fletcher, Harold, Arnold Howell, and Ruth Walker, *Mathematics in Modules* (Sydney, Australia: Addison-Wesley, 1973).

Goddard, T. R., and A. W. Grattidge, *Applied Mathematics Cards* (Huddersfield, England: Schofield & Sims Ltd., 1965; available from Mafex Associates, Willowdale, Ont.).

Huff, M. Elizabeth, Donald Irwin, *Activities in Geometry for Primary Pupils* (Don Mills, Ont.: Addison-Wesley, 1973).

Jenkins, Lee, and Peggy McLean, *It's a Tangram World* (San Leandro, Calif.: Educational Science Consultants, 1971).

Johnson, Donovan, *Paper Folding for the Mathematics Class* (Washington, D.C.: National Council of Teachers of Mathematics, 1957).

Mathex: Graphing and Probability No. 6; Numeration No. 7; Operations and Problem Solving No. 8; Geometry No. 9; Measurement No. 10 (Teacher's Resource Books and pupil pages) (Toronto, Ontario: Encyclopaedia Britannica Publications Ltd., 1970).

McLane, Lyn, William Perkins, and Norman Brule, *Independent Exploration Kit* (Belmont, Mass.: Concept Co., 1968).

Members of the Association of Teachers of Mathematics, *Notes on Mathematics in Primary Schools* (Cambridge, England: Cambridge University Press, 1967; available from Macmillan of Canada, Toronto).

National Council of Teachers of Mathematics, *Experiences in Mathematical Ideas*, Volume 1 and 2 (Washington, D.C.: National Council of Teachers of Mathematics, 1970).

Nuffield Mathematics Project: *Computation and Structure* 3 and 4; *Graphs Leading to Algebra* 2; *Probability and Statistics; Problems* — Green Set, Red Set; *Shape and Size* 3 (New York: John Wiley, 1967-1970).

Pearcy, J. F. F., and K. Lewis, *Experiments in Mathematics,* Stages 1, 2, and 3 (Boston: Houghton Mifflin, 1966; available from Thomas Nelson & Sons (Canada), Don Mills, Ont.).

Read, Ronald, *Tangrams — 330 Puzzles* (New York: Dover Publications, Inc., 1965; available from General Publishing, Don Mills, Ont.).

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Seymour, Dale, and Reuben Schadler, *Creative Constructions* (Palo Alto, Calif.: Creative Publications, 1968).

Swartz, Clifford, *Measure and Find Out,* Books 1, 2, and 3 (Glenview, Ill.: Scott, Foresman & Co., 1969; available from Gage Educational Publishing, Agincourt, Ont.).

Thomason, Mary, *Modern Math Games, Activities and Puzzles* (Belmont, Calif.: Fearon Publishers, 1970; available from Clarke, Irwin & Co., Toronto).

Turner, Ether, *Teaching Aids for Elementary Mathematics* (New York: Holt, Rinehart and Winston, 1966).

Wenninger, Magnus, *Polyhedron Models for the Classroom* (Washington, D.C.: National Council of Teachers of Mathematics, 1966).

Wirtz, Robert, Morton Botel, and B. G. Nunley, *Discovery in Elementary School Mathematics* (Chicago: Encyclopaedia Britannica Educational Corp., 1963).

Suppliers of Resources for Active Learning

Addison-Wesley (Canada) Ltd., Don Mills, Ontario

CCM School Materials, Inc., Chicago, Illinois

Childcraft Education Corp., New York, New York

Collier-Macmillan Canada Ltd. Don Mills, Ontario

Concept Co., Belmont, Massachusetts

Creative Publications, Palo Alto, California

Cuisenaire Company of America, Inc., New Rochelle, New York

Edmund Scientific Co., Barrington, New Jersey

Educational Supply Co., Toronto, Ontario

Educational Teaching Aids, Chicago, Illinois

Encyclopaedia Britannica Publications Ltd., Toronto, Ontario

Fitzhenry & Whiteside, Don Mills, Ontario

Gage Educational Publishing, Agincourt, Ontario

Gamco Industries Inc., Big Spring, Texas

Geyer Instructional Aids, Fort Wayne, Indiana

J. L. Hammett Co., Braintree, Massachusetts

D. C. Heath Canada Ltd., Toronto, Ontario

Herder and Herder, Inc., New York, New York

Holt, Rinehart and Winston of Canada Ltd., Toronto, Ontario

Kendrey Mfg. Co., Palo Alto, California

Lakeshore Equipment Co., San Leandro, California

James W. Lang, Mound, Minnesota

Lano Company, Ann Arbor, Michigan

LaPine Scientific Co., Chicago

Mafex Associates, Inc., Willowdale, Ontario

Math Media Inc., Danbury, Connecticut

McGraw-Hill Ryerson, Scarboro, Ontario

Metric Aids Ltd., Toronto, Ontario

Metric Association, Inc., Waukegan, Illinois

Midwest Publications Co., Inc., Birmingham, Michigan

Milton Bradley, Springfield, Massachusetts

Moyer-Vico Ltd., Weston, Ontario

Nasco, Fort Atkinson, Wisconsin

Responsive Environments Corp., Englewood Cliffs, New Jersey

Sargent-Welch Scientific Co. of Canada, Weston Ontario

Scholar's Choice, Stratford, Ontario

Selective Educational Equipment, Newton, Massachusetts

Sigma Division, Scott Scientific, Inc., Fort Collins, Colorado

TUF, Rowayton, Connecticut

Wff 'N Proof, New Haven, Connecticut

World Wide Games, Delaware, Ohio

Chapter and Page Lesson Notes

PAGES 2–3

Objective

Given a typical lesson in the Investigating School Mathematics *text, the child will be able to distinguish its three principal parts: Investigating the Ideas, Discussing the Ideas, and Using the Ideas.*

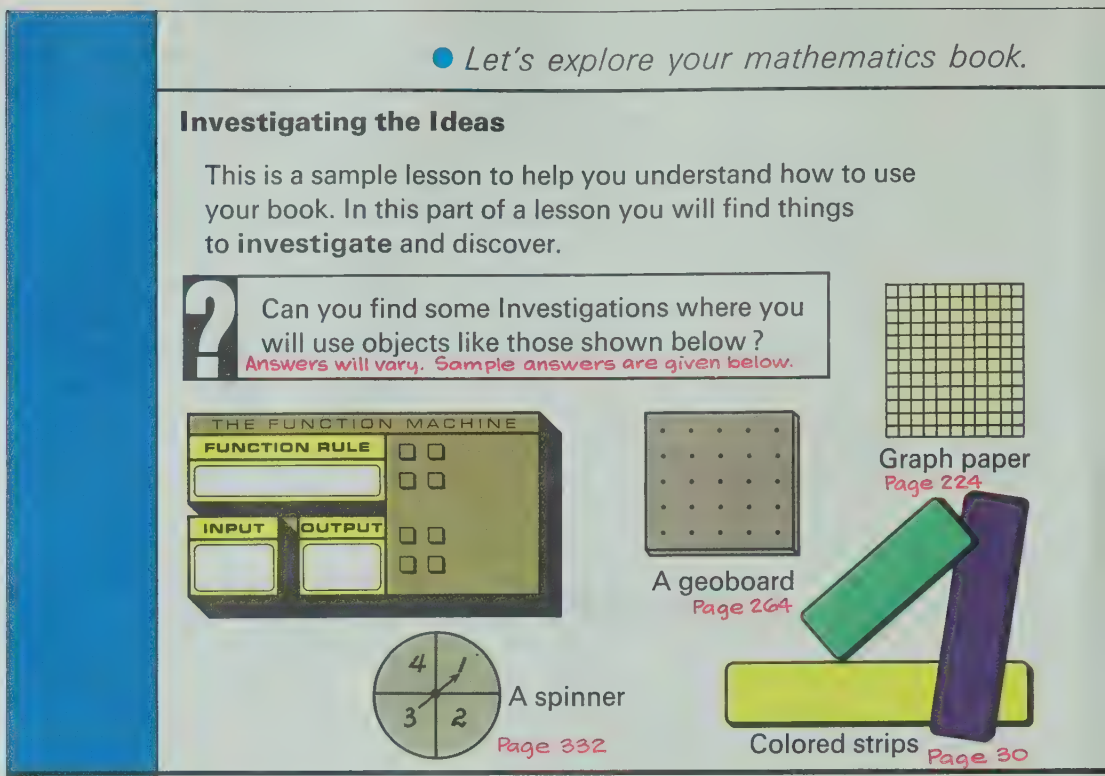
Preparation

This first step of the teaching plan for each lesson in the text is the briefest. However, it will often be the key to inspiring children with an enthusiastic approach to a lesson. In some lessons, it is essential to the Investigation; in other lessons, it simply serves as a review. In most cases, it should be limited to five minutes. Brisk and lively, it should ensure that each lesson is begun with a positive attitude.

For this introductory lesson, you might simply encourage the children to flip through the pages of the book, stopping to look at whatever interests them. After one or two minutes refer them to the title and author page and the table of contents, asking them which chapters they think they will enjoy the most. Finally, refer them to the first introductory lesson.

Investigation

As stated in the introductory material, the investigation presents the child with an opportunity to explore, with minimal teacher direction, the ideas presented. Sometimes this exploration consists of an investigation with concrete objects; at other times the child simply uses paper and pencil to explore ideas with number. In any case, this section should be totally child-centred. It is the child's opportunity to explore new ideas, or known ideas in a new way; it launches the lesson. The controlling consideration should be that in this section the child cannot "make a mistake"; since he will often learn as much from his "incorrect" approaches as from finding "correct" methods, he should be encouraged to explore things on his own.



● *Let's explore your mathematics book.*

Investigating the Ideas

This is a sample lesson to help you understand how to use your book. In this part of a lesson you will find things to **investigate** and discover.

? Can you find some Investigations where you will use objects like those shown below?
Answers will vary. Sample answers are given below.

Graph paper
Page 224

A geoboard
Page 264

A spinner
Page 332

Colored strips
Page 30

Discussing the Ideas

In this part of a lesson you will **discuss the ideas** of the lesson with your classmates and teacher. You will share your ideas with others. You will be getting ready to use the ideas.

1. Look through your book. What other kinds of objects will you use for the "Investigating the Ideas" sections?
Sample answers: Ruler, compass, protractor, counters, coins
2. Sometimes a full page is devoted to "Discussing the Ideas." Can you find a page like this? Sample answer: Page 24
3. Find a page called "Keeping in Touch." What do you think this means? Sample answer: Page 55. These pages review the important ideas presented in previous portions of the text.

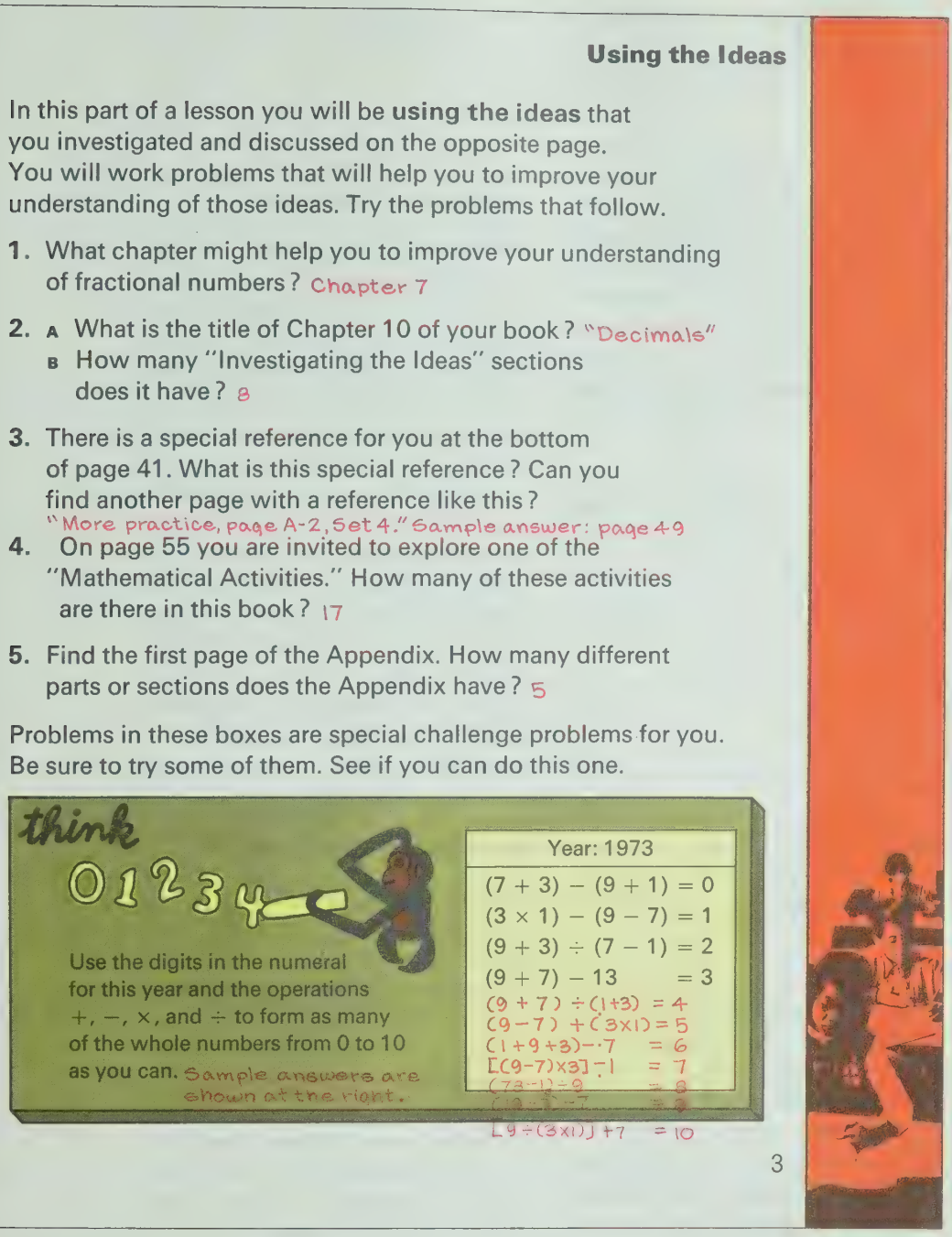
2

Read the introductory paragraph with the children, and point out the investigation question. Explain that, even though the text does not ask that they record their findings, keeping a record of their investigation results is important. Encourage the children to work independently, but you might allow discussion among small groups as the children work if that seems beneficial.

Discussion

The discussion section provides an opportunity for children to discuss the ideas they investigated and for you to guide the discussion so that the emphasis is on the main point of the lesson. It is in this section that you will sometimes want to demonstrate various computational skills or use visual materials to highlight a particular concept. The questions in the text help the children to focus on the most important concepts, but your skill in guiding discussion will be of inestimable value in this section.

Sometimes the discussion questions do not explicitly deal with the



Using the Ideas

In this part of a lesson you will be **using the ideas** that you investigated and discussed on the opposite page. You will work problems that will help you to improve your understanding of those ideas. Try the problems that follow.

1. What chapter might help you to improve your understanding of fractional numbers? Chapter 7
2. A What is the title of Chapter 10 of your book? "Decimals"
   B How many "Investigating the Ideas" sections does it have? 8
3. There is a special reference for you at the bottom of page 41. What is this special reference? Can you find another page with a reference like this?
   "More practice, page A-2, Set 4." Sample answer: page 49
4. On page 55 you are invited to explore one of the "Mathematical Activities." How many of these activities are there in this book? 17
5. Find the first page of the Appendix. How many different parts or sections does the Appendix have? 5

Problems in these boxes are special challenge problems for you. Be sure to try some of them. See if you can do this one.

think 0 1 2 3 4

Use the digits in the numeral for this year and the operations +, −, ×, and ÷ to form as many of the whole numbers from 0 to 10 as you can. Sample answers are shown at the right.

Year: 1973

$(7 + 3) - (9 + 1) = 0$
$(3 \times 1) - (9 - 7) = 1$
$(9 + 3) \div (7 - 1) = 2$
$(9 + 7) - 13 = 3$
$(9 + 7) \div (1 + 3) = 4$
$(9 - 7) + (3 \times 1) = 5$
$(1 + 9 + 3) - 7 = 6$
$[(9 - 7) \times 3] - 1 = 7$
$(7 \times 1) \div 9 = 8$
$[9 \div (3 \times 1)] + 7 = 10$

3

investigation. In such a case, it is recommended that you begin the discussion section with reference to the investigation question and then continue development of the topic with the questions in the book. For example, in this lesson, ask children to give some of their findings from the investigation. Then continue with exercise 1. Exercise 2 points out that some lessons do not have an investigation section but begin immediately with the discussion. As you discuss exercise 3, you might also point out the chapter review sections, which are titled *Reviewing the Ideas*.

Using the Exercises

The section in the child's text which is called *Using the Ideas* is referred to in the teaching notes as *Using the Exercises*. This section is intended to provide the children with an opportunity to use the ideas they have investigated and discussed. Frequently, however, you may wish to use a few of the exercises as basis for discussion. Exercises on these pages may be assigned selectively. It is rarely essential that a child do all of the exercises; on many pages, however, the exercises progress developmentally so children should do at least some parts of each exercise. Starred exercises are those which extend the concept of the lesson or give more difficult applications of it. They are not intended for all children.

Read the paragraph at the beginning of this section with the children and explain the purpose of the section. Assign the exercises as independent work.

Point out the *Think* problem at the bottom of the page, explaining that it is a special challenge. All children should be encouraged to try most such problems, although the more capable children will probably find them more appealing. When children have finished the exercises, you might want them to discuss their answers. Such a discussion at the end of the exercises will often serve as a summary or conclusion of the main points covered in the lesson.

PAGES 4–5

Objective

Given a set, some of whose elements have a common property, the child will be able to identify those elements which have the common property.

Preparation

To prepare for this lesson, you might simply ask the children to recall some geometric figures, pointing out various objects in the classroom which suggest such figures. For example, you might say, "I'm thinking of the set of objects with curved surfaces. Name an object in my set." Or, "I'm thinking of the set of objects with curved surfaces that are in this classroom. Name an object in my set." After a brief preparation such as this, explain to the children that in the investigation they will be studying a particular set.

Investigation

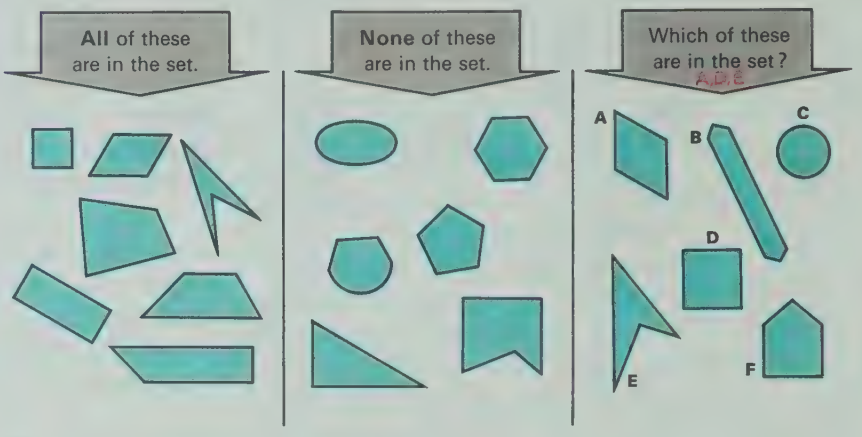
It is recommended that children work on this investigation independently. Most of them will be able to draw additional figures in the set quite readily. For those who do finish quickly, you might suggest that they think of a set and write up some questions of their own to be used later as a follow-up. Also, suggest that they think of ways of describing the set in the text. (This will be a topic of the discussion section.)

1 Sets, Logic, and Patterns

Can you name the set?

Investigating the Ideas

Here are some clues about a special set of geometric figures.



? Can you answer the question and draw some more figures that are in the set?

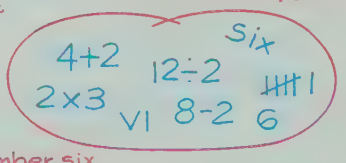
Any quadrilateral is acceptable.

Discussing the Ideas

1. How would you describe the special set of geometric figures in the Investigation? Sample answer: The figures are made up of four line segments.

2. What are some other sets of numbers or objects that you have studied in mathematics? Sample answers: Whole numbers, odd numbers, even numbers, prime numbers, geometric figures such as squares, circles, etc.

3. What is special about all the names or symbols in this set?



All are names for the number six.



4

Discussion

One of the most important ideas in this lesson is that, in order for a group or collection to be a *set*, one must be able to determine whether any given object is or is not in the set. A set is identified either by describing it or by listing its members. For example, the set in the investigation may be described as the set of all four-sided figures. Obviously, such a set cannot be identified by listing its members. Thus, a clear description is essential. However, the set whose members are the digits used in the base-ten system is a set which may easily be identified by listing its members: {1, 2, 3, 4, 5, 6, 7, 8, 9, 0}.

Notice that exercise 2 gives you an opportunity to discuss a variety of topics children have studied, such as fractional numbers, whole numbers, negative numbers, symmetric figures, and so on.

Using the Ideas

Answer the question and describe the set in each exercise.

1.

All of these are in the set.	**None** of these are in the set.	Which of these are in the set?
2, 4, 6, 16, 18, 20,	1, 3, 5 15, 17, 19	7, 8, 9, 10, 74 67, 128, 293

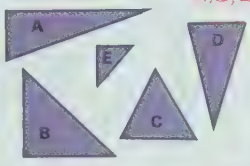
"All are multiples of 2," or "All are even numbers."

2.

All of these are in the set.	**None** of these are in the set.	Which of these are in the set?
$2+2$ $8\div2$ 2×2 IV $3+1$ $4+0$ $6-2$	$4+1$ $5-2$ $3+0$ $4\div2$ $16\div8$ 4×0	$5-1$ 1×4 $36\div9$ $4\div1$ $4-1$ $8\div4$ VI

"All are names for the number four."

3.

All of these are in the set.	**None** of these are in the set.	Which of these are in the set?
		A, B, E 

All are right triangles.

4.

All of these are in the set.	**None** of these are in the set.	Which of these are in the set?
$\frac{1}{2}$ $\frac{2}{4}$ $\frac{6}{12}$ $\frac{10}{20}$ $\frac{4}{8}$ $\frac{50}{100}$	$\frac{8}{4}$ $\frac{2}{5}$ $\frac{21}{41}$ $\frac{45}{85}$ $\frac{1}{3}$ $\frac{2}{1}$ $\frac{1}{20}$	$\frac{2}{6}, \frac{5}{10}, \frac{100}{200}$ $\frac{12}{6}$ $\frac{3}{6}$ $\frac{4}{9}$ $\frac{11}{21}$ $\frac{100}{200}$ $\frac{5}{10}$

All are members of the set of equivalent fractions $\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \cdots\}$".

★ **5.** Make up some exercises like those above and give them to a classmate to solve. Answers will vary.

5



Using the Exercises

Note that the sets used in the exercises on page 5 relate to the topics which the children will be studying all year. Therefore, after the children do this page independently, you might not only stress the answers to each question but also discuss the description of each set briefly as a review. For example, exercise 2 might remind the children of basic operations, and addition and multiplication facts for whole numbers. The star preceding exercise 5 indicates that this exercise is optional and might be used as a challenge for more capable children.

Assignments (page 5)

Minimum: 1–4. Average: 1–4. Maximum: 1–5

Mathematics

Although this lesson deals with classifying the elements of certain sets, only the language of sets is required, not the notation or symbolism of set theory.

The words *all*, *some*, and *none* are especially important in the lesson. In formal mathematical logic, these words are called *quantifiers*, since they specify, in a rather broad sense, the quantity or number of elements in a set. The word *all* is often used in discussing sets with infinitely many elements. Thus we may speak of "the set of *all* triangles" or "the set of *all* even numbers." The word *some*, as used in mathematics, means *at least one*. Thus, in reference to the set $\{1, 2, 3, 4, 5\}$, it is true that *some* of the numbers in the set are greater than 3. However, it would be false to assert that *some* of the numbers in the set are exactly divisible by 6, since none of the numbers in the set has this property. (It would be true to say that *none* of the numbers in the set is exactly divisible by 6.)

Basically, the lesson introduces certain concepts by means of examples and non-examples and gives the student the opportunity to discover the concepts through visual experiences. (For further discussion of these ideas, refer to *A Text for Teachers, Investigating Mathematics Learning:* Section III, in the appendix of this Teachers' Edition.)

Follow-up

Patterns which children might make throughout this chapter would provide material for an interesting bulletin board. For example, some children may follow the examples in the text to illustrate and display some clues for other sets so that other children can try to figure out which of the items are in each set.

Resources for Active Learning

Franklin Series: *Probability*, "Sets," pp. 5–8, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Workbook, page 1

PAGES 6–7

Objective

Given a set of attribute pieces, the child will be able to classify them according to the properties of the figures and the relationships among them.

Preparation

Materials

construction paper; scissors; circular discs or objects to be used as templates; ruler (Or, *Duplicator Masters*, page 61)

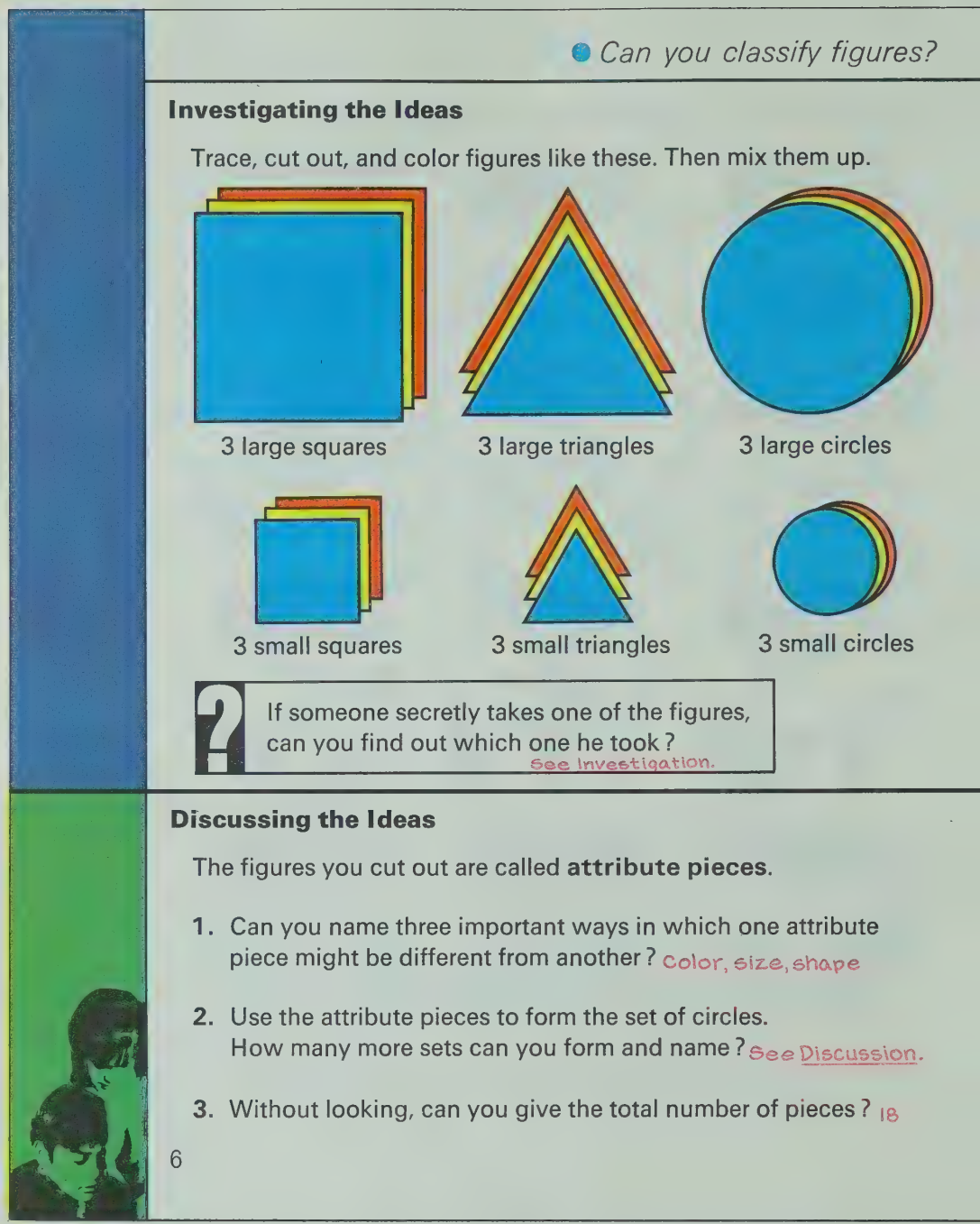
Before children can begin actual consideration of the investigation question, they must have the figures to work with. If attribute blocks or duplicated shapes are available, their use would save children the task of making them. However, the children should be able to make the squares by using rulers. Note that the bases of the triangles have the same lengths as the sides of the squares. Thus, the large triangle may be made simply by using a side of the large square and the midpoint of its parallel side to mark the three dots for the triangle. The only figure for which you might need to provide a model or template is the circle. (These figures should be retained for use in subsequent lessons.)

Investigation

One of the best ways to have children work with the attribute pieces is to have them work in pairs. For example, in this investigation, one child might secretly remove one of the figures and his partner could try to guess which figure is missing. You might encourage the children to do some of their thinking out loud.

After some practice, if, for example, the large red circle has been removed, a child might think: the 3 large triangles are there; the 3 small triangles are there; the 3 large squares are there; not all 3 of the large circles are there; the large red circle is missing.

It would be helpful to have the children repeat this activity several times.



Can you classify figures?

Investigating the Ideas

Trace, cut out, and color figures like these. Then mix them up.

3 large squares | 3 large triangles | 3 large circles

3 small squares | 3 small triangles | 3 small circles

? If someone secretly takes one of the figures, can you find out which one he took? See Investigation.

Discussing the Ideas

The figures you cut out are called **attribute pieces.**

1. Can you name three important ways in which one attribute piece might be different from another? color, size, shape
2. Use the attribute pieces to form the set of circles. How many more sets can you form and name? See Discussion.
3. Without looking, can you give the total number of pieces? 18

6

Discussion

To help the children answer the first discussion exercise, you might have them take a large red square and a small blue triangle. Then ask them to explain how these two pieces differ. Do this with several pairs of figures until children clearly see that two figures may differ in color, size, and shape.

In exercise 2 stress how various properties may be shared by a set of figures. Other sets the children may form are the set of blue figures, or the set of small figures, and so on. Encourage discussion of intersecting sets but do not belabor the terminology at this time. For example, the set of all blue pieces and the set of all circles intersect: the large and the small blue circles are in both sets.

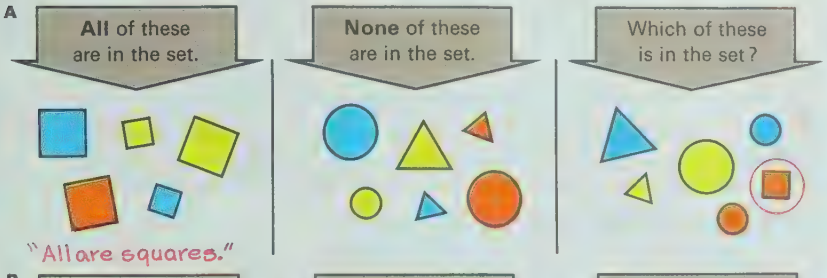
Using the Ideas

1. How many attribute pieces in each set?

A squares 6
B triangles 6
C circles 6
D small pieces 12
E large pieces 12
F red pieces 6
G yellow pieces 6
H blue pieces 6
I red squares 2
J small triangles 3
K small yellow pieces 3
L large blue triangles 1
M not-red pieces 12
N not-circle pieces 12
O small pieces with corners 6

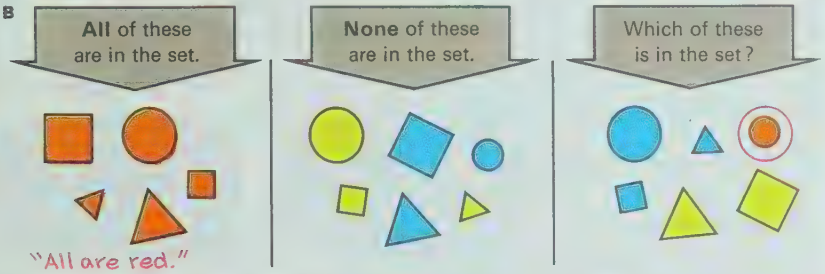
2. Answer the question and describe the set of pieces.

A



"All are squares."

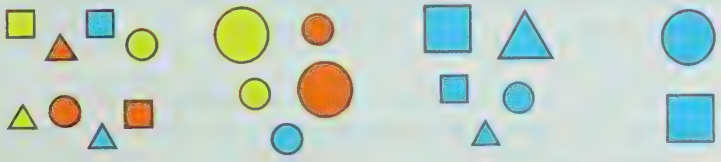
B



"All are red."

★**3.** One piece is missing from each set. Which piece is it?

A Small blue circle B Large blue circle C Large blue circle D Large blue triangle





7

Using the Exercises

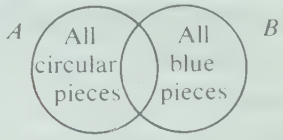
You might want the children to continue working with a partner as they do the exercises on page 7. Although exercise 1 appears to be asking only "how many," it actually provides the child with practice in classifying the figures. Note that in part O of exercise 1 the term "corners" refers to any vertex; that is, it is not restricted to square corners.

Assignments (page 7)

Minimum: 1, 2. Average: 1, 2. Maximum: 1–3.

Mathematics

The 18 attribute pieces that are utilized in this lesson can be used in many ways to illustrate some ideas of set theory and informal logic.



If A is the set of all circular pieces and B is the set of all of the blue pieces, then the *intersection* of A and B is the set consisting of the 2 blue circles. That is, $A \cap B =$ {small blue circle, large blue circle}.

The *union* of A and B ($A \cup B$) would consist of the set of all the pieces in A or in B or in both A and B. Note that set A consists of 6 circles and set B consists of 6 blue pieces but the union contains only 10 pieces, since the 2 blue circles are in both sets and can be counted only once.

If A is the set of all circular pieces, then a *subset* of A would be the set of all red circular pieces.

The idea of a *negation* in logic is involved when one of the pieces is described as being "not blue" or "not a square."

A *conjunction* is involved if a piece is described as being "red *and* square" or "small *and* triangular."

A *disjunction* occurs when a set is described as containing all of the pieces which are either "circular *or* blue." Note that this set is precisely $A \cup B$ described above. The set $A \cap B$ is the set of pieces that are "blue *and* circular."

Follow-up

The children may use the attribute pieces in many games and activities. For example, suggest that the children work as partners and take turns thinking of a set. When one child thinks of a set, his partner should select a piece and ask if this piece is in the set. After asking this about several pieces, he should be able to guess and describe the set his partner is thinking of.

Resources for Active Learning

Maths Mini-lab, Card 1, Selective Educational Equipment.

PAGES 8–9

Objective

Given any two of the attribute pieces, the child will be able to state the similarities and differences of the pieces in terms of size, shape, and color.

Preparation

Materials

attribute pieces from the previous lesson; envelopes

Although it would be suitable to begin this lesson immediately with the investigation, you might ask a few children to describe any games they made up with their attribute pieces. Use these brief descriptions to stimulate enthusiasm for the investigation.

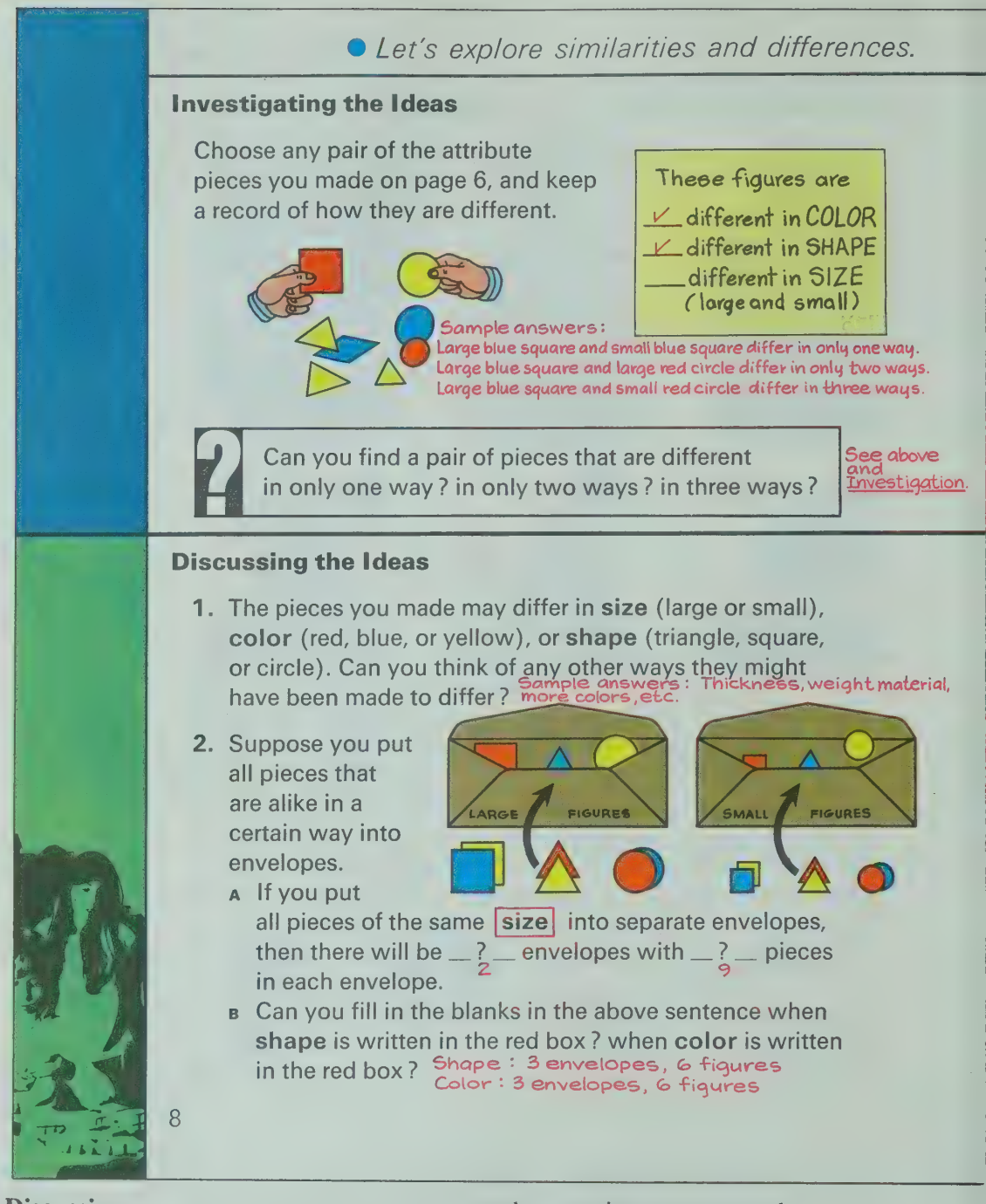
Investigation

You might have the children work either independently or in pairs. Remind them to record their findings. For example, you might suggest a chart such as the following:

Pair	Color	Shape	Size
large red square and large yellow circle	different	different	same

Note that the actual investigation question requires examination of pairs of figures. Thus, if children use the chart to record their findings, they will have the information they need to find the required pairs.

Even though systematic recording on a chart is suggested, the mood of the activity should be enthusiastic and investigative. The chart should only serve as a tool which, by aiding memory and preventing needless repetition, facilitates finding some of the required pairs of figures.



● *Let's explore similarities and differences.*

Investigating the Ideas

Choose any pair of the attribute pieces you made on page 6, and keep a record of how they are different.

These figures are
✓ different in COLOR
✓ different in SHAPE
___ different in SIZE
(large and small)

Sample answers:
Large blue square and small blue square differ in only one way.
Large blue square and large red circle differ in only two ways.
Large blue square and small red circle differ in three ways.

? Can you find a pair of pieces that are different in only one way? in only two ways? in three ways?

See above and Investigation.

Discussing the Ideas

1. The pieces you made may differ in **size** (large or small), **color** (red, blue, or yellow), or **shape** (triangle, square, or circle). Can you think of any other ways they might have been made to differ? Sample answers: Thickness, weight material, more colors, etc.

2. Suppose you put all pieces that are alike in a certain way into envelopes.

   A If you put all pieces of the same **size** into separate envelopes, then there will be __2__ envelopes with __9__ pieces in each envelope.

   B Can you fill in the blanks in the above sentence when **shape** is written in the red box? when **color** is written in the red box? Shape: 3 envelopes, 6 figures
Color: 3 envelopes, 6 figures

8

Discussion

Ask several children to name some pairs which they found in the investigation. There are many, many pairs of figures which the children may find. If some children are curious about how many pairs differ in one way, or two ways, or three ways, suggest that a few of them use a free-time period or time at home to try to figure out how many. Otherwise, such an inquiry would take a large amount of class time.

As children discuss exercise 1, point out that size, shape, and color are "attributes" of the figures. Point out that with these attribute pieces, a pair which differ in one way are the same in two ways, and a pair which differ in two ways are the same in one way. As examples of other attributes which might differ, children may suggest thickness; weight; kind of material (such as metal, wood, or plastic); translucent or opaque; smooth or rough surface; and so on. You might also discuss the fact that the attribute of size is relative to the pieces being used. Thus, an actual measurement is not needed to compare a large triangle with a small triangle; the relative difference in size is obvious.

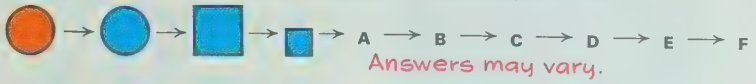
As you discuss exercise 2, have the children separate their attribute

Using the Ideas

1. The pieces in each pair differ in just **one way**. Write "**size**," "**shape**," or "**color**" to show the way in which they differ.

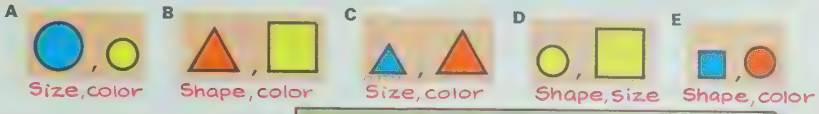


2. Each piece differs in just **one way** from the piece before it. Continue the "string" and name the pieces you use.



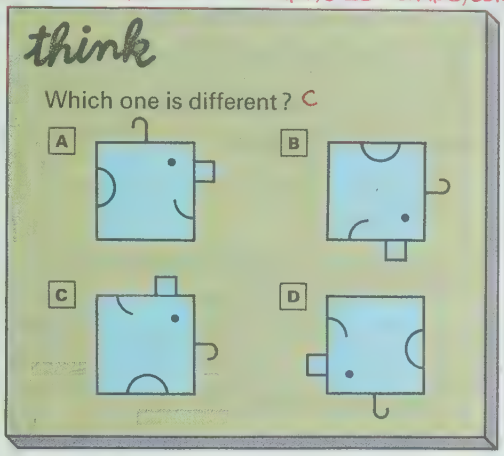
Answers may vary.

3. The pieces in each pair differ in just **two ways**. Tell in which ways they differ.



Answers may vary.

4. A Give 3 more pairs of pieces that differ in just one way.
   B Give 3 more pairs of pieces that differ in just two ways.
   C Give 3 pairs of pieces that differ in three ways.



5. Make a string of 6 pieces, each of which differs from the next one in A 1 way. B 2 ways. C 3 ways.

Answers may vary.

★ 6. Use all your attribute pieces to make a string of pieces that differ in A 1 way. B 2 ways. C 3 ways.

Answers may vary.

9

pieces into the categories being discussed. Distribute envelopes for this purpose, or simply have them sort the pieces into two groups. The exercise emphasizes that this particular set of attribute pieces deals with two sizes, three colors, and three shapes. Some children might find it interesting to know that the number of pieces in the final set can be determined by multiplying the number of different attributes used: 2 (sizes) × 3 (colors) × 3 (shapes) = 18.

Using the Exercises

You might assign these exercises as independent work or have the children work in pairs. Exercise 2 contains a kind of activity which can be expanded, as in exercises 5 and 6. It would be stimulating for children to do this activity as partners, taking turns adding appropriate figures to the "string."

You might use the *Think* problem to illustrate that differences and similarities are not always obvious.

Assignments (page 9)

Minimum: 1–3. Average: 1–5. Maximum: 1–6.

Follow-up

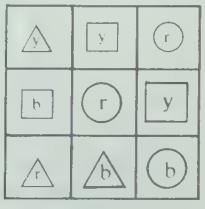
Continue to encourage children to create various rules for games with the attribute pieces. Note that it is usually best for them to work in pairs rather than in larger groups. You might suggest an activity such as the following.

For each player, make a cardboard playing board with nine squares, each 4-by-4 cm (so the board is 12-by-12 cm). Any of the 18 pieces may be chosen. The rules for positioning pieces are as follows: each piece must differ in one way from the piece(s) next to it horizontally, and differ in two ways from the piece or pieces next to it vertically. Each player fills his board according to the rules. Then players can compete for highest scores according to a chosen system. Points can be scored according to one of at least three score cards pulled before each player has filled his board. The different score cards will provide variety. If the card is kept hidden it becomes a game of chance; or, if it is shown, they can try to use pieces worth the highest points.

Sample score card:

All blue figures	15	All squares	20
All red figures	10	All yellow figures	15
All circles	5	All large figures	5

Sample game:



Score:

3 yellow figures	45
3 blue figures	45
3 red figures	30
3 squares	60
3 circles	15
4 large figures	20
Total:	215

Resources for Active Learning

Attribute Games and Problems, McGraw-Hill Ryerson [Suitable for use with this lesson and throughout the year].

PAGES 10–11

Objective

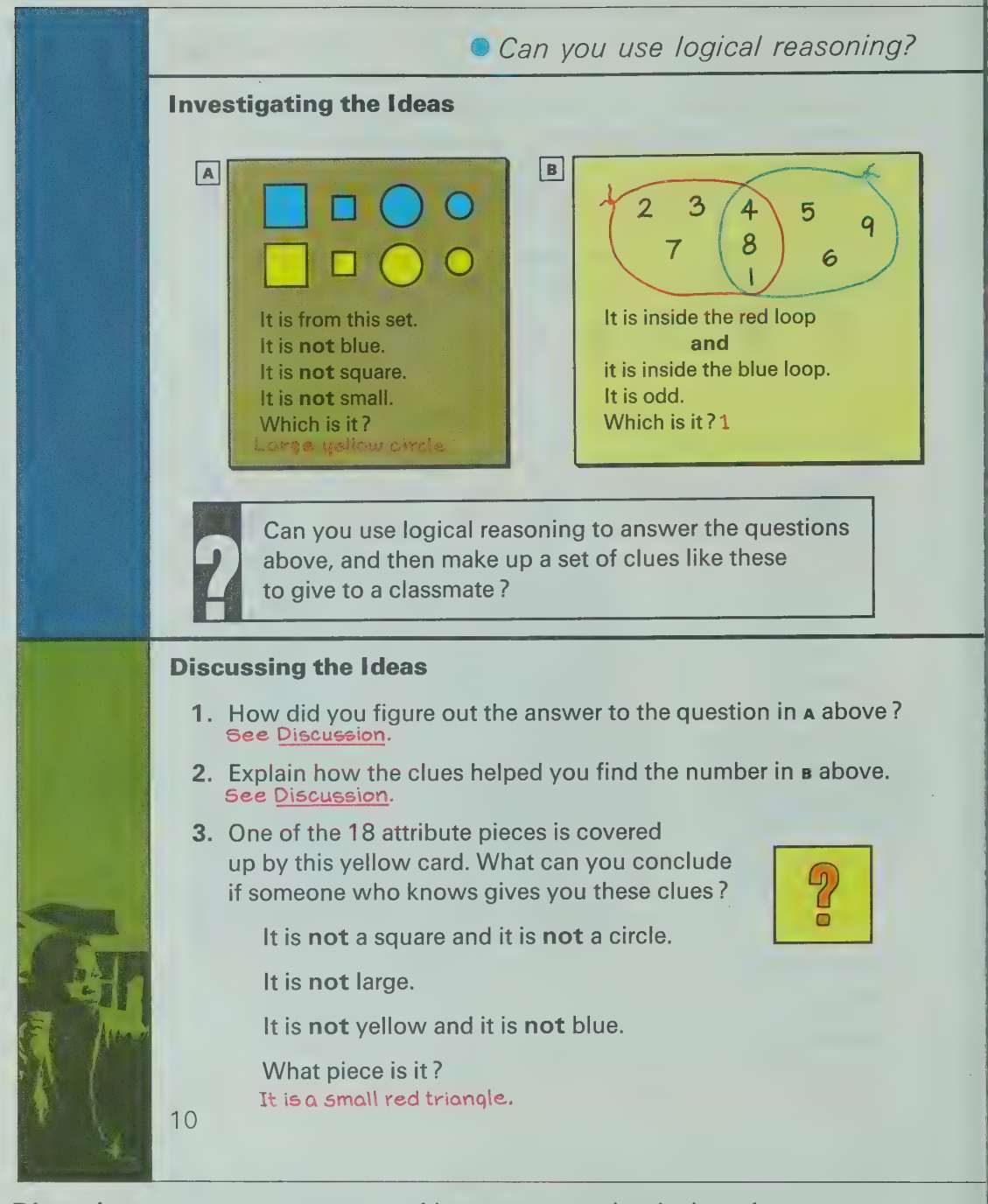
Given a set of elements and a description of one element within the set, the child will be able to select the element described.

Preparation

To prepare for this investigation, you might begin like this: Tell the children that you are thinking of some object in the classroom (such as your desk) and give hints using only statements that are negations, such as: "It is *not* alive. It is *not* hanging from a wall. It is *not* one of the student desks. What is it?" Use another example giving only clues which are conjunctions (that is, statements that are connected by "and"). You might even use the same object and describe it with statements such as: "It is inanimate *and* it is made of wood. It is the largest single piece of furniture in the room *and* it is often used with a chair. What is it?"

Investigation

Some children would benefit from working with a partner on this investigation. It would be helpful to have the children note that the examples in the book use the attribute pieces or numbers. According to children's needs, encourage them to try to describe a particular attribute piece or number in statements using "not" or "and." If they continue to work with the attribute pieces or with numbers, they should be able to use negations and conjunctions in a straightforward manner. However, they should eventually understand that, if they follow the patterns in the book, their descriptions should not combine negation and conjunction statements in the description of a single element. It is also necessary that each description which uses negations eliminate every property the piece or number shares with another. Similarly, descriptions which use conjunctions must include at least one property which, together with the other properties mentioned, distinguishes the piece or number from every other element in the set.



Discussion

For discussion exercises 1 and 2, encourage children to give their answers for parts A and B of the investigation and to explain their thinking for each. You might mention that the statements using "not" are called negations because they negate a property; they state which properties the element does *not* possess. Thus, by a process of elimination, the single element can be recognized. Similarly, you might discuss the fact that statements joined by the word "and" are called conjunctions. In exercise 3, point out that the statement "It is not a square and it is not a circle" combines two properties the item does not possess (a conjunction of two negations). The third statement in exercise 3 joins negations of two colors, leaving the only possible color to be the one omitted, namely red. See the follow-up section for further activities with this kind of exercise.

Using the Ideas

1. Each yellow clue card has the picture of one of these attribute pieces on the other side. Which piece is it?



A

It is **not** small.
It is blue.
It is **not** a square and **not** a circle.
Which is it?
Large blue triangle

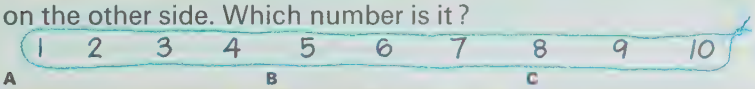
B

It is small.
It is **not** yellow and **not** blue.
It is **not** a circle and **not** a triangle.
Which is it?
Small red square

C

It is **not** large.
It is yellow.
It has **no** corners.
Which is it?
Small yellow circle

2. Each yellow clue card has one of these numerals written on the other side. Which number is it?



A

It is less than 5.
It is even.
It is **not** 4.
Which is it? 2

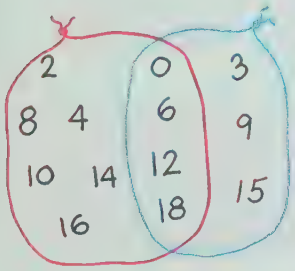
B

It is between 4 and 8.
It is **not** even.
It is **not** half of 10.
Which is it? 7

C

It is odd.
It is more than 5.
3 will "go into" it.
Which is it? 9

3. Each clue card describes one of the numbers shown in the loops. Which number is it?



A

It is **not** inside the red loop.
It is inside the blue loop.
It is less than 8.
Which is it? 3

B

It is inside the red loop **and** it is inside the blue loop.
It is between 8 and 15.
Which is it? 12

11

Using the Exercises

On page 11, children might want to use their own attribute pieces for exercise 1. With more advanced children, you might explain that "it is not a square or a circle" might have been used for "it is not a square and it is not a circle." When children finish exercises 2 and 3, you might want them to make up more clue cards of their own for their classmates to solve.

Assignments (page 11)

Minimum: 1, 2. Average: 1–3. Maximum: 1–3.

Mathematics

If p and q are statements, then the statement "p and q" is the *conjunction* of p and q. The *disjunction* of p and q is the statement "p or q." As an example, let p and q denote the following statements concerning the attribute pieces.

p: This piece is a triangle.
q: This piece is small.

Conjunction: This piece is a triangle *and* it is small. (p and q)
Disjunction: This piece is a triangle *or* it is small. (p or q)

Note that the conjunction and disjunction describe different subsets of the set of all the pieces.

If p is any statement, the statement "*not-p*" is the *negation* of p. Using the statement p in the example above, the negation of p can be stated as follows:

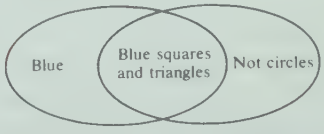
This piece is not a triangle.

In discussing the ideas of the lesson, the children need only develop intuitive ideas of the formal logic discussed here. It is not necessary that formal definitions of conjunction, disjunction, and negation be introduced.

Follow-up

Suggest that children group the attribute pieces according to categories which you write on the chalkboard. For example: (1) all blue pieces; (2) not blue pieces; (3) not circles and not blue pieces; (4) not circles and blue pieces.

This same kind of activity can be used with loops of yarn outlining the space for the pieces.



Resources for Active Learning

Math Activity Cards, E44, Macmillan.
Maths Mini-lab, Cards 135–136, Selective Educational Equipment.
Nuffield Project: *Problems*—Red Set, No. 10, Wiley
SMSG: *Puzzle Problems and Games Project*, pp. 13–20, Stanford University.

Duplicator Masters, page 1
Workbook, page 2

PAGES 12–13

Objective

Given the first three or four numbers of a numerical sequence, the child will be able to supply the next two or three numbers in the sequence, by interpreting dot patterns for the sequence.

Preparation

To help children understand what the investigation will require of them, you might write some familiar patterns on the chalkboard. For example:

0, 2, 4, 6, 8, . . .
(Begin with zero and add two to each successive number.)

0, 7, 14, 21, . . .
(Multiples of 7, or add seven to each successive number.)

Investigation

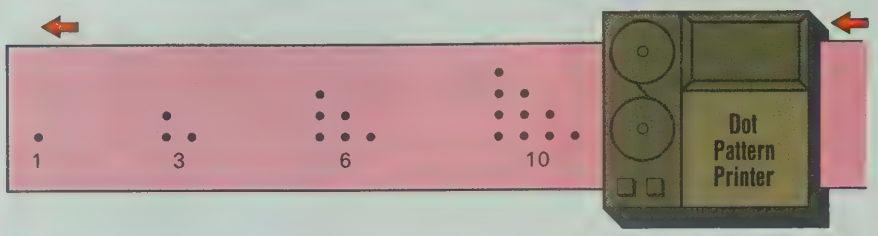
Since this investigation is brief, you might assign it as an independent activity and lead right into the discussion. You might challenge the children who finish quickly to find the next three numbers in a sequence you write on the board, such as:

1, 1, 2, 3, 5, 8, . . . (The next three numbers are 13, 21, 34; this is the Fibonacci sequence.)

or 1, 2, 4, 7, 11, 16, . . . (The next three numbers are 22, 29, 37.)

Can you find the pattern?

Investigating the Ideas



The **Dot Pattern Printer** printed the dot patterns above for the number sequence 1, 3, 6, 10, ▮, ▮, . . .



? Can you draw the next two patterns the "printer" will make and give the next two numbers in the sequence?

Discussing the Ideas

1. Why do you think the numbers in the sequence above are called triangular numbers? Can you give a triangular number greater than 30? 36, 45, 55, 66, etc.
   Sample answer: They can be represented by dots in a triangular array.
2. Here is an interesting pattern.
   - A What sum would equal the fifth triangular number? 1+2+3+4+5
     the sixth? 1+2+3+4+5+6
   - B Can you "see" these sums in the dot patterns for the triangular numbers?
     "Yes, first row has one dot, second row has two dots, and so on."

Triangular Numbers

First:	**1**
Second:	**3** = 1 + 2
Third:	**6** = 1 + 2 + 3
Fourth:	**10** = 1 + 2 + 3 + 4

3. A Why do you think the numbers 8, 15, and 24 are called rectangular numbers? They can be represented by rectangular arrays.
   B Can you name some other rectangular numbers?
   See Discussion.

12

Discussion

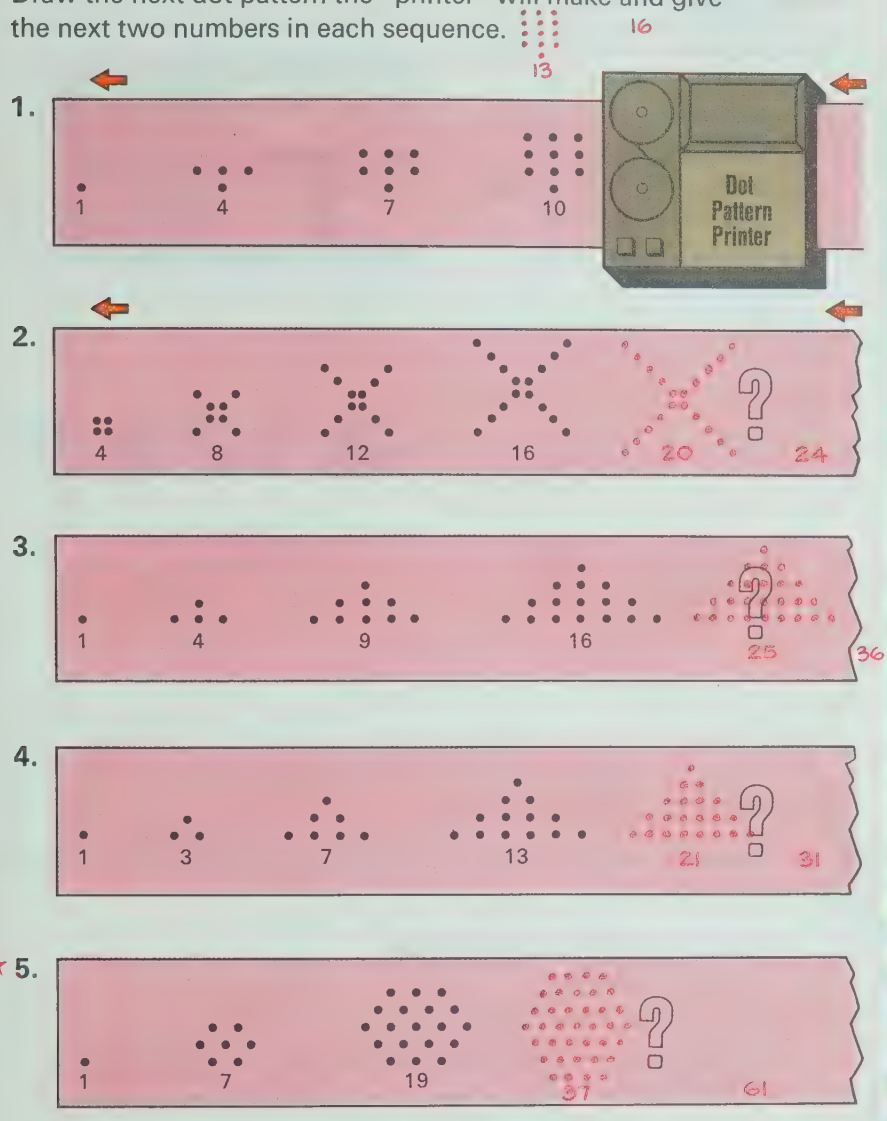
Use discussion exercises 1 and 2 to discuss how children found the next two patterns and numbers of the sequence in the investigation. Point out how the triangular dot pattern shows that each new row contains one more dot than the row preceding it. Arithmetically the sequence can be developed by adding the counting numbers, in order, to each previously formed number in the sequence. Thus, the sequence can be seen as

$^{+1}1, ^{+2}3, ^{+3}6, ^{+4}10, ^{+5}15, ^{+6}21, \ldots$.

Help the children think through the explanations suggested in exercise 2. In exercise 3, children consider how composite numbers or numbers that are multiples may be patterned in rectangular arrays. Note that some numbers such as 6, 10, 15, . . . are both triangular and rectangular numbers.

Using the Ideas

Draw the next dot pattern the "printer" will make and give the next two numbers in each sequence.



Using the Exercises

Assign these exercises as independent work, unless you prefer to have the children work in pairs. You might challenge more capable children not only to draw the dot pattern for the next two numbers of each sequence but also to try to figure out and explain the pattern arithmetically, as follows:

1. $1\ ^{+3}, 4\ ^{+3}, 7\ ^{+3}, 10\ ^{+3}, 13, \ldots$
   (Add 3 to each number.)
2. $4\ ^{+4}, 8\ ^{+4}, 12\ ^{+4}, 16\ ^{+4}, 20 \ldots$
   (Add 4 to each number.)
3. $1\ ^{+3}, 4\ ^{+5}, 9\ ^{+7}, 16\ ^{+9}, 25, \ldots$
   (Beginning with 3, add the odd numbers, in order, to each new term. Note that these numbers are all square numbers.)
4. $1\ ^{+2}, 3\ ^{+4}, 7\ ^{+6}, 13\ ^{+8}, 21, \ldots$
   (Beginning with 2, add the even numbers, in order, to each new term. Note that these numbers are all odd.)
5. $1\ ^{+6}, 7\ ^{+12}, 19\ ^{+18}, 37, \ldots$
   (Add multiples of 6, in order, to each new term)

Assignments (page 13)
Minimum: 1–4. Average: 1–4. Maximum: 1–5.

Mathematics

A *sequence* of numbers is a set of numbers given in a specific order. A sequence is finite if it contains a definite number of elements or terms. Many sequences in mathematics are infinite sequences. The set of even whole numbers (0, 2, 4, . . .) is an infinite sequence.

There are many different kinds of sequences since the only requirement of a sequence is that it have a first term, a second term, a third term, and so on. Some of the more interesting sequences in mathematics are those which are formed according to a definite rule or pattern. An *arithmetic sequence* is a sequence which has a fixed difference between every successive pair of numbers in the sequence. The sequence 3, 7, 11, 15, 19, . . . is an arithmetic sequence in which each term of the sequence is 4 more than the preceding term.

A *geometric sequence* is formed by starting with some number and then multiplying it by a fixed number to arrive at the next term. A sequence is a geometric sequence if there is a constant ratio between successive terms of the sequence. For example, the sequences 1, 3, 9, 27, 81, 243, . . . and 16, 8, 4, 2, 1, $\frac{1}{2}$, $\frac{1}{4}$, . . . are geometric sequences. The ratio between successive terms in the first sequence is 3, while in the second sequence it is $\frac{1}{2}$.

Resources for Active Learning

Applied Mathematics Cards. Group 3/1-2, Schofield and Sims. (Available from Mafex Associates)

Developmental Math Cards, J[1]15, K[1]21, L[1]19, Addison-Wesley.

Discovery, Section I, Activities 9-14, pp. 11–16, Encyclopaedia Britannica Educational Corp.

Nuffield Project: *Problems*—Green Set, No. 13, 47, Wiley.

Notes on Mathematics in Primary Schools, pp. 24–44, 47–61, Cambridge University Press.

Teaching Aids for Elementary Mathematics, "Fibonacci Numbers," pp. 36–37; "Number patterns," pp. 114–115, Holt, Rinehart and Winston.

Workbook, page 3

PAGES 14–15

Objective

Given a function rule which transforms a figure shown on an input-output card, the child will be able to apply the same rule to transform another input figure.

Preparation

To prepare for this lesson, you might introduce the topic of machines, such as computers, and spend just a few minutes mentioning various operations present-day machines can perform. For example, computers can run airplanes, show assimilated driving conditions for driver training, and so on. In this investigation, children will be studying a function machine.

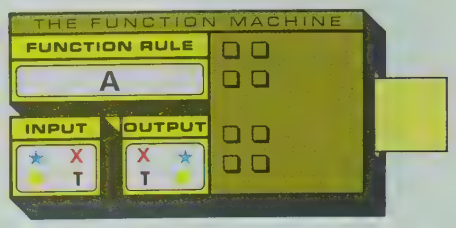
Investigation

In this investigation, children must study the differences between the input and the output of a function machine and figure out what rule has been applied. Then, after determining the output for a given input figure, they are challenged to make their own input figure. You might encourage children to make more than one input-output card. Perhaps you will want to extend the investigation by having them make up an input figure for which a classmate must select an appropriate output figure from three or four choices.

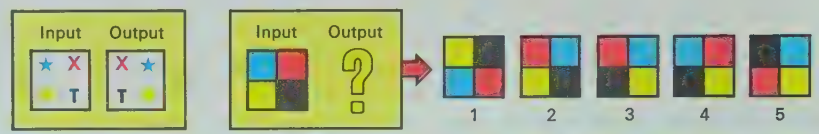
Let's explore figure patterns.

Investigating the Ideas

When you put a design in as the input of the function machine, the machine changes it in a certain way. Each time the machine changes a design, it prints a yellow input-output card to show what happened.



Rule **A**



When this design is the **input** this design is the **output**.

When this design is the **input**

which of these designs is the **output**? 3

Can you make an input-output card by using Rule **A** and a design of your own? Designs may vary. See Investigation.

Discussing the Ideas

1. A How would the machine above change this design? Circle would be on the left and square on the right.



   B Can you describe the way the machine changes any figure? Positions of designs are reversed horizontally. See Discussion.

2. Can you make up another interesting rule for the machine? Answers will vary.

14

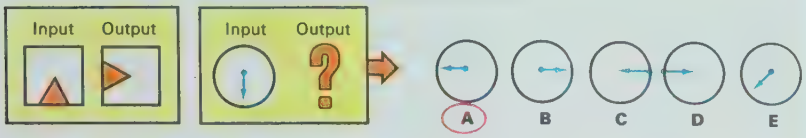
Discussion

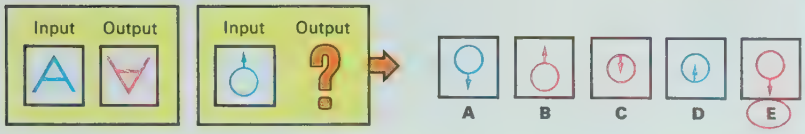
Discussion exercise 1 requires that children verbalize the function rule that was applied in the investigation. Accept any description which conveys the main idea. Some may say that the output design is the mirror image of the input design, with a vertical line between the two acting as a line of symmetry. Others may say that the design has been "flipped."

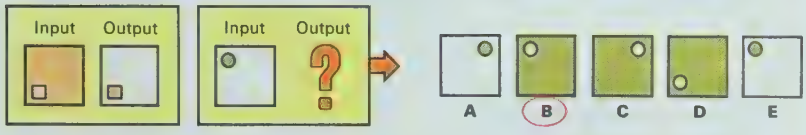
Exercise 2 gives children an opportunity to make up rules of their own. Have several children draw an input-output card on the chalkboard to illustrate the rule they suggest. You might have some volunteers draw their input-output card first and ask the class to explain the rule. Since this lesson should be treated with a light touch, you might turn it into a kind of "guess my rule" game: the child who guesses the rule can be the next one to draw designs for a new rule. However, the designs must be kept simple or too much time will be spent in drawing them.

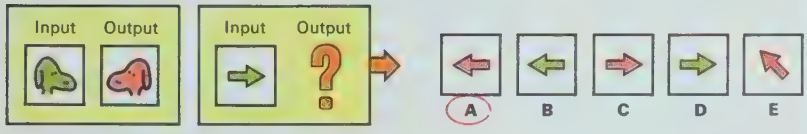
Using the Ideas

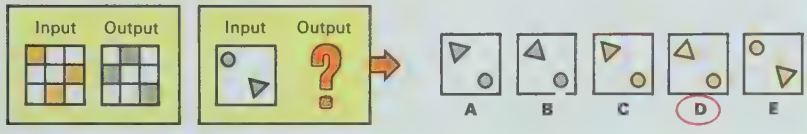
The first input-output card in each exercise shows what the machine does. Give the letter of the output for the second card.

1. 

2. 

3. 

4. 

5. 

15



Using the Exercises

Assign page 15 as independent work. Although children may arrive at an answer in various ways, the transformations they are dealing with are actually rotations, flips, and reflections. Also, for exercises 2–5, children should observe that color changes must be taken into consideration. You might challenge capable children to try to state in their own words the rule for each exercise. For example, the rule for the first exercise may be expressed as "a 90-degree rotation to the left." When children finish, encourage all to give a reason for each answer they chose.

Assignments (page 15)*
Minimum: 1–3. Average: 1–4. Maximum: 1–5.

Follow-up

Children might be interested in doing some research regarding patterns developed by Blaise Pascal in the famous "Pascal's triangle." Although the triangle is named for Pascal, it probably originated in China as early as 1100. In Western civilization this triangle appeared in print around 1527, almost a century before Pascal was born. However, the arithmetic triangle, as it is also called, has been given Pascal's name because of the extensive work Pascal did with it, particularly in the field of probability.

Encourage children not only to do some research on this triangle but to try and find patterns within it themselves. Besides relating the triangle to the triangular numbers studied on page 12, suggest that they find the numbers of the fifth row, or that they find the pattern made by the sum of the numbers in each row: 1, 2, 4, 8, 16, ...

1	First row
1 1	Second row
1 2 1	Third row
1 3 3 1	Fourth row
1 4 6 4 1	Fifth row

Duplicator Masters, page 2

PAGES 16–17

Objective

The child will demonstrate his ability to work with the concepts presented in this chapter.

Preparation

Materials

attribute pieces

You might want to use a short review in preparation for this lesson. For example, give a statement applicable to an attribute piece and ask the children to name pieces to which it might refer. For example, the statement, "It is large and it is blue" refers to all large blue pieces.

Reviewing the Ideas

1. How many? (Use your attribute pieces from page 6.)

A blue squares 2 C not yellow 12 E red and not square 4

B large circles 3 D small and not triangular 6 F large and blue 3

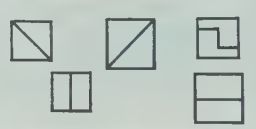
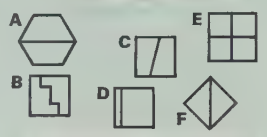
2. Name the attribute piece described for each part.

A
It is large.
It is red.
It is **not** a circle.
It is **not** a triangle.
Large red square

B
It is a square.
It is **not** yellow.
It is **not** large.
It is **not** red.
Small blue square

C
It is small.
It is **not** red.
It is **not** yellow.
It is **not** a triangle.
It is **not** a square.
Small red circle

3.

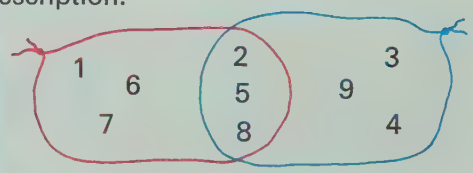
All of these are in the set.	**None** of these are in the set.	Which of these are in the set? B, C, F
		

4.

All of these are in the set.	**None** of these are in the set.	Which of these are in the set? 25, 30, 50, 75
15 5 40 80 35 70 95 45 90	2 44 86 12 29 93 8 1 37 3 72	25 16 50 64 36 48 4 75 30 91

5. Give the number that fits the description.

It is inside both loops.
It is an even number.
It is greater than 4.



16

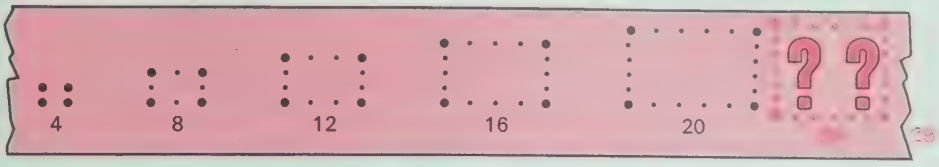
Discussion

Although pages 16 and 17 might be used for evaluation, keep in mind that they will simply indicate a child's relative grasp of the concepts touched in the chapter, rather than mastery of any reasoning skills. You might also use the exercises on these pages as discussion topics. Stress comparison of phrases which utilize the "is" and "is not" concepts and the "all" and "none" concepts. With exercises 6 to 8, stress the development of patterns.

The *Think* problem may stimulate much discussion. Allow the children time to discuss its main points. For the first part, discuss how the condition stated in the problem, together with the total weight given on each scale, determines the weight of the large blocks. For parts 2 and 3, be sure children see how the large block may be thought of as being replaced by the appropriate number of small blocks. Thus, in part 1, they might think: Since block *A* weighs 6 times as much as block *B*, we could replace *A* with 6 blocks of size *B*. Then each block *B* would weigh $56 \div 7$, or 8 grams. Hence, it is easy to see that block *A* weighs 6×8, or 48 grams.



6. Draw the next dot pattern and give the next two numbers.

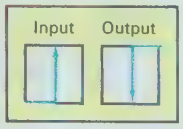


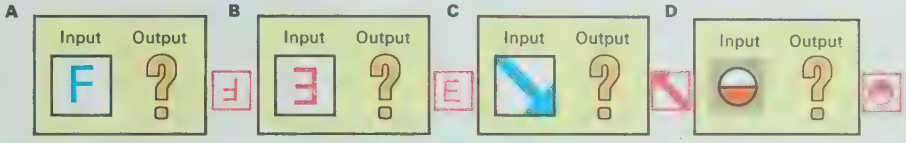
7. Name 5 pieces you would use to continue this string of "one-way" differences.



Answers may vary.

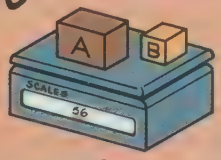
8. This card shows what a certain function machine does. Draw a picture of the output for each of the cards below.





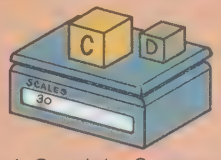
think Can you find the weight of each block in these problems?

1



Block **A** weighs 6 times as much as block **B**. Blocks **A** and **B** together weigh 56 grams.

2



Block **C** weighs 3 grams more than twice block **D**. Blocks **C** and **D** together weigh 30 grams.

3 Suppose block **E** and block **F** together weigh 85 grams and block **E** weighs 5 grams more than 3 times block **F**.

17

Follow-up

Since the Fibonacci series shown below has many interesting patterns related to it, you might have those who figure out its formation describe it to the class.

$1, 1^{+1}, 2^{+1}, 3^{+2}, 5^{+3}, 8^{+5}, 13^{+8}, \ldots$

Each new number in the sequence is formed by adding the two terms preceding it.

Children might also be interested in a little of the historical background of the man for whom the Fibonacci sequence is named. Fibonacci is really the "nickname" of Leonardo of Pisa (Fibonacci means son of Bonacci). He was the son of an Italian merchant and lived from 1180–1250. Since he studied under a Muslim teacher and travelled in Greece, Egypt, and Syria, he became familiar with the Hindu-Arabic numerals and used them before many Europeans adopted them. Encourage interested children to do further research in histories of mathematics or encyclopedias.

Workbook, page 4

General Objectives

To provide understanding of place-value concepts
To review and extend work with inequalities
To review and extend work with bases other than ten
To provide background for the introduction of decimals

A clear understanding of place value is one of the most important objectives of the *Investigating School Mathematics* series. Without such an understanding, there is little hope for meaningful comprehension of the algorithms of arithmetic and decimal notation. For this reason, the concepts of place value are treated carefully, beginning in Book 1 and continuing throughout the series.

Book 6 treats place-value concepts for whole numbers first and for decimal notation subsequently. In this chapter, place values through hundred millions are introduced, and inequalities are treated in such a way as to focus additional attention on important place-value concepts. Larger numbers are introduced, followed by a lesson on "rounding" large numbers. Powers of ten are used to introduce exponents, and scientific notation is introduced through work with exponents. The latter part of the chapter deals with different number bases. All of this material is provided to develop the children's understanding of basic concepts of place value.

Mathematics

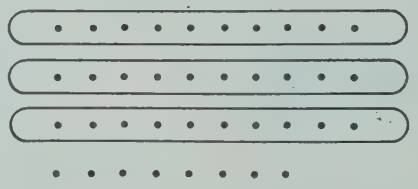
The symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 are *digits*. A *numeral* is any symbol that stands for a number. Thus, the symbol 4 is both a digit and a numeral. You might find it convenient to ask a child to *write* the "number" 1486 or refer to it as a "four-digit number." Strictly speaking, these are incorrect uses of the terminology. (You do not write the number, nor does the number have four digits. You write the symbol or the numeral, and it is the symbol or the numeral that has four digits.) The children will no doubt understand that, when you say, "Write the number," you want them to write the symbol for this number and that, when you call it a "four-digit number," you are referring to the symbol used to represent this number. For this reason, we consider both expressions acceptable. However, certain abuses of the number-numeral terminology are clearly objectionable and may confuse the children. Do not ask the children to add digits or numerals. They add numbers and write digits and numerals.

The important thing to remember in number-numeral terminology is to keep your language simple and meaningful to the children. Say *number* whenever you are uncertain whether you should say number or numeral. Avoid making an issue of these words with the children. If a child points to a symbol or numeral and calls it a number, do not criticize or correct this remark. Most children have an intuitive grasp of the difference between number and numeral, and a lengthy discussion of these ideas may serve only to confuse what was previously clear.

When we write the symbol 4357, the numerals 4, 3, 5, and 7 stand for 4000, 300, 50, and 7, respectively. This illustrates an important property of our numeration system: the utilization of place value. When we refer to place value, we mean simply that the number a digit represents depends on the place it occupies in the symbol.

Another property, which simplifies our calculations and makes it easy to learn arithmetic, is that we can represent any number by using only ten symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Each digit used by itself represents a single number; it is only when we write symbols greater than 9 that a given digit may stand for two or more numbers. Thus, in the numeral 636, one 6 stands for 600, and the other stands for 6.

Our place-value scheme has a base of ten, which means we group by tens. Given a collection of objects, we find out how many disjoint sets (sets that have no common elements) of ten can be formed. Consider the collection of dots shown in the illustration. There are 3 sets of ten, and 8 left over.



The importance of place value becomes evident when we write the numeral for this number of dots. Instead of writing "3 sets of ten, and 8 more," we write "38" and agree that the digit in the "second place" (3 in this case) represents sets of ten.

In order to represent numbers over 99, we need to group the sets of ten by tens. We then have sets of 10 tens, each of which we call one hundred. For example, we might have a collection of objects grouped as follows: 5 sets of one hundred, 3 sets of ten, and 7 more; we write 537.

Teaching the Chapter

Materials

Atlases
Colored strips
Encyclopedia sets
Other reference books suitable for finding large numbers (optional)

Slips of paper approximately 7 by 10 centimetres (5 per child)
Slips of paper approximately 10 by 15 centimetres (3 per child, if possible in three colors)

Vocabulary

abacus
base six
base ten
billion
digit
equals
equation
exponent
greater than
hundred million
hundred thousand
inequality
is not equal to
less than
million
numeral
operation
period
place value
powers of ten
quadrillion
quintillion
rounding
scientific notation
set
sextillion
ten million
ten thousand
thousand
trillion

Although grouping sets of objects is impractical for the study of large numbers in base ten, encourage children who appear to have difficulty with the first few lessons to use physical objects in demonstrating one-, two-, and even three-digit numerals. In the work with other bases, both the strips and sets of objects should be used freely, until children understand place-value concepts without physical objects.

The vocabulary list is quite long, inasmuch as it includes words which may be new to the children as well as many which may already be familiar. The majority of the words in the list should be familiar to nearly all the children. The words that are new, however, should be stressed so that they will be mastered by the end of the chapter.

Lesson Schedule

Plan to spend about two to two-and a half weeks on this chapter, depending upon the abilities and background of the children in your class. If they have had considerable experience in working with place value, you will be able to cover the material more rapidly.

Evaluation of Progress

The purpose of the material on base six in this chapter is to increase understanding of base ten and to challenge the children. Mastery is not expected. The principal value of teaching something unusual, such as base six, is that it arouses the children's interest and stimulates them to do better in arithmetic. Keep these objectives in mind as you teach this small section of the chapter.

Since you should not expect the children to master all of the topics concerning base six, do not be discouraged if some of them fail to understand the more difficult concepts. Most should understand the rudiments of grouping in other bases and the process of writing two-place numerals in base six. Treat this section with a light touch.

The review exercises on page 34 should serve as a guide for evaluating the children's progress. We stress again that while place value and inequalities are vital to a successful continuation of the text, topics such as base six and different types of numerals are included as interesting sidelights and should not play a significant role in your evaluation of the children's progress for this chapter. Encourage more capable children to make independent, in-depth explorations of these optional items if they are intrigued by them.

Resources for Active Learning

General Activities

Peas and Particles, McGraw-Hill Ryerson [Estimation and large numbers]
Franklin Series: *From Fingers to Computers*, "Finger Computation," pp. 5–17. "The Electronic Computer" (flow charts, exponents, expanded notation, binary system), pp. 60-98, Lyons and Carnahan (Available from McGraw-Hill Ryerson)
Modern Math Games, "Puzzles," pp. 47–48, Fearon (Available from Clarke, Irwin)
Nuffield Project: *Problems* — Green Set, No. 18, 44, Wiley

Manipulative Devices

Abacus or abacus board (Educational Teaching Aids; school supplier)
Cuisenaire Cubes, Squares, and Rods (Cuisenaire Co.)
Multibase Arithmetic Blocks (Educational Teaching Aids; Herder and Herder)
Multi-base Converter (Math Media; Selective Educational Equipment)

Commercial Games

Base Check Game (Lakeshore)
Dr. Nim (Childcraft; Gamco)
Equations (Creative Publications; Wff 'N Proof)
Radix (Lang)
Ranko (Midwest Publications)
Think-a-Dot (Childcraft, Cuisenaire Co.; Edmund Scientific)

PAGES 18–19

Objective

Given a numeral which has as many as 9 digits, the child will be able to read the numeral by recognizing and naming the periods.

Preparation

Materials

slips of paper approximately 10 by 15 centimetres (3 per child; if possible, each one a different color)

To prepare for this lesson, you might present the idea that each number is unique but can be symbolized in many ways. For example, for the number nine, these are but a few of the symbols which might be used:

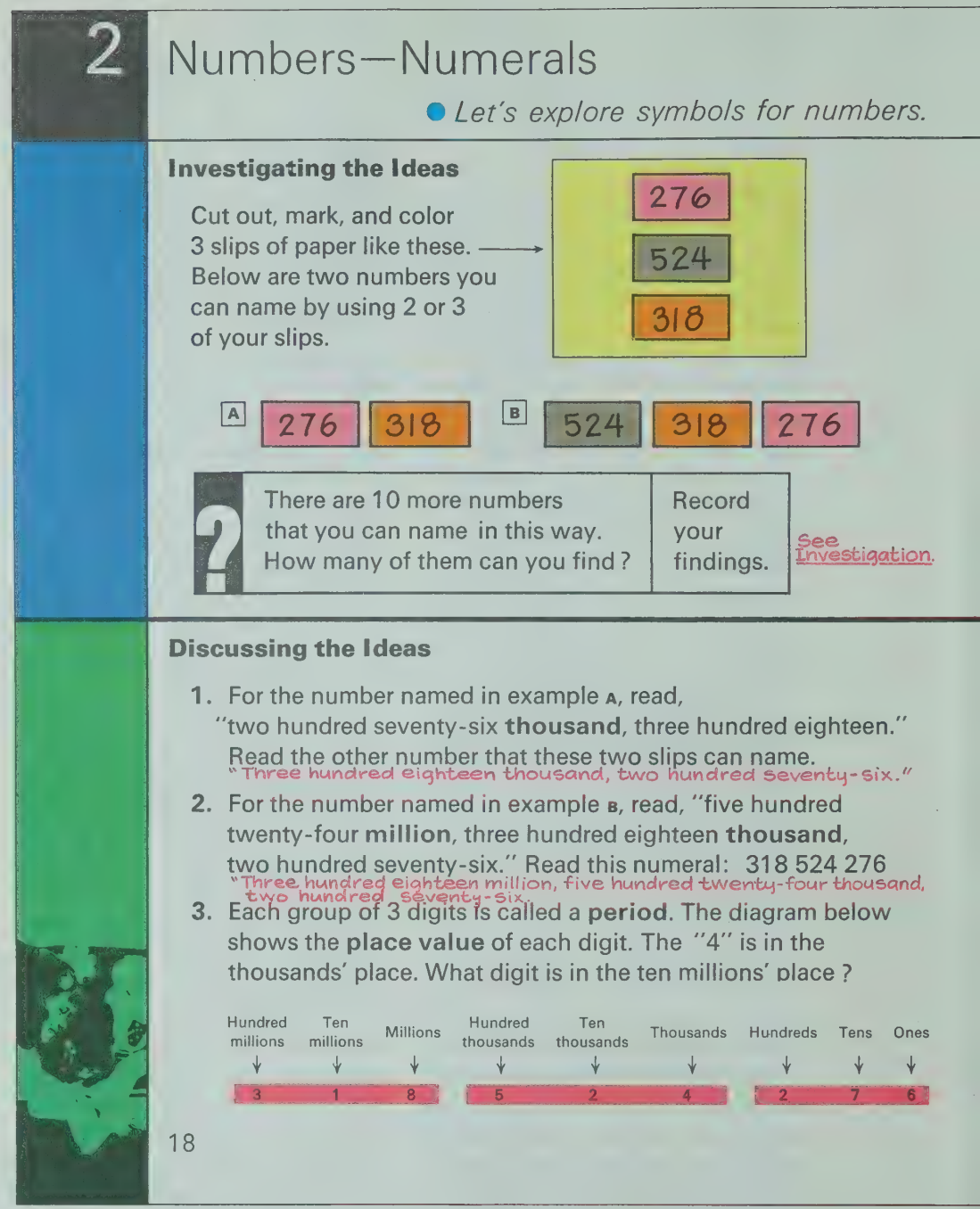
9 IX 200–191

 (rod numeral)

Investigation

Encourage children to work independently on this investigation. Remind them to record their work carefully in order to avoid repetition of a number. They should need little or no direction throughout the entire investigation. A total of 12 different numerals can be formed by using either 2 or 3 of the slips at a time. Besides the two shown in the text, they are as follows:

276 524
318 276
318 524
524 276
524 318
276 524 318
276 318 524
318 276 524
318 524 276
524 276 318



Discussion

After working through discussion exercises 1 and 2, have children read the other numbers which they recorded, explaining which color paper tells thousands and which tells millions. Then, as you discuss exercise 3, stress the convenience of grouping these large numerals into periods. Note that the three digits in each period are read as if they were simply hundreds, and then the name of the period is attached. Although it is important for children to realize each place value, they will not need these labels if they recognize the periods in each number.

To give children further practice with large numbers, you might have them do some counting in which they begin with a large number and count forward several times. For example, you might have them start with 9995 and count to 10 005, or start with 199 995 and count to 200 005.

Using the Ideas

1. In the numeral 534 896 201 tell what digit is in each of these places:

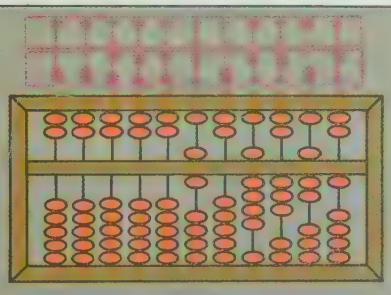
A ones' 1	D tens' 0	G hundreds' 2
B thousands' 6	E ten thousands' 9	H hundred thousands' 8
C millions' 4	F ten millions' 3	I hundred millions' 5

2. Starting at the right, the **first period** of the numeral 146 927 853 contains the digits 8, 5, and 3. Copy and complete these sentences about the numeral.
   - A The **second period** contains the digits __? __ 9, 2, and 7 and tells how many __? __. hundred thousands, ten thousands, and
   - B The **third period** contains the digits __? __ 1, 4, and 6 thousands and tells how many __? __. hundred millions, ten millions, and millions

3. Copy the sentence. Give the missing words and numbers.
   - A 687: The 8 in the __? __ place means 8 × ▮. tens; 10
   - B 687: The 6 in the __? __ place means 6 × ▮. hundreds; 100
   - C 395 687: The 5 in the __? __ place means 5 × ▮. thousands; 1000
   - D 395 687: The 9 in the __? __ place means 9 × ▮. ten thousands; 10 000
   - E 395 687: The 3 in the __? __ place means 3 × ▮. hundred thousands; 100 000
   - F 214 395 687: The 4 in the __? __ place means 4 × ▮. millions; 1 000 000
   - G 214 395 687: The 1 in the __? __ place means 1 × ▮. ten millions; 10 000 000
   - H 214 395 687: The 2 in the __? __ place means 2 × ▮. hundred millions; 100 000 000
   - I 214 395 687: The 7 in the __? __ place means 7 × ▮. ones; 1

think

This Chinese abacus (called a **suan-pan**) shows the number 609 371. Can you draw an abacus that shows 732 854 903? See above.





19

Using the Exercises

Have the children do the exercises on page 19 independently. You might point out that the chart at the bottom of page 18 would be helpful to them. If there are children who have difficulty understanding the basic concept of place value, work through these exercises with them. You might even want to have them use materials to group tens and hundreds, counting with them from numbers which span a decade or a hundred, such as 78 to 83; 89 to 92; 97 to 105; 190 to 210; and so on.

Sticks, pipe cleaners, or strips of paper bundled in groups of tens or hundreds can serve as counters.

When children finish, check exercise 3 carefully. Stress that because the 5 in 395 687 is in the thousands' place it means 5 thousands, that is, 5 × 1000, or 5000. Children may spend a long time figuring out how the Chinese abacus works. Let them wait a few days if necessary before discussing it. The wires stand for ones, tens, etc. A bead above the separating wood stands for 5, a bead below stands for 1.

Assignments (page 19)

Minimum: 1, 2. Average: 1–3. Maximum: 1–3.

Follow-up

Encourage children to pursue any interest they may have in the abacus. Some may do a little research to investigate various kinds of abacuses or other calculating devices. Others may want to construct a Chinese abacus, using simple materials such as a wooden frame, wire, and beads. Others may find it interesting to learn about the numerals used by various civilizations.

Resources for Active Learning

Franklin Series: *From Fingers to Computers*, pp. 18–35, Lyons and Carnahan [The abacus and counting boards]. (Available from McGraw-Hill Ryerson)

PAGES 20–21

Objectives

Given two numbers written in base-ten notation, the child will be able to compare them using the symbols <, >, or =. He will also be able to write a base-ten numeral in expanded notation.

Preparation

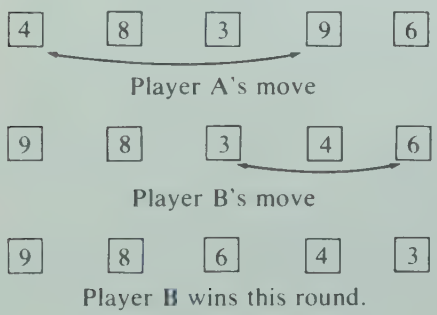
Materials

slips of paper approximately 7 by 10 centimetres (5 per child)

To introduce this lesson, you might simply ask the children to name some games in which the player or team with the highest score wins and some games in which the player with the lowest score wins. Explain to the children that in this investigation they will be playing a game in which the idea of highest or lowest will also determine the winner.

Investigation

You might want to read through directions of the game with some groups of children. Since each child might use 5 different digits, suggest that they initial the reverse sides of their slips of paper. The game is best suited for two players, although it would also be possible for three players to participate. Children use only one set of digits for each round, so they may take turns using each other's digits. Variations may be introduced, such as using 6 or 7 slips of paper; or changing the goal so that the person whose exchange makes the smallest number wins. A sample game in which the goal is the highest number might be as shown below.

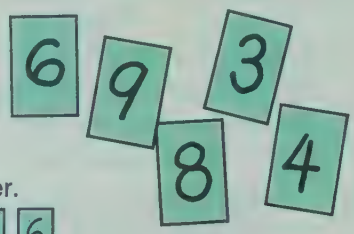


How are numbers compared?

Investigating the Ideas

Can you play the game LARGE, LARGER... LARGEST with a classmate? See Investigation.

Cut out five slips of paper and label them with different digits.



To play: 1. Mix the slips of paper and place them in a row to name a 5-digit number. Example: 4 8 3 9 6

2. Take turns **exchanging** the positions of any two digits so that after each exchange a larger number has been named.

3. The player whose exchange results in the largest possible number wins.

Discussing the Ideas

1. How can you tell when 5 different digits name the largest possible number? See Discussion.

2. What is the smallest number you could make with the 5 digits above? 34 689

3. In the example, the number is expressed in expanded notation.
Example:
$93\,864 = (9 \times 10\,000) + (3 \times 1000) + (8 \times 100) + (6 \times 10) + 4$

Can you use expanded notation to show these numbers?

A 9847 B 42 165 C 357 869 D 62 709

See Answers, T.E. page 20.



20

Discussion

Since the investigation in this lesson involves children in such instructive fun, you might want to begin some of the discussion exercises between rounds of the game.

Exercise 1 points out the right to left increase in the place value of base-ten numerals; thus, the largest digit must be in the place of the largest value, that is, furthest to the left, and the smallest digit furthest to the right. If players want the digits to name the smallest possible number, they must reverse the order, placing the smallest digit in the place of the largest value, as in 34 689.

When children have spent sufficient time with the game, discuss exercise 3 and point out how each numeral may be written as a sum of the values in each place-value position.

Answers, discussion exercise 3

A $9847 = (9 \times 1000) + (8 \times 100) + (4 \times 10) + 7$

B $42\,165 = (4 \times 10\,000) + (2 \times 1000) + (1 \times 100) + (6 \times 10) + 5$

C $357\,869 = (3 \times 100\,000) + (5 \times 10\,000) + (7 \times 1000) + (8 \times 100) + (6 \times 10) + 9$

D $62\,709 = (6 \times 10\,000) + (2 \times 1000) + (7 \times 100) + 9$

Using the Ideas

1. For each exercise, use expanded notation to represent each number.

A 436 C 6457 E 67 491 G 274 165
B 7243 D 5034 F 43 056 H 869 070

See Answers, T.E. page 21.

2. For "37 **is less than** 42" we write: $37 < 42$
For "56 **is greater than** 48" we write: $56 > 48$
Give the correct sign, $<$ or $>$, for each ◍.

A $348 \text{ ◍ } 328$ >
B $694 \text{ ◍ } 684$ >
C $462 \text{ ◍ } 439$ >
D $7296 \text{ ◍ } 7269$ >
E $5409 \text{ ◍ } 5390$ >
F $6843 \text{ ◍ } 68\,430$ <
G $27\,483 \text{ ◍ } 31\,120$ <
H $56\,257 \text{ ◍ } 56\,527$ <
I $83\,765 \text{ ◍ } 84\,101$ <

3. Give the correct sign ($>$, $<$, or $=$) for each ◍.

A $285 \text{ ◍ } 200+70+5$ >
B $368 \text{ ◍ } (3\times100)+(6\times10)+8$ =
C $7659 \text{ ◍ } (7\times1000)+(6\times100)+(4\times10)+9$ >
D $63\,742 \text{ ◍ } (6\times10\,000)+(3\times1000)+742$ =
E $59\,406 \text{ ◍ } (6\times10\,000)+(0\times1000)+406$ <
F $378\,941 \text{ ◍ } (3\times100\,000)+(78\times1000)+940$ >
G $8\,406\,000 \text{ ◍ } (8\times1\,000\,000)+(4\times100\,000)+(6\times10\,000)$ <

4. These exercises are about the number 342 817 695

A Rearrange the digits to name the largest number possible.
B Rearrange the digits to name the smallest number possible.
C Give the number 5 million greater.
D Give the number 42 million less.
★ E Give the number a half million less.
★ F What must be added to the number to reach 343 million?

See Answers, T.E. page 21.

think

I only have 6 digits,
And each one is the same.
My follower has seven.
Can you write
my name? 999 999

WHO AM I?



More practice, page A-1, Set 1

Using the Exercises

Assign the exercises on page 21 as individual work. If necessary use the exercises on expanded notation as a basis for further discussion. The exercises with inequalities are suitable for practice in reading large numerals as well as for the study of inequalities.

Attempt to give all the children a chance to read and solve the riddle in the *Think* problem before the correct answer is given. Even if only a few children solve the riddle, everyone should understand and benefit from a discussion of the correct answer.

Assignments (page 21)
Minimum: 1, 2. Average: 1–4D. Maximum: 1–4.

Follow-up

Suggest that children make variations of the game in the investigation. For example, besides using 6 or 7 slips and playing to get the smallest number, they might make variations such as playing to get the smallest odd number, or the smallest even number, or using only odd digits and back sides as zero, and so on. They might also devise a way of recording scores so that a player must win a certain number of rounds in order to win a game.

Resources for Active Learning

Franklin Series: *Making and Using Graphs and Nomographs,* "Graphing Number Sentences: Inequalities," pp. 25–34, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Answers, exercises 1 and 4

1.A $436 = (4 \times 100) + (3 \times 10) + 6$

B $7243 \times (7 \times 1000) + (2 \times 100) + (4 \times 10) + 3$

C $6457 = (6 \times 1000) + (4 \times 100) + (5 \times 10) + 7$

D $5034 = (5 \times 1000) + (3 \times 10) + 4$

E $67\,491 = (6 \times 10\,000) + (7 \times 1000) + (4 \times 100) + (9 \times 10) + 1$

F $43\,056 = (4 \times 10\,000) + (3 \times 1000) + (5 \times 10) + 6$

G $274\,165 = (2 \times 100\,000) + (7 \times 10\,000) + (4 \times 1000) + (1 \times 100) + (6 \times 10) + 5$

H $869\,070 = (8 \times 100\,000) + (6 \times 10\,000) + (9 \times 1000) + (7 \times 10)$

4.A 987 654 321
B 123 456 789
C 347 817 695
D 300 817 695
E 342 317 695
F 182 305

Duplicator Masters, page 3
Workbook, page 5
Skill Masters, page 3

PAGES 22–23

Objectives

Given the number one billion, the child will be able to think of it as 1000 million.

Given a number as large as sextillion, the child will be able to name the period of the number by referring to a chart of the names of some of the periods.

Preparation

To prepare for this lesson, you might briefly review products such as 10×10, 10×100, 10×1000, 100×10, 100×100, and 1000×10.

Alternatively, on the chalkboard write numerals for several numbers greater than one million and less than one billion and give the children an opportunity to read these numerals.

Investigation

The basis of each of the problems in this investigation is the introductory statement: "One billion is 1000 million." Except for problem 4, children need simply refer to this fact. For example, if 182 classes weigh 1 million kilograms, 182 × 1000 classes weigh 1000 million, or 1 billion, kilograms.

You might want to work through this first problem with some groups to demonstrate an approach applicable to the other parts.

Problem 4 involves a further step of changing 10 000 days to years. Remind the children that only estimates are expected.

● *How large is a billion?*

Investigating the Ideas

One billion is 1000 million.
We write: **1 000** 000 000

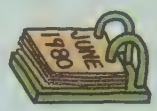


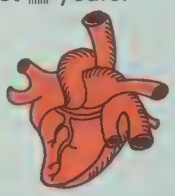
1. 182 classes, each containing 30 pupils, would weigh about one million kilograms. It would take ||| such classes to weigh one billion kilograms.

2. One million centimetres is 10 kilometres. One billion centimetres is a little more than one-quarter of the way round the earth. One billion centimetres is ||| kilometres.



3. One million minutes is almost 2 years. One billion minutes is almost ||| years.





4. If your heart beats 70 times each minute, it will take about 10 days for it to beat one million times and about ||| years for it to beat one billion times.

? Can you estimate each of the missing numbers above?
See Investigation.

Discussing the Ideas

1. See Discussion.

2. What number is:
   - A 1 billion more than 1 billion
   - B 2 billion more than 1 billion
   - C 1 billion more than 9 billion
   - D 5 billion more than 27 billion
   - E 1 million more than 1 billion
   - F 1 more than 1 billion
   - ★ G 1 less than 1 billion
   - ★ H one half of 1 billion

See Answers, T.E. page 22.

22

Discussion

Encourage volunteers to explain their answers. Accept any solution which a child can explain, and, of course, which yields a reasonable estimate.

As you discuss exercise 2, have children write the numbers on the chalkboard. For exercises 2C and 2D, you might want to refer to the diagram of the names of the periods on page 23. Help them find the period which comes after billions. You might use this diagram as a basis for further discussion.

Answers, discussion exercise 2

A 2 000 000 000 (two billion)
B 3 000 000 000 (three billion)
C 10 000 000 000 (ten billion)
D 32 000 000 000
E 1 001 000 000
F 1 000 000 001
G 999 999 999
H 500 000 000

Using the Ideas

1. The diagram shows the names of some of the periods of a very large number.

Sextillions	Quintillions	Quadrillions	Trillions	Billions	Millions	Thousands	
4 7 6 ,	5 8 7 ,	3 1 9 ,	6 0 8 ,	7 4 2 ,	8 7 1 ,	0 2 3 ,	9 6 4

A What digits are in the quintillions' period? 5, 8, and 7

B What digits are in the sextillions' period? 4, 7, and 6

2. The average distance from Earth to the sun is 149 565 801 km.
The largest period for this numeral is millions.
Give the largest period for each of the following numbers.

A Sun's diameter: 1 392 021 905 metres Billions

B Estimated remaining life of the sun:
100 000 000 000 000 000 seconds Quadrillions Sextillions

C Sun's mass: 1 969 000 000 000 000 000 000 000 million tonnes

D Recent estimate of Earth's age: 1 806 750 000 000 days Trillions

E Earth's mass: 5 794 588 715 000 000 000 thousand tonnes Quintillions

3. The nearest star beyond our sun is called **Proxima Centauri**.
It is about 40 trillion kilometres away. Write this numeral.
40 000 000 000 000

4. Sirius is the brightest star in the heavens. It is 80 trillion km away, and its mass has been calculated to be 45 077 sextillion tonnes. Write each of these numerals.
45 077 000 000 000 000 000 000 000 80 000 000 000 000

5. Which has the greater mass, the Earth or the sun? the sun

think

How much money would you lose if you traded a thousand million dollars for a million thousand dollars? None
Write the numeral for a million thousand. 1 000 000 000



Using the Exercises

Before children begin the exercises on page 23, point out that they are not required to memorize the names of these periods. You may find, however, that some of the children will want to memorize these names.

The various facts presented in exercise 2 may stimulate the children to discuss some of these ideas. Be sure to allow adequate time for such discussions, which are an important part of the children's experiences in arithmetic.

Assignments (page 23)*
Minimum: 1–5. Average: 1–5.
Maximum: 1–5.

Follow-up

The accompanying table of distances from the sun to the planets in our solar system furnishes other large numbers for the children to read and compare. Encourage the children to use these facts to write inequalities and to practice estimation and rounding. Trying to handle large, unwieldy numerals will give the class an appreciation of exponential notation when it is introduced.

Astronomers sometimes use the average distance from the earth to the sun (149 566 000 km) as the unit for measuring distance in our solar system. This Astronomical Unit (AU, as it is called) can be used to discuss the large figures in the table. For example:

1. Which planet is about 5 AU from the sun?
2. Which planet is approximately $\frac{3}{4}$ AU from the sun? Is its orbit inside or outside the earth's orbit?
3. Which planet is nearly 40 AU from the sun?

Planet	*Average Distance from the Sun (in kilometres)*
Mercury	58 019 000
Venus	108 134 000
Earth	149 566 000
Mars	227 784 000
Jupiter	777 793 000
Saturn	1 426 009 000
Uranus	2 867 605 000
Neptune	4 493 482 000
Pluto	5 907 702 000

Resources for Active Learning

Nuffield Project: *Computation and Structure* 4, "Large numbers . . .," pp. 50–58, Wiley.

PAGES 24–25

Objective

Given a number, the child will be able to round it to a specified place value, such as the nearest hundred, thousand, or million.

Preparation

To prepare for this lesson, you might review the meaning of the word "approximation." For example, look at the clock and give the approximate time in hours or half hours. Or, give an approximate length of the room in metres. Point out that "approximate" means "close to" or "near."



What does it mean to "round" a number?

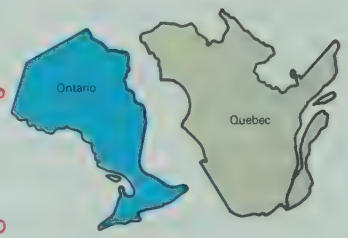
Discussing the Ideas

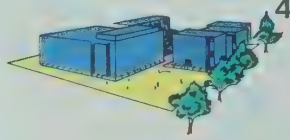
1. Study the chart below. Then give the missing words.
In a recent census* the population of Canada was given as 21 830 772. When less accurate information is needed, we often use a number that is "close" or approximately equal to the exact population. We say that the population is:

	21 830 770	(rounded to the nearest ten)
or	21 830 800	(rounded to the nearest hundred)
or	21 831 000	(rounded to the nearest thousand)
or	21 830 000	(rounded to the nearest ten thousand)
or	21 800 000	(rounded to the nearest hundred thousand)
or	22 000 000	(rounded to the nearest __?__) million
or	20 000 000	(rounded to the nearest __?__?__) ten million

2. Can you give a rule for rounding to a given place?
See Discussion.

3. Quebec and Ontario are the two largest provinces in Canada. The area of Quebec is 1 540 628 1 540 000 square kilometres. The area of Ontario is 1 068 546 square kilometres. Give each area rounded to the nearest ten thousand. 1 070 000





4. The Confederation building in Charlottetown cost $5 600 050. Give the cost:
   - A to the nearest ten thousand 5 600 000
   - B to the nearest million 6 000 000

5. There were 79 830 600 telephones in North America in a recent year. Give this number:
   - A to the nearest thousand 79 831 000
   - B to the nearest million 80 000 000

*June 1972

24

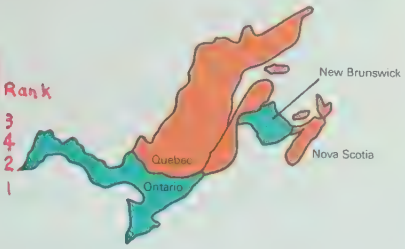
Discussion

Use discussion exercises 1 and 2 to develop the rule for rounding numbers to a particular place value. As you work through exercise 1, be sure children realize that each figure shown in the chart has been rounded directly from the number 21 830 772, above. Point out that when 5 or a number greater than 5 is the last digit before the place they are rounding to, they should round *up*. You might discuss the fact that the decision to round up when 5 is the last digit before the given place value is arbitrary, since 5 is actually just as close to the place value below as to that above. You might want to have the children answer exercises 3, 4, and 5 independently before discussing them.

Canada's First Four Provinces

In 1867 when Queen Victoria signed the British North America Act, Canada became a nation. At that time there were only four provinces: Nova Scotia, New Brunswick, Upper Canada (Ontario) and Lower Canada (Quebec). The territory which today constitutes the northern sections of Ontario and Quebec belonged at that time to the Hudson Bay Company, so that the area of each of these provinces was much smaller than it is now.

Province	Rank	Population 1871	Population 1971	Rank
Nova Scotia	3	390,000	788,960	3
New Brunswick	4	275,000	634,557	4
Quebec	2	1,200,000	6,027,764	2
Ontario	1	1,650,000	7,703,106	1



1. Make a chart, as in the above example. Show the population figures rounded off to the nearest thousand. See Answers, T.E. page 25.
2. A Rank the provinces according to the size of their populations in 1871.
   B Rank the provinces according to the size of their populations in 1971. See table above.
3. A Study the chart which you set up in Exercise 1. Which province has had the greatest increase in population between 1871 and 1971. Ontario
   B Which has had the least population increase between 1871 and 1971? New Brunswick
4. In 1971, which province had a population of 8 000 000, as rounded out to the nearest million? Ontario
5. In 1871, which province had a population of 1 000 000, as rounded out to the nearest million? Quebec
6. In the census of 1971, the territory with the smallest population was the Yukon, with 18 388 inhabitants, and Ontario had the largest population, with over seven million. If we round out the given figures to the nearest ten thousand, what is the difference between these two regions? 7 680 000



Using the Exercises

Before assigning the exercises on page 25, encourage children to discuss the maps and the table at the top of the page. Encourage the children to make observations concerning the fact that the provinces shown on the map are not the same sizes and shapes that they were when Canada became a nation. Then discuss the changes in population that have occurred since 1867.

This lesson should provide much more for the children than simply work with the rounding of large numbers. You might want to use this lesson as a springboard to other kinds of studies involving various geographical or historical facts.

Following the discussion, have the children do the exercises. When they have finished, allow time for checking papers.

Assignments (page 25)

Minimum: 1–3, oral. Average: 1–3. Maximum: 1–6.

Follow-up

Capitalize on any topic the children found particularly interesting. For example, if some show a particular interest in population facts, suggest that they find and classify the population of the world, Canada, their province, county, city or town, and school. Then they might round these figures to the nearest hundred, thousand, ten thousand, and hundred thousand. This information could easily be charted.

	Rounded to the nearest 100 000	10 000	1000	100
World				
Canada				
Province				
County				
City				
School				

Answers, Exercise 1, page 25

Province	Population 1871	Population 1971
Nova Scotia	390 000	789 000
New Brunswick	275 000	635 000
Quebec	1 200 000	6 028 000
Ontario	1 650 000	7 703 000

Workbook, page 6

PAGES 26–27

Objectives

Given a number which is an exact power of 10, the child will be able to write the number in exponent form, using 10 as the base.

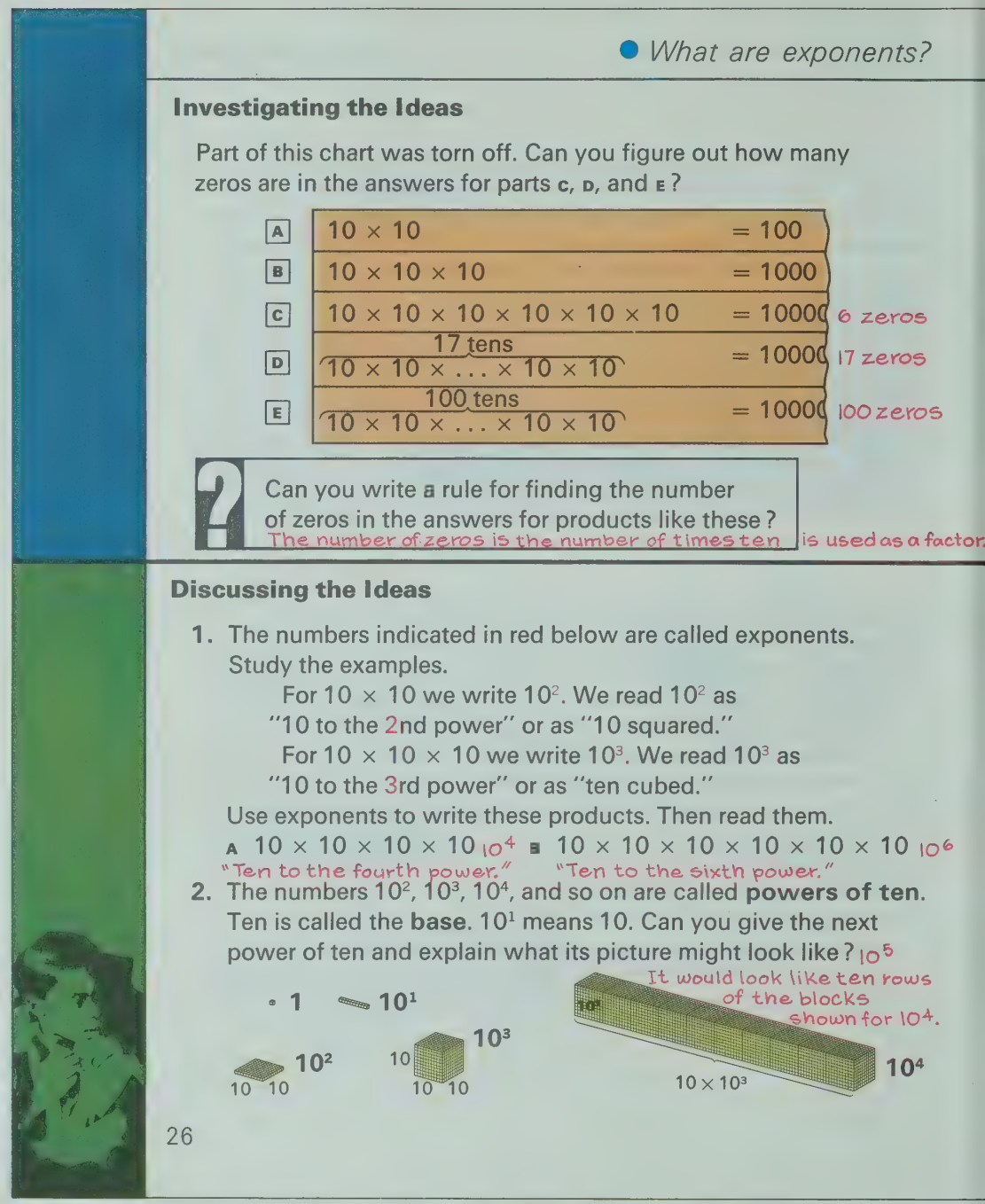
Given a base-ten numeral, the child will be able to write it in expanded notation using exponent form.

Preparation

You might plan a brief practice of rounding numbers. For example, list numbers on the chalkboard and ask children to round them to a specified place. However, such an activity should be kept brief—no more than five minutes.

Investigation

It would be appropriate for children to work in small groups on this investigation, helping each other find and write the rule they are challenged to discover. However, for most children it will be a relatively short investigation which you can use to lead into a discussion relating the number of zeros in the products to the exponent in each of the powers of ten you discuss. Children might be amused to learn that the number represented in Part E (ten as a factor 100 times) is called a googol.



● *What are exponents?*

Investigating the Ideas

Part of this chart was torn off. Can you figure out how many zeros are in the answers for parts C, D, and E?

A	10×10	$= 100$
B	$10 \times 10 \times 10$	$= 1000$
C	$10 \times 10 \times 10 \times 10 \times 10 \times 10$	$= 10000$ 6 zeros
D	$10 \times 10 \times \ldots \times 10 \times 10$ (17 tens)	$= 10000$ 17 zeros
E	$10 \times 10 \times \ldots \times 10 \times 10$ (100 tens)	$= 10000$ 100 zeros

Can you write a rule for finding the number of zeros in the answers for products like these?
The number of zeros is the number of times ten is used as a factor

Discussing the Ideas

1. The numbers indicated in red below are called exponents. Study the examples.

   For 10×10 we write 10^2. We read 10^2 as "10 to the 2nd power" or as "10 squared."

   For $10 \times 10 \times 10$ we write 10^3. We read 10^3 as "10 to the 3rd power" or as "ten cubed."

   Use exponents to write these products. Then read them.

   A $10 \times 10 \times 10 \times 10$ 10^4 "Ten to the fourth power." B $10 \times 10 \times 10 \times 10 \times 10 \times 10$ 10^6 "Ten to the sixth power."

2. The numbers 10^2, 10^3, 10^4, and so on are called **powers of ten**. Ten is called the **base**. 10^1 means 10. Can you give the next power of ten and explain what its picture might look like? 10^5

26

Discussion

You might begin the discussion by asking children if they would like to learn a shortcut way of writing a googol. After you study this page, help the children to figure out that a googol may be written 10^{100}.

As you read and study the discussion exercises with the children, point out that the small numeral beside 10 in the first exercise tells how many times 10 is to be used as a factor. That is, they can think of 10^2 as 10×10 or as 100. Emphasize that the exponent indicates the number of times the given number, in this case 10, is to be used as a factor.

Make certain the children see that an exponent does not indicate that the base, in this case, 10, is to be multiplied by the exponent, but rather that the base is to be multiplied by *itself* the number of times indicated by the exponent. Note also that 10^2 is often read "ten squared" and that 10^3 is often read "ten cubed."

As you discuss exercise 2, help children to see that 10^5 would be a single layer of ten rows each consisting of ten of the 10^3 blocks. Similarly, 10^6 would be a large block consisting of 10 layers each the size of the 10^5 layers. You might give the children an opportunity to discover, or encourage them to discover, why "squared" and "cubed" are used for these particular exponents. Of course, the idea has to do with area and volume, and you should help the children see this.

Using the Ideas

1. Give the missing numbers.

Example: $10^2 = 100$

A $10^3 =$ ||| 1000

B $10^4 =$ ||| 10 000

C $10^5 =$ ||| 100 000

D $10^6 =$ |||

E $10^7 =$ |||

F $10^8 =$ |||

G $10^9 =$ |||

D 1 000 000 E 10 000 000 F 100 000 000 G 1 000 000 000

2. Give the number for *a*.
Then give the product for *b*.

A $5 \times 10^3 = 5 \times a = b$ 1000; 5000

B $12 \times 10^3 = 12 \times a = b$ 1000; 12 000

C $37 \times 10^2 = 37 \times a = b$ 100; 3700

D $3 \times 10^4 = 3 \times a = b$ 10 000; 30 000

think



Suppose a book has 100 names listed in each column, with 5 columns per page. The book has 200 pages. How many books are needed to list all the people in the United States, if the population is 200 million?

220

3. Give the number for *a*. Then give the power of ten for *b*.

A $300 = 3 \times a = 3 \times b$ 100; 10^2

B $6000 = 6 \times a = 6 \times b$ 1000; 10^3

C $2700 = 27 \times a = 27 \times b$ 100; 10^2

D $90\,000 = 9 \times a = 9 \times b$ 10 000; 10^4

4. Find the products.

A 4×10^3 B 8×10^2 C 13×10^3 D 48×10^2 E 4×10^4 F 65×10^3 G 57×10^2 H 39×10^3 I 8×10^5 J 7×10^6

A 4000 B 800 C 13 000 D 4800 E 40 000 F 65 000 G 5700 H 39 000 I 800 000 J 7 000 000

5. Write each of the following as a product of a number between 1 and 10 and a power of ten.

Example: $700 = 7 \times 10^2$

A 900 9×10^2 B 8000 8×10^3 C 200 2×10^2 D 6000 6×10^3 E 70 000 7×10^4 F 90 000 9×10^4 G 20 000 2×10^4 H 600 000 6×10^5

6. Write each of the following as illustrated in the example.

Example: $362 = (3 \times 10^2) + (6 \times 10) + (2 \times 1)$

This is called expanded notation (exponent form).

A 457	D 2348	G 5042	J 9000
B 384	E 2459	H 6703	K 37 256
C 4265	F 6782	I 8006	L 79 384

See Answers, T.E. page 27.

More practice, page A-1, Set 2



Using the Exercises

Assign the exercises on page 27 as independent work. Notice that they are set up in a developmental way, progressing from the simple expression of the power of ten ($10^2 = 100$) to use of the powers of ten in expanded notation. For some groups of children, you will want to discuss this development step by step. For example, work through some particular expressions in a manner something like this:

$10^3 = 1000$

$7 \times 10^3 = 7 \times 1000 = 7000$

$7536 = 7000 + 500 + 30 + 6$

$= (7 \times 1000) + (5 \times 100) + (3 \times 10) + 6$

$= (7 \times 10^3) + (5 \times 10^2) + (3 \times 10) + 6$

Assignments (page 27)*

Minimum: 1–4. Average: 1–5. Maximum: 1–6.

Follow-up

If necessary, provide children with further practice using expanded notation. You might provide a worksheet similar to the following.

Match the numerals with the correct expanded form:

1. 285
2. 3782
3. 4265
4. 8971
5. 37 582
6. 7 608 423

a. $(8 \times 10^3) + (9 \times 10^2) + (7 \times 10) + 1$

b. $(7 \times 10^6) + (6 \times 10^5) + (8 \times 10^3) + (4 \times 10^2) + (2 \times 10) + 3$

c. $(2 \times 10^2) + (8 \times 10) + 5$

d. $(3 \times 10^4) + (7 \times 10^3) + (5 \times 10^2) + (8 \times 10) + 2$

e. $(4 \times 10^3) + (2 \times 10^2) + (6 \times 10) + 5$

f. $(3 \times 10^3) + (7 \times 10^2) + (8 \times 10) + 2$

Resources for Active Learning

Nuffield Project: *Computation and Structure* 4, "Indices" (exponents), pp. 59–64, Wiley.

Answers, exercise 6, page 27

A $457 = (4 \times 10^2) + (5 \times 10) + (7 \times 1)$

B $384 = (3 \times 10^2) + (8 \times 10) + (4 \times 1)$

C $4265 = (4 \times 10^3) + (2 \times 10^2) + (6 \times 10) + (5 \times 1)$

D $2348 = (2 \times 10^3) + (3 \times 10^2) + (4 \times 10) + (8 \times 1)$

E $2459 = (2 \times 10^3) + (4 \times 10^2) + (5 \times 10) + (9 \times 1)$

F $6782 = (6 \times 10^3) + (7 \times 10^2) + (8 \times 10) + (2 \times 1)$

G $5042 = (5 \times 10^3) + (4 \times 10) + (2 \times 1)$

H $6703 = (6 \times 10^3) + (7 \times 10^2) + (3 \times 1)$

I $8006 = (8 \times 10^3) + (6 \times 1)$

J $9000 = 9 \times 10^3$

K $37\,256 = (3 \times 10^4) + (7 \times 10^3) + (2 \times 10^2) + (5 \times 10) + (6 \times 1)$

L $79\,384 = (7 \times 10^4) + (9 \times 10^3) + (3 \times 10^2) + (8 \times 10) + (4 \times 1)$

Duplicator Masters, page 4
Workbook, page 7
Skill Masters, page 4

PAGES 28–29

Objective

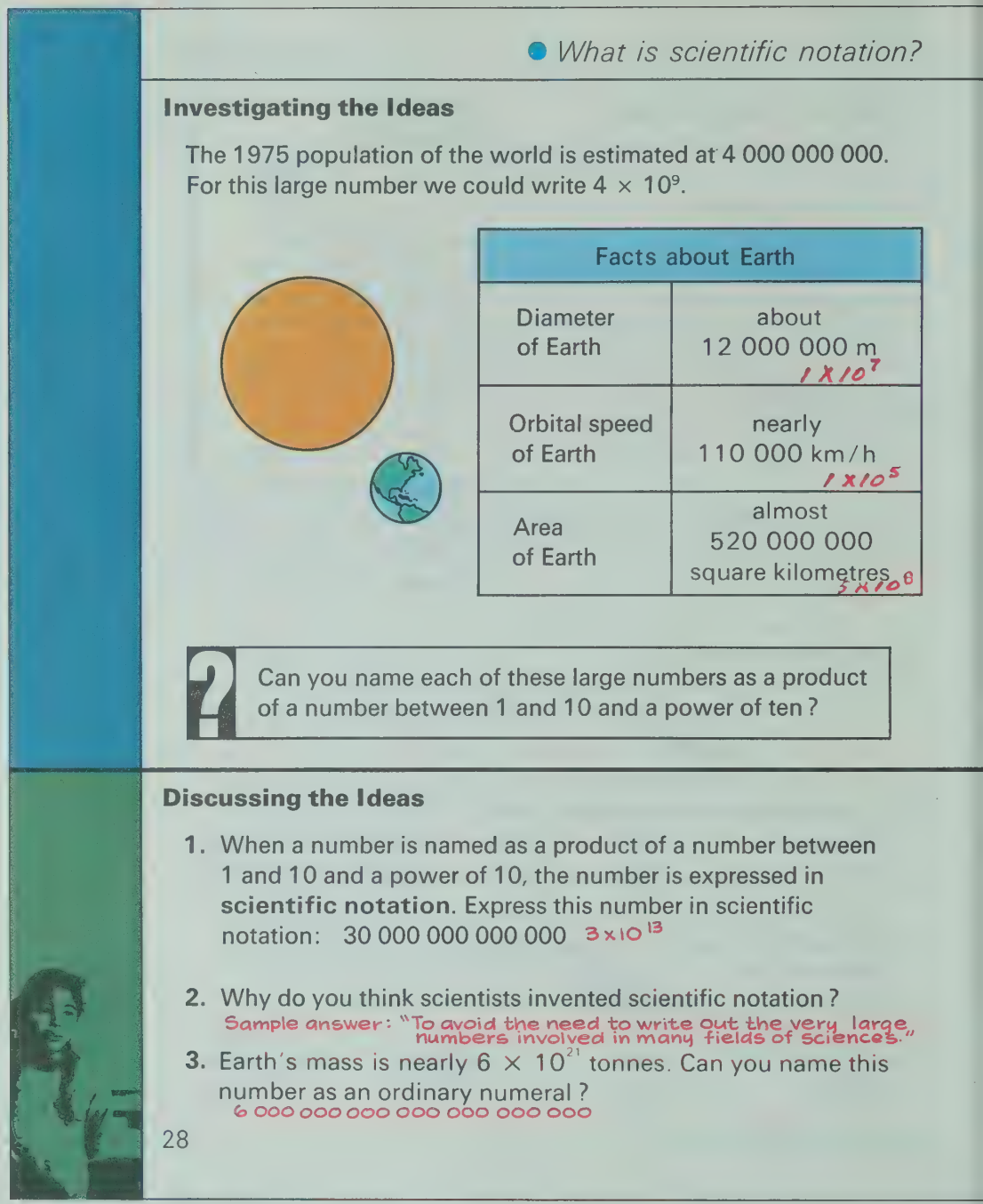
Given a relatively large number expressed in base ten, the child will be able to write it in scientific notation.

Preparation

To prepare for this lesson, it would be helpful to quickly review the manner of writing powers of ten. For example, write expressions such as 10^3, 1 000 000, 10^5, 10^6, 10 000, and 100 000 000 on the chalkboard and ask children to change them from an expression with exponents to the ordinary numeral or vice-versa.

Investigation

As for many investigations, you might want children to work together in groups of two or three. For children who finish very quickly, write other large numbers on the chalkboard or suggest that they themselves find large numbers and write them as products with powers of ten. However, remind these children that when writing these products they should first round a large number. For example, 92 912 000 is rounded to 93×10^6; but 93 000 000 might be rounded to 9×10^7.



● *What is scientific notation?*

Investigating the Ideas

The 1975 population of the world is estimated at 4 000 000 000. For this large number we could write 4×10^9.

Facts about Earth	
Diameter of Earth	about 12 000 000 m 1×10^7
Orbital speed of Earth	nearly 110 000 km/h 1×10^5
Area of Earth	almost 520 000 000 square kilometres 5×10^8

? Can you name each of these large numbers as a product of a number between 1 and 10 and a power of ten?

Discussing the Ideas

1. When a number is named as a product of a number between 1 and 10 and a power of 10, the number is expressed in **scientific notation**. Express this number in scientific notation: 30 000 000 000 000 3×10^{13}

2. Why do you think scientists invented scientific notation?
Sample answer: "To avoid the need to write out the very large numbers involved in many fields of sciences."

3. Earth's mass is nearly 6×10^{21} tonnes. Can you name this number as an ordinary numeral?
6 000 000 000 000 000 000 000

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Discussion

As you discuss the material in this section, use the examples from the investigation. Now that the children have had an opportunity to work with exponential notation involving powers of ten, you may want to elicit from them simple rules, such as, "The exponent helps us to count the number of zeros." Here is a case where it is not only important to understand just why 10^9 is 1 followed by nine zeros, it is also useful to know this convenient and easy rule. Work through other examples as needed until children realize how to use scientific notation.

Exercise 2 brings out an important aspect of this lesson by relating arithmetic to the important role it plays in science, particularly in the physical sciences.

Children may want to refer to the chart shown on page 23 for assistance in naming 6×10^{21} as an ordinary numeral.

Using the Ideas

1. Write the ordinary numeral for each exercise.

A 10^7 C 10^9 E 5×10^3 G 7×10^6 I 4×10^{12}
B 10^8 D 10^{10} F 8×10^2 H 6×10^4 J 8×10^{10}

See Answers, T.E. page 29.

2. Use scientific notation to name each of the following.

A 5000 5×10^3 B 7000 7×10^3 C 70 000 7×10^4 D 20 000 2×10^4
E 300 000 3×10^5 F 5 000 000 5×10^6 G 7 million 7×10^6
H 80 million 8×10^7 ★ I 6 sextillion 6×10^{21} ★ J 500 quadrillion 5×10^{17}

3. Use the chart and the diagram to help you understand and answer the questions.

Facts about the Universe
1 The diameter of our sun is about 1 392 000 km. It would take over a million Earths to make an object the size of the sun.
2 Earth is about 149 570 000 kilometres from the sun.
3 One of the two nearest stars, Alpha Centauri, is like our sun and is about 39 814 705 000 000 kilometres away.
4 There are probably 100 000 000 000 other stars in our galaxy, or group of stars.
5 As many as 500 000 000 000 other galaxies (groups of stars) have been detected by high-powered telescopes.

A Give the sun's diameter to the nearest hundred thousand km.
B Give the distance of Earth from the sun, rounded to the nearest 10 million kilometres. 150 000 000 km
C Give the distance to Alpha Centauri, rounded to the nearest 10 trillion kilometres. 40 000 000 000 000 km

4. Name each answer to exercise 3 in scientific notation.
A: 1×10^6 B: 2×10^8 C: 4×10^{13}
5. Use scientific notation to name the number of other galaxies. 5×10^{11}

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Follow-up

As suggested in the follow-up section on page 23, various distances between bodies in the solar system provide large numbers which are suitable for practice in both rounding numbers and using scientific notation. Encourage those interested or those who need further practice to make a chart showing the planets and their average distance from the sun, expressed as rounded numbers and in scientific notation. For example:

Mercury	58 019 000 → 60 000 000	→ 6×10^7
Venus	108 134 000 → 100 000 000	→ 1×10^8
⋮	⋮	

Using the Exercises

Before assigning exercises from page 29, you might want to use exercise 3 as a basis for further discussion.

The information provided under the table in this exercise is intended to give the children an idea of the tremendous size of the universe. That is, if the sun were the size of a pea, the earth would be only a tiny speck one metre away; and one of the nearest solar systems would be Alpha Centauri — 225 kilometres away!

Answers, exercise 1, page 29

A 10 000 000
B 100 000 000
C 1 000 000 000
D 10 000 000 000
E 5000
F 800
G 7 000 000
H 60 000
I 4 000 000 000 000
J 80 000 000 000

Assignments (page 29)*

Minimum: 1, 2. Average: 1–5.
Maximum: 1–5.

PAGES 30–31

Objective

Given a base-ten numeral, the child will be able to write it as a base-five numeral or a base-six numeral.

Preparation

Materials

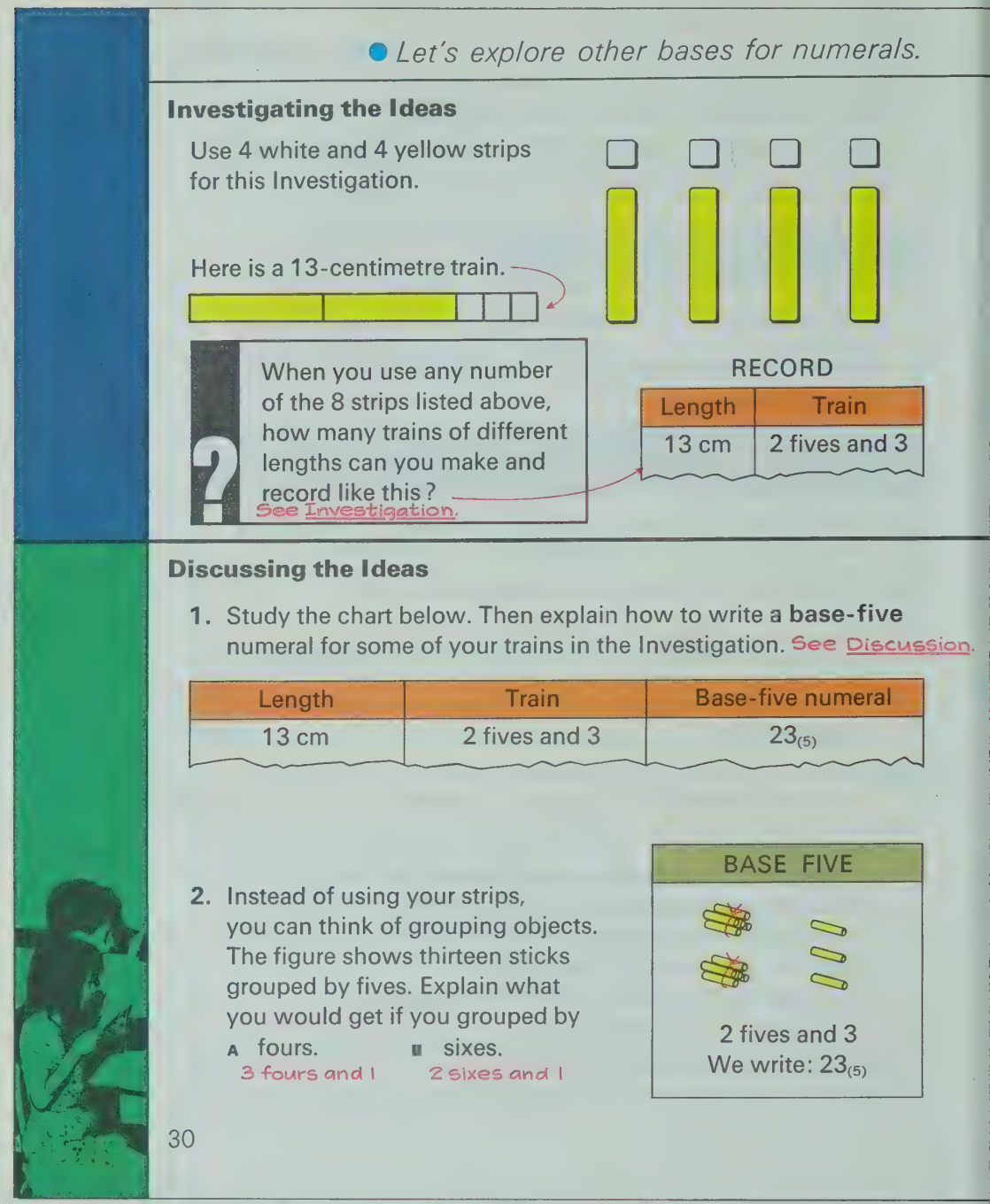
yellow strips (4 per child); white strips (4 per child)

Since the content of this lesson does not relate directly to that of previous lessons of this chapter, you might want to omit any preparation and begin immediately with the investigation.

Investigation

Because this investigation calls for the use of 4 yellow and 4 white strips, children are forced to think of numbers in groups of fives and ones. However, they are introduced to this idea through the familiar process of building trains. If some children have difficulty, simply ask them to pick up any number of the yellow and white strips and record the length of the train according to how many fives and ones they have. Note that trains of 1, 2, 3, or 4 centimetres in length will be recorded simply by 1, 2, 3, or 4 as in base ten. The notation for five, which is $10_{(5)}$, will be developed in the discussion, but when necessary, elicit from a child the observation that, with the strips he is using, a train of length 5 centimetres will be 1 yellow strip and should be recorded as 1 five. The lengths and trains that can be found are listed below.

1; 1	13; 2 fives and 3
2; 2	14; 2 fives and 4
3; 3	15; 3 fives
4; 4	16; 3 fives and 1
5; 1 five	17; 3 fives and 2
6; 1 five and 1	18; 3 fives and 3
7; 1 five and 2	19; 3 fives and 4
8; 1 five and 3	20; 4 fives
9; 1 five and 4	21; 4 fives and 1
10; 2 fives	22; 4 fives and 2
11; 2 fives and 1	23; 4 fives and 3
12; 2 fives and 2	24; 4 fives and 4



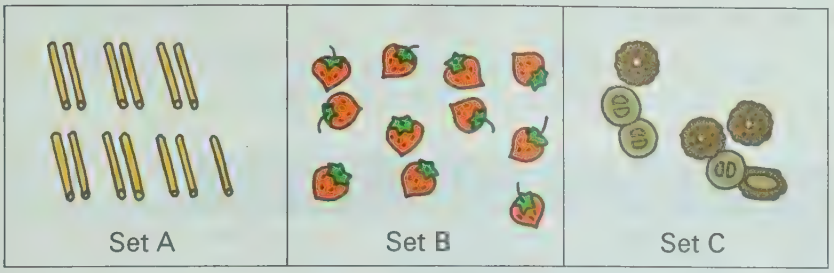
Discussion

The objective in the investigation is for children to think of numbers in groups of fives and ones. The intention in the discussion is for children to learn the notation of such grouping and to transfer this concept of grouping to other bases.

In discussion exercise 1, use children's examples to help them realize how numbers are written in base-five notation. Point out that the only digits appropriate in base five are 1, 2, 3, 4, and 0. Help them figure out how the number five or 1 five, should be written. When you discuss this numeral $10_{(5)}$, point out that the place value of the 1 is five and that this is indicated by the small subscript 5 written in parentheses. Thus, $10_{(5)}$ means 1 five and zero ones; it is read "one, zero, base 5."

Exercise 2 relates the concept of base to grouping objects rather than to strips. It also extends the notion of base to include bases other than five. You might want to demonstrate with groups of three, four, or six objects, asking children to record the groupings you show. Stress examples using base six in preparation for the following exercise set. For example, show three groups of six and 2 singles; ask children to record the number in

Using the Ideas



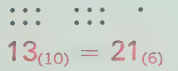
1. For sets A, B, and C above, give the base-ten numeral that tells how many objects are in each set. A 13 B 11 C 7

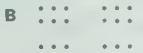
2. A How many groups of 6 sticks are in set A? 2
   B After all the possible groups of 6 are made, how many extra sticks are there? 1
   C Give the base-six numeral that tells how many sticks are in set A. $21_{(6)}$

3. Give the base-six numeral that tells how many objects are in set B; in set C. B $15_{(6)}$ C $11_{(6)}$

4. Since there is the same number of checkers whether we group by ten or six, we write $20_{(10)} = 32_{(6)}$. For each set, write an equation that uses the base-ten and base-six numeral for the number of dots. Exercise 4A is completed correctly.

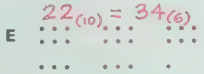


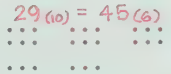
A 
$13_{(10)} = 21_{(6)}$

B 
$22_{(10)} = 34_{(6)}$

C 
$29_{(10)} = 45_{(6)}$

D 
$30_{(10)} = 50_{(6)}$

E 
$31_{(10)} = 51_{(6)}$

F 
$28_{(10)} = 44_{(6)}$

★ 5. Copy and complete each equation.

A $16_{(10)} = \text{▮▮}_{(6)}^{24}$ C $\text{▮▮}_{(10)}^{22} = 34_{(6)}$ E $30_{(10)} = \text{▮▮}_{(6)}^{50}$ G $28_{(10)} = \text{▮▮}_{(6)}^{44}$

B $\text{▮▮}_{(10)}^{17} = 25_{(6)}$ D $35_{(10)} = \text{▮▮}_{(6)}^{55}$ F $\text{▮▮}_{(10)}^{5} = 5_{(6)}$ H $\text{▮▮}_{(10)}^{11} = 15_{(6)}$

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base six. The correct numeral, $32_{(6)}$, is read "three, two, base six."

Using the Exercises

Note that the exercises on both pages 30 and 31 refer only to 2-digit numerals. Thus, with these exercises, children need only think in groups of sixes and ones, or in groups of tens and ones.

When the children have finished the exercises, correct them carefully, allowing time for further discussion. Although exercise 5 is starred, it would be helpful to ask those who tried it to explain how they arrived at their answer.

Assignments (page 31)*
Minimum: 1–4. Average: 1–4.
Maximum: 1–5.

Follow-up

To reinforce the concept of base, suggest other activities with the strips, similar to the investigation which introduced this lesson. For example, if you keep a file of task cards, add assignments similar to the following:

Use 3 purple strips and 3 white strips. Make as many trains as you can and record the train in two ways.	
Example:	
Strips in each train	Base-four notation
1 four and 3	$13_{(4)}$

Resources for Active Learning

Teaching Aids for Elementary Mathematics, pp. 39–53. Holt, Rinehart and Winston.

Modern Math Games, pp. 51–53, Fearon. (Available from Clarke, Irwin)

Workbook, page 8

PAGES 32–33

Objective

Given addition or multiplication problems with 1- or 2-digit base-six numerals, the child will be able to find the sums and products using base-six numerals.

Preparation

Materials

dark green strips (2 per child); white strips (5 per child)

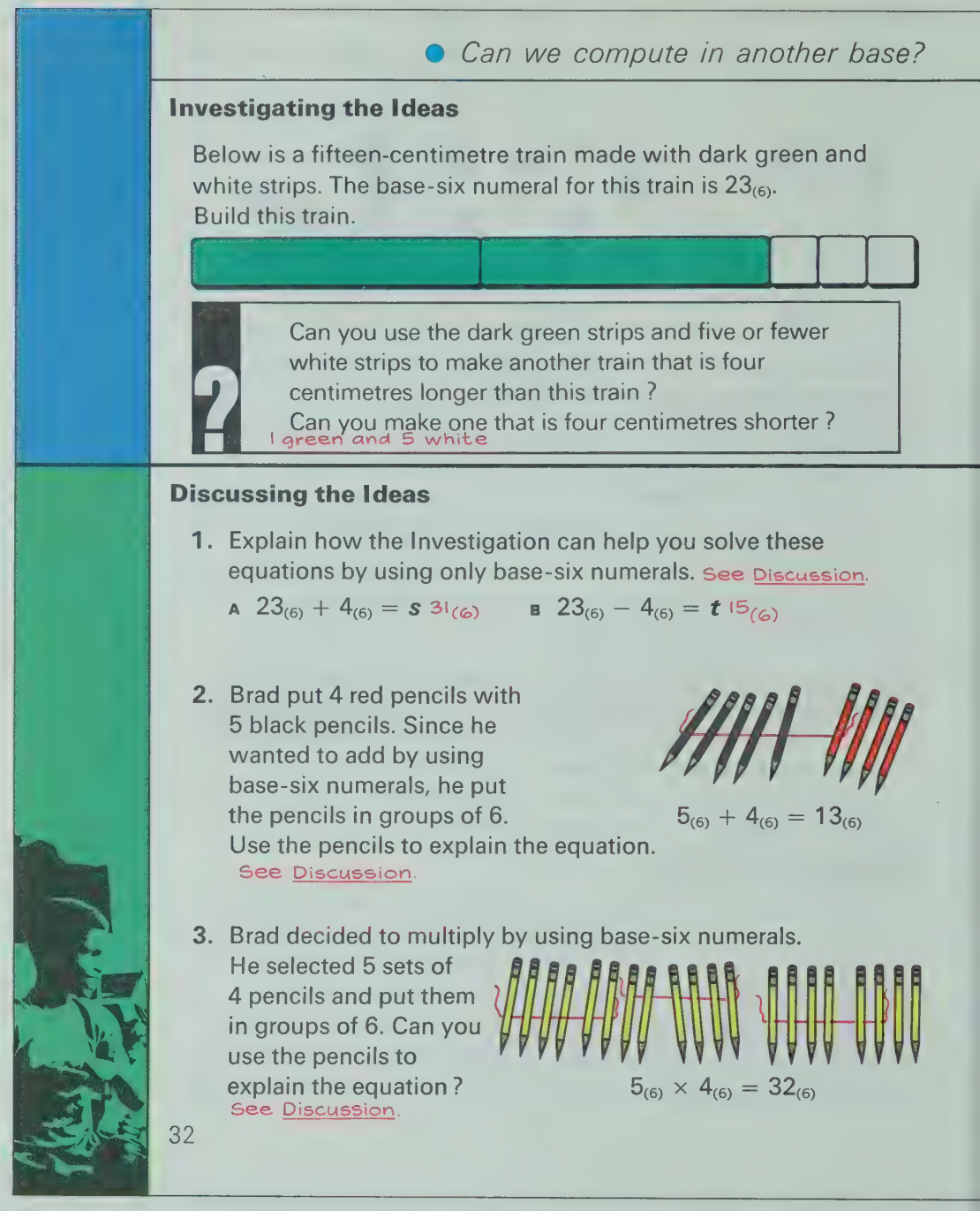
To prepare for this lesson, you might write several 2-digit numerals of bases four, five, and six on the chalkboard and ask children to explain what each numeral means. For example, if you write $13_{(4)}$, you might expect a child to explain that "one, three, base four" means that you have one four and 3, which is the same as 7 in base ten. Since most of the text work is with base six, you might stress 2-digit numerals in base six.

Investigation

Due to the restricted number of strips used in this investigation, the child is forced to use five white strips and a dark green strip. Thus, he works concretely with the regrouping process of computation in base six. As you observe children building the proper trains, you might ask some of them to use a base-six numeral to record each new train. However, this will be developed in the discussion section.

Give children adequate time to work on this investigation. If necessary, ask leading questions of any child who appears confused. For example, ask by how much he would lengthen the train if he added a dark green strip. Is there any way that he can use the dark green strip and still add only 4 centimetres to the train?

Throughout this lesson, keep in mind that there is no intention that children develop a speed skill in base six; the intent is for better understanding of the concept of base.



● *Can we compute in another base?*

Investigating the Ideas

Below is a fifteen-centimetre train made with dark green and white strips. The base-six numeral for this train is $23_{(6)}$.
Build this train.

?

Can you use the dark green strips and five or fewer white strips to make another train that is four centimetres longer than this train ?
Can you make one that is four centimetres shorter ?
1 green and 5 white

Discussing the Ideas

1. Explain how the Investigation can help you solve these equations by using only base-six numerals. See Discussion.

   A $23_{(6)} + 4_{(6)} = s$ $31_{(6)}$ B $23_{(6)} - 4_{(6)} = t$ $15_{(6)}$

2. Brad put 4 red pencils with 5 black pencils. Since he wanted to add by using base-six numerals, he put the pencils in groups of 6. Use the pencils to explain the equation.
   See Discussion.

$5_{(6)} + 4_{(6)} = 13_{(6)}$

3. Brad decided to multiply by using base-six numerals. He selected 5 sets of 4 pencils and put them in groups of 6. Can you use the pencils to explain the equation ?
   See Discussion.

$5_{(6)} \times 4_{(6)} = 32_{(6)}$

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Discussion

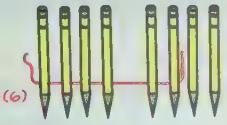
Ask those children who were able to make the trains to use a base-six numeral to describe the two trains they made. Relate the investigation activity to these numerals and to the equations in exercise 1. Help children realize that in base six, every six objects become a single group of six. Similarly, a single group of six may be replaced by six ones when necessary in subtraction.

Exercises 2 and 3 stress the same regrouping concept but provide opportunity for set demonstrations or work with physical objects such as pencils, sticks, and pipe cleaners. For example, have a child use sets of objects to explain exercise 2, first by putting a set of five and a set of four together, then by grouping sixes to show one group of six and three. Similarly, for exercise 3, it would be helpful to have a child (or as many children as materials allow) put together 5 groups of four pencils and then group them by sixes to show three groups of six and two.

Continue this type of demonstration for both addition and multiplication examples, using set demonstrations until you feel that the children understand the ideas.

Using the Ideas

1. You can use sets to help you find sums.
Solve: $4_{(6)} + 4_{(6)} = n_{(6)}$ $n = 12_{(6)}$



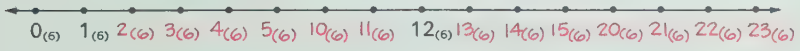
2. You can use your strips to help you find sums.
$4_{(6)} + 3_{(6)} = m_{(6)}$ $m = 11_{(6)}$



3. Find the sums. Use base-six numerals.

A	$4_{(6)} + 4_{(6)}$	$12_{(6)}$	D	$5_{(6)} + 1_{(6)}$	$10_{(6)}$	G	$5_{(6)} + 5_{(6)}$	$14_{(6)}$	J	$1_{(6)} + 5_{(6)}$	$10_{(6)}$
B	$3_{(6)} + 3_{(6)}$	$10_{(6)}$	E	$3_{(6)} + 5_{(6)}$	$12_{(6)}$	H	$3_{(6)} + 4_{(6)}$	$11_{(6)}$	K	$4_{(6)} + 0_{(6)}$	$4_{(6)}$
C	$2_{(6)} + 4_{(6)}$	$10_{(6)}$	F	$3_{(6)} + 2_{(6)}$	$5_{(6)}$	I	$5_{(6)} + 2_{(6)}$	$11_{(6)}$	L	$4_{(6)} + 5_{(6)}$	$13_{(6)}$

4. Copy this number line and label it with base-six numerals.



5. Find the sums and differences. Use base-six numerals.

A	B	C	D	E	F
$13_{(6)}$	$13_{(6)}$	$12_{(6)}$	$24_{(6)}$	$11_{(6)}$	$24_{(6)}$
$+2_{(6)}$	$+3_{(6)}$	$-5_{(6)}$	$+1_{(6)}$	$-4_{(6)}$	$+3_{(6)}$
$15_{(6)}$	$20_{(6)}$	$3_{(6)}$	$25_{(6)}$	$3_{(6)}$	$31_{(6)}$

6. Find the products. Use base-six numerals.

A	$3_{(6)} \times 3_{(6)}$	$13_{(6)}$	D	$4_{(6)} \times 2_{(6)}$	$12_{(6)}$	G	$4_{(6)} \times 3_{(6)}$	$20_{(6)}$	J	$4_{(6)} \times 4_{(6)}$	$24_{(6)}$
B	$2_{(6)} \times 2_{(6)}$	$4_{(6)}$	E	$2_{(6)} \times 4_{(6)}$	$12_{(6)}$	H	$5_{(6)} \times 5_{(6)}$	$41_{(6)}$	K	$5_{(6)} \times 1_{(6)}$	$5_{(6)}$
C	$3_{(6)} \times 2_{(6)}$	$10_{(6)}$	F	$3_{(6)} \times 4_{(6)}$	$20_{(6)}$	I	$3_{(6)} \times 0_{(6)}$	$0_{(6)}$	L	$2_{(6)} \times 3_{(6)}$	$10_{(6)}$

★ 7. Find the products. Use base-six numerals.

A	B	C	D	E	F
$11_{(6)}$	$12_{(6)}$	$13_{(6)}$	$21_{(6)}$	$25_{(6)}$	$30_{(6)}$
$\times 4_{(6)}$	$\times 4_{(6)}$	$\times 3_{(6)}$	$\times 2_{(6)}$	$\times 2_{(6)}$	$\times 2_{(6)}$
$44_{(6)}$	$52_{(6)}$	$43_{(6)}$	$42_{(6)}$	$54_{(6)}$	$100_{(6)}$

More practice, page A-2, Set 3



Using the Exercises

Allow those children who feel the need to do so to use sets of physical objects or their strips while working on these exercises. However, some children may find it easier to change the numeral to base ten, compute, and convert back to base six. For exercise 5, many will be able to regroup directly in base six, such as with

$$\begin{array}{r} {\scriptstyle 1} \\ 24_{(6)} \\ +\ 3_{(6)} \\ \hline 31_{(6)} \end{array}$$

4 + 3 = 7; in 7 there is one group of 6, which we add to the sixes' place, and 1.

Starred exercise 7 is intended as a challenge for more capable or interested children. You might suggest that they make a multiplication table for base six to help them work the problems. Some may again find it easier to change to base 10, compute, and change back to base six. Still others may multiply as in base ten, then simply regroup into sixes rather than into tens. For example:

$$\begin{array}{r} {\scriptstyle 1} \\ 25_{(6)} \\ \times\ 2_{(6)} \\ \hline 54_{(6)} \end{array}$$

$2 \times 5 = 10$; in 10 there is one six and four; $2 \times 2 = 4$; four sixes plus the one more six from 10 is 5 sixes.

Assignments (page 33)*

Minimum: 1–6. Average: 1–6.
Maximum: 1–7.

Follow-up

It would be helpful for the children to make addition and multiplication tables for base six. However, recall that such an activity should have as its purpose better understanding and development of the power skill in base six, rather than speed skill. Some children may want to use strips or sets of objects to build the tables.

	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	10
2	2	3	4	5	10	11
3	3	4	5	10	11	12
4	4	5	10	11	12	13
5	5	10	11	12	13	14

Base 6

Resources for Active Learning

Franklin Series: *Patterns and Puzzles*, "A Secret Code," pp. 90–92, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Mathex: Numeration No. 7, "Numeration Bases," pupil pages 15–22, Encyclopaedia Britannica Publications Ltd.

Duplicator Masters, page 5
Workbook, page 9

PAGES 34–35

Objective

The child will demonstrate his ability to work with the concepts presented in this chapter.

Preparation

In preparation for these pages, you might simply review the meaning of place value in base ten and in 2-digit numerals of other bases. You might also briefly review any concept which was troublesome to the children during their study of it, such as expanded notation with exponents.

Reviewing the Ideas

1. In the numeral 624 135 078 what digit is in the
   A thousands' place? 5 B millions' place? 4

2. Represent each number in expanded notation.
   A 56 (5×10)+(6×1) B 532 (5×100)+(3×10)+(2×1) C 8307 (8×1000)+(3×100)+(7×1) D 676 342 (6×100 000)+(7×10 000)+(6×1000)+(2×100)+(4×10)+(2×1)

3. Give the correct sign (<, >, or =) for each ◍.
   A 6328 ◍ 6000 + (6 × 100) + 28 < B 462 ◍ 400 + 500 + 2 <
   C 654 + 325 ◍ 644 + 325 > D 317 + 408 ◍ 307 + 418 =

4. In a recent year, the United States, with 5 894 536 kilometres of graded roads, had more roads than any other country. Give this information rounded to the nearest:
   A ten kilometres 5 894 540. B hundred kilometres 5 894 500 C thousand kilometres 5 895 000
   D ten thousand kilometres 5 890 000 E million kilometres 6 000 000

5. Write the ordinary base-ten numeral for:
   A 10^3 1000 B 10^4 10 000 C 10^5 100 000 D 10^6 1 000 000 E 8×10^5 800 000 F 34×10^3 34 000

6. Find the products.
   A 10×10^2 1000 B $10^2 \times 10^2$ 10 000 C $10^2 \times 10^3$ 100 000

7. Write the number that is 10 more than one billion. 1 000 000 010

8. Write the largest possible 6-digit number that has no two digits alike. 987 654

9. Represent each number by using expanded notation (exponent form).
   A 463 B 6489 ★C 8 740 684
   See Answers, T.E. page 35.

★10. Round 3 568 394 to the nearest million. Then write it in scientific notation. 4 000 000; 4×10^6

think

Egyptian numerals:
| = one 𓆼 = one thousand
∩ = ten 𓁨 = one million
9 = one hundred
The Egyptians did not use place value. Give the base-ten numeral for each of these.

1. 𓆼 ∩ | | 3. 𓁨 𓁨 9 9
2. 𓁨 𓆼 9 ∩ | 4. 𓁨 ∩

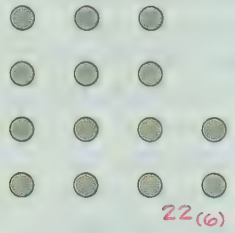
Discussion

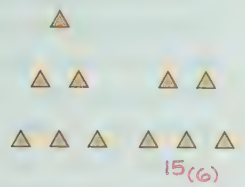
If you choose to use these pages as evaluation material, assign the exercises as independent work. You might prefer to use them as a basis for review with the class. In either case, allow time to discuss the children's responses. For example, in exercise 3, you might point out that they need not write the left side of the statements in expanded notation nor perform the operations indicated. They might simply compare the number in each place value, beginning with the position of greatest value, until they find an inequality or see that the numbers are equal. As children work exercise 4, be sure that they use the number given in the exercise, 5 894 536, for each of the 5 parts of the problem. Since the exercises on page 35 all deal with base-six numerals, you might use this page as a basis for discussion.

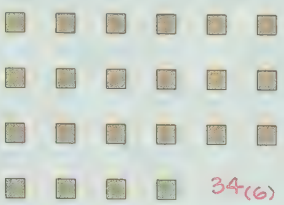
As you discuss exercises 11, 12, and 13, it would be helpful to show on an overhead projector groups of dots, first arranged in sixes, then arranged in groups of 6 sixes, as illustrated in the next column.

Explain that since there are 3 groups of 6 × 6 (6 sixes), 2 groups of 6, and 4 extra dots, the base-six numeral for the number in this set is $324_{(6)}$.

11. For each set, write a base-six numeral for the number of objects in the set.

A  $22_{(6)}$

B  $15_{(6)}$

C  $34_{(6)}$

12. Complete each sentence.

A 15 is 2 sixes and __? 3.
B 21 is __? 3 sixes and 3.
C 10 is 1 six and __? 4.
D 35 is __? 5 sixes and 5.
E 18 is 3 sixes and __? 0.
F 29 is __? 4 sixes and 5.

13. Write the base-six numeral for each base-ten numeral.

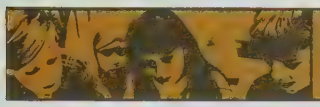
A 15 $23_{(6)}$ B 21 $33_{(6)}$ C 10 $14_{(6)}$ D 35 $55_{(6)}$ E 18 $30_{(6)}$ F 29 $45_{(6)}$

14. Find the sums and differences. Use base-six numerals.

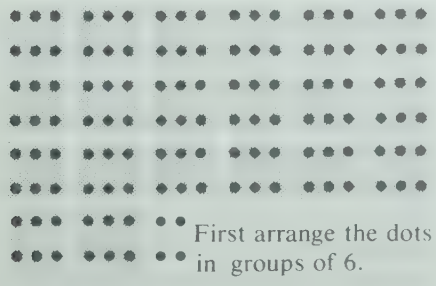
A $3_{(6)} + 4_{(6)} = 11_{(6)}$	B $5_{(6)} + 5_{(6)} = 14_{(6)}$	C $12_{(6)} + 14_{(6)} = 30_{(6)}$	D $15_{(6)} + 23_{(6)} = 42_{(6)}$	E $25_{(6)} + 22_{(6)} = 51_{(6)}$
F $5_{(6)} - 2_{(6)} = 3_{(6)}$	G $11_{(6)} - 4_{(6)} = 3_{(6)}$	H $12_{(6)} - 5_{(6)} = 3_{(6)}$	I $20_{(6)} - 13_{(6)} = 3_{(6)}$	J $44_{(6)} - 25_{(6)} = 15_{(6)}$

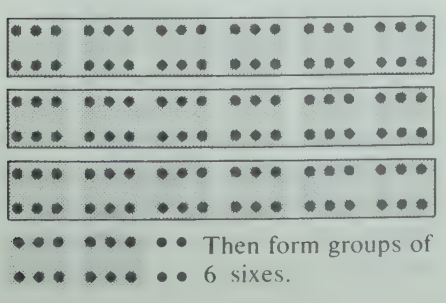
15. Find the products. Use base-six numerals.

A $2_{(6)} \times 5_{(6)}$ $14_{(6)}$ B $3_{(6)} \times 2_{(6)}$ $10_{(6)}$ C $4_{(6)} \times 3_{(6)}$ $20_{(6)}$ D $5_{(6)} \times 3_{(6)}$ $23_{(6)}$



35


First arrange the dots in groups of 6.


Then form groups of 6 sixes.

Follow-up

As a follow-up to the *Think* problem, you might have the children who worked on it write Egyptian numerals of their own choosing, and show them to the class, explaining their base-ten meaning. The children may also enjoy making posters to display Egyptian numerals. Some children might like to devise addition or multiplication problems in this notation to trade with one another.

Answers, exercise 9, page 34

9.A $463 = (4 \times 10^2) + (6 \times 10) + (3 \times 1)$

B $6489 = (6 \times 10^3) + (4 \times 10^2) + (8 \times 10) + (9 \times 1)$

C $8\,740\,684 = (8 \times 10^6) + (7 \times 10^5) + (4 \times 10^4) + (6 \times 10^2) + (8 \times 10) + (4 \times 1)$

Resources for Active Learning

Teaching Aids for Elementary Mathematics, p. 85, Holt, Rinehart and Winston [Crossnumber puzzles].

Workbook, page 10

General Objectives

To review the basic operations for whole numbers
To provide experiences in solving equations
To review basic facts of addition, subtraction, multiplication, and division
To develop some basic concepts of functions
To stress the inverse relation between addition and subtraction and between multiplication and division
To review basic principles for addition and multiplication
To provide experiences with word problems
To provide work with special products and quotients

The first two lessons of this chapter provide an overall review of the four basic operations, and of the related basic principles. This is followed by material emphasizing the inverse relation between addition and subtraction and between multiplication and division.

Subsequent material provides work with functions and the function machine, particularly as related to the basic operations. Skills in working with special products and quotients are then developed. These special skills mainly involve working with products, such as multiplying by 10 and 100, and multiplying by multiples of 10 and 100, and related quotients. Along with this work on special products and quotients, application of the basic principles is stressed. For example, the use of the distributive principle is emphasized in relation to its role in finding products involving a factor having two or more digits.

Finally, work with word problems and a variety of situations involving the writing of equations and basic skills is provided.

Mathematics

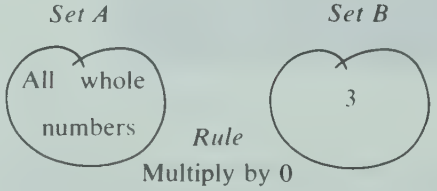
The concept of a function is one of the most important ideas in mathematics, yet it can be understood on an intuitive level very early in the child's experiences. Rather than present a precise mathematical definition of function, we give examples and point to some significant features.



Multiply by 2
and add 1
$(2x + 1)$

If we take a number from set A, for example 7, and apply the rule, we get exactly one number in set B. Hence, we generate the set of pairs $(3, 7)$, $(7, 15)$, $(5, 11)$, $(12, 25)$, $(40, 81)$. One vital feature of these pairs is that for each first number *there is only one second number.*

Here is another example:



Multiply by 0
and add 3
$(0\ x + 3)$

Some of the pairs of numbers are $(17, 3)$, $(2, 3)$, $(1, 3)$, $(0, 3)$, $(285, 3)$, ... Notice that, although every second number is 3, it remains true that, given any first number, *we get only one second number* (in this case, 3).

In summary, we have a set and a rule for each function. When we apply the rule to an element of the set, we get just *one* answer. Thus, for each function, we have a set of ordered pairs, no two of which have the same first element.

Since a large part of this chapter deals with basic facts and since we rely upon the inverse relation between operations to develop subtraction and division facts, one of the most important mathematical concepts of the chapter is this inverse relation. The following definitions state the relation for addition and subtraction and for multiplication and division.

Definition of subtraction:

> Let a, b, and c be whole numbers such that $a + b = c$. The number a is the difference $c - b$, and the number b is the difference $c - a$. In symbols, $a = c - b$, $b = c - a$.

Definition of division:

> If a, b, and c are whole numbers such that $b \neq 0$, and $a \times b = c$, then $a = c \div b$.

Notice that in the definition of subtraction we define an addend as a particular difference, and in the definition of division we define a factor as a particular quotient. Material such as that on pages 40 and 41 is derived from this inverse relation between the operations. That is, we find differences by thinking about missing factors.

Teaching the Chapter

Materials

Cards, 10 by 15 centimetres, at least 7 per child
Slips of paper, approximately 10 by 15 centimetres, 7 per child

Vocabulary

add
addend
associative (grouping) principle
commutative (order) principle
difference
distributive (multiplication-addition) principle
divide
equals
equation
factor
function
minus
multiply
operation
order
plus
parentheses
principles
product
quotient

repeated addition
repeated subtraction
solution
solve
subtract
sum
times

Since there is a considerable amount of review material in this chapter, the vocabulary list is quite long. You will find that the children are familiar with most of the words in this list. However, you should see that each word is carefully stressed as it occurs in the lessons in the chapter. The words *commutative, associative,* and *distributive* are given as names for the principles; however, if you choose, you may continue to use the words *order, grouping,* and *multiplication-addition,* respectively, for these principles.

Lesson Schedule

Plan to cover the material in this chapter in about two or two and a half weeks. Of course, the precise amount of time required for this chapter will depend largely upon the background and abilities of your children.

Evaluation of Progress

It is vitally important that the children acquire mastery of the basic addition and multiplication facts and the associated subtraction and division facts before they begin the computing work in a subsequent chapter. Likewise, the work on special products and quotients is essential to the successful completion of Chapter 5. This leads to the two important criteria for evaluating children's achievement in the chapter: mastery of the basic facts and comprehension of the concepts presented. You can easily determine from their success with appropriate exercises whether or not the children know the basic facts. It is much more difficult to evaluate whether or not the children understand the concepts. Specifically, do they understand the work with functions and the inverse relationship between pairs of operations? Also, do they understand how the basic principles are utilized to determine the special products and quotients presented at the end of the chapter? Evaluation of these understandings can best be made on a day-to-day basis, while evaluation of the basic skills can be made easily by means of a proper selection of test questions.

You should note that, although this chapter is oriented toward work with functions, the actual arithmetic material is really quite easy. This chapter deals almost exclusively with basic facts and rather simple products and quotients using the basic facts. You may simply think of working with functions as a supplement to the work with basic facts, equation solving, and story problems.

Do not make a great issue of the notation used for functions. Simply remark to the children that the f in expressions such as $f(8)$ helps them think about the idea of a function and how the function rule may be applied to the number 8 in order to give the value for $f(8)$.

Resources for Active Learning

General Activities

[Games]

Developmental Math Cards, J4-9, K4-4, L4-5, Addison-Wesley

Independent Exploration: Activity Cards, Concept Co.

Madison Project: *Independent Exploration Material,* Math Media

Mathex: Operations and Problem Solving No. 8 "Skill-Drill Games," pp. 1–6 (pupil pages 1–6), Encyclopaedia Britannica Publications Ltd.

Modern Math Games, pp. 5–10, Fearon (Available from Clarke, Irwin)

Notes on Mathematics in Primary Schools, pp. 17–21, 80–86, 95–103, Cambridge University Press (Available from Macmillan of Canada)

SMSG: *Puzzle Problems and Games Project*, pp. 97–109, Stanford University

[Napier's Rods (Bones)]

Experiments in Mathematics, Stage 2, pp. 30–31, Houghton Mifflin (Available from Thomas Nelson & Sons)

Franklin Series: *From Fingers to Computers,* pp. 38–45; *Patterns and Puzzles*, pp. 86–89, Lyons and Carnahan (Available from McGraw-Hill Ryerson)

Mathex: Operations and Problem Solving No. 8, pp. 19–23 (Pupil pages 22–24), Encyclopaedia Britannica Publications Ltd.

[Slide Rules]

Experiments in Mathematics, Stage 2, pp. 32–33, Houghton Mifflin (Available from Thomas Nelson & Sons)

Franklin Series: *From Fingers to Computers*, pp. 47–56, Lyons and Carnahan (Available from McGraw-Hill Ryerson)

Mathex: Operations and Problem Solving No. 8, pp. 23–24, Encyclopaedia Britannica Publications Ltd.

[Problem Cards (for use throughout the year)]

Senior Balancing, McGraw-Hill Ryerson

Manipulative Devices

"Invicta" Math Balance (Math Media; Selective Educational Equipment)

Papy Minicomputer (Macmillan)

SEE Calculator (Selective Educational Equipment)

Commercial Games

[The first three resources are games that teach functions.]

Hindu Pyramid Puzzle (Tower of Hanoi) (Creative Publications; World Wide Games)

Independent Exploration: Math Games and Puzzles Kit (Concept Co.)

Shuttle Puzzle (Math Media; World Wide Games)

Heads Up (Creative Publications; Hammett; Math Media)

Krypto (Creative Publications; Edmund Scientific)

Math Match (Creative Publications)

Numo (Midwest Publications)

Orbiting the Earth (Scott Foresman)

Quinto (Hammett; Selective Educational Equipment)

Real Numbers Game (Creative Publications; Wff 'N Proof)

TUF (Creative Publications; Cuisenaire Co.; TUF)

Twin-Choice (Holt, Rinehart and Winston)

The Winning Touch (CCM School Materials; school supplier)

PAGES 36–37

Objective

Given appropriate problems requiring the use of addition, subtraction, multiplication, and division, the child will be able to identify the operation applicable to each problem.

Preparation

Since this investigation challenges children to recall each of the basic operations, a specific preparation is not necessary. Encourage children to begin work on the investigation right away.

Investigation

For this investigation, it would be suitable for children to work in groups of two or three. The general purpose of the investigation is to review the meaning of the operations: addition, subtraction, multiplication, and division. Encourage each child to find at least one equation for each figure, even though some illustrations suggest more than one operation. For example, in illustration D a child might think of 4 sets of 5, 5 sets of four, or 20 separated into such sets, or even of adding 4 five times. He might, therefore, write $4 \times 5 = 20$, $5 \times 4 = 20$, $20 \div 5 = 4$, $20 \div 4 = 5$, $4 + 4 + 4 + 4 + 4 = 20$, or $5 + 5 + 5 + 5 = 20$.

If any child appears to be confused, guide him indirectly. For example, ask him if he could combine the items, compare them, show how they are grouped, find how many he could buy, and so on.

3 Whole Number Concepts

● *How much do you remember?*

Investigating the Ideas

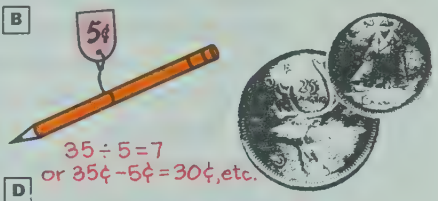
Each figure below suggests at least one equation.

A



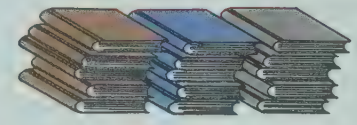
7+2=9
or 7−2=5, etc.

B



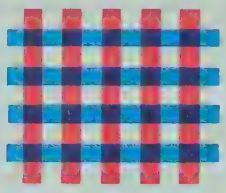
35÷5=7
or 35¢−5¢=30¢, etc.

C



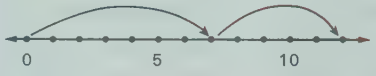
3×5=15
or 15÷3=5, etc.

D



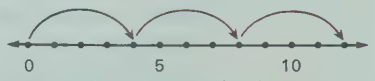
4×5=20
or 20÷5=4, etc.

E



7+5=12

F



3×4=12, or 4+4+4=12

? Can you write one or more equations for each figure?
See sample equations above and Investigation.

Discussing the Ideas

1. Can you tell which numbers are addends, which are sums, and which are differences in your equations? See Discussion.
2. Find the factors, products, and quotients in your equations. See Discussion.
3. Make up word problems for figures B and C.

Samples: B If you had 35¢ and bought a pencil for 5¢, how much would you have left?
C If there are 3 stacks of books with 5 books per stack, how many altogether?

36

Discussion

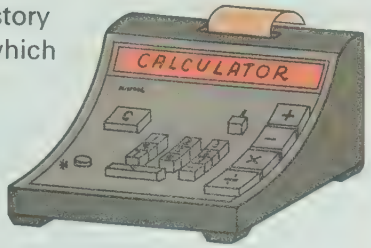
Use each investigation illustration as a basis for discussion, so that the meaning of each operation is reviewed. Accept any equation which a child can justifiably relate to the illustration. For example, in illustration A, one child might write $7 + 2 = 9$, thinking of combining dots and squares. For this equation, point out that 7 and 2 are addends, and 9 is the sum. However, another child might write $7 - 2 = 5$, thinking how many more dots there are than squares. In this case, point out that the 5 is the difference.

Similarly, for illustration F, one child might write $4 + 4 + 4 = 12$ and another, $3 \times 4 = 12$. Point out that, in the equation $3 \times 4 = 12$, 3 and 4 are called factors, and 12 is called the product of 3 and 4. In the related division equation, $12 \div 4 = 3$, 3 is called the quotient.

Encourage discussion of several word problems for B and C, and for other illustrations as well. Such an activity will serve as an introduction to the problems on page 37.

Using the Ideas

No numbers are given in these short story problems. Use **A, S, M,** or **D** to tell which operation (**A**ddition, **S**ubtraction, **M**ultiplication, or **D**ivision) you would use to solve each problem. For a problem where two operations are needed, say subtraction and division, you should answer **S, D.**



1. Corn: ▮ ears per plant. ▮ plants. How many ears ? M
2. Wheat: ▮ dollars per bag. Oats: ▮ dollars per bag. How much more per bag is the wheat than the oats ? S
3. Bought a bike for ▮. Sold it for ▮. Sold it for how much less ? S
4. ▮ cookies in each box. ▮ cookies in all. How many boxes of cookies ? D
5. Triangle sides: ▮ cm, ▮ cm, and ▮ cm. What is the perimeter ? A (M also possible if sides same length)
6. Rectangle: length ▮ cm, width ▮ cm. What is the area ? M
7. Went ▮ kilometres. Then went ▮ more km. How far ? A
8. Bought ▮ metres of material for ▮ cents. How much for one metre ? D
9. ▮ pads of paper. ▮ sheets on each pad. Used ▮ sheets. How many sheets left ? M,S
10. Jim had ▮ cents. Tom had ▮ times as much as Jim. How much did they have together ? M,A
11. ★ Had ▮ cents. Spent ▮ cents. Then spent all the rest on ▮ ping-pong balls. How much per ball ? S,D
12. ★ ▮ litres and ▮ millilitres of milk. How many 250 ml bottles of milk ? M,A,D



Using the Exercises

Assign the exercises on page 37 as independent work. An understanding of the basic operations which were reviewed in the discussion should lead directly to the ability to apply them to word problems.

Make sure children observe that they are to use the letters A, S, M, and D for the operations: addition, subtraction, multiplication, and division. Point out also that, if more than one operation is involved in a given problem, they should use as many letters as are needed to tell which operations would be used to solve the problem. Notice that exercises 11 and 12 are starred, indicating that they present a special challenge.

Assignments (page 37)

Minimum: 1–10. Average: 1–10. Maximum: 1–12.

Follow-up

To provide children an opportunity for further practice with operations and principles and for review of basic facts, you might devise a game similar to the following.

Prepare 7 to 10 cubes by printing the digits 1 through 9 on the faces. On the faces of the two other cubes print operational signs (+, −, ×, ÷). On at least one cube, print an equals sign on each face. Thus, each set of cubes should contain 7 to 10 digit cubes, 2 operational cubes, and 1 equality cube. Children may play individually or in groups of as many as 4 players; they may also play in teams of two players each. The player whose turn it is rolls all the cubes. His challenge then is to arrange the cubes so that the faces which appear on the top side form an equation, and to do so within a certain time limit, say, two minutes. Various scorekeeping methods are possible. For example, a player might receive 2 points for each cube he was able to use. Or the score may be the sum of the face values of each cube used correctly. The first player with 100 points wins.

Sample throw:

7, 7, 2, 4, 2, 3, 4, =, +, −

Possible equations:

with no place value allowed:

$7 - 7 + 2 = 2$ 14 points

with place value allowed:

$7 - 7 + 42 = 42$ 18 points

Resources for Active Learning

A Cloudburst, Vol. 2 and 3, 1500 series, Midwest Publications.

Workbook, page 11

PAGES 38–39

Objective

Given addition and multiplication equations, the child will be able to solve them by applying the basic principles of addition and multiplication: the zero principle, the one principle, the associative and commutative principles, and the distributive principle.

Preparation

Since there is no investigation in this lesson, you might introduce the lesson by writing the numbers 0 and 1 on the chalkboard. Ask children to explain anything special which they remember about these numbers, and have them write equations for all to see as examples of what they point out. Focus attention on the equations which relate to the zero and one principles described in the text.



Let's review the basic principles.

Discussing the Ideas

1. A Solve the equation.
   $987 + 0 = n$ 987
   B The equation illustrates the zero principle for addition. State this principle in your own words.
   "The sum of zero and any number is that number."

2. A State the 1 principle for multiplication. How is it like the zero principle for addition?
   "The product of 1 and any number is that number."
   B Give several examples.
   17+0=17
   17×1=17

3. A Solve the equation:
   $863 + 92 = 92 + n$ 863
   B The equation illustrates the commutative principle for addition. State this principle in your own words.
   "Changing the order of addends does not change the sum."

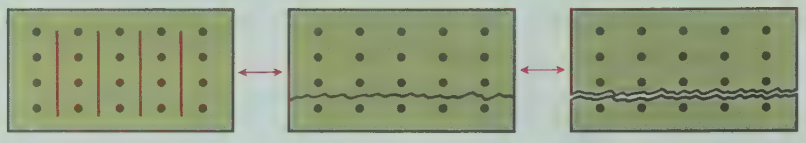
4. A What is the commutative principle for multiplication?
   "Changing the order of factors does not change the product."
   B Can you give examples?
   6×7=7×6

5. A The equation below shows the associative principle for addition.
   Solve the equation: $18 + (82 + 37) = (18 + 82) + n$ (Grouping) 37
   B Does this principle deal with the order of addends or the grouping of addends?

6. A Is there an associative principle for multiplication? Yes
   B Give a numerical example of the principle. (5×2)×2=5×(2×2)

7. A Solve these equations.
   $27 + 58 + 73 = 73 + 27 + n$ 58
   $25 \times 38 \times r$ 4 $= 38 \times 4 \times 25$
   B Can you explain what principles can help you solve the equations without pencil and paper? Comm. and assoc. principles for addition and comm. and assoc. principles for multiplication.

8. The figure illustrates the distributive principle
   Solve the equation.



$5 \times 4 = 5 \times (3 + 1) = (5 \times 3) + (5 \times n)$
1

38

Discussion

Although this page may be used as an independent activity, you might find it helpful to read and study the material in this section with the children. You may choose to continue using the words *order* and *grouping* for the names of the principles rather than introducing or stressing the words *commutative* and *associative*. You should keep in mind that the really important points here are the ideas and the fact that the two principles together allow the *rearrangement* of addends and factors in any way that is convenient. Once the children thoroughly understand that the commutative and associative principles work together to allow them to rearrange factors and addends, there is seldom any need at the elementary school level to stress these principles individually.

Using the Ideas

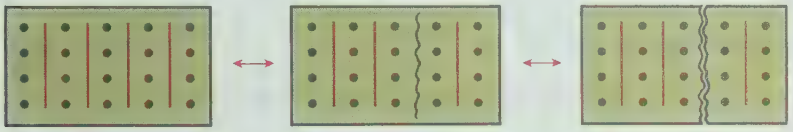
1. Show as many different ways of arranging the three addends 17, 25, and 39 in an addition problem as you can.
   Examples: (17 + 39) + 25; (39 + 25) + 17
   (17+25)+39; 17+(39+25); 25+(17+39); 39+(25+17); etc.

2. Copy and complete this multiplication table. Try to avoid doing any computing.

×	37	84	97	59
59	2183	4956	5723	3481
97	3589	8148	9409	5723
84	3108	7056	8148	4956
37	1369	3108	3589	2183

3. Solve each equation.
   Then give the name of the principle involved.
   A $76+n=76$ 0 C $53\times101=101\times n$ 53 E $796+487=n+796$ 487
   B $68\times1=n$ 68 D $4742\times n=4742$ 1
   A Zero B "1" C Order D "1" E Order

4. Study the display. Then give the number for n.



5×4 = $(3+2)\times4$ = $(3\times4)+(2\times n)$

5. Solve the equations.
   A $68+27=n+68$ 27
   B $(84+26)+13=84+(n+13)$ 26
   C $74\times n=61\times74$ 61
   D $(n\times32)\times15=89\times(32\times15)$ 89

6. Solve the equations.
   A $5\times12=n\times(10+2)$ 5
   B $6\times74=6\times(n+4)$ 70
   C $8\times21=8\times(20+n)$ 1
   D $7\times56=7\times(n+6)$ 50

think

How many ways can you show a sum of 8 by using two or more addends?



Using the Exercises

In exercise 4, on page 39, note that we break apart the first factor and multiply by 4, whereas, in discussion exercise 8 on page 38, we break apart the second factor and multiply each part by 5. One of the important things to stress here is that, either way, we get the same product we would have if we had simply multiplied 5×4. The fact is that, no matter how we consider this set of dots, we still have the same number of dots at the end. The set of dots simply illustrates the basic principle and the idea of breaking a number apart before we multiply.

You may find it helpful to provide additional demonstrations for the children who have not yet grasped the multiplication-addition principle.

Assignments (page 39)
Minimum: 1–4. Average: 1–6.
Maximum: 1–6.

Mathematics

For all whole numbers a, b, and c:

Addition	Multiplication
1. $a+b=b+a$	1. $a\times b=b\times a$
2. $(a+b)+c=a+(b+c)$	2. $(a\times b)\times c=a\times(b\times c)$
3. $a+0=a$	3. $a\times0=0$, $a\times1=a$
4. $a\times(b+c)=(a\times b)+(a\times c)$	

1. Order (commutative) principle
2. Grouping (associative) principle
3. Zero and one principles
4. Distributive principle

In the elementary school, the most important consequences of these principles are the generalizations which arise from them: the commutative and associative principles together allow us to rearrange addends in addition and factors in multiplication.

Once these generalizations are drawn and children have learned to rearrange addends and factors freely, there is little need to continue stressing the commutative and associative principles separately. However, since it is important that children understand that the rearranging is possible because of these principles, we will continue to reinforce this fact periodically.

The distributive principle is, in a sense, a generalization of the relation between repeated addition and multiplication. For example:

$3\times4=(1+1+1)\times4=4+4+4$
$2\times6=(1+1)\times6=6+6$
$4\times5=4\times(1+1+1+1+1)$
$=4+4+4+4+4$

Resources for Active Learning

Franklin Series: *Patterns and Puzzles*, "Magic Stars," pp. 27–29, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Math Activity Cards, D38, Macmillan

Mathex: Operations and Problem Solving No. 8, pp. 6–7 (pupil pages 10–15), Encyclopaedia Britannica Publications Ltd.

Nuffield Project: *Problems*—Red Set, No. 6, Wiley.

Teaching Aids for Elementary Mathematics, pp. 102–107, Holt, Rinehart and Winston.

PAGES 40–41

Objective

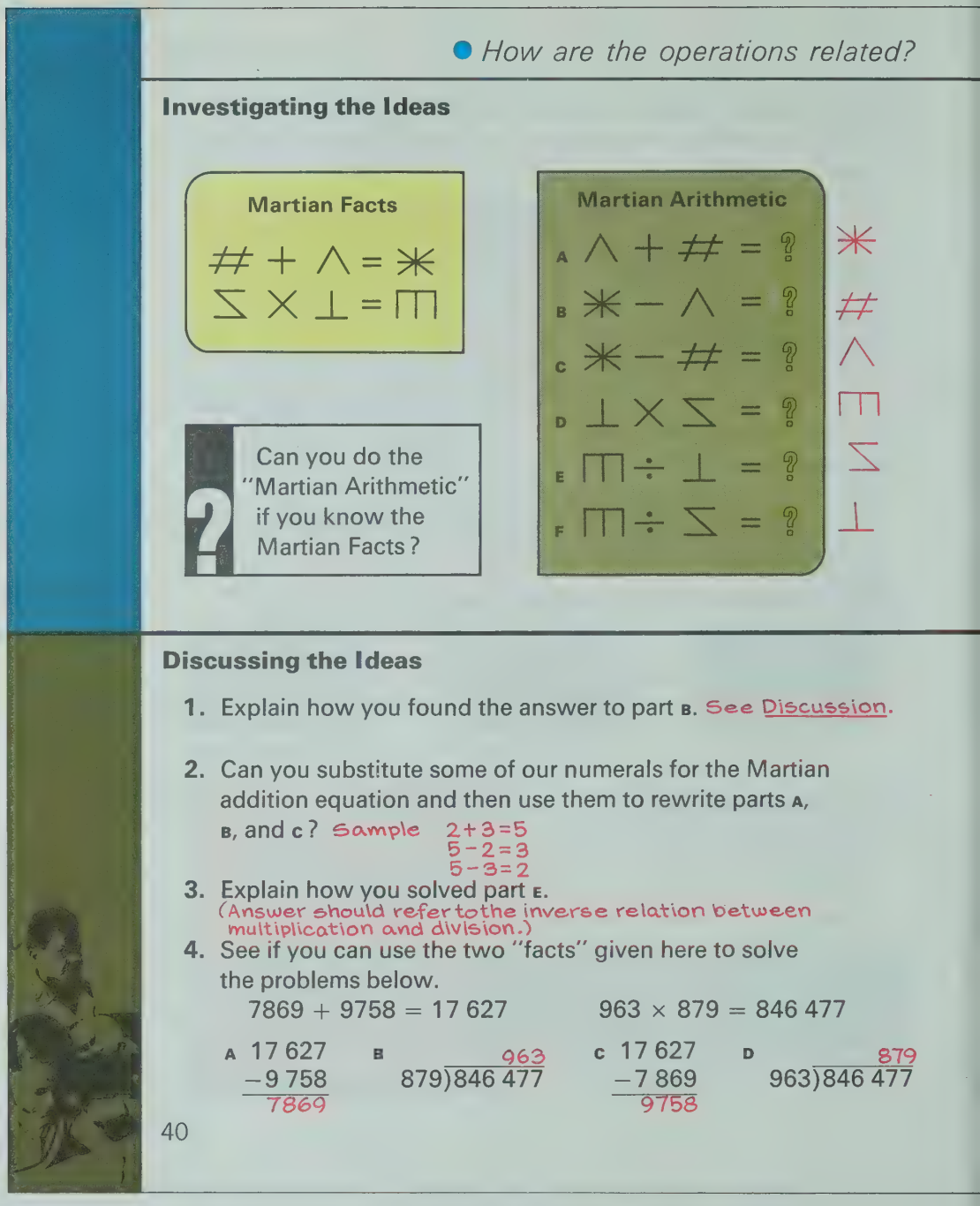
Given subtraction or division equations, the child will be able to solve them by applying his understanding of the inverse operations to find the missing addend or missing factor.

Preparation

To introduce the investigation, you might mention to the children that the arithmetic they have been studying is not the only kind of arithmetic there is. Furthermore, the basic principles do not always apply to every kind of arithmetic. But in the investigation of this lesson, they will be exploring an arithmetic in which the basic principles do apply.

Investigation

Have the children work directly from the text, independently or with a partner. Suggest that they record the equations as well as the answers. Note that the solutions to these equations involve either the commutative (order) principle or the inverse relations of addition and subtraction or multiplication and division. However, let children observe this on their own without your pointing it out; these topics will be discussed in the next section.



● *How are the operations related?*

Investigating the Ideas

Martian Facts

+ ∧ = ✳

Σ × ⊥ = ⊓

Martian Arithmetic

A ∧ + # = ?

B ✳ − ∧ = ?

C ✳ − # = ?

D ⊥ × Σ = ?

E ⊓ ÷ ⊥ = ?

F ⊓ ÷ Σ = ?

✳ # ∧ ⊓ Σ ⊥

? Can you do the "Martian Arithmetic" if you know the Martian Facts?

Discussing the Ideas

1. Explain how you found the answer to part B. See Discussion.
2. Can you substitute some of our numerals for the Martian addition equation and then use them to rewrite parts A, B, and C? Sample 2+3=5, 5−2=3, 5−3=2
3. Explain how you solved part E. (Answer should refer to the inverse relation between multiplication and division.)
4. See if you can use the two "facts" given here to solve the problems below.

7869 + 9758 = 17 627 963 × 879 = 846 477

A 17 627 − 9 758 = 7869

B 963 ; 879)846 477

C 17 627 − 7 869 = 9758

D 879 ; 963)846 477

40

Discussion

Have volunteers write the completed investigation equations on the chalkboard, explaining how they decided on an answer. Discussion exercise 1 refers to the inverse relation between subtraction and addition; in other words, subtraction "undoes" addition. As you discuss the investigation, help children relate the solutions of equations A and D to the order principle, and the solutions of equations B, C, E, and F to the inverse relations between addition and subtraction, and between multiplication and division. If the children do not make the observation, you might point out the fact that the Martian symbols ⊥ and Σ cannot correspond to our 0, since division by 0 is undefined. Even Martians would find that division by 0 is impossible.

To solve exercise 4, the children should apply their understanding of the inverse relations.

It would be helpful to show equations such as $n \times 5 = 40$ and $40 \div 5 = n$, pointing out that the missing factor in the first equation is the quotient in the second equation. The inverse relation of addition and subtraction should be shown similarly. Such examples will help to prepare children for page 41.

Using the Ideas

1. Find the missing addend in the addition equation. Then copy the subtraction equation, and give the correct difference.

A $n + 8 = 14$	C $c + 3 = 11$	E $r + 9 = 17$	G $x + 7 = 13$
$14 - 8 = n$ 6	$11 - 3 = c$ 8	$17 - 9 = r$ 8	$13 - 7 = x$ 6
B $a + 9 = 15$	D $b + 4 = 12$	F $t + 8 = 16$	H $s + 5 = 14$
$15 - 9 = a$ 6	$12 - 4 = b$ 8	$16 - 8 = t$ 8	$14 - 5 = s$ 9

2. Find the missing factor in the multiplication equation. Then copy the division equation, and give the correct quotient.

A $n \times 5 = 30$	C $c \times 7 = 42$	E $r \times 9 = 63$	G $x \times 6 = 54$
$30 \div 5 = n$ 6	$42 \div 7 = c$ 6	$63 \div 9 = r$ 7	$54 \div 6 = x$ 9
B $a \times 4 = 28$	D $b \times 8 = 40$	F $t \times 4 = 32$	H $s \times 9 = 0$
$28 \div 4 = a$ 7	$40 \div 8 = b$ 5	$32 \div 4 = t$ 8	$0 \div 9 = s$ 0

3. Solve the equations.

A $6 + 7 = a$ 13	I $14 = 9 + b$ 5	Q $13 - d = 10$ 3
B $35 \div 7 = k$ 5	J $56 \div j = 7$ 8	R $40 \div 8 = n$ 5
C $15 = 10 + n$ 5	K $s - 7 = 3$ 10	S $(36 - 9) + 9 = n$ 36
D $g \times 4 = 24$ 6	L $8 \times i = 40$ 5	T $(72 \div 9) \times 9 = c$ 72
E $16 - 8 = f$ 8	M $i = 8 + 7$ 15	U $(75 - 46) + b = 75$ 46
F $9 \times c = 18$ 2	N $7 \times 6 = x$ 42	V $(56 \div 8) \times b = 56$ 8
G $3 + k = 12$ 9	O $5 + x = 10$ 5	W $(s - 68) + 68 = 75$ 75
H $56 \div 7 = h$ 8	P $72 \div y = 8$ 9	X $(63 \div n) \times 9 = 63$ 9

think



Joe's age is three times Susan's. Four years from now Joe will be twice as old as Susan. How old is Joe now? 12

4. A Does the equation $n \times 0 = 7$ (F F P) have a solution? No

B Does the equation $n \times 0 = 0$ (F F P) have **just one** solution? No

C Since $7 \div 0 = n$ (P F F) has no solution and $0 \div 0 = n$ (P F F) has many solutions, we never divide by 0.

More practice, page A-2, Set 4

Using the Exercises

If necessary, observe with the children how exercises 1 and 2 on page 41 deal with pairs of equations such that the missing addend in one equation is the difference in another; or that the missing factor in one equation is the quotient in another as in the examples which follow.

You have found this difference: A A S $n + 5 = 14$

S A A $14 - 5 = n$ when you find this addend.

You have found this quotient: F F P $n \times 4 = 28$

P F F $28 \div 4 = n$ when you find this factor.

When children have finished the exercises, check their work carefully. Stress the information included in exercise 4 concerning 0 in division.

Assignments (page 41)

Minimum: 1, 2. Average: 1–4. Maximum: 1–4.

Solution, Think, page 41

Encourage the children to solve the *Think* problem by trial and error. One of the surprising things about a problem like this is the fact that it is not immediately apparent that Joe's age will not always be three times Susan's. It is fairly significant to the children that Joe's age, though at one time three times Susan's, will someday be only twice Susan's. After the correct answer to this problem is given (12) the children should see that Joe's age is, in fact, three times Susan's but that in four years it will be just twice Susan's.

Mathematics

This lesson stresses the point that finding a difference is equivalent to finding a missing addend and that finding a quotient is equivalent to finding a missing factor. These ideas are directly related to the meaning, or definition, of subtraction and division. We define subtraction below and observe that the definition of division is similar to that for subtraction, except that in division we do not divide by zero. We define the number $a - b$ as the number that adds to b to give a. That is,

$$(a - b) + b = a.$$

Thus, to find the difference $a - b$, we find the number which when added to b gives a.

Similarly, if $b \neq 0$, we define the number $a \div b$ to be the number which when multiplied by b gives a. That is, $(a \div b) \times b = a$.

Resources for Active Learning

Nuffield Project: *Computation and Structure* 3, "Addition," pp. 2–13; "Multiplication," pp. 24–60, Wiley.

Duplicator Masters, page 6
Workbook, page 12
Skill Masters, page 6

PAGES 42–43

Objectives

Given an input number and a rule, the child will be able to find the output number.

Given an output number and a rule, the child will be able to find the input number.

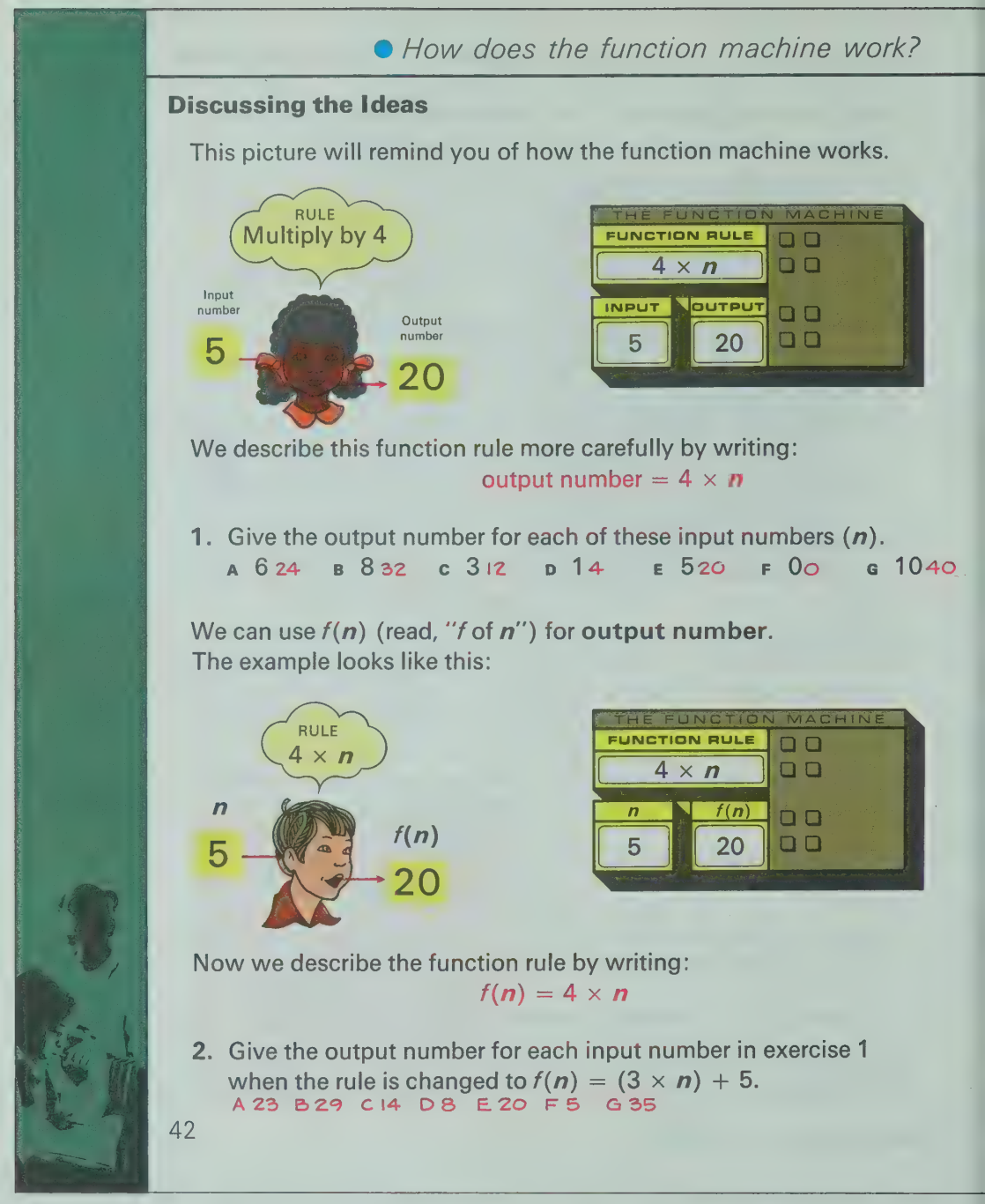
Preparation

To prepare for this lesson, conduct a short session of oral practice with simple addition and multiplication facts. For example, you might use mental chain games similar to the following:

"Start with 56... Divide by 7... Multiply by 3... Divide by 6... Add 8... What's your answer?" (12)

"Start with 12... Divide by 2... Add 7... Add 2 more... Divide by 3... Add 6... What's your answer?" (11)

"Start with 36... Divide by 4... Multiply by 8... Subtract 2... Divide by 10... What's your answer?" (7)



How does the function machine work?

Discussing the Ideas

This picture will remind you of how the function machine works.

We describe this function rule more carefully by writing:

output number = 4 × *n*

1. Give the output number for each of these input numbers (*n*).
   A 6 24 B 8 32 C 3 12 D 1 4 E 5 20 F 0 0 G 10 40

We can use $f(n)$ (read, "*f* of *n*") for **output number**.
The example looks like this:

Now we describe the function rule by writing:

$f(n) = 4 \times n$

2. Give the output number for each input number in exercise 1 when the rule is changed to $f(n) = (3 \times n) + 5$.
   A 23 B 29 C 14 D 8 E 20 F 5 G 35

42

Discussion

As you discuss this page be sure to relate the picture of the child hearing the number 5 and saying the number 20 to the picture of the function machine with input number 5 and output number 20. Point out that *n* on the screen represents the input number. In the second picture, point out that the expression $f(n)$ (read: "*f* of *n*") has replaced the phrase "output number." Thus, the input number is represented by *n*, and the output number by $f(n)$, so we can show the complete function rule as $f(n) = 4 \times n$.

As a part of the discussion, you might play the "What's My Rule" game. For example, give the input numbers, apply a rule, and, after a few examples, have children supply the output number for you.
Example:

$4 \rightarrow 32$
$3 \rightarrow 24$
$9 \rightarrow 72$
$6 \rightarrow ?$
$7 \rightarrow ?$
$5 \rightarrow ?$

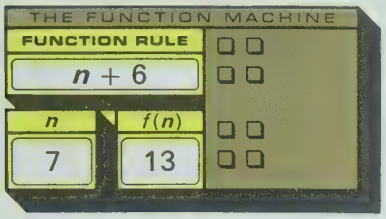
Or, use a grid:

4	3	9	6	7	5	2	8
32	24	72	?	?	?	?	?

In any case, keep the activity lively, using a variety of rules.

Using the Ideas

Use the rule shown on the function machine and give the missing numbers in exercise 1. Then think about the function machine with the rule given in each of the other exercises, and give the missing numbers.



1.

Function Rule: $n + 6$		
	n	$f(n)$
A	7	13
B	9	15
C	5	11

2.

Function Rule: $n + 9$		
	n	$f(n)$
A	5	14
B	6	15
C	7	16
D	8	17
E	9	18

3.

Function Rule: $n \times 5$		
	n	$f(n)$
A	9	45
B	8	40
C	7	35
D	6	30
E	5	25

4.

Function Rule: $n + 8$		
	n	$f(n)$
A	4	12
B	2	10
C	7	15
D	9	17
E	8	16

5.

Function Rule: $n \times 6$		
	n	$f(n)$
A	7	42
B	8	48
C	5	30
D	6	36
E	9	54

6.

Function Rule: $(3 \times n) + 2$		
	n	$f(n)$
A	2	8
B	5	17
C	4	14
D	3	11
E	6	20

7.

Function Rule: $4 \times (n + 5)$		
	n	$f(n)$
A	3	32
B	0	20
C	4	36
★ D	1	24
★ E	2	28



Follow-up

There are various ways to adapt the "What's My Rule" game which was mentioned in the discussion section. For example, you can have a child invent a simple operation rule and next have the other children give him a number. Then he gives them the output number which results from application of this rule.

This game simply duplicates the material illustrated on page 42. The child pictured at the top of the page is thinking of the rule "Multiply by four," so for the number 5, he says the number 20. The children will catch on to this game quickly, so you should change the rule frequently.

Resources for Active Learning

Inquiry in Mathematics via the Geoboard, "Functions," Geocards 7/1–8/3; 30/1–32/3, Walker. (Available from Fitzhenry & Whiteside)

Math Activity Cards, "Tower of Hanoi," E46, Macmillan.

SMSG: *Puzzle Problems and Games Project*, "Linear Function Games," pp. 79–95, Stanford University.

Workbook, page 13

Using the Exercises

If children are not familiar with the function tables, work through exercise 1 with them. Then assign the remaining exercises as independent work. Note that in exercises 4 and 5 children will have to use the inverse operation when they have been given the output number, $f(n)$, instead of the input number, n.

Use function tables such as these to provide further practice for those needing it. You might make duplicator masters or simply write others on the chalkboard.

Assignments (page 43)
Minimum: 1–5. Average: 1–7C.
Maximum: 1–7.

PAGES 44–45

Objectives

Given a function rule and an input number, the child will be able to apply the rule and find the output number.

The child will be able to express a given repeated addition problem as a multiplication problem, and conversely.

Given a set of input-output numbers, the child will be able to give the function rule for each number pair.

Preparation

Material

cards, 10 by 15 centimetres (at least 7 per child)

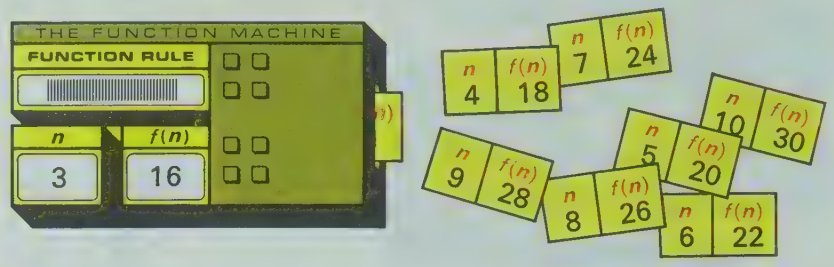
You might use a brief game of "What's My Rule" as a warm-up activity and as a review of basic facts. For example, ask the children to give you a 1-digit number. Then, silently apply your rule and give the output number. When children guess your rule, they should volunteer the output number for any input number given. Since this activity is suggested as a warm-up, keep it brief and brisk.

Investigation

Be sure children understand the n and $f(n)$ notation at the top of the function machine. You might want them to help each other guess the function rule illustrated in the investigation (double n and add 10, or $2n + 10$). However, encourage each child to invent his own function rule and make his own set of cards.

Investigating the Ideas

This function machine prints a card for each input-output pair. Can you guess the rule for the set of cards shown? f(n)=(2×n)+10



? Can you invent your own function rule and make a set of cards for it? | See if your classmates can guess your rule.

Rules and cards will vary. See Investigation.

Discussing the Ideas

1. Use multiplication to find a rule for this set of cards. f(n)=n×3

n	f(n)
6	18

n	f(n)
4	12

n	f(n)
9	27

n	f(n)
1	3

n	f(n)
0	0

n	f(n)
5	15

2. What addition rule could you use for the cards in exercise 1? f(n)=n+n+n

3. What multiplication rule would give the same set of cards as this rule? $f(n) = n + n + n + n + n + n$ f(n)=n×6

4. What addition rule would give the same set of cards as this rule? $f(n) = 4 \times n$ f(n)=n+n+n+n



Discussion

Allow children to share some of the rules they invented for the investigation. Then use them as a basis for discussion, to lead into the exercises in the text. Discussion exercises 1 and 2 bring out the relation of repeated addition and multiplication. Exercises 3 and 4 show a more generalized approach to the same relation. Help children realize that $n + n + n + n$ and $4 \times n$ are simply different ways of expressing the same number. Continue to stress the idea that multiplication may be thought of as repeated addition.

Using the Ideas

A function rule is given for each row of cards in exercises 1, 2, and 3. Give the missing numbers.

1. $f(n)=8\times n$

	A		B		C		D	
	n	$f(n)$	n	$f(n)$	n	$f(n)$	n	$f(n)$
	6	48	3	24	5	40	0	0

2. $f(n)=n\times 5$

	A		B		C		D	
	n	$f(n)$	n	$f(n)$	n	$f(n)$	n	$f(n)$
	10	50	0	0	9	45	8	40

3. $f(n)=n\times n$

	A		B		C		D	
	n	$f(n)$	n	$f(n)$	n	$f(n)$	n	$f(n)$
	6	36	5	25	7	49	10	100

Give the function rules for exercises 4, 5, and 6.

4. $f(n)=$ 2×n

A		B		C		D	
n	$f(n)$	n	$f(n)$	n	$f(n)$	n	$f(n)$
7	14	9	18	0	0	50	100

5. $f(n)=$ 10×n

A		B		C		D	
n	$f(n)$	n	$f(n)$	n	$f(n)$	n	$f(n)$
6	60	7	70	1	10	0	0

6. $f(n)=$ (3×n)+3

A		B		C		D	
n	$f(n)$	n	$f(n)$	n	$f(n)$	n	$f(n)$
3	12	6	21	9	30	0	3

7. What is the "addition rule" that corresponds to:
   - A $f(n)=5\times n$ n+n+n+n+n
   - B $f(n)=9\times n$ n+n+n+n+n+n+n+n+n
   - C $f(n)=6\times n$ n+n+n+n+n+n

8. What is the "multiplication rule" that corresponds to:
   - A $f(n)=n+n+n+n+n+n$ 6×n
   - B $f(n)=n+n+n+n$ 4×n
   - C $f(n)=n+n+n+n+n+n+n$ 7×n

think

A pen cost a dollar more than an eraser.
Together they cost one dollar and ten cents.
How much was the eraser? $.05





45

Using the Exercises

Although children have worked exercises with function rules in the text, their independent work has been with the function tables, so take care that they understand the card notation used here. Note that exercises 7 and 8 again stress the relation between multiplication and repeated addition.

Those who finish early can work on the *Think* problem. You may choose to give all the children an opportunity to work on this problem before having the correct answer given to the entire class. The point of the problem is that an immediate guess of $1.00 for the pen and 10 cents for the eraser is incorrect since in that case the pen would cost only $.90 more than the eraser. The children who make a quick guess will be likely to miss the problem, while those who think it through will observe that $1.05 and $.05 is the correct answer. All of the children should easily understand this once the correct answer is given.

Assignments (page 45)
Minimum: 1–3. Average: 1–8.
Maximum: 1–8.

Follow-up

Encourage children to exchange the input-output cards which they made in the investigation. You might use these as part of a file of task or assignment cards which can be expanded throughout the year.

Resources for Active Learning

Discovery, Section 11, Units 4/5, 11/3, 12/2, Encyclopaedia Britannica Educational Corp. ["What's My Rule" problems].

PAGES 46–47

Objective

Given a multiplication problem in which at least one factor is a multiple of ten, the child will be able to find the product.

Preparation

If the children seem to need and benefit from oral review, use the "What's My Rule" game or mental chain games to provide practice with basic facts. Some sample mental chain games follow.

"Start with 63... Divide by 7... Multiply by 6... Add 2... Divide by 7... What's your answer?" (8)

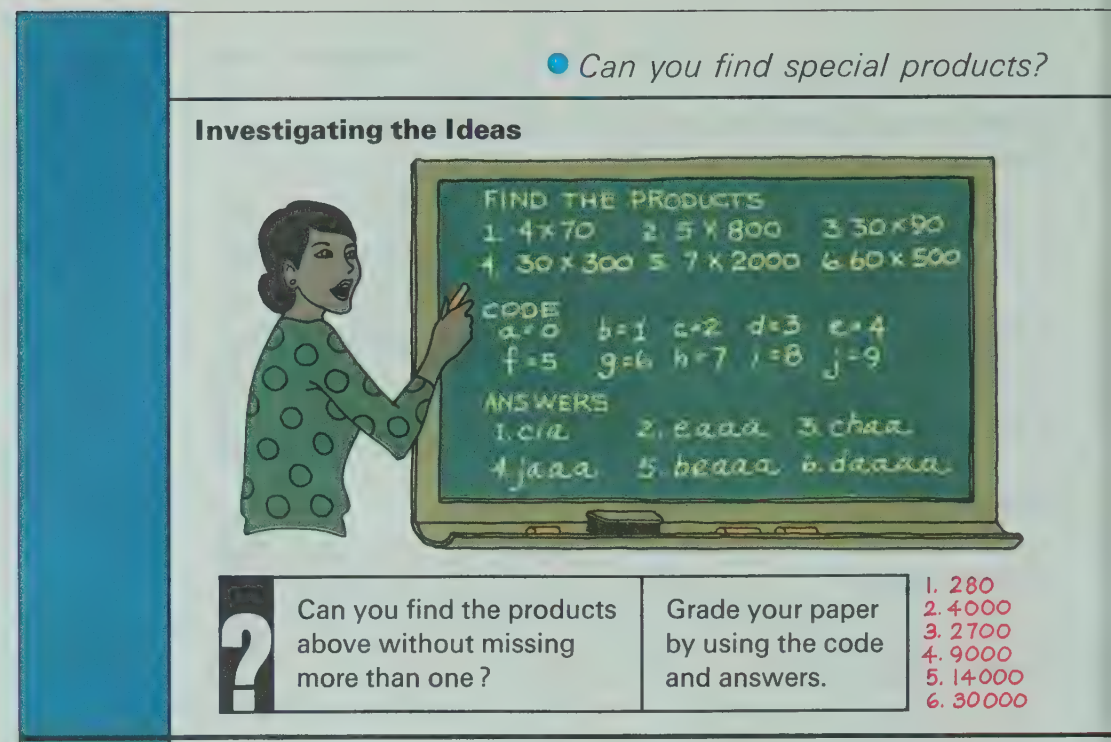
"Start with 64... Divide by 8... Multiply by 3... Add 1... Divide by 5... What's your answer?" (5)

"Start with 81... Divide by 9... Multiply by 5... Subtract 3... Divide by 7... What's your answer?" (6)

"Start with 40... Divide by 8... Multiply by 7... Add 1... Divide by 9... What's your answer?" (4)

Investigation

Encourage children to work independently on this investigation. Remind them to record their answers carefully so that they can check them with the coded answers. Children should need little or no guidance for this investigation.



Can you find special products?

Investigating the Ideas

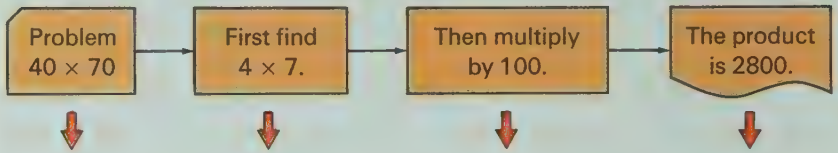
Can you find the products above without missing more than one?

Grade your paper by using the code and answers.

1. 280
2. 4000
3. 2700
4. 9000
5. 14000
6. 30000

Discussing the Ideas

1. Explain why you multiply by 100 in the flow chart below.



$40 \times 70 = 28 \times 100 = 2800$

"The factors 40 and 70 were regrouped as 4×7 and 10×10 (or 100)."

2. Give and explain flow chart steps for each of these products.

A 20×800 $20 \times 800 = 16 \times 1000 = 16\,000$

B 6×4000 $6 \times 4000 = 24 \times 1000 = 24\,000$

3. Use the flow chart to correct any mistakes you made in the Investigation.

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Discussion

Use discussion exercises 1 and 2 to guide the children through a discussion of the illustrated flow chart before checking the answers to the problems in the investigation. Point out how the factors 40 and 70 are broken down and how the basic commutative and associative principles which allow regrouping are applied:

$$40 \times 70 = 4 \times 10 \times 7 \times 10$$
$$= (4 \times 7) \times (10 \times 10)$$

Similarly, in exercise 2:

$$20 \times 800 = 2 \times 10 \times 8 \times 100$$
$$= (2 \times 8) \times (10 \times 100)$$

and $6 \times 4000 = (6 \times 4) \times 1000$

Before proceeding to exercise 3, have volunteers give the numerical answers to the problems in the investigation. Encourage those who answered all of them correctly to proceed directly to the exercises on page 47. You may want to discuss a few more examples with some of the children.

Using the Ideas

1. Find the products.

A 10×10 100 B 10×100 1000 C 100×100 10 000

2. Give the number for *a*; then give the number for *b*.

A $15 \times 10 = a$ (150) $\rightarrow 3 \times 50 = b$ 150
B $42 \times 10 = a$ (420) $\rightarrow 6 \times 70 = b$ 420
C $24 \times 10 = a$ (240) $\rightarrow 4 \times 60 = b$ 240
D $18 \times 100 = a$ (1800) $\rightarrow 3 \times 600 = b$ 1800
E $45 \times 100 = a$ (4500) $\rightarrow 5 \times 900 = b$ 4500
F $28 \times 100 = a$ (2800) $\rightarrow 4 \times 700 = b$ 2800
G $35 \times 100 = a$ (3500) $\rightarrow 50 \times 70 = b$ 3500
H $48 \times 100 = a$ (4800) $\rightarrow 60 \times 80 = b$ 4800
I $27 \times 100 = a$ (2700) $\rightarrow 30 \times 90 = b$ 2700
J $36 \times 100 = a$ (3600) $\rightarrow 90 \times 40 = b$ 3600

3. Find the products.

A 20×80 1600
B 60×40 2400
C 40×90 3600
D 70×20 1400
E 30×30 900
F 50×60 3000
G 7×200 1400
H 5×600 3000
I 6×400 2400
J 20×300 6000
K 30×800 24 000
L 30×500 15 000
M 40×400 16 000
N 70×300 21 000
O 6×7000 42 000
P 8×2000 16 000
Q 80×9000 (720 000)
R 50×8000 (400 000)

4. Read the function rule for these cards. Then give the missing numbers.

$f(n) = n \times 30$

A

n	$f(n)$
80	[illegible]

2400

B

n	$f(n)$
[illegible]	180

6

C

n	$f(n)$
600	[illegible]

18 000

5. Give a function rule for this set of cards. $n \times 700$

n	$f(n)$
2	1400

n	$f(n)$
4	2800

n	$f(n)$
0	0

n	$f(n)$
1	700

n	$f(n)$
9	6300

n	$f(n)$
40	28 000

think

ROMAN NUMERALS

Give the products in Roman numerals.

1 = I
5 = V
10 = X
50 = L
100 = C
500 = D
1000 = M

1. XX × LM
2. XXX × XL
3. L × LX (MCC)
4. CC × VM
5. C × XL
6. D × III (MMMM)

(MMM) (MD)



47

Using the Exercises

Assign the exercises on page 47 as independent work, and when the children have finished, allow time for further discussion and checking papers. If some children notice the pattern that the number of zeros in the product equals the total number of zeros in the factors, have them share this discovery with the others. In the next lesson, this pattern will be developed in relation to the exponents of powers of ten.

The most difficult part of the *Think* problem is the writing of products in Roman numerals. Most children will be able to find the products using Hindu-Arabic numerals. You may choose to use this problem to stimulate a discussion of Roman numerals, or simply as a challenge for more capable children.

Assignments (page 47)

Minimum: 1–3. Average: 1–4. Maximum: 1–5.

Follow-up/Combo

A game similar to bingo will help the children review multiplication facts and special products. The object of the game is to cover five products in any row, column, or diagonal as the caller shows and calls out the combinations on flash cards. Winners call "Combo" and verify the combinations from the caller's master sheet.

To make the game, give the children 10-by-12 centimetres tagboard cards, and have them rule 60 one-centimetre boxes on each side. Show them how to fill in "Combo" and "Free"; then write products at random on one side for Combo I, and special products (multiples of ten) on the other side for Combo II (see illustration below). Players will need cork or paper discs to use in covering the numerals.

C	O	M	B	O
12	35	72	48	63
24	56	18	64	36
81	16	Free	32	20
45	27	49	28	14
6	42	15	21	54

Combo I Card

C	O	M	B	O
240	4800	150	360	2100
400	2800	120	2400	270
2500	250	Free	140	2500
630	720	540	560	6400
7200	8100	210	1600	210

Reverse side of Combo I Card

PAGES 48–49

Objective

Given a multiplication or division problem in which at least one of the factors is a multiple of ten, the child will be able to find the product or quotient by applying his understanding of the inverse relation between multiplication and division.

Preparation

Materials

slips of paper, approximately 8-by-12 centimetres (7 per child)

It would be appropriate to have the children begin immediately with this investigation. However, if you prefer a specific preparation, you might choose to review products of multiples of ten as a warm-up activity.

Investigation

The straightforward instructions of this investigation leave little need for teacher guidance. However, you might want to remind the children to carefully record their equations.

Let's explore special products and quotients.

Investigating the Ideas

Make 4 slips of paper with these numerals → 2, 60, 1800, 30

and 3 slips with these signs. → ÷, =, ×

?	How many equations can you make with your slips?	Record each equation you find by writing it on paper.

$2 \times 30 = 60$ $60 \div 2 = 30$ $30 \times 60 = 1800$ $1800 \div 30 = 60$ $1800 \div 60 = 30$
$30 \times 2 = 60$ $60 \times 30 = 1800$

Example: 60 ÷ 30 = 2

Discussing the Ideas

1. Can you give two multiplication and two division equations by using the numbers 20, 70, and 1400? $20 \times 70 = 1400$, $70 \times 20 = 1400$, $1400 \div 20 = 70$, $1400 \div 70 = 20$
2. Find the missing factors and quotients.
   - A Since $a \times 70 = 2800$, we know that $2800 \div 70 = b$. $b = 40$
   - B Since $a \times 4 = 2400$, we know that $2400 \div 4 = b$. $b = 600$
   - C Since $a \times 300 = 2100$, we know that $2100 \div 300 = b$. $b = 7$
   - D Since $a \times 60 = 4200$, we know that $4200 \div 60 = b$. $b = 70$
3. Finding the product $(7 \times 10^1) \times (6 \times 10^2)$ is much like finding the product 70×600. The exponents help you "keep track" of the zeros. Explain each example and give the missing exponent.

A $70 \times 600 = 42\,000$
$(7 \times 10^1) \times (6 \times 10^2) = 42 \times 10^{\square}$ $\square = 3$

B $800 \times 3000 = 2\,400\,000$
$(8 \times 10^2) \times (3 \times 10^3) = 24 \times 10^{\square}$ $\square = 5$

48

Discussion

One of the main objectives of this lesson is to stress how an understanding of the inverse relation of multiplication and division can be helpful in finding special products and quotients. The equations formed by the children in the investigation and in discussion exercise 1 may be used to illustrate this point. Exercise 2 further illustrates how this relationship may be applied to find missing factors in order to arrive at quotients in division problems.

Exercise 3 introduces the use of exponents as a method of finding accurate products that are multiples of ten. First review with the children how multiples of ten can be expressed in exponent notation. Then help them see how the exponents in the factors can help them find the total number of zeros in the product. You might express this rule simply by explaining that the number of zeros in the product equals the combined number of zeros in the factors and that the exponents simply help to keep track of them.

Using the Ideas

1. Find the quotients.

A Since $6 \times 40 = 240$, we know that $240 \div 6 = a$. 40
B Since $5 \times 90 = 450$, we know that $450 \div 90 = r$. 5
C Since $7 \times 800 = 5600$, we know that $5600 \div 7 = x$. 800
D Since $9 \times 600 = 5400$, we know that $5400 \div 600 = m$. 9
E Since $80 \times 30 = 2400$, we know that $2400 \div 80 = c$. 30
F Since $30 \times 40 = 1200$, we know that $1200 \div 40 = n$. 30
G Since $70 \times 20 = 1400$, we know that $1400 \div 20 = t$. 70
H Since $60 \times 90 = 5400$, we know that $5400 \div 60 = q$. 90

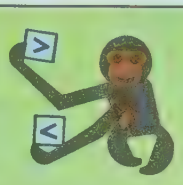
2. In each exercise, give the numbers for a and b.

A $10 \times 10 = a$ 100
$10^1 \times 10^1 = 10^b$ 2
B $10 \times 100 = a$ 1000
$10^1 \times 10^2 = 10^b$ 3
C $10 \times 1000 = a$ 10 000
$10^1 \times 10^3 = 10^b$ 4
D $100 \times 100 = a$ 10 000
$10^2 \times 10^2 = 10^b$ 4
E $50 \times 700 = a$ 35 000
$(5 \times 10^1) \times (7 \times 10^2) = b \times 10^3$ 35
F $300 \times 40 = a$ 12 000
$(3 \times 10^2) \times (4 \times 10^1) = 12 \times 10^b$ 3
G $30 \times 8000 = a$ 240 000
$(3 \times 10^1) \times (8 \times 10^3) = 24 \times 10^b$ 4
H $700 \times 900 = a$ 630 000
$(7 \times 10^2) \times (9 \times 10^2) = 63 \times 10^b$ 4

3. Find the products and quotients.

A 7×60 420
B $180 \div 20$ 9
C 80×20 1600
D $240 \div 40$ 6
E $2400 \div 80$ 30
F 70×70 4900
G $3600 \div 60$ 60
H $4200 \div 60$ 70
I 30×90 2700
J $3500 \div 50$ 70
K $4500 \div 90$ 50
L 5×900 4500
M 80×30 2400
N $1800 \div 30$ 60

think



Give the correct sign (< or >) for each ◯

1. 100×10^9 ◯ 10^{12} <
2. 10^{100} ◯ 100×10^{10} >
3. $10^{10} \times 10^{10}$ ◯ 10^{100} <
4. 10^{100} ◯ 100^{10} >

★**4.** Find the quotients.

A $27\,000 \div 90$ 300
B $56\,000 \div 80$ 700
C $320\,000 \div 40$ 8000
D $210\,000 \div 300$ 700

More practice, page A-3, Set 5

Using the Exercises

Assign the exercises on page 49 as independent work. When the children have finished, check the exercises with them, and continue to stress the inverse relation of multiplication and division. Point out again how finding the quotient in a division equation is the same as finding the missing factor in the related multiplication equation.

The *Think* problem is intended primarily for those children who finish the exercises quickly.

Assignments (page 49)
Minimum: 1, 3. Average: 1–3.
Maximum: 1–4.

Mathematics

The *multiplication law* for exponents, used informally in this lesson, may be stated as follows:

> If n is any non-zero number and a and b are integers, then $n^a \times n^b = n^{a+b}$.

In finding products of numbers which are expressed in scientific notation, the multiplication law for exponents and the associative and commutative laws for multiplication can be applied.

Example:

$$\begin{aligned}(3 \times 10^4) \times (2 \times 10^3) &= (3 \times 2) \times (10^4 \times 10^3)\\ &= 6 \times 10^{4+3}\\ &= 6 \times 10^7\\ &= 60\,000\,000\end{aligned}$$

Follow-up

Some children may respond to written exercises which demonstrate patterns, such as:

Find the products.

$3 \times 8 =$ ____

$3 \times 80 =$ ____ $30 \times 80 =$ ____
$3 \times 800 =$ ____ $300 \times 80 =$ ____
$30 \times 8 =$ ____ $30 \times 800 =$ ____
$300 \times 8 =$ ____ $300 \times 800 =$ ____

Find the products.

$(3 \times 10^1) \times (8 \times 10^1) = 24 \times 10^2$
$= 2400$
$(3 \times 10^2) \times (8 \times 10^2) = 24 \times 10^4$
$=$ ____
$(3 \times 10^1) \times (8 \times 10^3)$
$=$ ____ $\times$ ____ $=$ ____
$(9 \times 10^4) \times 3 = 27 \times 10^4$ $=$ ____

$(9 \times 10^3) \times (3 \times 10^1)$
$=$ ____ $\times$ ____ $=$ ____
$(9 \times 10^2) \times (3 \times 10^3)$
$=$ ____ $\times$ ____ $=$ ____

Duplicator Masters, page 7
Workbook, page 14
Skill Masters, page 7

PAGES 50–51

Objective

Given a multiplication problem with one 2-digit factor, the child will be able to find the product by applying the distributive principle.

Preparation

As a warm-up activity, you might use the "What's My Rule" game to review multiplication and division facts and multiples of ten. Or, you might use some mental chain games similar to the following:

"Start with 56... Divide by 8... Multiply by 20... What's your answer?" (140)

"Start with 63... Divide by 9.. Multiply by 4... And 2... Multiply by 3... What's your answer?" (90)

"Start with 48... Divide by 6... Multiply by 5... Multiply by 7... What's your answer?" (280)

"Start with 20... Multiply by 4... Add 1... Divide by 9... Multiply by 50... What's your answer?" (450)

Investigation

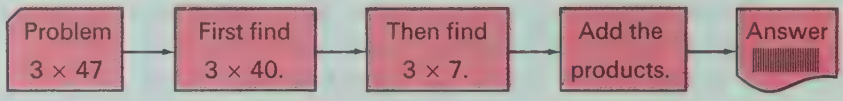
Explain to the children that both of the flow charts in the investigation show steps for finding the product 3 × 47. First, they should determine whether the steps lead to the correct answer. Then, they should make flow charts for other examples. You might suggest products such as 5 × 83, or 3 × 56.

● *Let's use the distributive principle to help us find products.*

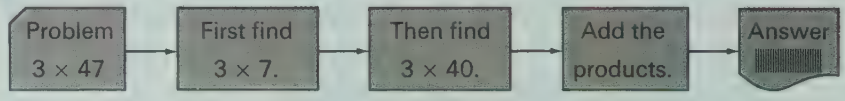
Investigating the Ideas

Cindy and Carl made flow charts to show the steps in finding 3 × 47.

Cindy's flow chart:



Carl's flow chart:



Will Cindy's and Carl's flow charts give the correct answer to the problem? Find the answer. Yes; 141

? Can you make a flow chart that will show the steps for multiplying a 2-digit number by a 1-digit number that you choose? Examples will vary. See Investigation.

Discussing the Ideas

1. Explain how Cindy and Carl used the distributive principle to find 3 × 47. See Discussion.

2. Find products ***a*** and ***b***. Then give the product for ***c***.

   A $3 \times 50 = a \xrightarrow{150} 3 \times 54 = c$ 162
   $3 \times 4 = b \xrightarrow{12}$

   B $5 \times 30 = a \xrightarrow{150} 5 \times 34 = c$ 170
   $5 \times 4 = b \xrightarrow{20}$

   C $7 \times 20 = a \xrightarrow{140} 7 \times 24 = c$ 168
   $7 \times 4 = b \xrightarrow{28}$

   D $6 \times 30 = a \xrightarrow{180} 6 \times 35 = c$ 210
   $6 \times 5 = b \xrightarrow{30}$

   E $4 \times 70 = a \xrightarrow{280} 4 \times 73 = c$ 292
   $4 \times 3 = b \xrightarrow{12}$

   F $8 \times 40 = a \xrightarrow{320} 8 \times 42 = c$ 336
   $8 \times 2 = b \xrightarrow{16}$

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Discussion

As you discuss exercise 1, you might want to have the children refer back to pages 38 and 39 to review the meaning of the distributive principle. Point out the convenience of "breaking apart" a 2-digit factor into tens and ones.

As you work through exercise 2 with the children, have them explain each example. Be sure that they include the step in which they must add the partial products. For example:

$3 \times 50 = 150$
$3 \times 4 = 12$
$3 \times 54 = 150 + 12 = 162$

Using the Ideas

1. Find the numbers for a and b. Then find the product for c.

A $4 \times 20 = a^{80}$, $4 \times 1 = b^{4}$, $4 \times 21 = c^{84}$

B $5 \times 30 = a^{150}$, $5 \times 5 = b^{25}$, $5 \times 35 = c^{175}$

C $6 \times 40 = a^{240}$, $6 \times 3 = b^{18}$, $6 \times 43 = c^{258}$

D $7 \times 60 = a^{420}$, $7 \times 3 = b^{21}$, $7 \times 63 = c^{441}$

E $4 \times 90 = a^{360}$, $4 \times 7 = b^{28}$, $4 \times 97 = c^{388}$

F $5 \times 80 = a^{400}$, $5 \times 3 = b^{15}$, $5 \times 83 = c^{415}$

2. Find the products. Write only the answers.

A	3×21^{63}	E	7×21^{147}	I	7×32^{224}	M	3×18^{54}	Q	9×22^{198}
B	6×14^{84}	F	4×36^{144}	J	5×36^{180}	N	4×41^{164}	R	4×43^{172}
C	5×32^{160}	G	3×45^{135}	K	4×24^{96}	O	8×32^{256}	S	6×27^{162}
D	6×12^{72}	H	6×24^{144}	L	2×76^{152}	P	6×72^{432}	T	7×36^{252}

3. Find the product for a; then find the product for b. Write only the answers.

A $5 \times 32 = a^{160} \rightarrow 50 \times 32 = b^{1600}$

B $6 \times 24 = a^{144} \rightarrow 60 \times 24 = b^{1440}$

C $7 \times 32 = a^{224} \rightarrow 7 \times 320 = b^{2240}$

D $6 \times 72 = a^{432} \rightarrow 60 \times 720 = b^{43\,200}$

E $3 \times 45 = a^{135} \rightarrow 30 \times 450 = b^{13\,500}$

F $8 \times 32 = a^{256} \rightarrow 800 \times 320 = b^{256\,000}$

4. Find the products. Write only the answers.

A	40×36^{1440}	E	20×76^{1520}
B	70×21^{1470}	F	70×36^{2520}
C	50×36^{1800}	G	40×41^{1640}
D	30×21^{630}	H	90×22^{1980}

think

Find the missing number pairs.

Products ✕ Differences −

	24	6	✕	4	2
A	24	8	✕	3	5
B	40	8	✕	5	3
C	42	7	✕	6	1
D	35	7	✕	5	2
E	27	9	✕	3	6
F	54	9	✕	6	3



Using the Exercises

Note that exercise 1 on page 51 is similar to the second discussion exercise on page 50. Exercise 2 requires only the answers; it is not intended that children write out the long method of multiplication. Encourage the faster children to write only the answers and the slower children to write two products, as they did in exercise 1. For example, in exercise 2F, the children can write the products 4×30 and 4×6 and add these two products to get the final product. Many of the faster children will be able to do this without the aid of pencil and paper.

After those children who wish to have tried the *Think* problem, you might have them explain how they found the missing pairs.

Assignments (page 51)

Minimum: 1–3. Average: 1–4. Maximum: 1–4.

Mathematics

The distributive principle is vital to the development of the multiplication algorithm. For example, if we wish to find a product such as 3×45, we apply the distributive principle and think:

$$\begin{aligned} 3 \times 45 &= (3 \times 40) + (3 \times 5) \\ &= 120 + 15 \\ &= 135 \end{aligned}$$

This is a simple example of the application of the distributive principle in multiplication. We could also demonstrate that the distributive principle is used in finding a product such as 376×42. The chief difference would be in the size of the number and in the number of times that we would apply the principle to arrive at the correct product.

Follow-up

Other tables similar to that in the *Think* problem would provide children with interesting and worthwhile practice. Some may enjoy making tables of their own for others to work on. Examples of possibilities are shown below.

Sums + Products ✕

	7	3	4	12
A	9			20
B	8			15
C	10			24
D	12			35
E	12			20

Products ✕ Differences −

	16	4	4	0
A	9			0
B	36			0
C	4			0
D	81			0
E	49			0

Products ✕ Quotients ÷

	32	8	4	2
A	27			3
B	16			4
C	18			2
D	25			1
E	3			3

PAGES 52–53

Objective

Given a word problem whose solution requires one or two steps, the child will be able to write and solve an equation for the problem.

Preparation

To prepare for this lesson, you might exhibit an equation such as $8 + 7 = n$ on the chalkboard; then have several volunteers offer word problems that could be expressed by this equation. For example, a child might ask, "How much money does Ron have if he starts with 8 cents and someone gives him 7 cents?" Encourage a variety of different kinds of word problems for the equation.

Investigation

If you followed the preparation suggestions given above, explain to the children that they have been working with problems which require one operation to solve. Introduce the idea that the solution of some problems requires two or more operations. Pose the question of how such steps might be written in equation form: do they think that two steps might be described in one equation? Suggest that they study the investigation in their texts.

Many children will need help in thinking of a problem which uses two steps. You might suggest one of the following for those who need help, but ask them to write their own equation for the problem.

If 5 is multiplied by 9, and 12 is added to the product, what's the answer? [$(5 \times 9) + 12 = n$]

If 32 is divided by 8, and 7 is added to the quotient, what's the answer? [$(32 \div 8) + 7 = n$]

If 4 is multiplied by 6, and 10 is subtracted from the product, what's the answer?

[$(4 \times 6) - 10 = n$]

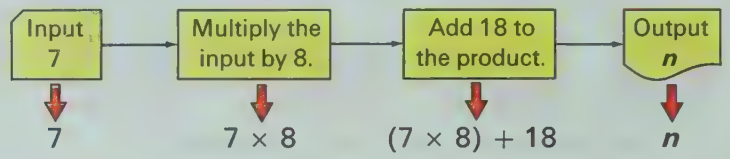
● *Can you write and solve equations?*

Investigating the Ideas

Study the problem, its flow chart, and its equation.

Problem: If 7 is multiplied by 8 and 18 is added to the product, the result is what number? 74

Flow Chart:



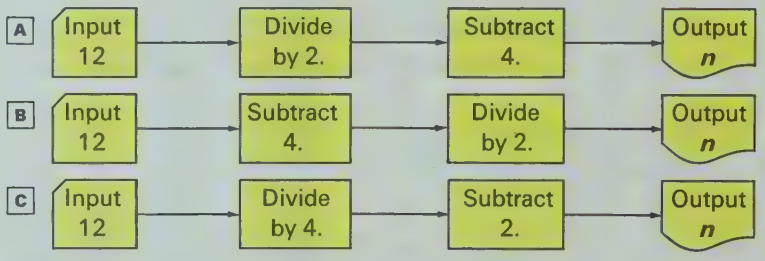
Equation: $(7 \times 8) + 18 = n$

? Can you write a problem like this one, then make a flow chart, and write an equation for it? See Investigation.

Discussing the Ideas

1. A What is the output number for the Investigation flow chart? 74
   B What is the solution to the equation? 74

2. Which flow chart below is correct for this problem? B
   Problem: If 4 is subtracted from 12 and the difference is divided by 2, the result is what number? 4



3. Explain how to use the correct flow chart to write an equation for the problem in exercise 2. (12-4)÷2=n

52

Discussion

Use exercise 1 to discuss the example in the investigation and the other equations the children wrote. Explain the function of the parentheses in two-step equations: the operation in the parentheses should be performed before the other operation. Give children an opportunity to study exercise 2 before asking for a response from them. Help them relate flow chart B to the word problem and use it to write the equation:

$(12 - 4) \div 2 = n$

Using the Ideas

Write and solve an equation for each exercise.

1. What number is 8 more than the product of 6 and 2?
   Answer: $(6 \times 2) + 8 = n$
   $n = 20$
2. Beginning with 48, what is the result if you divide by 6 and then multiply by 7?
   $(48 \div 6) \times 7 = n \quad n = 56$
3. If 9 is multiplied by 7 and 6 is subtracted from the product, what is the result?
   $(9 \times 7) - 6 = n \quad n = 57$
4. The sum of 4 and 5 is multiplied by 8. What is the resulting number?
   $(4 + 5) \times 8 = n \quad n = 72$
5. The difference between 15 and 8 is multiplied by 9. What is the product?
   $(15 - 8) \times 9 = n \quad n = 63$
6. Find the product of 7 times the sum of 8 and 2.
   $(8 + 2) \times 7 = n \quad n = 70$
7. If you divide 54 by 6, then add 1, and multiply this sum by 8, what is the result?
   $[(54 \div 6) + 1] \times 8 = n \quad n = 80$
8. What is the quotient when you divide the sum of 68 and 4 by 9?
   $(68 + 4) \div 9 = n \quad n = 8$
9. If you multiply 10 by itself, then subtract 10 from this product and divide the result by 10, what is the final number?
   $[(10 \times 10) - 10] \div 10 = n \quad n = 9$
10. ★ Find the number that is 7 less than the product of 8 times the sum of 6 and 3.
    $[(6 + 3) \times 8] - 7 = n \quad n = 65$

think



Copy each equation and give the correct sign for each ◍.

1. (6 ◍ 3) ◍ 7 = 25
2. 3 ◍ (7 ◍ 4) = 31
3. 5 ◍ (6 ◍ 4) = 10
4. (18 ◍ 3) ◍ 4 = 2

Find numbers for **a**, **b**, and **c** so that the equation is a true statement. Answers will vary. Examples are shown.

1. $a \times (b + c) = 56$
   $7 \times (3 + 5) = 56$
2. $(a \div b) - c = 2$
   $(16 \div 8) - 0 = 2$
3. $a - (b \times c) = 0$
   $10 - (5 \times 2) = 0$

More practice, page A-3, Set 6

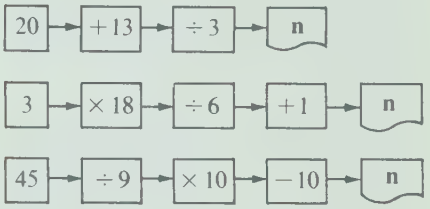
Using the Exercises

Encourage the children to work independently on the exercises on page 53. Remind them that, even if they can give the answer without writing an equation, they should record their steps in equation form. Some children may be interested in making flow charts as well as writing equations for these exercises. After children have completed the exercises, it would be helpful to use several of them as a basis for discussion.

Assignments (page 53)
Minimum: 1–9, oral. Average: 1–9.
Maximum: 1–10.

Follow-up

To provide further practice with equations, you might have children write equations for flow charts which you put on the chalkboard.



You might also have them write equations for word problems like the ones below.

Find the number that is 2 more than 6 times 5.
$[(6 \times 5) + 2 = n]$

Find the number that is 8 less than 16 times 2.
$[(16 \times 2) - 8 = n]$

Find the number that is 3 times the difference of 7 and 2.
$[3 \times (7 - 2) = n]$

Find the number that is 5 more than the quotient 45 divided by 9.
$[(45 \div 9) + 5 = n]$

Resources for Active Learning

Franklin Series, *Patterns and Puzzles*, "Operations" (a game), p. 93, Lyons and Carnahan. (Available from McGraw Hill Ryerson)
Nuffield Project: *Computation and Structure* 3, "Truth-sets," pp. 14–18, Wiley.

Duplicator Masters, page 8
Workbook, page 15

PAGES 54–55

Objective

The child will demonstrate his ability to work with the concepts presented in this chapter.

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

Briefly review any material studied so far in the text which the children have found particularly difficult. Encourage the children to decide which topics need the greatest amount of review. You might choose to develop these at greater length as you discuss page 55.

1. Solve the equations.

A $8 + 7 = a$ 15 D $24 \div 6 = t$ 4 G $17 - b$ 8 $= 9$ J $6 \times s$ 8 $= 48$

B $6 + n$ 7 $= 13$ E $15 - 7 = d$ 8 H $8 \times 9 = m$ 72 K $7g + 7 = 14$

C $9 \times 3 = c$ 27 F $7r \times 6 = 42$ I $42 \div 7 = x$ 6 L $48 \div 6 = q$ 8

2. List the missing numbers for each table.

Function Rule: $n \times 40$

	n	$f(n)$
A	60	2400
B	30	1200
C	80	3200
D	9	360

Function Rule: $n \times 70$

	n	$f(n)$
E	10	700
F	4	280
G	30	2100
H	40	2800

Function Rule: $n \div 60$

	n	$f(n)$
I	1800	30
J	4800	80
K	1200	20
L	6000	100

3. Function rule: $f(n) = (7 \times n) + 8$. Give the number for:

A $f(3)$ 29 B $f(6)$ 50 C $f(20)$ 148 D $f(50)$ 358 E $f(10)$ 78 F $f(80)$ 568

4. Find the numbers for a, b, and c.

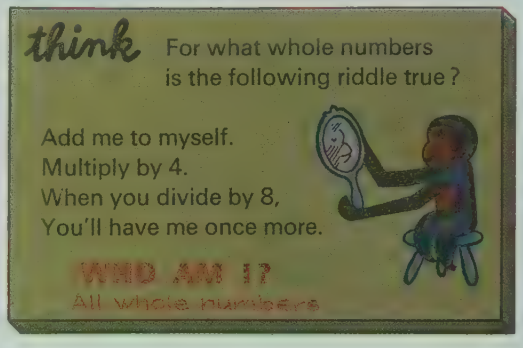
A $5 \times 70 = a$ 350 $\rightarrow 5 \times 73 = c$ 365
$5 \times 3 = b$ 15

B $9 \times 40 = a$ 360 $\rightarrow 9 \times 42 = c$ 378
$9 \times 2 = b$ 18

5. Write and solve an equation for each part.

A If 17 is subtracted from 23 and the difference is multiplied by 5, what is the product? $(23-17) \times 5 = 30$

B 14 is added to itself and the sum is divided by 4. What is the quotient? $(14+14) \div 4 = 7$



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Discussion

Page 54 may be used as an evaluative instrument or as a basis for discussion to review various topics. You may want to explain the use of the $f(n)$ symbol in exercise 3 by pointing out that $f(3)$ means that 3 is the number that is to replace n in the given equation: $f(3) = (7 \times 3) + 8$. When children have finished the exercises, correct their answers carefully with them in order to help clear up any misunderstandings.

You might challenge some of the children to write an equation for the *Think* problem on page 54:

$$[(n + n) \times 4] \div 8 = n$$

If they write and solve the equation in the following form they will see that the riddle is true for any whole number.

$$\frac{2n \times 4}{8} = n$$

Keeping in Touch with

Addition
Multiplication
Place value
Inequalities

1. Write each number in expanded notation.

A 826 C 1032 E 34 576 G 469 506
B 2745 D 52 280 F 80 365 H 9 037 260

See Answers, T.E. page 55.

2. Give the correct sign (<, >, or =) for each ◍.

A 78 263 ◍ 70 000 + (8 × 1000) + (2 × 10^2) + 60 + 3 =
B 5 293 648 ◍ 5293 × 10^2 >
C 387 + 256 ◍ 387 + 246 >
D 9203 + 6152 ◍ 9203 + 6252 <
E 6327 + 5461 ◍ 6337 + 5461 <

3. Find the products.

A $10^1 \times 10^2$ 1000 B $10^2 \times 10^2$ 10 000 C $10^2 \times 10^3$ 100 000 D $10^3 \times 10^1$ 10 000

4. Write the ordinary base-ten numeral for each exercise.

A 7×10^2 700 C 6×10^3 6000 E 38×10^5 3 800 000 G 60×10^2 6000 I 600×10^1 6000
B 3×10^4 30 000 D 25×10^1 250 F 43×10^4 430 000 H 160×10^3 160 000 J 750×10^4 7 500 000

5. Round these population figures (for a recent year) to the nearest million.

A Canada: 21 681 000 22 000 000 B Mexico: 48 313 438 48 000 000

6. Give the missing numbers.

Power of ten	Ordinary base-ten numeral
10^4	A 10 000
B 10^3	1000
10^5	C 100 000
10^7	D 10 000 000
E 10^8	100 000 000

think

These counting sequences are in base six. Give the missing numerals.

1. 3, 4, ◍, ◍, ◍, 12, 13 (5, 10, 11)
2. 34, 33, 32, ◍, ◍, ◍, 24 (31, 30, 25)
3. 424, 425, ◍, ◍, ◍, 433 (430, 431, 432)
4. 52, 53, ◍, ◍, ◍, 101, 102 (54, 55, 100)



You are invited to explore

ACTIVITY CARD 2
Page 356

55

Using the Exercises

Page 55 may be assigned as independent work or used as a cumulative review. Exercises 1 and 4 may appear to be similar, but it is not intended that the notation in exercise 1 includes the use of exponents.

Answers, exercise 1, page 55

A 826 = (8 × 100) + (2 × 10) + 6

B 2745 = (2 × 1000) + (7 × 100) + (4 × 10) + 5

C 1032 = (1 × 1000) + (3 × 10) + 2

D 52 280 = (5 × 10 000) + (2 × 1000) + (2 × 100) + (8 × 10)

E 34 576 = (3 × 10 000) + (4 × 1000) + (5 × 100) + (7 × 10) + 6

F 80 365 = (8 × 10 000) + (3 × 100) + (6 × 10) + 5

G 469 506 = (4 × 100 000) + (6 × 10 000) + (9 × 1000) + (5 × 100) + 6

H 9 037 260 = (9 × 1 000 000) + (3 × 10 000) + (7 × 1000) + (2 × 100) + (6 × 10)

Workbook, page 16 (Use with text page 54.)

General Objectives

To review some basic concepts of geometry
To introduce standard geometric notation
To review congruence of segments, angles, and triangles
To provide experience with simple ruler and compass constructions
To introduce parallel lines and perpendicular lines
To review linear measure
To introduce measurement of angles
To provide experience in finding the sum of the angles of a polygon
To strengthen the children's understanding of symmetry
To strengthen the children's spatial perception
To contrast the study of space geometry with the study of plane geometry
To investigate the five regular polyhedrons

The first two lessons of the chapter review basic concepts of plane geometry and space figures. The material is designed to help the children understand the relationship between the physical world and the ideas of geometry: we only think about the points, lines, rays, and segments of geometry, whereas we actually touch or see the things in the physical world which represent these geometric ideas.

To help children interpret pictorial representations of 3-dimensional objects, a lesson which involves drawing space figures is provided. A study of plane geometric figures; a study of ruler-and-compass construction; and important ideas of congruence of segments, angles, and triangles are presented. The technique of using a protractor to measure angles is studied and children are led to understand that measurement of an angle is basically the counting of a unit angle.

Finally, space figures are studied in relation to plane figures. This is done through a study of the five regular polyhedrons and an introduction to cross sections of certain basic space figures.

Mathematics

In this chapter, we present a view that appeals to children's intuition. Because children have a feeling for such ideas as "nearer" and "farther" and "exactly as far apart as," we leave these basic concepts undefined and build on them. Using these basic, undefined concepts, then, two segments are defined as being congruent if the end points of one are exactly as far apart as the end points of the other. With this definition for congruent segments, we can move to a definition for congruent angles, using the same basic concepts except that, this time, we use particular points on the rays, and we define these as corresponding points. Hence, the definition for congruent angles is stated as follows.

> Two angles are congruent if corresponding points on their rays are equally far apart (if the segments determined by the two points are congruent).

Before presenting this definition to the children, the idea of corresponding points must be developed. When congruence of segments and angles is established, it is easy to define congruence of triangles. We simply observe that two triangles are congruent if they can be matched so that the pairs of angles and segments are congruent.

The subject of space geometry concerns sets of points that may or may not lie in one plane. The work provided in this chapter contrasts space geometry with plane geometry through a highly intuitive approach. The five regular space polyhedrons or solids are termed *regular* because in each figure all of the faces are congruent polygons. Patterns for constructing several solids are included to provide a basis for relating concepts of surface area to concepts of plane area, and to help the children think of space figures as hollow.

We define *edges*, *faces*, and *vertices* by using a cube as a model. The concept of a *cross section*, the intersection of a space figure and a plane, is also presented intuitively by asking the children to think of slicing solids and looking at the edges of their cuts.

Teaching the Chapter

Materials

Cardboard for use under compass point
Compass
Cube, 1 per child
Glue or paste
Heavy paper for constructing space figures and making templates of polygons
Models of the five regular polyhedra or patterns for their construction
Modelling clay
Overhead projector
Paper fasteners
Protractor
Ruler, centimetre, 1 per child
Scissors
Set of geometric space figures, if available
Tape
Tracing paper
Transparent protractor for use on overhead projector

Vocabulary

angle
angle sum
bisect
centimetre
compass
cone

congruent
construction
cross section
cube
cylinder
degree
dodecahedron
edge
face
geometric illusions
hexagon
icosahedron
isosceles
length
octahedron
parallel
parallelogram
pentagon
perpendicular
plane
point
polygon
polyhedron
prism
protractor
quadrilateral
ray
rectangle
regular polygon
regular polyhedron
right angle
segment
skew lines
space geometry
sphere
square pyramid
straightedge
symmetrical figures
symmetry
tessellate
tessellations
tetrahedron
trapezoid
triangle
unit
vertex

It is important for each child to have his own compass, ruler, and protractor; an uncalibrated straightedge (rather than an ordinary ruler) may be preferable for the work on a non-metric basis. We also recommend that children use either a cardboard backing or a thick pad of newspaper to keep from marring their desk tops while using their compasses.

In some of the investigations, you will want children to use paper heavier than tracing paper so that they can better fold or manipulate the figures. To transfer a figure onto heavy paper, first have the children place the tracing paper over the illustration and carefully mark its corners. Next puncture a hole with a compass or sharp pencil at each of these marked corners. Place the tracing paper over the heavy paper and with pencil mark the corners through the holes onto the heavy paper. Discard the tracing paper; use a straightedge and pencil to connect the marks; and, finally, cut out the figure.

The children should not be required to memorize all of the technical words such as icosahedron or dodecahedron. However, some of the other words—angle, bisect, congruent, parallel, and perpendicular—are vital to their study of geometry. Therefore, the children should attempt to understand and remember their meanings.

Lesson Schedule

Plan to spend about three weeks on this chapter, adjusting this suggested schedule according to the needs, abilities, and interests of your children. Since some of them may find this chapter one of the more stimulating ones, you may want to allot extra time to encourage this interest.

Evaluation of Progress

In evaluating the children's achievement in this chapter, you should not put great emphasis upon the mechanical skills of constructing or drawing figures. The children should acquire a general understanding of congruence for segments, angles, and triangles; and they should be familiar with the basic notation for segments, rays, and angles.

We emphasize that one of the chief objectives of the chapter is to give the children a general feeling for the study of geometry rather than to teach them specific facts, notation, or names.

Resources for Active Learning

General Activities

Creative Constructions, Creative Publications

Experiments in Mathematics, Stage 1, pp. 26–29; Stage 2, pp. 42–45; Stage 3, pp. 36–39, Houghton Mifflin. (Available from Thomas Nelson & Sons Ltd)

Franklin Series: *Patterns and Puzzles*, pp. 36-43, 49-59, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Geometry in the Classroom, pp. 123–127, 161–180, Holt, Rinehart and Winston of Canada

Line Designs, Creative Publications
[Polyhedra Nets and Geodesic Structures]

Mathex: Geometry No. 9, pp. 3–6 (pupil pages 5–12), Encyclopaedia Britannica Publications Ltd.

Modern Math Games, pp. 39–42, Fearon (Available from Clarke, Irwin)

Polyhedron Models for the Classroom, NCTM
[Tangrams]

Experiments in Mathematics, Stage 1, pp. 6-7, Houghton Mifflin. (Available from Thomas Nelson & Sons Ltd.)

Franklin Series: *Patterns and Puzzles*, pp. 15-18, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Tangrams: 330 Puzzles, Dover Publications

Manipulative Devices

Clear acrylic cubes (Selective Educational Equipment)

Geoboards (Addison-Wesley)

Geo-D-Stix (Childcraft; Cuisenaire Co.)

Geometric figures and solids (Milton Bradley; school supplier)

Geo Strips (Mafex Associates; Math Media)

LaPine Sage Kit (LaPine Scientific)

Metre Stick (Geyer Instructional Aids)

Optical Illusion Kit (Edmund Scientific)

Pattern Blocks (Selective Educational Equipment; McGraw-Hill Ryerson)

Polyhedron (construction) Kit (Gamco; Lano)

Space Spider (Childcraft; LaPine Scientific; Nasco)

Straw Polyhedra Kit (Creative Publications)

Tangrams—cards and pieces McGraw-Hill Ryerson)

"Think-Sticks" (Edmund Scientific)

True Equal-Volume Geometric Solids Set (Edmund Scientific)

Commercial Games

Configurations (Hammett; Wff 'N Proof)

Polyhedron Rummy (Scott Foresman)

PAGES 56–57

Objective

Given the words, symbols, or pictures for point, ray, line, segment, plane, angle, *and* triangle, *the child will be able to identify each.*

Preparation

To prepare for this lesson, write the words *point*, *ray*, *segment*, and *line* on the chalkboard. Ask the children to mention something they can remember about each, because they will be using the terms in the investigation. However, do not try to define them (*point* and *line* are usually taken as undefined terms) or even to give adequate descriptions. Simply use the words to lead into the investigation.

Investigation

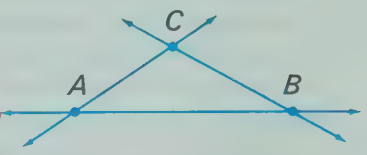
Suggest to the children that they study the drawing and see how many figures they can find from the list you have written on the chalkboard. Suggest that they identify the figures specifically; that is, instead of listing a term such as *line segment*, they should list a specific line segment, such as line segment AB. Point out that the correct symbols for these figures will be reviewed in the discussion. Here they should simply try to find as many different figures as they can.

4 Geometry and Measurement I

● *Let's look at some basic geometric figures.*

Investigating the Ideas

Study this drawing.

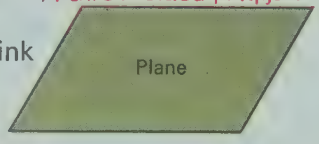


Possibilities include: Points A, B, C; lines AB, AC, BC; segments AB, AC, BC; rays AC, CA, AB, BA, BC, CB; angles CAB, ABC, ACB; triangle ABC

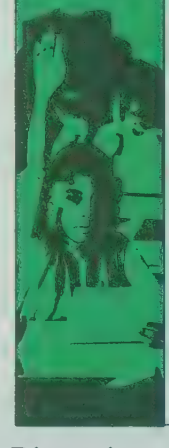
How many different geometric figures can you find and name in the drawing? See Investigation.

Discussing the Ideas

1. To name line AB we write $\overleftrightarrow{AB}$. How many other lines did you find and name? $\overleftrightarrow{BC}$, $\overleftrightarrow{AC}$
2. To name segment AC we write $\overline{AC}$. What other segments did you name in the drawing above? $\overline{AB}$, $\overline{BC}$
3. A ray starts at a point on a line and contains all the points on one half of the line. Ray BC (symbol: $\overrightarrow{BC}$) is an example. How many other rays did you find? $\overrightarrow{AC}$, $\overrightarrow{CA}$, $\overrightarrow{AB}$, $\overrightarrow{BA}$, $\overrightarrow{CB}$
4. A How many angles are shown in the drawing? 12
   B Explain which angle is meant by the symbol $\angle ACB$? See Discussion.
5. Did you name a triangle in the drawing? What is a triangle? $\triangle ABC$ A three-sided polygon.
6. A plane is a flat surface that extends endlessly in each direction. Do you think that any three points that are not in a line can always be in one plane? Yes See Discussion.



7. Describe some physical objects that remind you of points, lines, segments, rays, angles, triangles, and planes. Examples will vary. See Discussion.

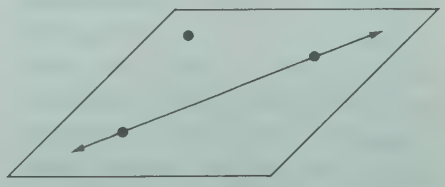


56

Discussion

Use exercises 1 through 5 to discuss the figures the children found in the drawing. Also, review the correct symbols for each. In discussing exercise 4B, point out that it is important to list three letters for each angle of the triangle; since there are really 4 angles at each point, the symbol $\angle C$ would not be clear.

As you discuss exercise 6, refer to various surfaces to help children develop an intuitive idea of plane. Be sure they realize, however, that a surface such as the floor represents only a part of a plane since a plane extends indefinitely in all directions. Also help the children see that any two points determine a line, and a third point not on that line determines a plane.



As children describe objects for exercise 7, accept any suggestions which they can reasonably explain.

Using the Ideas

Match each picture with the name of the geometric figure it suggests, with the drawing of the figure, and with the symbol for the figure. For exercise 1, write C, H, Q.

We see ↓

1. 
C,H,Q

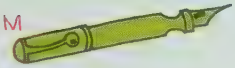
2. 
F,J,R

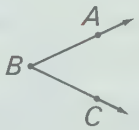
3. 
A,I,N

4. 

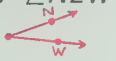
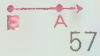
B,L,P

5. 
E,G,O

6. 
D,K,M

We think	We draw	We write
A line	G 	M A
B angle	H 	N $\overleftrightarrow{AB}$
C segment	I 	O $\overline{AB}$
D point	J 	P $\angle ABC$
E ray	K $\cdot A$	Q $\overline{AB}$
F triangle	L 	R $\triangle ABC$

7. Draw and label a figure for each symbol.

A $\overline{PQ}$ 
B $\angle NEW$ 
C $\overleftrightarrow{ST}$ 
D $\overleftrightarrow{GQ}$ 
E $\triangle PQR$ 
F $\overrightarrow{BA}$ 



Using the Exercises

An important part of page 57 is devoted to helping the children understand the relationship between physical objects and geometrical concepts. Be sure to explain that the figures described in this lesson are all *plane geometric figures*—figures that lie on one flat surface, or plane—and that later in the chapter they will study *space figures*—figures made up of sets of points that do not necessarily lie on one flat surface, or in one plane.

Encourage children to work independently. You might also suggest that they make a chart to show the figures described by some of the objects they thought of during the discussion.

Assignments (page 57)
Minimum: 1–6. Average: 1–7.
Maximum: 1–7.

Mathematics

In most formal treatments of geometry, beginning usually at the high school level, the words *point*, *line*, and *plane* are taken as the primitive, or undefined, words of geometry. Since these words are taken as undefined, assumptions or postulates concerning them must be carefully stated. For example, one postulate might be stated as, "Any two different points lie on exactly one line." A second postulate might be, "There is exactly one plane containing any three points that are not all on the same line."

Other geometric figures are defined in terms of these undefined words. Formal definitions for some of the figures in this lesson may be stated as follows:

A *segment* consists of two points A and B and all the points on a line between A and B.

A *ray* from a point R consists of R and all the points on a line through R that are on one side of R.

An *angle* is the set of points on two rays from the same point.

A *triangle* consists of the set of points on the segments determined by three points not on the same line.

Resources for Active Learning

Geometry in the Classroom, pp. 82–88, Holt, Rinehart and Winston of Canada [Basic geometric figures].

Experiments in Mathematics, Stage 1, p. 52; Stage 2, p. 58, Houghton Mifflin [Optical illusions]. (Available from Thomas Nelson & Sons Ltd)

Math Activity Cards, "Euler's Relation," D14, Macmillan.

Workbook, page 17

PAGES 58–59

Objectives

Given appropriate materials and patterns for certain space figures, the child will be able to construct models of the figures.

Given pictures of certain physical objects, the child will be able to match them with the corresponding space figures.

Preparation

Materials

geometric solids (at least one set; *optional*) paper suitable for tracing and folding, and tape or glue (or *Duplicator Masters*, page 71)

To prepare for this lesson, it would be helpful to briefly review the idea of a plane. For example, ask the children to name surfaces in the room which suggest to them the idea of a plane. They may mention the floor, each wall, their desk tops, the chalkboard, and so on.

Investigation

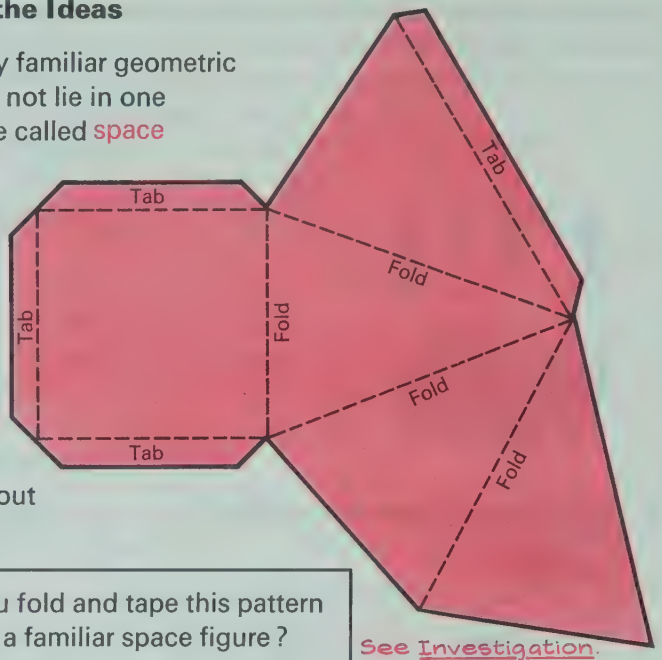
In this investigation, children move from the study of plane figures to the study of figures which do not lie in one plane, namely, space figures. Rather than have children trace the investigation pattern, you might duplicate copies of the pattern on paper of a heavier weight. Since the purpose of this investigation is simply to use a pattern to form a space figure, any method children use for this purpose would be suitable.



● *Let's explore space (figures).*

Investigating the Ideas

There are many familiar geometric figures that do not lie in one plane. They are called space figures.



Trace and cut out this pattern.

? Can you fold and tape this pattern to form a familiar space figure?

See Investigation.

Discussing the Ideas

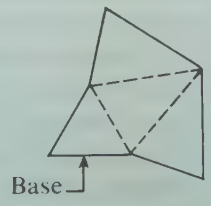
1. Does the space-figure model you made remind you of any physical objects you have seen? Do you know the name of the space figure? See Discussion.
2. How many faces, edges, and vertices does your space figure have? 5 faces, 8 edges, 5 vertices
3. The pattern above was made from plane figures. Do you know the names of these figures? Triangles and a rectangle (in this instance, a square)
4. Can you make or describe a pattern that could be used to make a cube? Patterns may vary but should be made up of 6 congruent squares.

58

Discussion

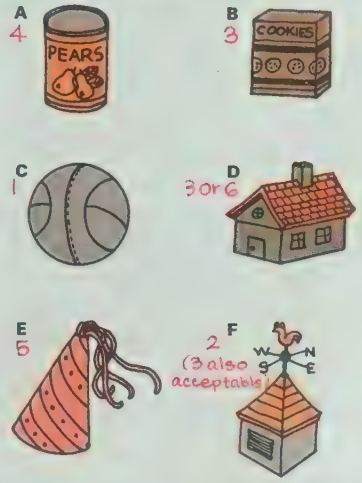
Point out to the children that since the figure they made for the investigation has a rectangle for a base, the figure is called a rectangular pyramid. As you discuss exercise 2, emphasize the relation of points, segments, and planes. For example: each face suggests a portion of a plane; the edges suggest line segments; and the intersections of the edges suggest points that are vertices of the space figures.

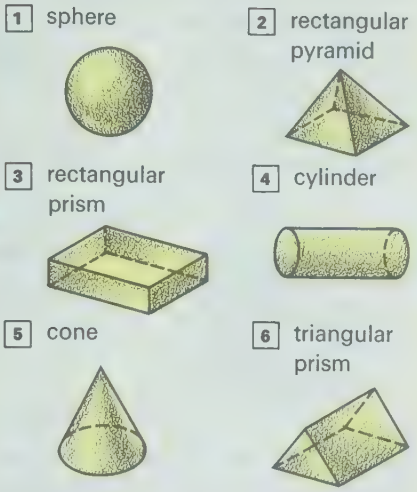
The lateral faces of a pyramid are triangular regions, but the shape of the base can vary. For example, the base of the pyramid in the investigation is a square. The pattern for a triangular pyramid would include only triangles, since the base is also a triangle.



Using the Ideas

1. Match each physical object on the left with one of the geometric figures pictured on the right.



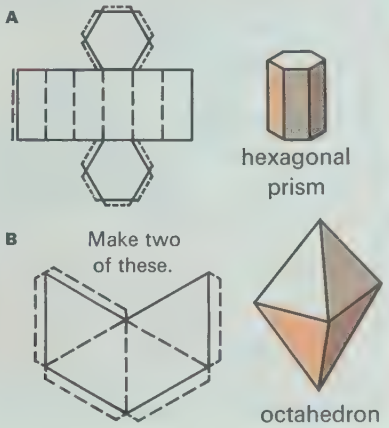


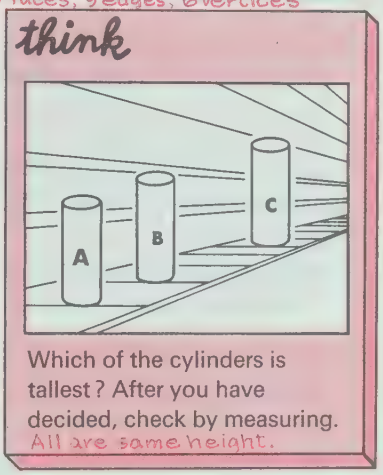
2. How many faces, edges, and vertices does each figure have?

A rectangular prism
6 faces, 12 edges, 8 vertices

B triangular prism
5 faces, 9 edges, 6 vertices

★ **3.** Make models of these figures.







59

Using the Exercises

Assign the exercises on page 59 as independent work. If possible, have sets of geometric solids available so that children can examine them freely. Have children check their answers for exercise 1 carefully to make sure they interpret correctly the three-dimensional drawings.

The patterns in starred exercise 3 are intended for children who are capable or interested enough to enlarge the patterns and construct the figures.

Note: If the children make the hexagons for the prism 4 centimetres an edge, and the vertical faces 4-by-8 centimetres, the figures will be easier to complete. If they make the equilateral triangles 7 centimetres on a side, with flaps for pasting, the octahedrons will be easier to duplicate. Be sure they make two copies of the pattern for the octahedron.

Assignments (page 59)
Minimum: 1. Average: 1, 2.
Maximum: 1–3.

Follow-up

If commercial sets of geometric solids are not available, the children might enjoy making solids from modelling clay or plastic foam. For example, to make a cone, a rectangular prism, and a right circular cylinder, carefully pack modelling clay into a cone-shaped paper cup, the end of a rectangular box (such as the kind toothpaste comes in), and a small juice can.

Resources for Active Learning

Experiments in Mathematics, Stage 1, "Regular Polyhedra," pp. 24–25, Houghton Mifflin. (Available from Thomas Nelson & Sons Ltd.).

Math Activity Cards, "Point of View," E19, Macmillan.

Nuffield Project: *Shape and Size* 3, "Polyhedra," pp. 55–59, Wiley.

PAGES 60–61

Objective

Given a simple space figure, the child will be able to draw pictures showing top, front, and side views of the figure and to draw pictures of simple space figures.

Preparation

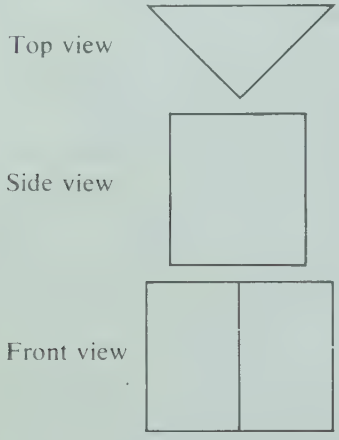
Materials

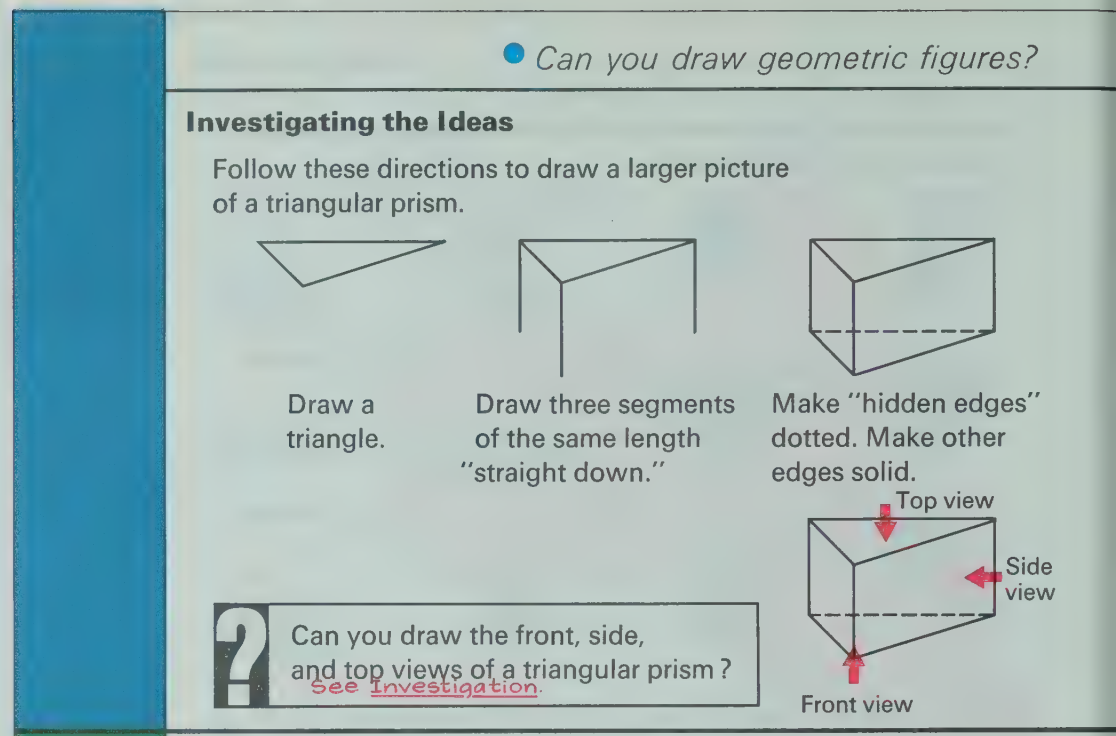
model of a triangular prism

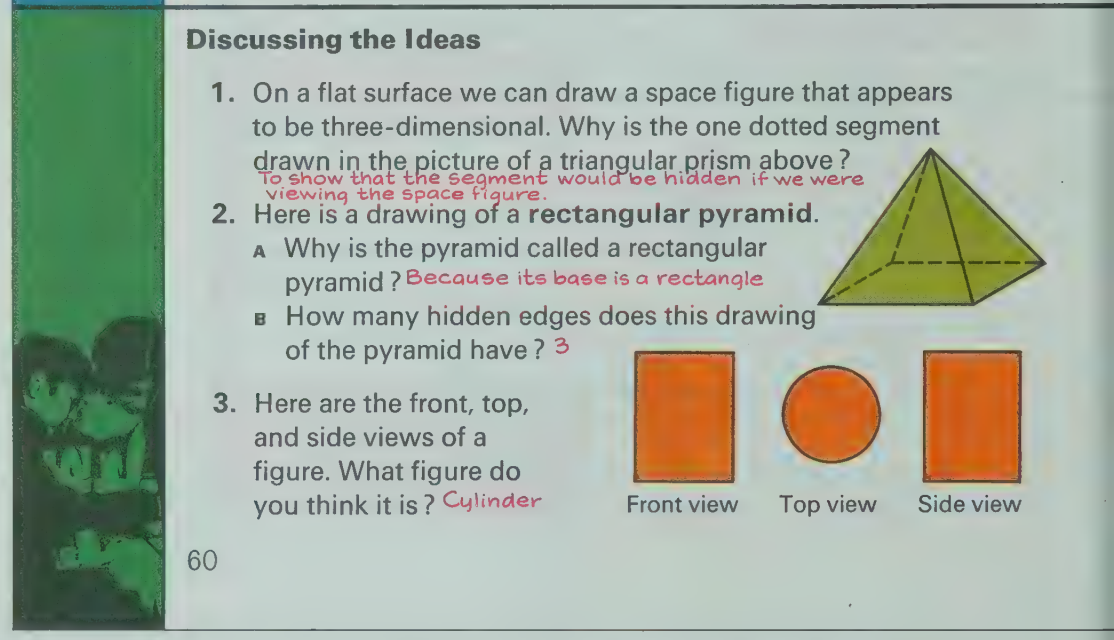
If a set of geometric solids is available, use it to review identification of the shapes and proper use of the terms. Show each solid and have children try to identify it.

Investigation

It would be most helpful to have a fairly large model of a triangular prism to exhibit. Allow the children to use not only the directions in the text, but also to walk around freely and inspect the displayed model. In an activity such as this, children may draw the views incorrectly but the discussion section should help them correct any views they have misunderstood. The views for the triangular prism are as shown below.







Discussion

One of the main purposes of this lesson is to help build the children's spatial perception. Skill in visualizing front, side, and top views of space figures will enhance such perception. Point out that the dotted lines are important in the drawings, because without them the hidden part of the figure may be misinterpreted. For example, if only the side view of the triangular prism is shown, the fact that the figure is a triangular prism is not at all apparent.

In exercise 2, children may need help in explaining how to draw the rectangular pyramid. Help them understand that the base should be shown as a parallelogram. Then the edges can be filled in.



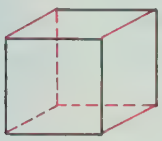
As you discuss exercise 3, you might have children examine the front, side, and top views of familiar geometric shapes until they can determine which each of them is.

Throughout this discussion, it would be helpful not only to display various solids but also to show various steps of the drawings on the overhead projector.

Using the Ideas

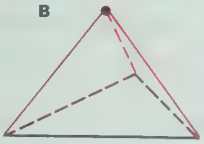
1. Trace and complete the drawing of each figure.
Be sure to show the hidden edges with dotted segments.

A



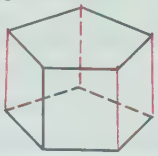
cube

B



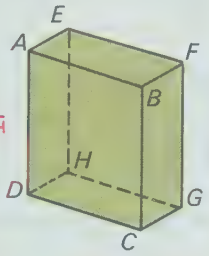
triangular pyramid

C



pentagonal prism

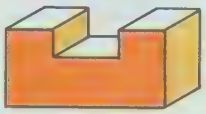
2. The vertices of this rectangular prism have been labelled.



A Name the edges that are "hidden." $\overline{EH}$, $\overline{DH}$, $\overline{GH}$

B The front face is named *ABCD*.
Name the faces of the prism that are "hidden." AEHD, EFGH, DHGC

3. This object was pictured in a draftsman's handbook. Can you draw front, top, and side views of the object?

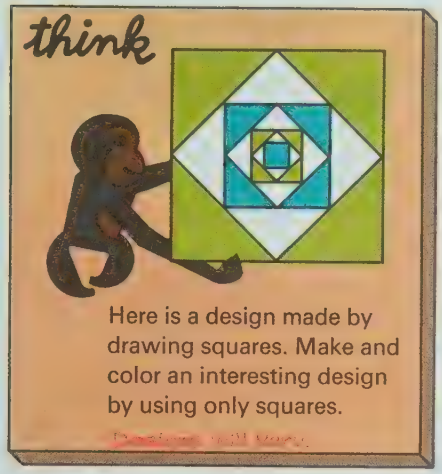




Front



Top Side



Here is a design made by drawing squares. Make and color an interesting design by using only squares.

★4. Choose an object, such as a birdhouse, a desk, or a table, and make a drawing of it. Then show the front view, top view, and side view.
Drawings will vary.

★5. Find out what an octahedron is. Make a drawing of one.

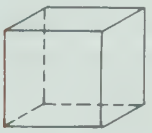




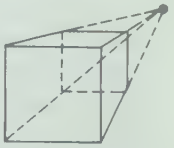
61

Mathematics

The type of drawings discussed in this lesson have the technical name of *isometric drawings*. In making an isometric drawing of a cube, for example, parallel edges are drawn parallel and equal in length. Edges which are inclined to the drawing surface are equally shortened.



An isometric drawing of a cube



A perspective drawing of a cube

In contrast, in a *perspective drawing* of a cube, the edges which are inclined to the drawing surface are drawn as if lying on rays emanating from a single point, called the vanishing point.

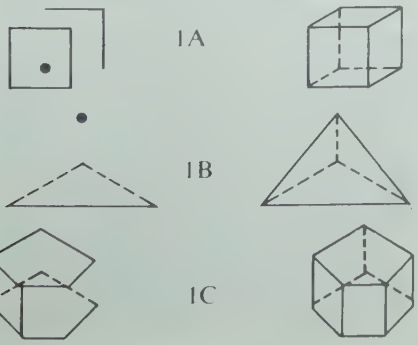
Follow-up

It would be appropriate to correlate some art activities with this lesson. Suggest that children find an object in which they can identify the shape of a rectangular prism. Then, ask them to draw its front, side, and, if possible, top view. Suitable objects would be buildings, trucks, boxes, and shelves. You might also suggest that they make a perspective drawing of the object. Some may need guidance for this activity.

Duplicator Masters, page 9
Workbook, page 18

Using the Exercises

After children have completed the exercises on page 61, it would be helpful to discuss the correct drawings while showing them on an overhead projector. For example, discuss the following steps used to draw the figures in exercise 1.



Assignments (page 61)
Minimum: 1, 2. Average: 1–3.
Maximum: 1–5.

PAGES 62–63

Objective

Given parallel or perpendicular lines, the child will be able to recognize and identify them.

Preparation

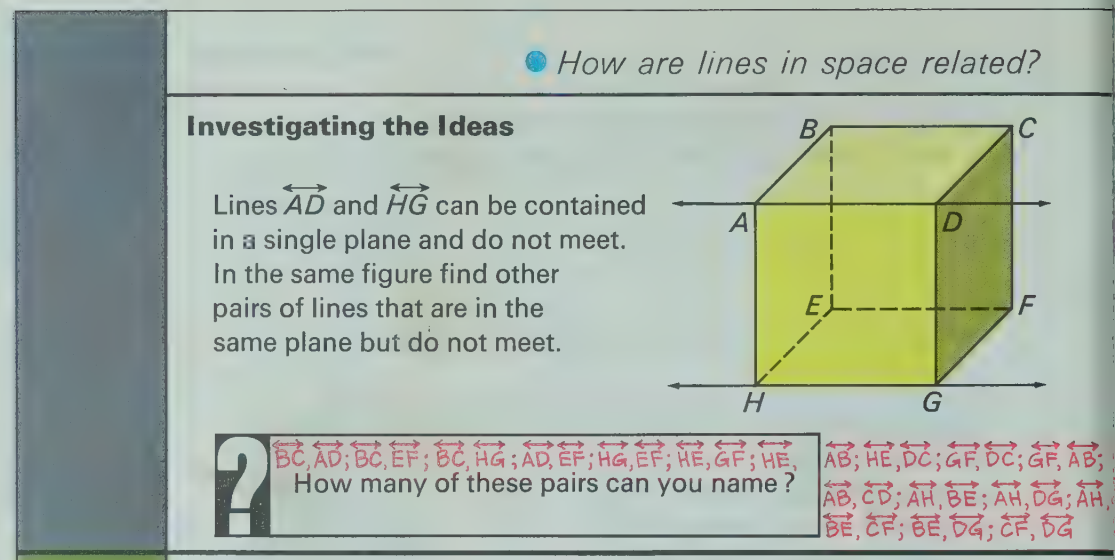
Materials

cubes, 1 per child (optional)

If children do not have individual cubes to examine, you might display a model cube and have children identify various vertices and edges on it. Remind them that each face may be thought of as part of a plane. In this way, they should be better prepared to work from the pictorial representation in their texts.

Investigation

Encourage the children to study their cubes and the pictorial representation in the investigation. They need simply record pairs of lines to indicate which ones they think are parallel. Some children may not have difficulty in finding pairs of lines which belong to a common face. But if they quickly claim that they have found all there are, hint to them that they might not be considering all possible planes. For example, $\overleftrightarrow{BC}$ and $\overleftrightarrow{HG}$ are in the same plane and do not meet.



How are lines in space related?

Investigating the Ideas

Lines $\overleftrightarrow{AD}$ and $\overleftrightarrow{HG}$ can be contained in a single plane and do not meet. In the same figure find other pairs of lines that are in the same plane but do not meet.

How many of these pairs can you name?

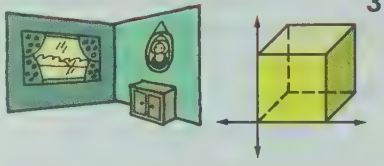
$\overleftrightarrow{BC}, \overleftrightarrow{AD}; \overleftrightarrow{BC}, \overleftrightarrow{EF}; \overleftrightarrow{BC}, \overleftrightarrow{HG}; \overleftrightarrow{AD}, \overleftrightarrow{EF}; \overleftrightarrow{HG}, \overleftrightarrow{EF}; \overleftrightarrow{HE}, \overleftrightarrow{GF}; \overleftrightarrow{HE}, \overleftrightarrow{AB}; \overleftrightarrow{HE}, \overleftrightarrow{DC}; \overleftrightarrow{GF}, \overleftrightarrow{DC}; \overleftrightarrow{GF}, \overleftrightarrow{AB}; \overleftrightarrow{AB}, \overleftrightarrow{CD}; \overleftrightarrow{AH}, \overleftrightarrow{BE}; \overleftrightarrow{AH}, \overleftrightarrow{DG}; \overleftrightarrow{AH}, \overleftrightarrow{BE}, \overleftrightarrow{CF}; \overleftrightarrow{BE}, \overleftrightarrow{DG}; \overleftrightarrow{CF}, \overleftrightarrow{DG}$

Discussing the Ideas

1. Two lines which are in the same plane and do not meet, or intersect, are parallel lines. To show that $\overleftrightarrow{AD}$ is parallel to $\overleftrightarrow{HG}$, we write $\overleftrightarrow{AD} \parallel \overleftrightarrow{HG}$. Show how to write statements for other lines that are parallel to $\overleftrightarrow{AD}$ in the figure above.
$\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$, $\overleftrightarrow{AD} \parallel \overleftrightarrow{EF}$

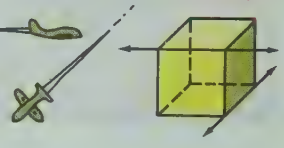
2. A long, straight section of railroad tracks may remind you of parallel lines. Name some other things that remind you of parallel lines.
Power lines, outer edges of highways, etc.





3. When two lines intersect and form a "square corner," they are perpendicular to each other. Name some things that remind you of perpendicular lines.
Corners of books and papers street corners, etc.

4. Lines in space that do not intersect and are not parallel are skew to each other. Show how to hold two rulers or two pencils to represent skew lines.
See Discussion.





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Discussion

As you discuss the investigation, explain the meaning of intersecting lines and parallel lines. It would be helpful to display the following:



Intersecting lines



Parallel lines

You might use the edges of the cube to point out lines which intersect as well as lines which are parallel. There are 18 possible pairs of parallel lines on the cube, if order is disregarded. That is, $\overleftrightarrow{BC}$ *is parallel to* $\overleftrightarrow{AD}$ *and* $\overleftrightarrow{AD}$ *is parallel to* $\overleftrightarrow{BC}$ are statements which describe one pair of parallel lines. Explain how these statements may be symbolized:

$$\overleftrightarrow{BC} \parallel \overleftrightarrow{AD}$$

"Line BC is parallel to line AD."

As you discuss exercise 1, continue to use the cube as a basis of discussion. For example $\overleftrightarrow{AH}$ is perpendicular to $\overleftrightarrow{HG}$. You might show the symbolized statement for this: $\overleftrightarrow{AH} \perp \overleftrightarrow{HG}$ (but do not stress the symbol; emphasize the concept of being perpendicular).

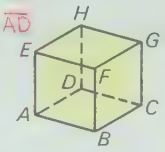
Encourage children to describe other examples that suggest parallel, perpendicular, or skew lines.

Using the Ideas

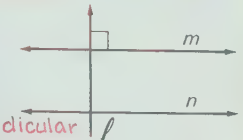
1. A Give all the edges that are parallel to $\overline{EH}$. $\overline{FG}, \overline{BC}, \overline{AD}$

   B Give all the edges that are perpendicular to $\overline{AB}$. $\overline{EA}, \overline{FB}, \overline{BC}, \overline{AD}$

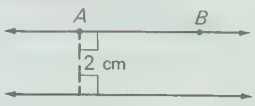
   C Give the edges that are neither parallel nor perpendicular to $\overline{BC}$. $\overline{EF}, \overline{HD}, \overline{HG}, \overline{EA}$



2. A Lines m and n are parallel to each other. Line m and line ℓ are perpendicular. What do you think about n and ℓ? Perpendicular to each other.



   B We use a segment that is perpendicular to two parallels to measure the distance between them. If we find that the lines are 2 cm apart at point A, what do you think we will find at any other point, such as B? The two lines are 2 cm apart at any other point.

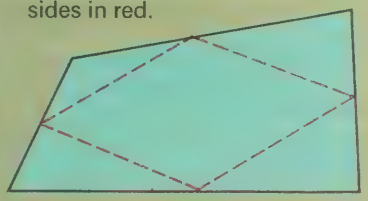


3. Lines m and n are straight.
   Are they parallel? Yes
   Why or why not? $\overline{AB}$ and $\overline{CD}$ are the same length.



think

Draw any 4-sided figure and connect the midpoints of the sides in red.



Do this with five 4-sided figures of different shapes. What can you discover about the red figures? They are parallelograms.

★ 4. Start with a line AB and use the idea in exercise 2B and the corner of your book to help you draw two parallel lines.



See Using the Exercises.

★ 5. Show how to draw an accurate picture of two parallel lines by using only your ruler.

63

Using the Exercises

When the children have finished the exercises on page 63, allow time for discussion. For instance, in exercise 1B, point out that perpendicular lines can be contained in a single plane; thus $\overline{HD}$ is not perpendicular to $\overline{AB}$ since there is no single plane which can contain them both. Exercises 2 and 3 are based on the fact that the perpendicular distance between two parallel lines is always the same. Although exercise 5 is starred, you might want all children to see that an easy way to construct parallel lines is simply to draw a line on each side of the ruler.

Assignments (page 63)

Minimum: 1, 2. Average: 1–3. Maximum: 1–5.

Follow-up

Encourage children to find pictures which suggest parallel or perpendicular lines. For example, they might find a picture or advertisement in a magazine and use colored pencils or marking pens to mark segments which suggest pairs of parallel or perpendicular lines.

Mathematics

The definition of parallel lines given in this lesson states: *Two lines are parallel if they are in the same plane and do not intersect.* There is a certain subtlety in this definition that sometimes eludes children. Some may think that it would be sufficient to say that any two lines which do not intersect must be parallel. However, if we consider all the lines in space there are certain pairs of lines which do not intersect and yet are not parallel. Such lines are called skew lines.

Physical examples of such lines are easily demonstrated. The line of intersection of the front wall and ceiling of a classroom and the line of intersection of side wall and the floor of the same room illustrate a pair of skew lines. Thus the requirement that parallel lines lie in the same plane is vital to the definition.

Resources for Active Learning

Geometry in the Classroom, "Parallelism," pp. 99–100, Holt, Rinehart and Winston of Canada.

Mathex: Geometry No. 9, "Lines in Space," pp. 7–9, Encyclopaedia Britannica Publications Ltd.

Nuffield Project: *Shape and Size* 3, ". . . parallels and angles," pp. 9–10, Wiley.

PAGES 64–65

Objective

Given a compass and a ruler, the child will be able to construct a line segment, an angle, a right angle, a triangle, and the bisector of an angle.

Preparation

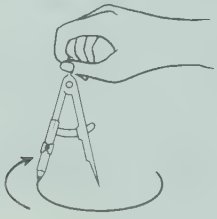
Materials

ruler; compass; cardboard (for use under compass point)

If possible you might show some sample designs constructed with a ruler and compass and then colored. Since even very simple designs can look intricate and pleasing, children should enjoy seeing the possibilities of what this lesson will present.

Investigation

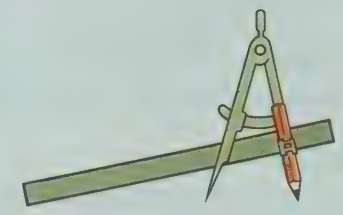
Encourage children to explore ways of using their compass. Demonstrate how the compass can be manipulated easily by holding the little handle on top and turning it while moving the wrist in a circle.

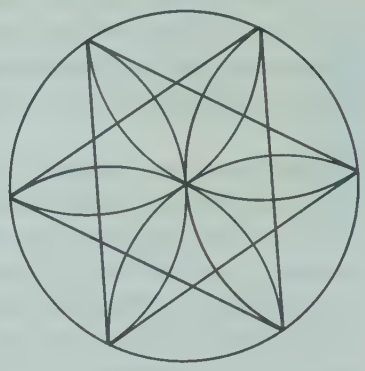


Explain how a small piece of cardboard under the paper on which they are working will prevent the compass point from sticking into their desk tops. When they have been able to reproduce the design, suggest that they create others of their own. Since one of the main purposes of the investigation is to help children develop a facility in using the compass, this kind of free-play with designs can be very helpful.

● *Can you construct geometric figures?*

Investigating the Ideas





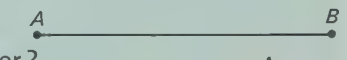
Can you make this design with only a compass and a ruler?

See Investigation.

Discussing the Ideas

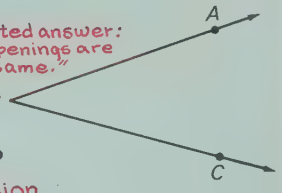
To **construct** a geometric figure we use only a compass and the edge of a ruler. No guessing or measuring is allowed.

1. How could you **construct** a segment that is just as long as $\overline{AB}$ on your paper?



See Discussion.

2. A What does it mean to say that two angles are "the same size"?

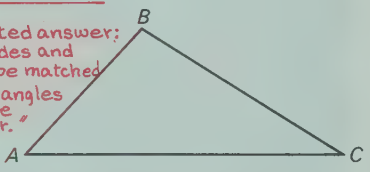


Accepted answer: "The openings are the same."

   B How could you **construct** an angle the same size as $\angle ABC$ on your paper?

See Discussion.

3. A What does it mean to say that two triangles are "the same size and shape"?



Accepted answer: "The sides and angles of one can be matched with the sides and angles of the other."

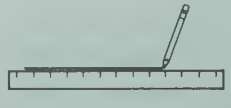
   B How could you **construct** a triangle the same size and shape as $\triangle ABC$ on your paper?

See Discussion.

Discussion

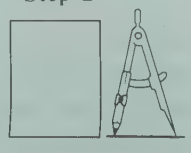
The important point in discussion exercise 1 is that the ruler is not to be used to measure the segment. Rather, the edge of the ruler is simply the guide for drawing a straight line segment. Then the compass is to be used to copy the segment $\overline{AB}$ from the text.

Step 1



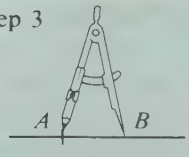
Use ruler to draw a segment.

Step 2



Use the compass to find the length of $\overline{AB}$.

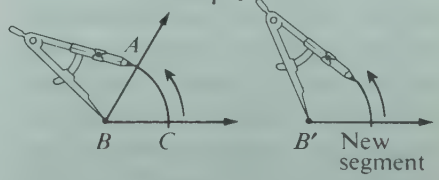
Step 3



With the compass, mark the points A and B on the segment.

Similarly, in exercise 2, the ruler is used only as a straightedge.

Step 1



Mark the new segment with an arc the same distance from B.

Using the Ideas

See Using the Exercises.

1. Construct a segment that is just as long as $\overline{EF}$.



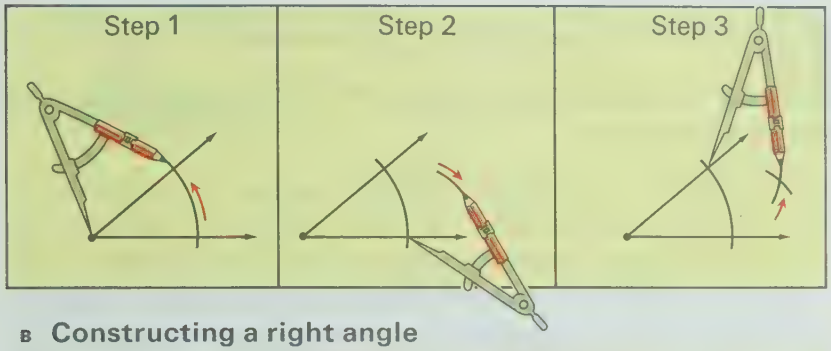
2. Construct an angle that is the same size as $\angle JKL$.



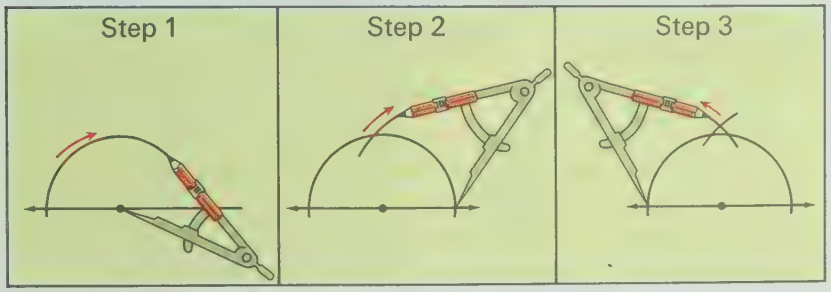
3. Construct a triangle that is the same size and shape as $\triangle PQR$.

4. To "bisect" means to divide into two parts of the "same size." Study the first three steps and think about how to complete the construction in step 4. Then do each construction.

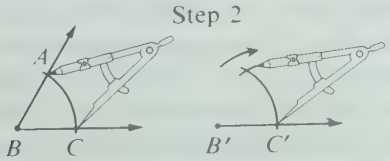
A Bisecting an angle



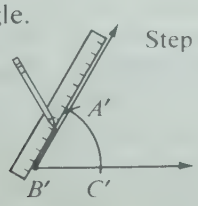
B Constructing a right angle



65



Set the compass to match the distance from C to A. Mark A' on the new angle.



Use the ruler to draw the ray $B'A'$. Encourage children to use these steps to find different ways to construct the triangle in exercise 3.

Using the Exercises

Encourage children to try the constructions on page 65 on their own. Afterward, it would be helpful to give the children an opportunity to express why they think their methods of construction give the desired results. Such explanations should not be construed as proof, but rather as intuitive thought about the constructions.

Before assigning exercise 4, you might want to discuss the meaning of *bisect* as explained in the text.

Assignments (page 65)

Minimum: 1–3. Average: 1–4. Maximum: 1–4.

Mathematics

Geometric figures which can be constructed using only the compass and an unmarked straightedge are sometimes called *Euclidean constructions*. In the classical geometry of Euclid, only these two tools were permitted and the ancient Greek mathematicians were absorbed by problems involving the geometric figures which could be constructed by using these two devices. One problem, which was unresolved by the Greeks, was whether any angle could be divided into 3 congruent parts (trisected) using only straightedge and compass. It was not until 1837 that Pierre Wantzel proved that it is impossible to trisect any given angle with only the straightedge and compass.

Another problem which puzzled the ancient Greeks was whether certain regular polygons could be constructed with straightedge and compass alone. They were acquainted with constructions of most of the simple regular polygons, including the regular pentagon. This problem was resolved by Carl Frederich Gauss (1777–1855). Among the many important things that Gauss discovered was a way of constructing a regular 17-sided polygon. In addition, he was able to prove exactly which regular polygons were constructible and which were not.

Follow-up

Encourage children to use their compasses to make interesting circle designs similar to the kind they explored in the investigation. When such designs are colored, they will make attractive displays.

Resources for Active Learning

Geometry in the Classroom, "The Construction of Triangles," pp. 108–114, Holt, Rinehart and Winston of Canada.

Workbook, page 19

PAGES 66–67

Objective

Given two segments, two angles, or two triangles, the child will be able to determine if the figures in each pair are congruent by using tracing paper or by using their compasses.

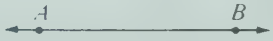
Preparation

Materials

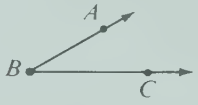
tracing paper; compass (1 per child)

To prepare for this lesson you might conduct a brief review of the symbols used for various geometric figures. For example, write the following symbols on the chalkboard or show them on an overhead projector, and ask children to read them:

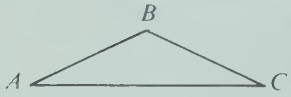
$\overline{AB}$ Line segment AB



$\angle ABC$ Angle ABC



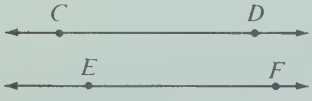
$\triangle ABC$ Triangle ABC



$\overrightarrow{AB}$ Ray AB



$\overline{CD} \parallel \overline{EF}$ Line segment CD is parallel to line segment EF.



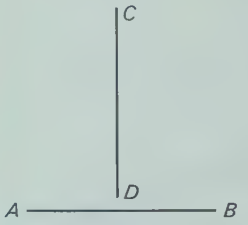
Investigation

Encourage children to find ways of checking their answers for each question. For example, suggest that they may want to use various materials such as a ruler, a compass, tracing paper, and anything else they think would be helpful. Encourage initiative in the methods children devise, even if you realize a particular method some choose is long and tedious. Later, by comparing methods with each other, they will find the method that suits them best.

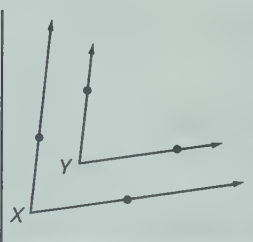
When are two geometric figures congruent?

Investigating the Ideas

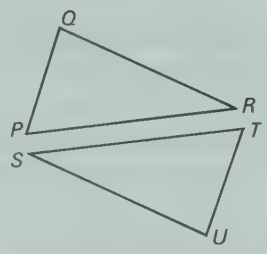
Answer each of these questions just by looking.



Is $\overline{AB}$ longer than, shorter than, or just as long as $\overline{CD}$?
Just as long



Is $\angle X$ smaller than, larger than, or the same size as $\angle Y$?
Same size



Is $\triangle PQR$ the same size and shape as $\triangle UTS$? Yes

? Can you find a way to check your answers?

Discussing the Ideas

1. Two segments (like $\overline{AB}$ and $\overline{CD}$) are **congruent** if their end points are equally far apart. We write, "$\overline{AB} \cong \overline{CD}$," and read, "Segment AB is congruent to segment CD." How did you check to see if $\overline{AB}$ is congruent to $\overline{CD}$ above? See Discussion.
2. Two angles (like $\angle X$ and $\angle Y$) are **congruent** if "corresponding points" on their rays are equally far apart. We say, "Angle X is congruent to angle Y." We write, "$\angle X \cong \angle Y$." How could you use tracing paper or a compass to see if $\angle X$ above is congruent to $\angle Y$? See Discussion.
3. Two triangles (like $\triangle PQR$ and $\triangle UTS$) are congruent if the parts (angles and segments) of one can be matched with the parts of the other. How could you use tracing paper to see if $\triangle PQR \cong \triangle UTS$? Trace one triangle and place it over the other to see whether the segments and angles of one can be matched with the segments and angles of the other.



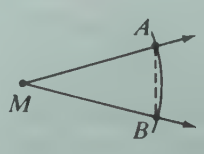
Discussion

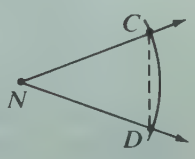
In discussion exercise 1, stress the meaning of congruent segments and the symbol $\cong$ ("is congruent to"). Give children an opportunity to describe the methods they used in the investigation.

As you discuss exercise 2, draw angles on the chalkboard and use a compass to show the meaning of corresponding points.

Observe that the idea of congruent angles depends upon the basic notions of congruent segments; that is, by first establishing the general idea of corresponding points on the rays of a pair of angles, we are able to define *less than*, *greater than*, and *congruence* for angles. The use of corresponding points allows us to return to the basic notions of *closer together*, *equally far apart*, or *farther apart*.

The segments indicated on the rays of the angles which follow are all congruent. We say that C and D are the points on $\angle CND$ that correspond to points A and B on $\angle AMB$.





Using the Ideas

1. For each exercise, trace one segment and place it over the other segment. Tell whether the two segments are congruent.

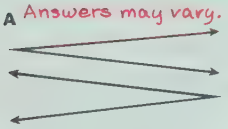
A Yes

B No

C Yes

2. Trace each pair of angles to tell whether or not they are congruent. See Using the Exercises.

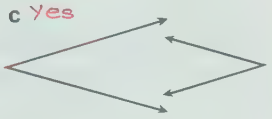
A Answers may vary.



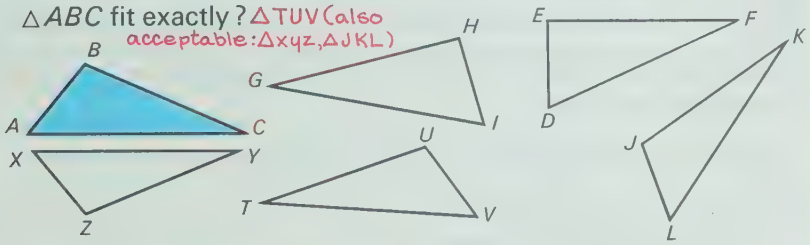
B Yes



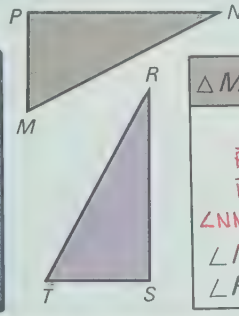
C Yes



3. Trace $\triangle ABC$. By sliding, turning, or flipping the tracing paper, on which of the other triangles can you make $\triangle ABC$ fit exactly? $\triangle TUV$ (also acceptable: $\triangle XYZ$, $\triangle JKL$)



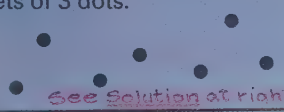
4. Give the missing segments and angles in the table. You may want to trace $\triangle MPN$ and move it to help you match the segments and angles.



$\triangle MPN \cong$	$\triangle TSR$
$\overline{MP} \cong$	$\overline{TS}$
$\overline{PN} \cong$	$\overline{SR}$
$\overline{MN} \cong$	$\overline{RT}$
$\angle NMP \cong$	$\angle RTS$
$\angle MPN \cong$	$\angle TSR$
$\angle PNM \cong$	$\angle SRT$

think

Arrange 7 dots so you can draw straight lines through 6 different sets of 3 dots.



See Solution at right.

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The idea of congruent triangles as treated in exercise 3 is based on the notion of congruent segments and congruent angles. As you discuss the congruence of $\triangle PQR$ and $\triangle UTS$, point out that $\overline{PR}$ corresponds to $\overline{ST}$; $\overline{PQ}$ corresponds to $\overline{UT}$; and $\overline{QR}$ corresponds to $\overline{SU}$. Similarly, $\angle P \cong \angle T$; $\angle Q \cong \angle U$; and $\angle R \cong \angle S$.

If some children exhibit difficulty with the use of the compass, have them use the tracing-paper method.

Using the Exercises

On page 67, you might have the more able children use the compass to test for congruence. However, suggest the tracing-paper method for most children. Note that in exercises 2 and 3 children's answers may vary, depending upon the accuracy of their tracing or measurement.

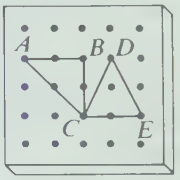
As you discuss the completed exercises, stress the meaning of congruence for the segments, angles, and triangles.

Assignments (page 67)

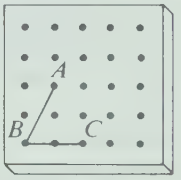
Minimum: 1–3. Average: 1–4.
Maximum: 1–4.

Follow-up

Now that children are prepared to determine the congruence of segments, angles, and triangles, suggest that they construct one of these figures on a geoboard and try to build another that is congruent to the first. Or, challenge them with problems similar to the following:



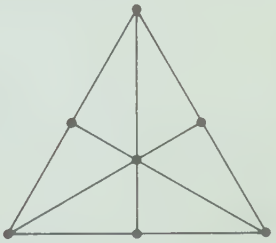
Is $\triangle ABC \cong \triangle CDE$?



How many angles congruent to $\angle ABC$ can you build on the geoboard?

Solution, Think, page 67

The most likely solution that children will find for the *Think* problem is shown below.



Resources for Active Learning

Freedom to Learn, "Congruence and Symmetry," pp. 153–154, Addison-Wesley.

Geometry in the Classroom, "Congruence," pp. 89–94, Holt, Rinehart and Winston of Canada.

Nuffield Project: *Problems* – Red Set, No. 19, Wiley.

Workbook, page 20

PAGES 68–69

Objective

Given a line segment, the child will be able to find its length in centimetres.

Preparation

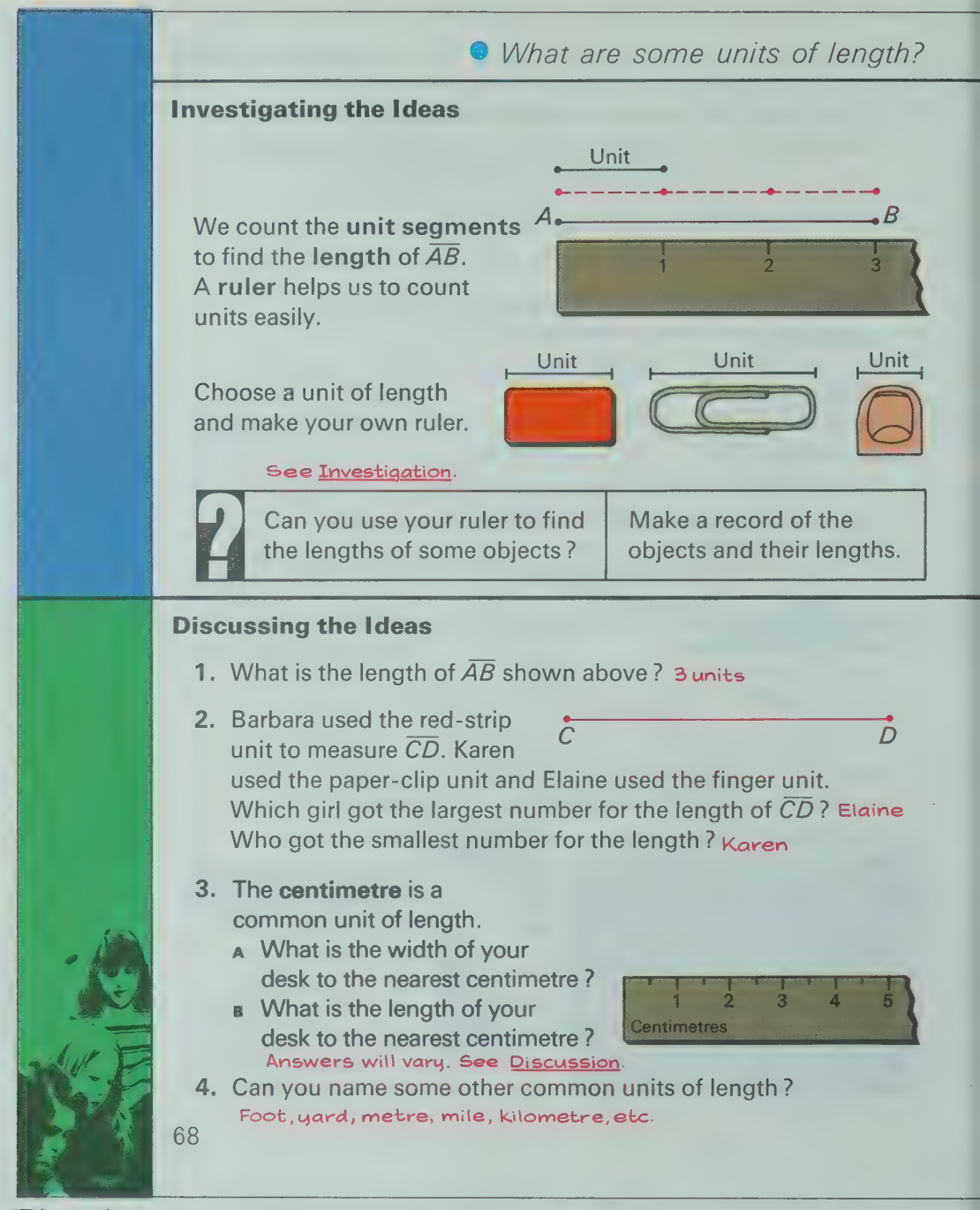
Materials

construction paper; paper fasteners; overhead projector (optional)

Since the investigation for this lesson may need more teacher-direction than usual, you might begin immediately with the text.

Investigation

If children have studied from the previous books in this series, the concept of building a measuring instrument from a particular unit should not be new to them. However, you might find it helpful to discuss the material at the top of the page before children begin to work on their own. With some groups of children, you may even want to guide them in making the ruler so that they will understand how a measuring tool is built from a unit.



What are some units of length?

Investigating the Ideas

We count the **unit segments** to find the **length** of $\overline{AB}$. A **ruler** helps us to count units easily.

Choose a unit of length and make your own ruler.

See Investigation.

Can you use your ruler to find the lengths of some objects?	Make a record of the objects and their lengths.

Discussing the Ideas

1. What is the length of $\overline{AB}$ shown above? 3 units

2. Barbara used the red-strip unit to measure $\overline{CD}$. Karen used the paper-clip unit and Elaine used the finger unit. Which girl got the largest number for the length of $\overline{CD}$? Elaine Who got the smallest number for the length? Karen

3. The **centimetre** is a common unit of length.
   - A What is the width of your desk to the nearest centimetre?
   - B What is the length of your desk to the nearest centimetre?

Answers will vary. See Discussion.

4. Can you name some other common units of length?
Foot, yard, metre, mile, kilometre, etc.

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Discussion

Use exercise 1 to discuss the rulers children made in the investigation. Ask for the length of $\overline{AB}$ in terms of the unit shown and in terms of the other units children chose to use for their rulers in the investigation. As you discuss exercises 1 and 2, help children generalize that the smaller the unit used in measuring length the greater will be the number given as the length. For exercise 3, give the children more practice in measuring "widths" and "lengths" in centimetres. They could measure objects and articles in the classroom: table tops, chalkboards, sheets of paper, etc. Let them read out the measurements.

Using the Ideas

1. Measure each of these segments to the nearest centimetre.

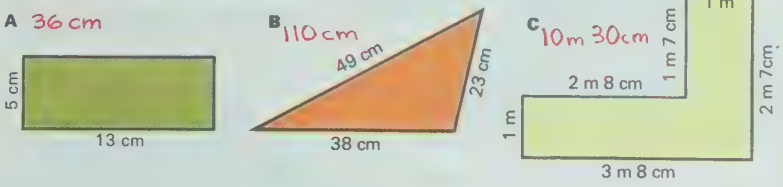
A 3 or 4 cm B 7cm

C 12 cm

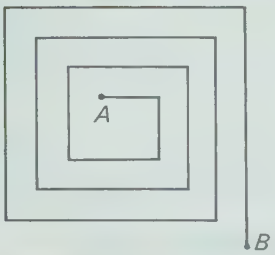
D 10 cm

2. Find the length of each segment above to the nearest half centimetre. A. $3\frac{1}{2}$ cm B. $7\frac{1}{2}$ cm C. $12\frac{1}{2}$ cm D. $10\frac{1}{2}$ cm

3. The **perimeter** of a figure is the sum of the lengths of its sides. Find the perimeter of each figure below.



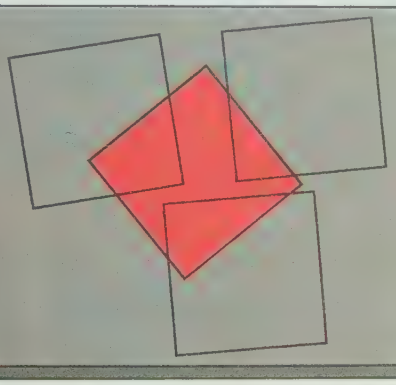
★ 4. Find the length of the path from *A* to *B* in centimetres. 35 cm



think

The figure shows three "clear" squares overlapping a red square of the same size. What is the largest number of "clear" squares that can overlap the red square if the "clear" squares cannot overlap? 7

See Using the Exercises.



More practice, page A-4, Set 7



Follow-up

To review the centimetric unit of measurement, ask the children to match pictures with the centimetre unit to measure the height, length, and width dimensions of the object pictured.

Resources for Active Learning

Applied Mathematics Cards, "Length," Group 3/4, Schofield and Sims. (Available from Mafex Associates)

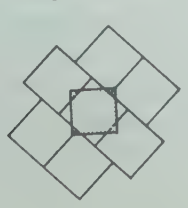
Mathex: Measurement No. 10, "Standard Units," pp. 2–6 (pupil pages 12–22), Encyclopaedia Britannica Publications Ltd.

Workbook, page 21

Using the Exercises

Before assigning the exercises on page 69 as independent work, you might review the meaning of perimeter. Be sure each child has a centimetre ruler.

For those who wish to try the *Think* problem, suggest that they cut out and use two-centimetre squares to see if they can find an answer. They may be surprised to find that 7 squares is the answer.



Assignments (page 69)

Minimum: 1–3. Average: 1–3. Maximum: 1–4.

PAGES 70–71

Objective

Given an angle, the child will be able to find its measure in degrees by using a protractor.

Preparation

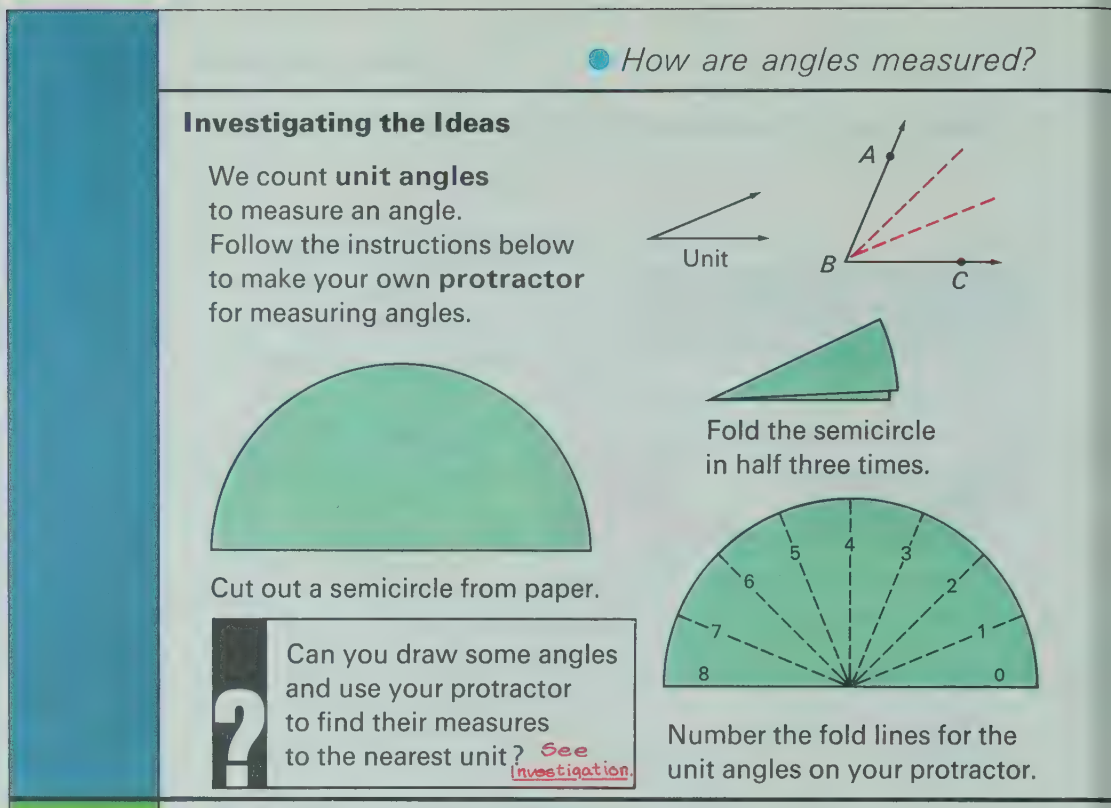
Materials

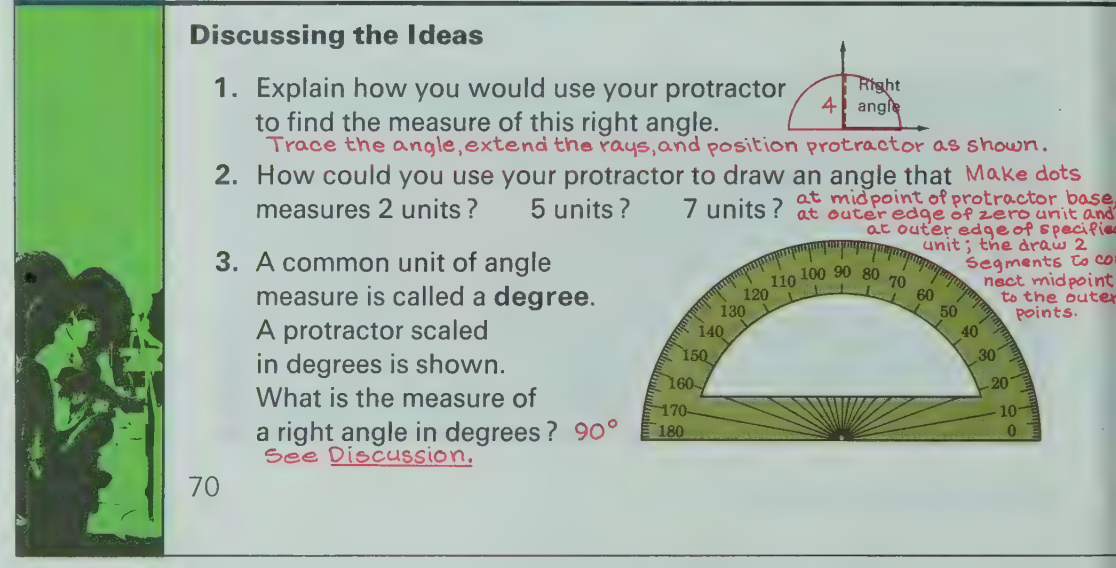
construction paper, scissors, and compass (or, *Duplicator Masters*, page 68); overhead projector, transparent protractor

Review the idea that a measuring instrument is made from marking some number of the chosen unit onto the device. Thus the ruler is marked with a number of centimetres. Point out to the children that in this investigation they will have an opportunity to make an instrument which will enable them to measure angles.

Investigation

Unless you provide the children with duplicated patterns, they may need directions for making the semicircle required in the investigation. If so, direct them to use their compass and make a circle that has a diameter about 8 or 9 centimetres. They should then fold this circle in half and cut out a semicircle. This semicircle in turn should be folded in half three times as indicated in the text. Move around the room as the children work, guiding those who need help. You might also distribute papers on which you have duplicated angles which the children can measure.

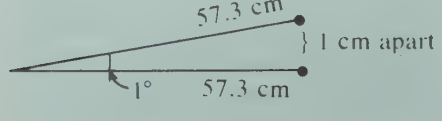




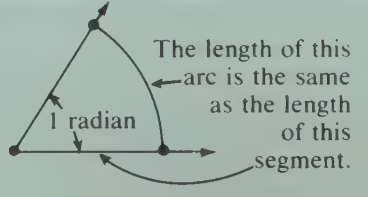
Discussion

During your discussion, introduce and carefully explain the fact that a common unit of angle measure is called a *degree*. Use the protractor in discussion exercise 3 to explain that a degree is $\frac{1}{360}$ of a circle. Thus, a right angle, which is the angle in a quarter of a circle, measures 90°.

To show the size of a degree, cut a string 114.6 cm long. Fasten it at its midpoint and ends so that its ends are 1 centimetre apart.

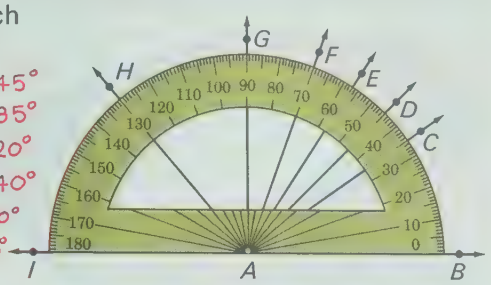


To show the use of a standard protractor, use an overhead projector with a transparent protractor as suggested for discussion of the exercises on page 70. Also explain that another unit of angle measure is the *radian*. This unit is such that the length of any arc is the same as the distance of the arc from the vertex of the angle. A radian is about 57°.



Using the Ideas

1. A protractor with degree units has been placed over some angles. $\angle BAC$ measures 35° (35 degrees). Give the measure of each angle below.



A	$\angle BAD$ 45°	G	$\angle DAG$ 45°
B	$\angle BAE$ 57°	H	$\angle DAH$ 85°
C	$\angle BAF$ 70°	I	$\angle FAG$ 20°
D	$\angle BAG$ 90°	J	$\angle GAH$ 40°
E	$\angle BAH$ 130°	K	$\angle GAI$ 90°
F	$\angle CAD$ 10°	L	$\angle FAI$ 110°

2. Use a protractor to find the degree measure of these angles.



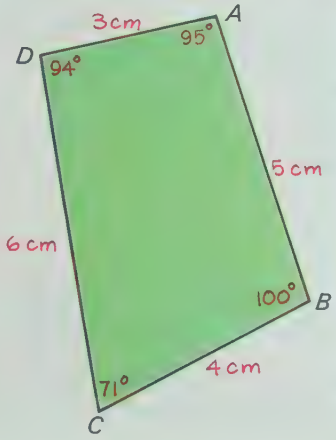
(also acceptable: 34° or 36°)

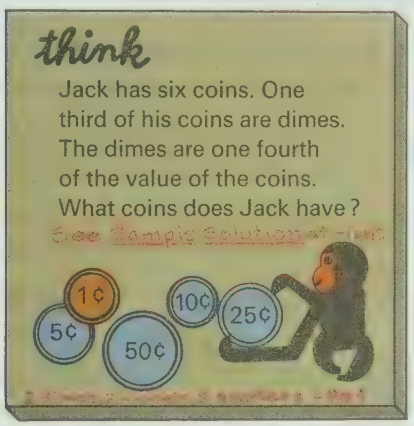
3. Draw angles that have the following measures.

A 25° B 72° C 105° D 143° E 14°

★ 4. Find the degree measure of each angle. Find the length of each segment in centimetres.

(Angle measures may vary slightly.)





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Follow-up

Suggest to more capable children that they make the constructions suggested below.

Angles of these measures:
30° 45° 60° 90° 120°

Triangles with the following angles:
90°, 45°, 45°
60°, 60°, 60°
90°, 60°, 30°
70°, 60°, 50°

Sample solution, Think, page 71

Expect reasoning similar to this from the children who solve the *Think* problem. "One third of the 6 coins are dimes, so 2 of Jack's coins are dimes. These 2 dimes are worth $\frac{1}{4}$ of the total of Jack's coins, so Jack must have 4×20, or 80 cents, altogether. The 2 dimes are worth 20 cents, so the other 4 coins are worth 60 cents. Therefore, the other 4 coins must be 2 quarters and 2 nickels."

Resources for Active Learning

Math Activity Cards, "Angles," E10, Macmillan.

Developmental Math Cards, "Measuring Angles," J-2, Addison-Wesley.

Duplicator Masters, page 10
Workbook, page 22

Using the Exercises

Children should use a protractor of standard measure for the exercises on page 71. Encourage independent work, but give assistance to those who need it. An overhead projector is an excellent aid for teaching children how to use a standard protractor. In particular, point out that after they have aligned the middle point of the protractor at the vertex of the angle and the edge of the protractor with one ray, they must read from the scale where zero meets this ray.

Assignments (page 71)
Minimum: 1–3. Average: 1–3.
Maximum: 1–4.

PAGES 72–73

Objectives

Given isosceles, equilateral, *and* right triangles, acute angles, *and* obtuse angles, *the child will be able to identify each figure. He will also be able to find sums of the measures of angles of polygons and discover that the sum of the measures of the angles of any triangle is 180°.*

Preparation

Materials

compass; tracing paper; paste; scissors

If necessary, review the correct use of the protractor by showing a few brief examples of angle measure on the overhead projector or on the chalkboard. Otherwise, begin immediately with the investigation.

Investigation

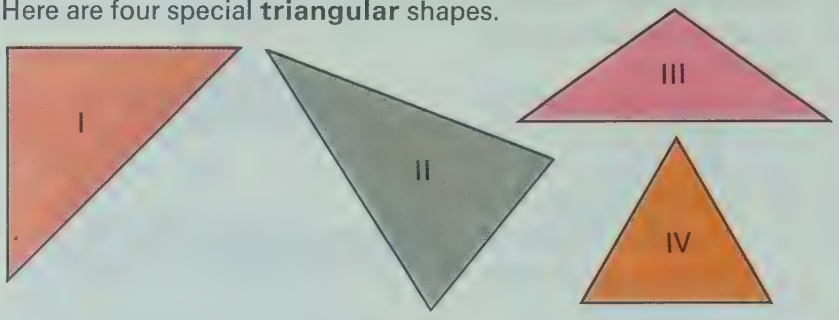
Make sure the children realize that they must cut out more than one of each triangle. For example, they need a total of 6 I's, 4 III's, and so on.

One of the main purposes of this investigation is to review the various polygons. Suggest to the children that they label each polygon after they have put it together. You might refer them to the glossary in the back of the text (pages A27–A30) for a review of the definition of each term.

● *Let's explore polygons.*

Investigating the Ideas

Here are four special **triangular** shapes.



Here are ways to use these shapes to make other **polygons.**

Two I's → **square** □ Two II's → **parallelogram** ▱
Two III's → **rhombus** ◇ Four I's → **rectangle** ▭
One II, two III's → **pentagon** ⬠ Six IV's → **hexagon** ⬡

? Can you trace, cut out, and paste the triangular shapes to form the polygons? See Investigation.

Discussing the Ideas

1. An **isosceles** triangle has at least two congruent sides. Which triangles above are isosceles? All
2. An **equilateral** triangle has all sides congruent. Which triangle above is equilateral? IV
3. A **right triangle** has a 90° angle. Which triangle is a right triangle? I
4. An **acute** angle is less than 90°. An **obtuse** angle is greater than 90°. Which triangles have acute angles? All obtuse angles? III
5. Describe each polygon you made. Can you think of other ways to use the triangles to make the polygons?
   Example: Two II triangles and two III triangles can be used to form a parallelogram.

72

Discussion

Throughout this discussion, you will want to stress the meaning of terms presented and relate them to the triangles used in the investigation. Have children use a compass or ruler to find congruent sides, and a protractor to measure the angles. Triangle I is an isosceles right triangle, that is, one of its angles is a right angle and two of its sides have the same length. Triangles II and III are isosceles triangles, so each has two equal angles opposite the congruent sides. Triangle IV is an equilateral triangle, which implies also that all of its angles have the same measure.

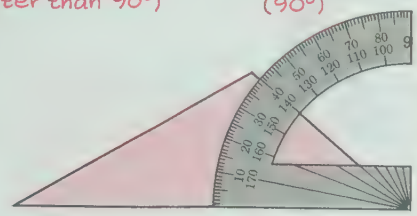
In discussing exercise 4, ask children to find the measure of the angles of each triangle, and also ask them to find the sum of the measures of the three angles in each. Although allowance for error in measurement must be made, this sum should be 180°. When you discuss this, be sure the children understand that when we speak of the "sum of two angles" we mean the sum of the degree measures of the two angles. It is not the angles that are being added here but their degree measures.

Let children experiment with using the triangles to form other polygons.

Using the Ideas

Constructions will vary for exercises 1–4.

1. Use your compass to help you draw each of the following.
   - A isosceles triangle (At least 2 congruent sides)
   - B equilateral triangle (3 congruent sides)
   - C right triangle (One 90° angle)
2. Use your protractor to help you draw:
   - A an acute angle (Less than 90°)
   - B an obtuse angle (Greater than 90°)
   - C a right angle (90°)
3. A Draw a triangle large enough so that you can measure each angle easily. What is the sum of the measures of the three angles? 180°



   B Try this again with a different triangle. What do you think might be true about the sum of the three angle measures for any triangle? It will be 180°.
4. Draw a four-sided polygon (**quadrilateral**) on your paper. Make it large enough so that you can measure the angles easily. What is the sum of the four angle measures? 360°

think

Trace this tangram square and cut it into the pieces shown.



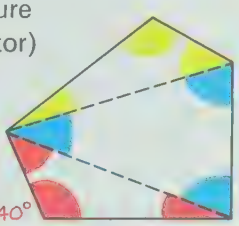
How many of the polygons named in the Investigation on page 72 can you make with all 7 tangram pieces? All of them.

★5. A What is the sum of the angles of each triangle? 180°



   B What is the sum of the angles of the quadrilateral? 360°

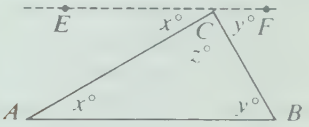
★6. Use this figure (no protractor) to find the sum of the angles of this pentagon. 540°





Mathematics

The fact that the sum of the measures of any triangle is 180° is easy to prove by applying some elementary theorems of geometry. Suppose ΔABC is any triangle and that the measures of angles A, B, and C are x, y, and z degrees respectively.



Let $\overline{EF}$ be a line through C parallel to $\overline{AB}$. Then the measure of $\angle ECA$ is $x°$ and the measure of $\angle FCB$ is $y°$. But the measure of the straight angle ECF is clearly 180°, hence $x° + y° + z° = 180°$, which is the sum of the measures of the angles of ΔABC.

Every convex polygon of n sides can be divided into $n - 2$ triangles by constructing segments from one of the vertices. It then follows that the sum of the measures of the angles of any convex polygon is $(n - 2) \times 180°$.

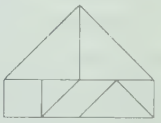
Solutions, Think, page 73

These diagrams are provided only for your benefit. The solution for the square is shown in *Think*.

Rectangle



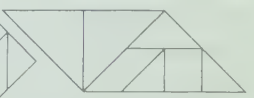
Pentagon



Hexagon



Parallelogram



Using the Exercises

Remind children to refer to the definitions given on page 72 as they do the exercises on page 73. When children have worked on exercise 4, you might discuss the two triangles and the quadrilateral together with the fact that the six angles of the two triangles make up the four angles of the quadrilateral. When the children see this, they will probably be able to say that the sum of the angles of the quadrilateral is the same as the sum of the angles of the two triangles, or 360 degrees.

Give interested children an opportunity to discuss exercise 6, which requires that they recognize the three triangles and the fact that the five angles of the pentagon include all nine angles of the three triangles. Recognizing these facts will lead them naturally to realize that the sum of the degree measures of the pentagon must be three times 180°.

The activity suggested in the *Think* problem may be extended over a few days. It is possible to make all of the polygons on page 72.

Assignments (page 73)

Minimum: 1–4. Average: 1–4. Maximum: 1–6.

Resources for Active Learning

Applied Mathematics Cards, Group 2/21–22; Group 3/18–19, Schofield and Sims. (Available from Mafex Associates)

Experiments in Mathematics, Stage 1, "Paper Folding," pp. 18–19; "Triangles . . .," pp. 20–21, Houghton Mifflin. (Available from Thomas Nelson & Sons Ltd.)

Mathex: Geometry No. 9, "Angles and Triangles," pp. 25–32 (pupil pages 35–38), Encyclopedia Britannica Publications Ltd.

Nuffield Project: *Shape and Size* 3, ". . . 2-D frameworks," pp. 2–8, Wiley.

PAGES 74–75

Objective

Given a symmetrical figure, the child will be able to identify at least one line of symmetry by folding the figure.

Preparation

Materials

tracing paper; scissors

Conduct a short review of some of the topics covered so far in the chapter (congruent figures, angle measure, uses of the compass and the protractor). You might also go over the names of common polygons, such as those which will be explored in this lesson. For example, hold up cutouts of various polygons and ask children to identify each. Skill in using the names of the polygons will facilitate discussion later in the lesson.

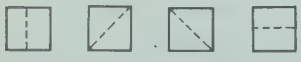
Investigation

Stress with the children the fact that accuracy in tracing and cutting is particularly important here since they will be investigating symmetric halves of the figures. Also, remind them to record for each polygon the ways in which they were able to fold it in half. Point out that there are other ways to fold these figures in half, but only those shown yield symmetric halves.

Equilateral triangle



Square



Regular pentagon



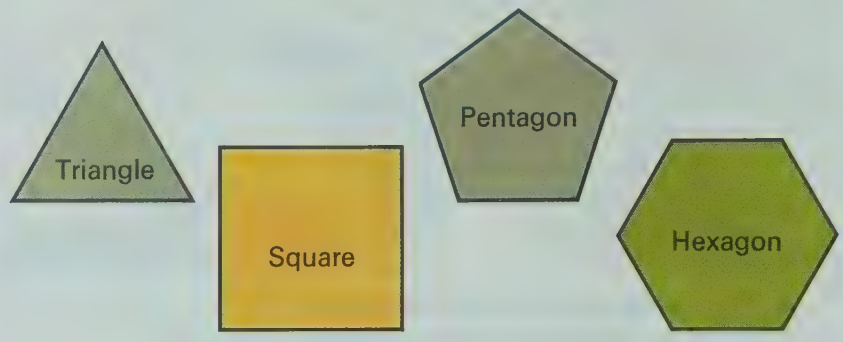
Regular hexagon



How many lines of symmetry does a polygon have?

Investigating the Ideas

Trace and cut out each of these regular polygon shapes.



? In how many ways can you fold each figure so one half fits exactly upon the other half?

See Investigation.

Discussing the Ideas

1. A figure has a **line of symmetry** if the fold on that line makes two halves of the figure match exactly. How many lines of symmetry does each figure above have?
   See Investigation.
2. A Can you name some other types of triangles and tell how many lines of symmetry each figure has? Isosceles - 1 Scalene - 0
   B Can you name some other quadrilaterals and tell how many lines of symmetry each one has?
   Rectangle - 2; rhombus - 2; many other quadrilaterals - 0
3. A **regular polygon** has all sides congruent and all angles congruent. If you know how many sides a regular polygon has, can you tell how many lines of symmetry the polygon has?
   Yes; see Discussion.
4. Figures with at least one line of symmetry are called **symmetrical**. Can you describe a familiar symmetrical figure?
   Answers will vary. Samples: kite, rectangular sheet of paper, etc.

74



Discussion

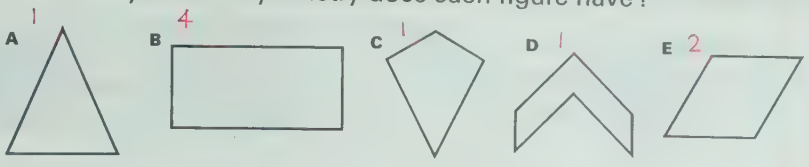
It would be helpful to use large demonstration polygons made from construction paper or other suitable material so that children can point out the lines of symmetry for each polygon. As you discuss the lines of symmetry, ask the children if they can recognize something about the sides of each polygon (each figure has congruent sides). Also point out that each line of symmetry *bisects*, or divides into two equal parts, an angle or a side.

As you discuss exercise 2, help the children realize that an isosceles triangle has a line of symmetry, but many quadrilaterals have none.

Since all of the figures children studied in the investigation were regular polygons, refer to them as you discuss exercise 3.

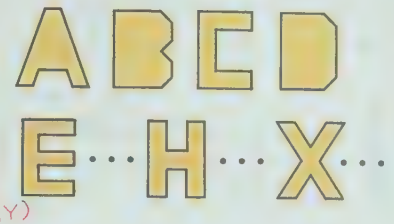
Using the Ideas

1. How many lines of symmetry does each figure have?



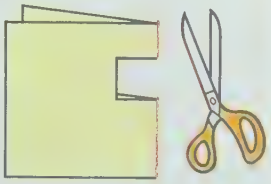
isosceles triangle | rectangle | "kite" | "chevron" | rhombus

2. Think about special polygons that are the outlines of the 26 letters of the alphabet. How many of these shapes have one or more lines of symmetry? 16 (A,B,C,D,E,H,I,K,M,O,T,U,V,W,X,Y)



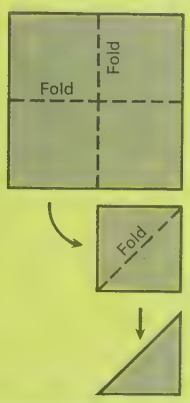
See Using the Exercises.

3. Suppose you fold a piece of paper and cut out the figures shown below. Draw and name the figure that is formed when you unfold each piece.



think

Cut out a square and fold it twice to form a small square. Fold the small square to form a triangle. Can you cut off one corner of the triangle so that it unfolds to form a square?



A 
Cut out a square. Rectangle

B 
Cut out an isosceles right triangle. Square

C 
Cut out an isosceles trapezoid. Hexagon

D 
Cut out this quadrilateral. Pentagon

More practice, page A-4, Set 8



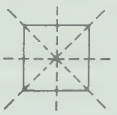
Mathematics

Line symmetry is also called *reflection symmetry*. When a mirror is placed along a line of symmetry, the reflected image falls directly on the other half of the figure.

Another type of symmetry is *rotational symmetry*. This type of symmetry is illustrated below.



If we imagine rotating the figure a half turn with the centre of rotation at point O, the figure would look exactly the same as before the rotation. Observe that a square, as well as many other figures, has both reflection and rotational symmetry.



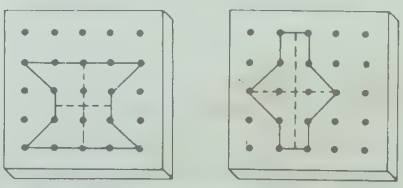
Another kind of symmetry is *translational symmetry*. This kind of symmetry exists for certain repeating shapes which extend indefinitely in all directions in a plane. The pattern of repeating congruent parallelograms shown below exhibits translational symmetry.



If each point of the pattern is "slid" or translated in the same distance and direction as shown by the arrow, the pattern will appear to be exactly the same as before the translation.

Follow-up

If geoboards are available, you might have children try to form symmetrical figures on the geoboard.



Using the Exercises

You might suggest that children do some of the exercises on page 75 together and share their results. As they work, point out correct use of the vocabulary.

A figure is said to be symmetrical if it has at least one line of symmetry. Such a figure, when drawn on paper, can be folded so that one half of the figure will fit exactly upon the other half. You might point out the term *isosceles trapezoid* and explain that it refers to a figure that has one pair of opposite sides parallel and a pair of non-parallel congruent sides.

Note that children's answers for exercise 2 may vary because certain letters that have lines of symmetry in some type-face styles may be asymmetric in other styles.

Assignments (page 75)

Minimum: 1, 3. Average: 1–3. Maximum: 1–3.

Resources for Active Learning

Inquiry in Mathematics via the Geo-board, Geo-cards 10/1–9, Walker. (Available from Fitzhenry & Whiteside Ltd.)

Nuffield Project: *Shape and Size* 3, pp. 48–54, 60–63. Wiley.

PAGES 76–77

Objective

Given an appropriate polygon, the child will be able to use the polygon to tessellate a plane.

Preparation

Materials

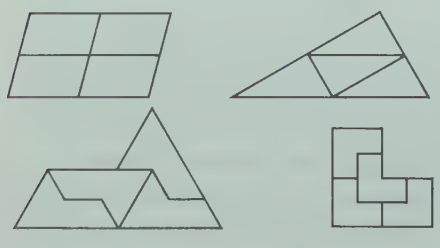
tracing paper; heavy paper for constructions (optional); scissors

To prepare for this lesson, simply remind the children that a regular polygon has congruent sides and congruent angles. For example, a rectangle that is not a square is a polygon with congruent angles, but it is not a regular polygon. Some of the figures most suitable for tessellations are the regular polygons.

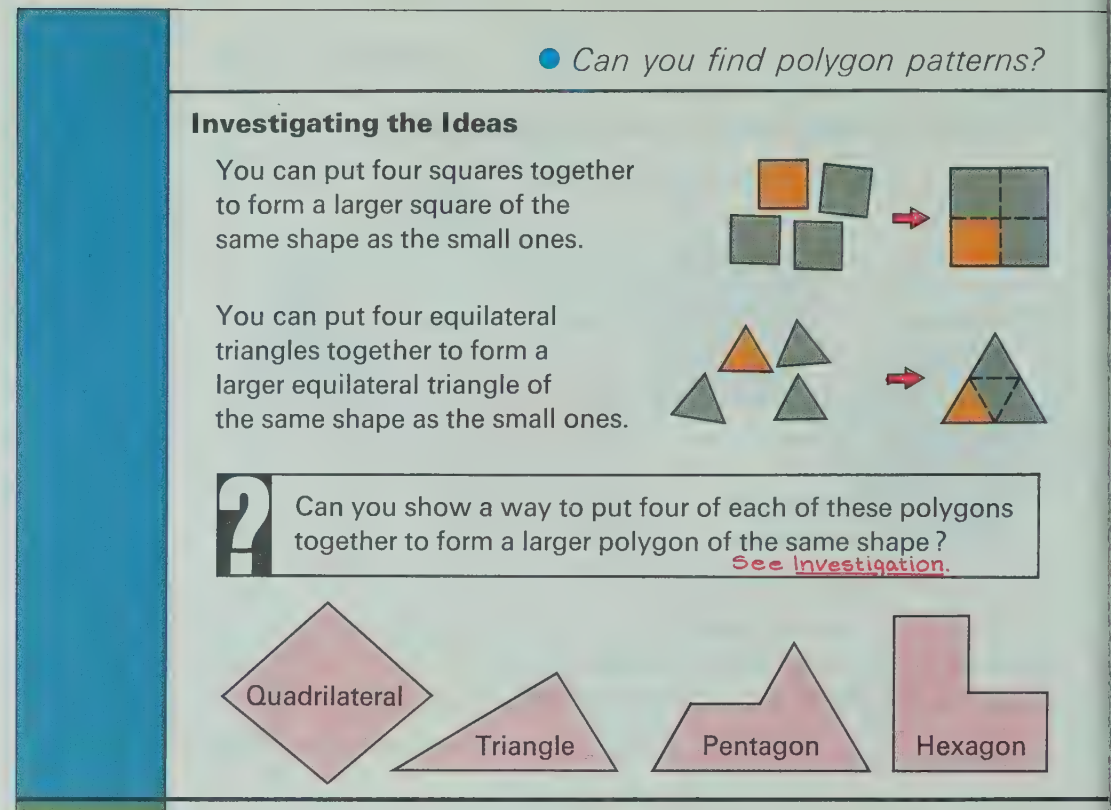
Investigation

Since children would find it easier to work with polygons made of sturdy paper such as tagboard, you might show them how to use the tracing paper to transfer the shape to the stiffer paper. Have them trace and mark the corners of the figure on the tracing paper. Then, a sharp pencil may be used to make a hole in the tracing paper at each of these points through which a pencil mark can be made onto the stiffer paper. Finally, these marks can be connected with pencil and ruler, to form the shape on the cardboard.

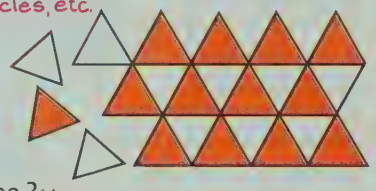
Allow children freedom in sharing ideas about an investigation such as this, but encourage independent thinking. The speed with which a few children find the configurations should not discourage the others. Help them to realize that four of each of the new shapes they form can again be used to make a similar but larger figure.



Some children might want to continue building a particular shape until materials are no longer practical.







Discussing the Ideas

1. Two geometric figures that have the same shape are said to be **similar** to each other. Give examples of similar figures.
   Two rectangles, two squares, two circles, etc.
2. Here is a pattern formed by fitting together equilateral triangles.
   - A Imagine that you had as many triangles as you needed. Could you completely cover any surface? Yes
   - B Patterns which can "cover any surface" are called **tessellations**. Explain why we can be sure that each polygon above could make a tessellation. Sample answer: "Because they fit together to form larger shapes with no 'holes' in them."

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Discussion

Discussion exercise 1 contains an intuitive description of similar figures. In this discussion, there is no need for a more mathematical definition. Expect the children to use the word "similar" with an intuitive meaning, such as "having the same shape." It would be helpful to point out that congruent figures have the same size and shape; similar figures have the same shape, not necessarily the same size.

Since children find tessellating (or tiling) the plane an enjoyable activity, allow them sufficient time to explore tessellations made with the equilateral triangles. You might allow them to experiment with designs which cover the plane and require more than one shape. If a tessellation is made with a regular polygon, it is called a regular tessellation. Regular tessellations are possible with the equilateral triangle, the squares, and the regular hexagon. Since the polygons in the investigation can cover a plane surface without any holes, they can be used to tessellate the plane, although the tessellation formed is not a regular tessellation.

Using the Ideas

1. Make a cardboard cut out of this regular hexagon. Then you can draw around it to make pictures of hexagons. Can you use regular hexagons to make a tessellation? Yes

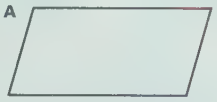
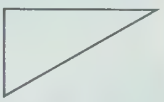


2. You have seen that you can make tessellations with at least three regular polygons. Which ones? Equilateral triangles, squares, hexagons

3. Complete exercise 1 by using this pentagon. Do you think tessellations can be made with any of the other regular polygons? No

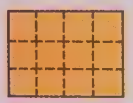


★ 4. Can you make and color two different tessellations with each of these figures? See Sample answers, T.E. page 77.

A  B 

think

If you use twelve squares for each rectangle, you can form rectangles of three different shapes.



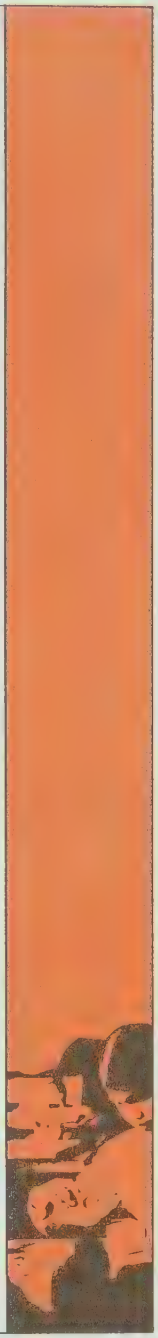






What is the smallest number of squares you can use to form rectangles of four different shapes? 24 (1×24, 2×12, 3×8, 4×6)

77



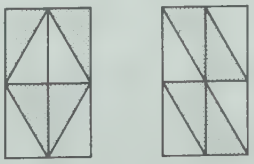
Using the Exercises

Assign the exercises on page 77 to coincide with any other tessellation activities in which the children are interested. Exercise 3 is designed to help children discover that the regular pentagon will not tessellate a plane. The only regular tessellations possible are by means of the equilateral triangle, the square, and the regular hexagon. If more than one regular polygon is used, other tessellations are possible. For example, the plane can be completely covered or tiled with regular octagons and squares.

Encourage children to try the *Think* problem. Help them to realize that a trial-and-error method is often a very satisfactory way of solving such problems.

Sample answers, Exercise 4

A 

B 

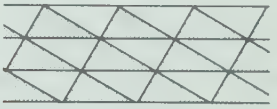
Assignments (page 77)

Minimum: 1, 2. Average: 1–3. Maximum: 1–4.

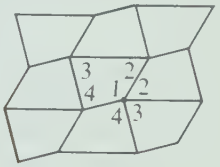
Mathematics

Tessellations of the plane are also called by the more intuitive term "tilings of the plane."

It is possible to tessellate, or tile, a plane with any triangle. The proof of this statement uses the fact that the sum of the measures of the angles of any triangle is 180°. It is



also possible to tessellate the plane with any quadrilateral. All one needs to do is to be certain that all four angles of each quadrilateral have a common vertex at the point where the quadrilaterals meet.



In general, irregular polygons with more than 4 sides do not tessellate, but one can find many special shapes which have more than 4 sides which do tessellate the plane.

Regular tessellations are tessellations formed by using one of the regular polygons. Regular tessellations can be made only from equilateral triangles, squares, and hexagons.

Follow-up

Encourage children to make various tessellations, coloring their designs as well.

Resources for Active Learning

Experiments in Mathematics, Stage 1, pp. 14–15; Stage 2, pp. 8–9, Houghton Mifflin. (Available from Thomas Nelson & Sons Ltd.)

Geometry in the Classroom, pp. 75, 149–161, Holt, Rinehart and Winston of Canada.

Mathex: Geometry No. 9, pp. 53–56, Encyclopaedia Britannica Publications Ltd.

Notes on Mathematics in Primary Schools, pp. 131–142, Cambridge University Press. (Available from Macmillan)

Nuffield Project: *Shape and Size* 3, pp. 27–28, 32–40, Wiley.

Workbook, page 23

PAGES 78–79

Objectives

Given a polyhedron, the child will be able to recognize whether or not it is a regular polyhedron.

Given patterns for some polyhedrons, the child will be able to construct models of the polyhedra.

Preparation

Materials (optional)

patterns of regular polyhedrons to be run off on Manila or other heavy paper (*Duplicator Masters*, pages 70, 72)

models of regular polyhedrons

If you have models of regular polyhedrons, have the children examine them. Ask if they think it would be difficult to make them. It would also be suitable for the children to construct one of the regular polyhedrons from duplicated patterns. If each child constructs one of the polyhedrons, you would have a sufficient number of each to enable children to examine all of the five figures.

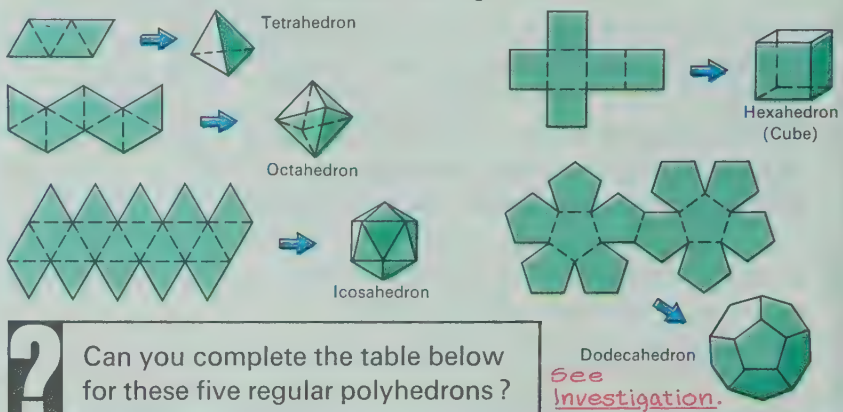
Investigation

Whether or not you have actual models of the polyhedrons for the children to examine, the pictorial representations should be clear enough for children to interpret. The number of faces of each polyhedron can be seen clearly from the patterns. However, they may have difficulty counting the number of vertices and edges, even if they have models to examine. Caution children against counting some edges or vertices twice; if they realize that the pictures of the dodecahedron and the icosahedron show exactly one half of the figure, they might find it easier to count the edges and vertices from the picture. If some children feel that their counting is not reliable, and you have no models or duplicated patterns available, suggest that they enlarge the pattern themselves. Since each pattern uses only one regular polygon, the child can simply trace the equilateral triangle or the regular pentagon from the previous lesson and build his own model.

Let's look at space figures made from regular polygons.

Investigating the Ideas

The ancient Greeks discovered that there are only five space figures with faces that are congruent regular polygons. Here is how models of these solids might be made.



? Can you complete the table below for these five regular polyhedrons?

See Investigation and Discussion.

Name of the polyhedron	Name of the face polygon	Number of faces	Number of vertices	Number of edges

Discussing the Ideas

1. How can you easily find the number of faces that each of the five regular polyhedrons has?
2. If "poly-" means "many," "tetra-" means "four," "hexa-" means "six," and "octa-" means "eight," then what do you think "dodeca-" and "icosa-" mean? "twelve" "twenty"
3. For each polyhedron, find the sum of the number of faces and the number of vertices. How does this sum compare with the number of edges? Can you write a formula about what you have found? See Discussion.

78



Discussion

Encourage children to discuss how they counted faces, edges, and vertices of each polyhedron and the difficulties they encountered. Point out that the pictures of the patterns may be used to count the number of faces. Relate the name of each polyhedron to the number of faces it has.

In exercise 3, refer children to the chart they made in the investigation. (See table in the next column.)

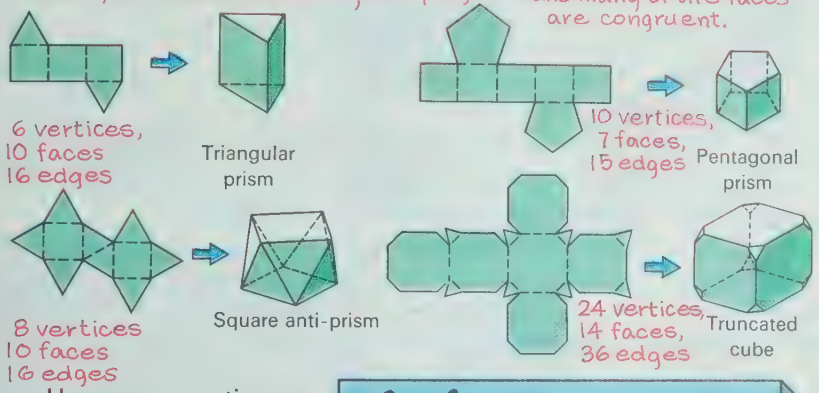
Lead children to the generalization that the sum of the faces and vertices is equal to the number of edges plus 2 ($F + V = E + 2$).

Polyhedron Name of Face	No. of Faces	No. of Vertices	No. of Edges
Tetrahedron Equilateral triangle	4	4	6
Hexahedron Square	6	8	12
Octahedron Equilateral triangle	8	6	12
Dodecahedron Regular pentagon	12	20	30
Isocahedron Equilateral triangle	20	12	30

Using the Ideas

1. A Draw a pattern that could be used to make a tetrahedron. Make it different from the pattern on page 78. Sample:
   B Draw another pattern for a cube. Sample:

2. Each face of a regular polyhedron is a regular polygon. Each face of a regular polyhedron is congruent to every other face. Here are some polyhedrons.
   A Why are they not regular polyhedrons? Faces are not all congruent.
   B How are they like regular polyhedrons? Many of the faces are regular polygons and many of the faces are congruent.

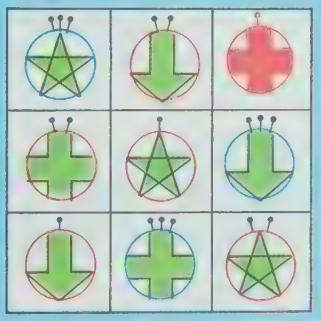


3. A How many vertices, faces, and edges does each polyhedron in exercise 2 have? See above.
   B If a polyhedron had 7 faces and 7 vertices, how many edges would it have? 12

★ 4. Make a model of at least one of the polyhedrons in exercise 2. Models will vary

think

Draw the missing figure.



79

Using the Exercises

Assign the exercises on page 79 and correlate them with other polyhedron activities. For example, you might have the children build straw polyhedrons. Straw polyhedrons may be constructed by using straws and connecting them at the vertices with pipe cleaners. Play dough or clay may also be used to connect the straws at the vertices. If straws are not available, simply roll a magazine page around a pencil to make a substitute straw. Pipe cleaners and tape are also suitable. If you prefer, encourage the children to do starred exercise 4 so that each child chooses a figure, enlarges the pattern, and then constructs it.

Assignments (page 79)*
Minimum: 1–3. Average: 1–3.
Maximum: 1–4.

Mathematics

The five regular polyhedrons were known to the early Greek mathematicians; they were called the Platonic solids in honor of Plato. These are the only regular polyhedrons that exist.

Another class of polyhedrons are the semiregular polyhedrons, or Archimedean solids. In these polyhedrons, the faces consist of two or more regular polygons. An example of such a polyhedron is the truncated cube illustrated in the student text on page 79. There are 13 Archimedean solids. Some students may be interested in doing some research on their names and making models of some of the simpler ones.

The most spectacular and beautiful polyhedrons are the stellated or star polyhedrons. Patterns for these polyhedrons may be found in reference books.

Follow-up

Innumerable construction activities might serve as extensions of this lesson. You might, for example, want the children to construct polyhedrons and use them to make mobiles. Some capable children might be challenged by the problem of determining the surface areas of the polyhedrons in relation to their weights so that they can figure out how to arrange the polyhedrons in such a way that the mobile will balance.

Children interested in science might do some research to see if the regular polyhedrons occur in nature.

You might also challenge some children to find the most rigid or sturdiest polyhedron. For example, ask them to construct the sturdiest bridge they can, using one of the polyhedrons. (Just as the triangle is the most rigid plane figure, the regular tetrahedron is the basic rigid polyhedron.)

Resources for Active Learning

Geometry in the Classroom, pp. 15–19, 45, 46, 63–75, Holt, Rinehart and Winston of Canada.
Mathematics in Modules, SK9, Addison-Wesley.

PAGES 80–81

Objective

Given a picture of a plane intersecting a space figure, the child will be able to name or draw the cross section formed by the plane and space figure.

Preparation

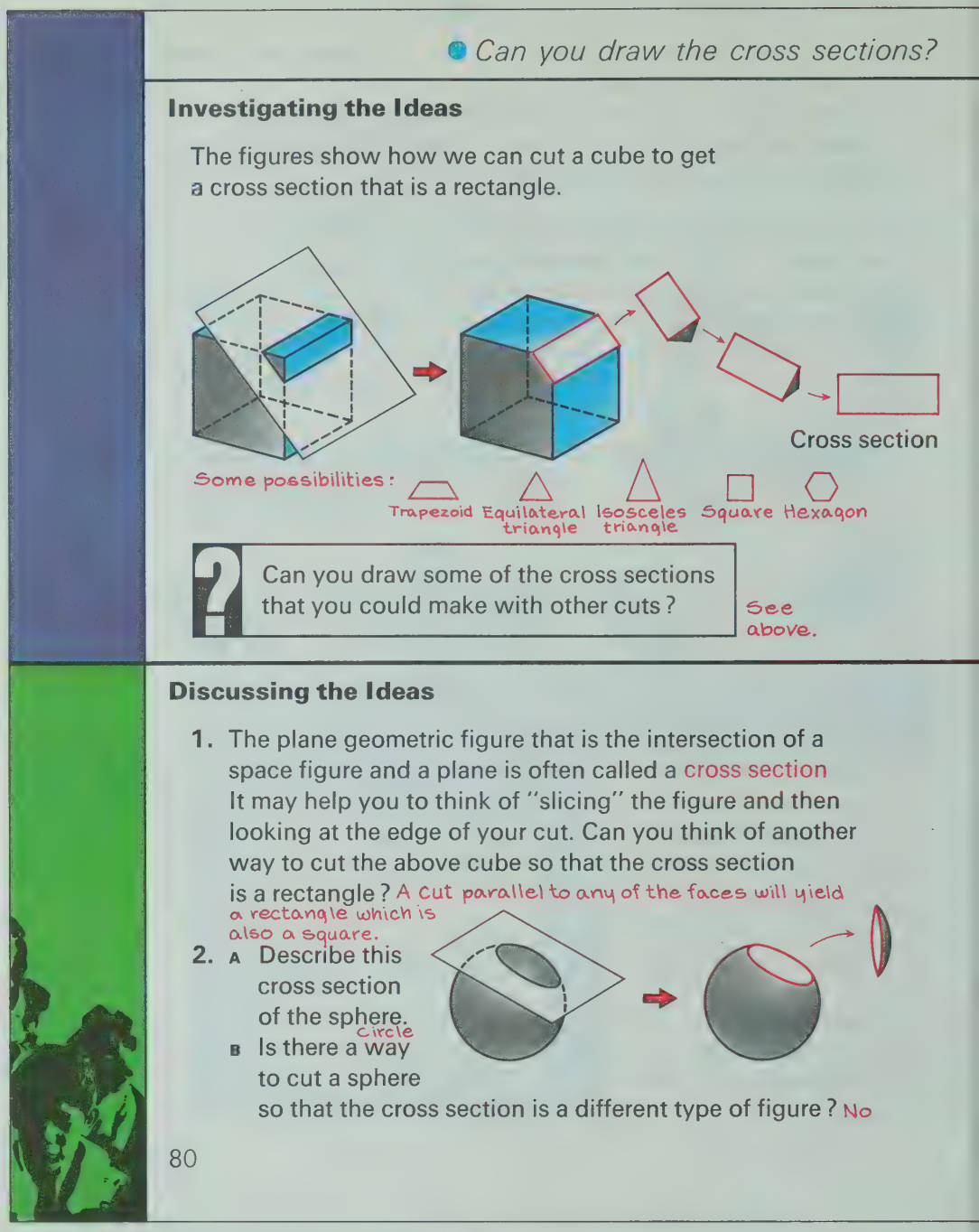
Materials

models of geometric solids or modelling clay to construct substitutes

Since cross sections are thought of as the intersection of a space figure and a plane, you might want to review the idea of a plane. For example, refer to the planes suggested by the floor and walls in the classroom. Point out that the only kind of figures which are contained in a plane are two-dimensional figures such as the circle, triangle, square, and other polygons. The space figures or polyhedra which they studied in the previous lesson cannot be contained in a single plane; they are three-dimensional figures.

Investigation

Although it is intended that the children picture the cross sections from studying the pictorial representations in their text, you might provide cubes made of modelling clay or plastic foam. Children who find it necessary may then actually cut a cube to find the outline of the cross section. If children have difficulty making their drawings, refer them to page 61, exercise 1A, for help in getting started. Do not require or expect the children to make accurate drawings. If any children become frustrated in their efforts, simply ask them to explain with one of the model cubes how they would cut it and then encourage them to draw only the outline of the cross section, that is, the plane figure resulting from the cut.



Can you draw the cross sections?

Investigating the Ideas

The figures show how we can cut a cube to get a cross section that is a rectangle.

Can you draw some of the cross sections that you could make with other cuts? See above.

Discussing the Ideas

1. The plane geometric figure that is the intersection of a space figure and a plane is often called a cross section. It may help you to think of "slicing" the figure and then looking at the edge of your cut. Can you think of another way to cut the above cube so that the cross section is a rectangle? A cut parallel to any of the faces will yield a rectangle which is also a square.

2. A Describe this cross section of the sphere. circle

   B Is there a way to cut a sphere so that the cross section is a different type of figure? No

80

Discussion

Use as many actual models as are available, to help children picture the cross section cuts. To help children think of a plane cutting a solid figure, speak of such physical objects as a knife and suggest that they think of using a knife to cut off the cross section.

Remind the children that the pictures are an attempt to represent, on a two-dimensional surface, the ideas concerning three-dimensional geometry. One of the purposes of this kind of lesson is to give the children an opportunity to think about and try to visualize the cross sections without actually seeing the space figure cut. Thus, if you do use actual space figures in connection with this lesson, you should give the children an opportunity to work with the drawings and the pictures, in addition to actually seeing the cross sections.



Using the Exercises

Have the children attempt the exercises on page 81. Notice that exercises 7, 8, and 9 are intended primarily for the more able children. For some, you may need to observe that the drawing in exercise 9 represents a cube being cut by a plane in a certain peculiar way. It is interesting to note that the cross section in exercise 9 is a regular hexagon.

When the children have finished the exercises, allow time for discussion and checking of papers. At this time, if you have actual space figures that are cut like those depicted, it will be stimulating to demonstrate them to the children and check them with the cross sections that the children have drawn on their papers.

Assignments (page 81)*

Minimum: 1–6. Average: 1–6.
Maximum: 1–9.

Follow-up

Many commercial packages are space figures with some portion cut off. It may be interesting to follow up this lesson by showing the children such packages and having them relate these to various ideas presented here. Some of the children may wish to collect samples and label them for a classroom display.

Resources for Active Learning

Franklin Series: *Making and Using Graphs and Nomographs*, pp. 75–78, Lyons and Carnahan [A parabola and multiplication]. (Available from McGraw-Hill Ryerson)

Experiments in Mathematics, Stage 3, "Sections through a cube," pp. 32-33, Houghton Mifflin. (Available from Thomas Nelson & Sons Ltd.)

PAGES 82–83

Objectives

The child will demonstrate his ability to work with the concepts presented in this chapter.

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

Materials

centimetre ruler; protractor

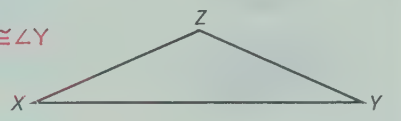
Review any of the material in the chapter which children found particularly difficult. For example, review the method of measuring angles by using a protractor. Or, review the basic concepts and symbols treated in the first lesson.

Reviewing the Ideas

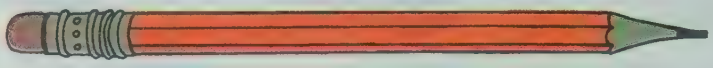
1. Which pair of segments are congruent? $\overline{AB} \cong \overline{EF}$



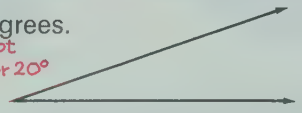
2. An isosceles triangle has a pair of congruent angles. $\triangle XYZ$ is isosceles.
   - A Which two angles are congruent? $\angle X \cong \angle Y$
   - B Which pair of sides are congruent? $\overline{XZ} \cong \overline{ZY}$



3. Find the length of the pencil
   - A to the nearest centimetre. 12 cm
   - B to the nearest half centimetre. $11\frac{1}{2}$ cm

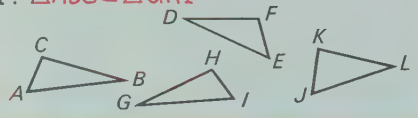


4. A Find the measure of this angle in degrees.
   B Construct an angle congruent to this angle. Accept 18°, 19°, or 20°

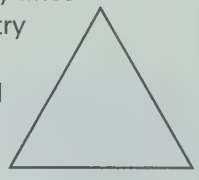


5. Is the sum of the angles of a triangle 90°, 100°, 180°, or 200°? 180°

6. A Which two triangles are congruent? $\triangle ABC \cong \triangle GHI$
   B Complete each statement.
   $\angle A \cong \angle I$ $\angle B \cong \angle G$ $\angle C \cong \angle H$
   $\overline{AB} \cong \overline{GI}$ $\overline{BC} \cong \overline{GH}$ $\overline{AC} \cong \overline{HI}$



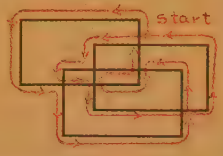
7. A How many lines of symmetry does an equilateral triangle have? 3



   B Can you tessellate a plane with an equilateral triangle? Yes

think

Draw a figure like the one below. Now trace over the entire figure by drawing a path that does not cross itself or retrace itself.



82

Discussion

Page 82 may be used as an evaluative instrument, or you might work through the exercises together as a class review. One of the important points to stress is the concept of congruence. Help children realize that we may speak of the length of two segments as being equal but we speak of such segments as being congruent. Since length is a number, we can speak of equality with regard to length, but when we refer to the geometric figures themselves, we speak of congruence. Note that although the chapter review may be used as an evaluative instrument, its purpose is not to test the child's retention of geometric concepts and vocabulary. Rather, it should indicate to you the child's development of intuition about the material presented and of basic skill in handling a compass and ruler.

Page 83 may be assigned as an independent review. However, if you prefer, you might use it as the basis of a review lesson on expanded notation, inverse relations, and multiples of ten. In any case, be sure to give children an opportunity to check their work and discuss any troublesome exercises.

Keeping in Touch with

Addition
Subtraction
Multiplication
Division
Place value
Principles

1. Give the number for n.

A $10\,000 = 10^n$ 4
B $100\,000\,000 = 10^n$ 8
C $1000 \times 100\,000 = 10^n$ 8
D $4000 \times 60\,000 = 24 \times 10^n$ 7
E $30\,000 \times 90\,000 = 27 \times 10^n$ 8

think

Take 18 toothpicks (or small sticks) and arrange them to form 9 triangles of the same size.



See Solution, T.E. page 83.

2. Solve the equations.

A $a + 7 = 15$ 8
B $14 - n = 8$ 6
C $5 \times 9 = r$ 45
D $c \times 8 = 48$ 6
E $s - 6 = 7$ 13
F $12 \div y = 6$ 2
G $42 \div 7 = d$ 6
H $f + 9 = 18$ 9
I $7 \times x = 49$ 7
J $8 + 9 = a$ 17
K $q \div 3 = 7$ 21
L $10 - b = 0$ 10

3. Solve the equations.

A $5 \times 27 = (5 \times 20) + (5 \times n)$ 7
B $6 \times 284 = (6 \times 200) + (6 \times n) + (6 \times 4)$ 80

4. Find the products and quotients.

A 10×100 1000
B 20×700 14 000
C 50×60 3000
D 800×40 32 000
E 56×1000 56 000
F 200×300 60 000
G 5000×200 1 000 000
H 760×20 15 200
I $480 \div 60$ 8
J $4800 \div 60$ 80
K $4800 \div 600$ 8
L $480 \div 6$ 80
M $2400 \div 8$ 300
N $3700 \div 10$ 370
O $3700 \div 100$ 37
P $2600 \div 200$ 13

5. Solve the equation. $6 + 6 + 6 + 6 + 6 = n \times 6$ 5

6. Solve the equation. $(6 \times 30) + (6 \times 7) = 6 \times n$ 37

7. Solve:

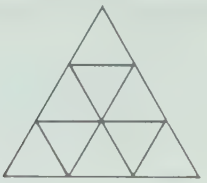
A 347×4 = 1388
B 24×39 = 936
C $3948 + 2765$ = 6713
D $8\overline{)344}$ 43
E $16\overline{)592}$ 37
F $8403 - 1679$ = 6724



You are invited to explore

ACTIVITY CARD 3
Page 356

Sample solution, Think, page 83



Workbook, page 24
(Use with text page 82)

CHAPTER 5 | Computing

Pages 84–123

General Objectives

To review the addition and subtraction algorithms and to develop skill in using them
To extend the development of the multiplication algorithm
To develop skill in working with the multiplication algorithm
To extend development of the division algorithm
To develop skill in working with the division algorithm
To provide additional experience with estimation
To provide additional experiences with the concept of average
To provide a variety of experiences with word problems
To provide work in computation involving dollar notation

The first lessons of this chapter are devoted to work with the addition and subtraction algorithms. Material is provided to help the children review the concepts involved in the algorithms; then, practice on the algorithms is followed by experiences with word problems. Following this, the multiplication algorithm is reviewed and extended, and experiences with appropriate word problems are again provided. Additional work involving exponents is also included in the work with the multiplication algorithm. The concept of exponents is extended to include powers of numbers other than ten. This work is included with the multiplication algorithm, since the finding of large products is involved.

Following the lessons on multiplication, the division algorithm is developed, first by using the number concepts underlying division; then, toward the end of the chapter, material is provided to help the children work with digits in the quotient and divisor to develop shortcut methods. The technique for finding averages is integrated into the material dealing with the division algorithm and into appropriate word-problem sets as well. Dollar and cent notation is then studied in relation to the multiplication and division algorithms.

The chapter concludes with the usual chapter and cumulative reviews.

Mathematics

The work with the addition, subtraction, and multiplication algorithms in this chapter requires no new mathematical concepts. Rather, the new material is based primarily on mathematical concepts which were developed earlier in the program. For example, the associative and commutative principles for addition, and the generalization about rearranging addends derived from combining them, allow us complete freedom to group when adding in columns.

For the multiplication algorithm, the distributive principle is most important. In multiplying, we are faced with the necessity of finding various products and then adding these products to get the final product. The distributive principle governs this procedure.

Our basic definition of division (if $a \times b = c$ and $b \neq 0$, then $a = c \div b$) must be extended to allow for remainders in division. Thus, we must define $c \div b$ for the case for which there is no whole number a such that $a = c \div b$. There are two choices: We can redefine division to allow for non-zero remainders, or we can allow for the possibility that a, b, and c are fractional numbers. Consider the following definition.

> If a, b, c, and d are whole numbers such that $(a \times b) + d = c$ and $b \neq 0$ and $d < b$, then for $c \div b$, a is the quotient and d is the remainder.

Notice in this definition that, if $d = 0$, we have the original definition. In other words, $a \times b = c$ simply implies $c \div b = a$. If we wanted to make a statement of equality with regard to $c \div b$ when $d \neq 0$, we would be involved with fractional numbers, and we would have

$$c \div b = a + \frac{d}{b}.$$

An example of this would be $14 \div 3$.

$$\begin{array}{r} 4 \\ 3\overline{)\,14} \\ \underline{12} \\ 2 \end{array} \qquad 14 \div 3 = 4 + \tfrac{2}{3} \text{ or } 4\tfrac{2}{3}$$

In our work with fractional numbers, we shall return to this idea and establish that non-zero remainders give a quotient that is not a whole number.

You will note that we are considering two different ways of thinking about quotients: as whole numbers with a number called the remainder related to the quotient (studied in the present chapter) and as rational numbers such that the quotient times the divisor equals the dividend (studied in a subsequent chapter).

Teaching the Chapter

Materials

Catalogues, newspaper advertisements
Current almanacs, encyclopaedias, or other reliable resources
Graph paper
Local airline or highway maps
Overhead projector (if available) and transparencies for the study of algorithms

Vocabulary

arithmetic mean
average
regrouping
time, rate, and distance

Since there is little new mathematical content in this chapter, the vocabulary list is quite short. This is a chapter devoted primarily to the development of skills and the utilization of mathematical concepts that have been developed previously.

Lesson Schedule

In a chapter as long as this, care should be taken not to let the time schedule become too flexible. Because of the large amount of material covered and the emphasis upon skills, it would be quite easy to spend excessive time on the work. Some two-page lessons are obviously a fairly easy day's work; others clearly will require more than a single day. There may be places in the chapter where you will choose to cover more than two pages in a single day. However, as a general rule, you will find that the work in this chapter is quite intense and that you will not want to cover more than two pages in one day, in order to leave time for explanatory development. In some instances, you may choose to spend two, or even three, days on a single lesson. For average classes, however, four to five weeks are recommended as sufficient time for the chapter. Obviously, if your children are very slow, you may choose to spend slightly longer. Or, if you have a very able class, you may choose to work through this chapter in considerably less than four weeks.

Evaluation

Evaluation of children's progress and achievement in a chapter like this is relatively simple. It is quite easy to determine whether or not the children are able to work with a given algorithm. To test whether or not they understand the algorithm is another matter. Naturally, we feel that it is extremely important for the children to understand the algorithms, and certainly, understanding will lead toward mastery of any algorithm. We therefore suggest that you try to evaluate understanding of the algorithms on a day-to-day basis, and conduct written tests to evaluate whether or not the children are actually able to perform the necessary computations. The children's ability to work with word problems should, of course, be part of your considerations in evaluation.

A considerable portion of the chapter is devoted to the division algorithm. However, please note that our general approach to the division algorithm centres on an understanding of the ideas and work with numbers, rather than emphasizing quotient and divisor figures. We therefore urge you to permit wide flexibility in the way the children work with these algorithms, especially the division algorithm. For example, some of the less able children may have mastered some of the simpler ideas of the division algorithm by making guesses for the quotient and subtracting certain multiples of the divisor until they find a remainder that is less than the divisor, and then adding their estimates to get the final quotient. It is better to allow them to continue using this method than to force them into using a shortcut which they do not understand. The shortcut is of value only to children who can handle it. It should not be treated as something to be memorized by all children. Such an approach would tend to frustrate and confuse the slower children, who may have gained some understanding of the simpler exercises involving the concepts. Therefore, be sure to treat shortcuts according to the abilities of the children.

A chapter review is provided on pages 120 and 121, while pages 122 and 123 offer a cumulative review. These pages may be used strictly for review or to evaluate children's progress, as you see fit.

Resources for Active Learning

General Activities

Maths Mini-lab, Card 50 (Selective Educational Equipment)

[The following resources contain puzzles involving algorithms.]

Mathex: Operations and Problem Solving No. 8 "A Subtraction Puzzle," pupil page 16, Encyclopaedia Britannica Publications Ltd.

Modern Math Games, pp. 46, 54–56, Fearon (Available from Clarke, Irwin)

[The following resources contain many investigations using pendulums.]

Developmental Math Cards, K[1]19, L[2]7, Addison-Wesley

Math Activity Cards, D30, Macmillan

Pendulums, McGraw-Hill Ryerson

Manipulative Devices

Pedometer and compass (Edmund Scientific; Math Media)

Racemaster Stopwatch (Edmund Scientific; Math Media)

SEE Calculator (Selective Educational Equipment)

Surveyor's Chain (Selective Educational Equipment)

Thermometer (Edmund Scientific; Nasco)

Commercial Games

Equations (Creative Publications; Wff 'N Proof)

Math Match (Creative Publications)

Playing Card Number Games, Whole Numbers (Heath)

PAGES 84–85

Objective

Given addition and subtraction exercises, the child will be able to find the sums and differences by applying standard addition and subtraction algorithms.

Preparation

Lead the children in a short oral practice session on basic addition facts. For example, give them an opportunity to work mentally with finding sums of three or four 1-digit numbers. Have children who are capable find the sums without pencil and paper. The children should attempt to hold partial sums in their heads as they add groups of 1-digit numbers. If time permits, following this practice, review the fact that, in addition, because of the order and grouping principles, addends can be rearranged in any way that is convenient.

Investigation

This investigation consists of a short self-test which the children can correct themselves. Be careful to observe which children are finished more quickly and which ones seem to have difficulty. For those who finish quickly, you might put the following "What's My Rule" grid on the chalkboard.

c	13	19	7	?	?	?	?	?
a	5	7	3	6	11	9	8	2
b	11	15	7	?	?	?	?	?

Challenge the children to find the missing numbers and express the rule for rows b and c in an equation.

$$b = (2 \times a) + 1$$
$$c = (3 \times a) - 2$$

5 Computing

How are your adding and subtracting skills?

Investigating the Ideas

Let's check your adding and subtracting skills.

1. 47 +35 82	**2.** 97 +75 172	**3.** 216 +438 654	**4.** 876 +359 1235	**5.** 462 657 +536 1655
6. 93 −47 46	**7.** 543 −127 416	**8.** 824 −569 255	**9.** 605 −386 219	**10.** 7003 −3586 3417

? Can you find the sums and differences above without missing more than one?

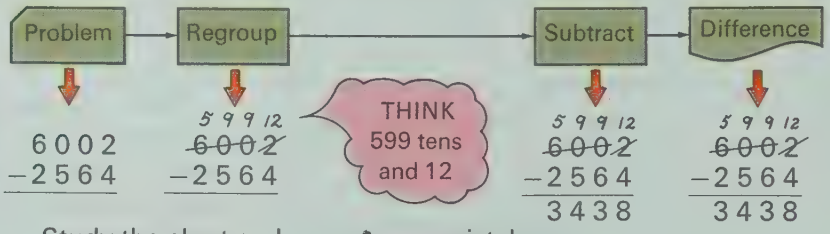
Use this code to grade your own paper:

a = 0 c = 2 e = 4 g = 6 i = 8
b = 1 d = 3 f = 5 h = 7 j = 9

Answers: **1.** ic **2.** bhc **3.** gfe **4.** bcdf **5.** bgff
6. eg **7.** ebg **8.** cff **9.** cbj **10.** debh

Discussing the Ideas

1. If you missed the subtraction problem with two zeros, this flow chart may help you correct it.



Study the chart and correct your mistakes.

2. Can you make a flow chart for Investigation problem 9?
See Discussion.

84

Discussion

It would be helpful as a review to have one or two children explain their method of solving investigation exercises 4 or 5, and 7 or 8. Many children will benefit from a careful discussion of exercise 1. The key step is the regrouping step. In exercise 2, they should think, for investigation problem 9, that 605 is the same as 59 tens and 15. If necessary, provide other regrouping problems for those who need reinforcement. Examples such as the following would be helpful:

726 → 71 tens and 16
812 → 80 tens and 12
2501 → 249 tens and 11
7102 → 709 tens and 12

709 tens and 12 may be regrouped again to 69 hundreds, 19 tens, and 12.

Using the Ideas

1. If you need practice in adding and subtracting, find these sums or differences.

A 28 +57 = 85	B 65 +93 = 158	C 78 +69 = 147	D 239 +446 = 685	E 968 +875 = 1843	F 6472 +3869 = 10341
G 56 75 +81 = 212	H 934 369 +548 = 1851	I 675 87 +948 = 1710	J 562 + 78 + 843 + 7651 = 9134		
L 86 −49 = 37	M 80 −52 = 28	N 96 −39 = 57	O 732 −415 = 317	P 653 −287 = 366	Q 9247 −1638 = 7609
R 906 −238 = 668	S 5037 −1699 = 3338	T 8007 −5679 = 2328	U 562 − 371 = 191	W 8076 − 1384 = 6692	
			V 6003 − 706 = 5297	X 7000 − 256 = 6744	

2. Give the number for the point above each letter.



736 786 795



536 545 595

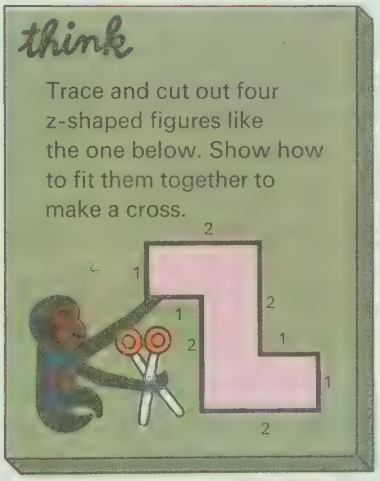
★ **3.** In each problem, some of the digits are covered with screens. In each exercise, give all possible digits that could be under the red screens.

A 4▮ + 3▮ = ▮▮ 7,8

B ▮9▮ ▮8▮ + ▮2▮ = ▮▮▮ 9,0,1

C ▮4▮ − 8▮ = ▮▮ 6,5

D ▮00▮ − ▮04▮ = ▮▮▮ 9



See Solution, T.E. page 85.

More practice, page A-5, Set 9

Follow-up

Magic squares intrigue children while providing them with addition and subtraction practice. Suggest that more capable children find the missing numbers in the following squares. (Rows, columns, and diagonals should give the same sum.)

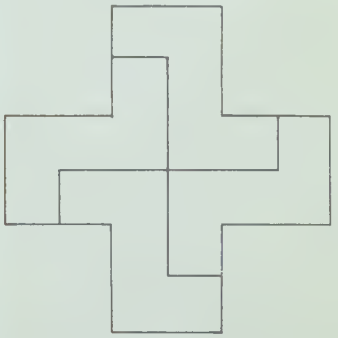
16	2	3	
	11	10	
9		6	
	14		1

(34)

25	11		22
	20	19	
	16		21
13			10

(70)

Solution, Think, page 85



Using the Exercises

Assign the exercises on page 85 according to the needs of the group. With more capable groups, you might suggest that they time themselves, or that they estimate their answers before they compute. With other groups, allow ample time for them to finish the assigned exercises.

All would benefit from an explanation of the starred exercise by those children who did it. Point out that regrouping may or may not be involved. For example, the digit covered by the red screen in exercise 3A may be 7 or 8; in exercise 3B, if there happened to be a 9 in each one's place, the digit in the red screen might be 1 (of 21), or it might be 0 or 9.

Assignments (page 85)

Minimum: 1. Average: 1, 2.
Maximum: 1–3.

Resources for Active Learning

Discovery, Section I, Activity 4, pp. 4–6, Encyclopaedia Britannica Educational Corp.

(See also, "Resources for Active Learning," page 39, for magic squares and other curious problems.)

Duplicator Masters, page 11
Workbook, page 25
Skill Masters, page 11

PAGES 86–87

Objective

Given word problems, the child will be able to solve them by applying the addition and subtraction algorithms.

Preparation

Mental chain games can provide a quick, basic review. Here are a few samples:

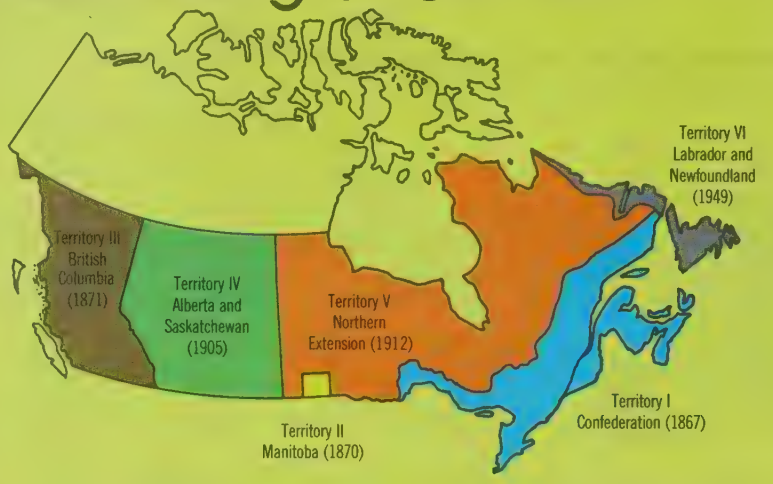
"Start with 24. . . Add 7. . . Subtract 3. . . Add 2. . . Subtract 10. . . What's your answer?" (20)

"Start with 35. . . Add 4. . . Subtract 11. . . Add 2. . . Subtract 10. . . What's your answer?" (20)

"Start with 40. . . Subtract 9. . . Add 1. . . Subtract 8. . . Add 1. . . Subtract 6. . . What's your answer?" (19)

"Start with 6. . . Add 7. . . Subtract 8. . . Add 9. . . Subtract 10. . . Add 11. . . What's your answer?" (15)

The Making of Canada



1. When Prince Edward Island entered Confederation, Territory I had been established for only six years. In what year did Prince Edward Island enter Confederation ? 1873
2. How long after Territory I did the last territory, Territory VI, enter Confederation ? 82 years
3. How long was it after 1873 before a new territory was added ? 32 years
4. The provinces which formed Territory 1 spread 45 years northward in 1912. How long had this area been a part of Confederation ?
5. In which territory is your province ? How long has it been a member of Confederation ?
6. In 1869, Canada bought the Northwest Territories. How long ago was that ? 105 years
7. In 1873, A. MacKenzie became prime minister of Canada. He was 51 years old. He lived to the age of 70. Give the year of his birth and the year of his death. 1822; 1892

★ 8. The Canadian Pacific Railway Co. was established in 1880 and inaugurated its first transcontinental railway 19 years after the signing of the British North America Act. How long had the Canadian Pacific been established at that time ? 6 years

86

Discussion

Study the maps on pages 86 and 87 with the children and give them an opportunity to discuss the various facts presented on them. Point out that, although both are maps of Canada, they both contain differing, unrelated information. Explain that the information on them will be needed for the solutions of the problems on each page.

Notice that exercises 8 and 9 on page 86 are intended primarily for the faster children. Exercise 9 might stimulate interest in other facts which can be found in an encyclopaedia, history book, or almanac. Encourage children to pursue such interest whether it seems related to mathematics or not.

Vacation Trips



Route	Distance
From Cape Breton to Halifax	428 (km)
From the Bay of Fundy to Moncton	69
From 1000 Islands to Toronto	265
From Georgian Bay to Toronto	142
From Mt. Riding to Winnipeg	253
From Banff to Calgary	105
From Jasper to Edmonton	327

1. Use the map and the chart to give the distance
   - A From Regina to Jasper National Park by way of Edmonton ? 1140 km
   - B From Toronto to Mt. Riding by way of Winnipeg ? 2264 km
   - C From Montreal to the Cape Breton Highlands by way of Halifax ? 1604 km
   - D From Ottawa to Banff by way of Calgary ? 3661 km
   - E From Moncton to the Georgian Bay Islands by way of Toronto ? 1742 km
   - F From Vancouver to Mt. Riding by way of Winnipeg ? 2177 km
   - G From St. John to the Thousand Islands by way of Toronto ? 1240 km

★2. Make use of the map to indicate the shortest distance
   - A From Vancouver to Georgian Bay. 4583 km
   - B From St. John to Mt. Riding. 3769 km
   - C From Winnipeg to the Bay of Fundy. 3680 km

★3. Prepare a trip going from your home to one of the points on the map. Give the distance covered.

87

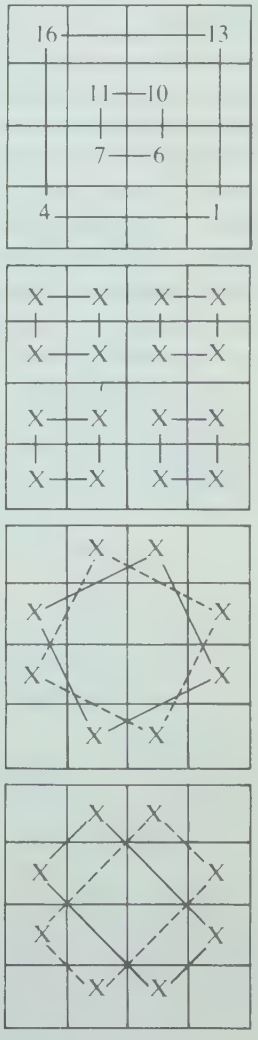
Using the Exercises

For those who try starred exercise 4, page 87, you may find it necessary to give the distance from your city to a particular point on the map such as Toronto or Regina, so that the children can plan their trip from there. Give them considerable leeway in planning their own trips and in deciding what route to take. However, point out that you want them to restrict their plans to routes given on the map. Of course, you might decide to deviate from this direction, but expect many variations if you do.

Assignments (page 87)
Minimum: 1. Average: 1, 2.
Maximum: 1–4.

Follow-up

If the children were successful with the magic squares in the follow-up for pages 84 and 85, duplicate a worksheet of either completed square, repeated several times. Use the same numbers in the same positions. Ask the children to find "magic" patterns other than the row, column, and diagonal patterns, so that each group of four numbers has the magic sum for that square. See the examples in the accompanying illustrations. In some, the numbers are omitted to emphasize the patterns, but the children will have to experiment with the actual numbers to discover the patterns. Some classes have found as many as 87 variations.



Resources for Active Learning

Mathex: Operations and Problem Solving No. 8, "Rallying Addition," pupil pages 17–18, Encyclopaedia Britannica Publications Ltd.

PAGES 88–89

Objective

Given addition and subtraction problems using money notation, the child will be able to find the sums or differences.

Preparation

As a warm-up activity, you might put some problems on the board with screened-out digits and ask the children to explain what digits the screen could be replacing. For example, write:

9 x	x 0 x	x 6 x
− 6 x	−x 5 x	−x 7 x
▮▮ x	x ▮▮ x	x ▮▮ x

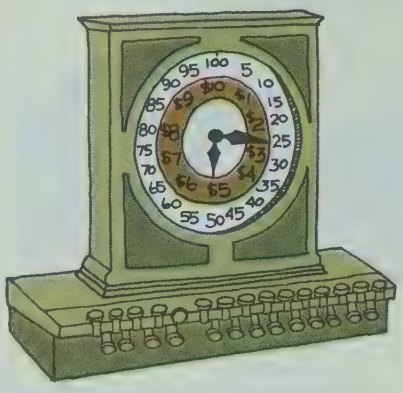
(Answers, left to right: 3 or 2, 5 or 4, 9 or 8)

Investigation

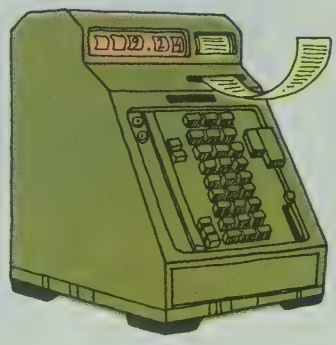
One way children might do this investigation would be to subtract an initial amount from one of the total amounts and split the difference into two parts. For example, if an item of $1.79 was purchased as a part of a $5.20 purchase, two other items must total $3.41. Encourage children to work independently on this investigation. If some finish more quickly than others, suggest that they find prices of four items which might have been purchased with these amounts.

● *Can you find money totals and differences?*

Investigating the Ideas



One of the first cash registers



A modern cash register

? Can you give three items whose prices would add up to the total shown on each cash register?

Answers will vary. See Investigation.

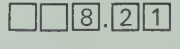
Discussing the Ideas

1. Explain how to read the total amount on the old cash register.
   Small hand indicates $5 and large hand indicates 25¢, so total is $5.25.
2. Modern cash registers show the amount for each purchase, the total amount, and how much change the buyer should receive.
   - A Suppose a cash register rings up these two purchases. Explain how to find the total it would show. $13.78
   - B Explain how to find what the machine will show for your change if you pay with a 20-dollar bill. $20-13.78=$6.22
   - C Explain how to find the amount of your purchase if this is the amount of change you get from a 10-dollar bill. $10.00-8.21=$1.79









Change:

88

Discussion

Ask children to give the prices of their items as they discuss how to read both cash registers. Give them an opportunity to draw analogies between the old-fashioned cash register and the hands of a clock, and the way in which money is represented on the newer cash register. Use exercise 2 to discuss the total amount of a purchase, the amount of money used to purchase it, and the change returned to the buyer. It would be helpful to present other, similar examples. You might do so in chart form as shown in the next column.

	Items	Total	$ Given to Clerk	Change
A	7.26			
	3.15	____	$10 + $5	_____
B	12.42			
	7.43	____	$20.00	_____
C	6.51			
	2.99	____	$10.00	_____

Using the Ideas

1. Find the total amounts.

A	B	C	D	E
$2.98	$9.82	$5.68	$15.67	$117.88
3.65	6.54	6.47	83.47	654.92
7.82	7.29	9.98	16.59	846.98
$14.45	$23.65	$22.13	$115.73	$1619.78

2. Find the differences in the amounts.

A	B	C	D	E
$5.67	$4.82	$19.21	$765.42	$9.37
2.91	3.46	12.59	186.50	6.54
$2.76	$1.36	$6.62	$578.92	$2.83

F	G	H	I	J
$8.00	$20.00	$100.00	$10.00	$5.00
1.29	16.98	37.98	.75	.98
$6.71	$3.02	$62.02	$9.25	$4.02

3. Use dollar notation to write each amount.

A	125¢ $1.25	C	1298¢ $12.98	E	5¢ $0.05	G	50¢ $0.50
B	700¢ $7.00	D	95¢ $0.95	F	10¢ $0.10	H	25¢ $0.25

Short Stories Money

1. Had 1 dollar 25 cents. Spent 79 cents. How much left? $46

2. Had $12.53. Earned 5 dollars and a quarter. How much now? $17.78

3. Had 10 dollars. Spent 3 dollars. Then spent $2.79. How much left? $4.21



4. 15 dollars in the bank. Withdrew $10.98. How much left in the bank? $4.02



5. $27.35 in the bank. Deposited $3.95. How much in the bank now? $31.30

6. Had a quarter. Earned 50 cents. Spent 35 cents. How much left? $0.40

7. Had a dollar. Spent 49 cents. Lost a quarter. How much left? $0.26

8. Had 5 dollars. Spent $2.98. Earned $1.25. How much now? $3.27

Using the Exercises

Ask children to do the exercises. When they have finished, you might allow time to have several parts of exercises 1 and 2 put on the chalkboard. After these exercises have been explained, give the children an opportunity to discuss the Short Stories and how they were able to arrive at their answers.

Assignments (page 89)

Minimum: 1–3. Average: 1–3, and Short Stories. Maximum: 1–3, and Short Stories.

Follow-up

Children might be intrigued by the palindromic property of numbers. A number is "palindromic" if it reads the same from either end. For example, "11011" is palindromic. If any number is chosen, the digits are reversed, and the numbers added, then the digits in the sum are reversed and added again, and the process is repeated, a palindromic number will eventually occur as a sum. For the number, 158, a palindromic number will appear after 3 steps.

```
  158
+ 851      (reverse order and add)
 1009
+ 9001     (reverse order and add)
 10010
+ 01001    (reverse order and add)
 11011
```

Other numbers which will give this same palindromic sum after three steps are 257; 356; 455; 554; 653; 752; 851; 950. You might challenge some children to find numbers which give a palindromic sum after only one or two steps, such as 3254.

Resources for Active Learning

Math Activity Cards, "Palindromes," E39, Macmillan.

Mathematics in Modules, WN19, Addison-Wesley.

Duplicator Masters, page 12
Workbook, page 26
Skill Masters, page 12

PAGES 90–91

Objective

Given a multiplication problem with a 1-digit factor the child will be able to find the product.

Preparation

Since this lesson is principally concerned with the child's ability to use the multiplication algorithm rather than his skill with basic facts, a short practice with multiplication facts would be helpful. Use any suitable short oral drill for this purpose.

Investigation

Again, children are presented with a small self-test which they can correct on their own. For those who finish quickly, you might provide a "What's My Rule" grid on the chalkboard:

b	19	31	43	?	?	?	?	?
a	3	5	7	2	6	8	9	?
c	25	43	61	?	?	?	?	?

Ask children not only to fill in the blanks but also to try to write the rule in equation form.

[Rules: $b = (6 \times a) + 1$;
$c = (9 \times a) - 2$].

How are your multiplying skills?

Investigating the Ideas

Let's check your multiplying skills.

1.	2.	3.	4.	5.
65	96	287	709	5978
×3	×8	×4	×7	×9
195	768	1148	4963	53802

Can you find these products without missing any of them?
See Investigation.

Use the code on page 84 to check your answers.
Answers: 1. bjf 2. hgi 3. bbei 4. ejgd 5. fdiac

Discussing the Ideas

1. Study the steps for multiplying 576 × 4 in the charts below. Then correct any mistakes you made in the Investigation.

4 × 6 = 24	4 × 70 = 280	4 × 500 = 2000	Add
A 576 ×4 24	576 ×4 24 280	576 ×4 24 280 2000	576 ×4 24 280 2000 2304
B ^{2}576 ×4 4	$^{3\,2}$576 ×4 04	$^{3\,2}$576 ×4 2304	You did the adding as you did each step.

Multiply ones. Multiply tens and add tens from the ones' step. Multiply hundreds and add hundreds from the tens' step.

2. Explain each step of the shortcut shown in chart B.
See above.

90

Discussion

Have a few children present examples from the investigation on the chalkboard. However, before discussing them, you might find it helpful to review the distributive principle by presenting examples such as

$$3 \times 56 = (3 \times 50) + (3 \times 6).$$

Even though for larger products the use of the distributive principle is much more complex than in this illustration, the ideas are essentially the same. As you work through examples step by step, point out the use of the distributive principle. Carefully work through the flow charts in exercise 1. Example A should be stressed so that the children will clearly understand each step. It is hoped that children at this level will become proficient in the use of the shortcut method shown in example B.

Using the Ideas

1. Find the products.

A 56 × 3 = 168	B 72 × 6 = 432	C 59 × 5 = 295	D 62 × 8 = 496	E 47 × 6 = 282
F 159 × 9 = 1431	G 328 × 6 = 1968	H 527 × 2 = 1054	I 806 × 7 = 5642	J 564 × 4 = 2256
K 392 × 7 = 2744	L 564 × 8 = 4512	M 306 × 7 = 2142	N 530 × 9 = 4770	O 654 × 5 = 3270
P 1326 × 3 = 3978	Q 5804 × 6 = 34 824	R 6001 × 5 = 30005	S 7643 × 8 = 61144	

2. Find the product for ***a***. Then give the product for ***b***.

A $6 \times 84 = a \xrightarrow{504} 60 \times 84 = b$ 5040

B $7 \times 53 = a \xrightarrow{371} 70 \times 53 = b$ 3710

C $6 \times 364 = a \xrightarrow{2184} 60 \times 364 = b$ 21 840

D $7 \times 156 = a \xrightarrow{1092} 70 \times 156 = b$ 10 920

3. Find the products.

A 48 × 70 = 3360	B 63 × 80 = 5040	C 57 × 30 = 1710	D 69 × 50 = 3450	E 846 × 90 = 76 140	F 562 × 30 = 16 860

4. Find the products.

A 500 × 3 = 1500	B 400 × 7 = 2800	C 200 × 9 = 1800	D 600 × 8 = 4800
E 800 × 6 = 4800	F 700 × 5 = 3500	G 600 × 9 = 5400	H 300 × 4 = 1200

5. Estimate the products. Then find the products and check your estimates.

A 599 × 3 = 1797	B 807 × 6 = 4842	C 1087 × 4 = 4348

think

Find these products.

142 857 × 2 = 285 714	142 857 × 3 = 428 571	142 857 × 4 = 571 428
142 857 × 5 = 714 285	142 857 × 6 = 857 142	

What pattern can you find in these products? Guess what this product will be.

Find the product. 142 857 × 7 = 999 999

Were you correct?

See Using the Exercises.



Follow-up

Place the following multiplication problems on the chalkboard or on duplicated worksheets. (The products are shown for your convenience.)

102 564 × 4 = (410 256)	128 205 × 4 = (512 820)	153 846 × 4 = (615 384)
179 487 × 4 = (717 948)	205 128 × 4 = (820 512)	230 769 × 4 = (923 076)

After the children have found the products, challenge them to try to see why the products are unusual and how they might have written the products without multiplying, once they see the pattern. Examination of the products will reveal that the answers can be found by transposing the units' digit to the hundred thousands' place while "sliding" the original digits one place to the right.

Resources for Active Learning

Franklin Series: *Making and Using Graphs and Nomographs,* "Nomographs: Multiplication with Whole Numbers," pp. 71-72, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

[The resources below deal with multiplication patterns.]

Experiments in Mathematics, Stage 1, pp. 56-58, Houghton Mifflin. (Available from Thomas Nelson & Sons Ltd.)

Mathex: Operations and Problem Solving No. 8, "Patterns in Multiplication," pp. 8–12 (pupil pages 19–20), Encyclopaedia Britannica Publications Ltd.

Nuffield Project: *Computation and Structure* 3, pp. 48–55, Wiley.

Using the Exercises

Assign the exercises on page 91 as independent work. You might point out the relationships between the pairs of equations in exercise 2. The children should observe that since they can find 6×84, they can certainly find 60×84, for this product would be just ten times as great. When the children finish, help them resolve any points of difficulty.

Encourage all the children to do the *Think* problem. Suggest also that they look for patterns in these products:

142857 × 8	142857 × 9

Some patterns for the *Think* problem:

$142857 \times 2 = 285714$

$142857 \times 3 = 428571$

$142857 \times 4 = 571428$

Assignments (page 91)

Minimum: 1, 3. Average: 1–4.
Maximum: 1–5.

Duplicator Masters, page 13
Workbook, page 27
Skill Masters, page 13

PAGES 92–93

Objective

Given two numbers whose numerals have at least 2 digits, the child will be able to find their product.

Preparation

To prepare for this lesson, use mental chain games to review basic multiplication facts:

"Start with 54... Divide by 9... Multiply by 4... Divide by 8... What's your answer?" (3)

"Start with 63... Divide by 7... Multiply by 4... Subtract 1... Divide by 5... What's your answer?" (7)

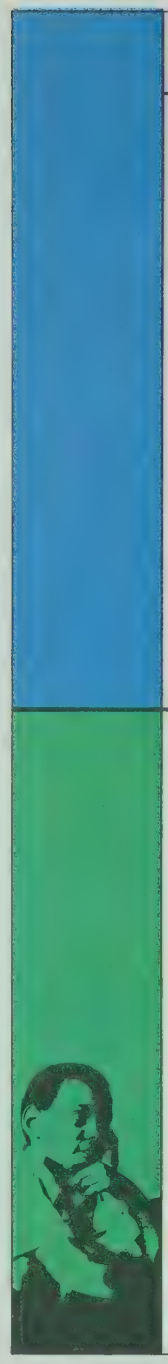
"Start with 8... Multiply by 5... Add 2... Divide by 6... Multiply by 4... What's your answer?" (28)

"Start with 48... Divide by 8... Multiply by 5... Add 2... Divide by 8... What's your answer?" (4)

Investigation

After the children have studied the explanation in their text, they should choose to use one of the methods shown. Encourage them to be able to give a reason as to why they chose the method they used. Since many sixth graders will be familiar with the multiplication algorithm, you may have many children who chose Nancy's method. You might have those who finish early work the following problems, using their chosen method.

36×75 (= 2700)
28×56 (= 1568)
49×72 (= 3528)



Can you multiply large numbers?

Investigating the Ideas

How do you find 38 × 47? Three students gave these methods.

47 you could add up 38 forty sevens
47
47
47
47
+
Jan.

47 47 47
×8 ✚ ×30 ×38
376 ✚ 1410 1786

Break the 38 apart. First find 8 forty-sevens, then 30 forty-sevens, and then add to get the final answer. Ted

47
×38
376
1410
1786

Use the distributive principle as Ted did, but write it the easy way, like this. Nancy

Can you use one of these methods to find 26 × 34?
Choices will vary. See Investigation.

Discussing the Ideas

1. Why are Ted's and Nancy's methods more useful than Jan's?
They are shorter and faster.
2. A Explain how Ted's and Nancy's methods are alike.
B How do these methods use the distributive principle? See Discussion.
3. Study the flow chart and use it to find another product. Choices will vary.

Problem	Multiply 8 × 47.	Multiply 30 × 47.	Add.	Product is 1786.
47 ×38	47 ×38 376	47 ×38 376 1410	47 ×38 376 1410 1786	47 ×38 376 1410 1786

92

Discussion

Children should see that even though multiplication can be thought of as repeated addition, Jan's use of this method is clearly more tedious than Nancy's algorithmic method. As you discuss exercise 2, be sure that the children see the relationship between finding each of these products a step at a time and the use of the distributive principle. They should observe that they begin by multiplying 47 × 8, then multiply 47 × 30, and then add these two products. In exercise 3, work through several examples using the flow chart. Point out the regrouping which is necessary in each step. For example, in the first step, the 5 tens in 56 must be added to the product 8 × 40.

$$\begin{array}{r} 47 \\ \times 38 \\ \hline 376 \end{array}$$

Similarly, in the second step, the 2 tens in 21 must be added to the product 30 × 40. Stress the method in exercise 3 according to the needs of your class.

Using the Ideas

1. Find the products.

A 36 × 48 = 1728	B 59 × 27 = 1593	C 78 × 35 = 2730
D 26 × 39 = 1014	E 342 × 87 = 29754	F 647 × 98 = 63406
G 826 × 48 = 39648	H 392 × 64 = 25088	I 822 × 41 = 33702

2. For each exercise, give the products for ***a*** and ***b***.

A $6 \times 827 = 4962 \rightarrow 60 \times 827 = a$ 49 620; $600 \times 827 = b$ 496 200

B $4 \times 391 = 1564 \rightarrow 40 \times 391 = a$ 15 640; $400 \times 391 = b$ 156 400

C $7 \times 285 = 1995 \rightarrow 70 \times 285 = a$ 19 950; $700 \times 285 = b$ 199 500

3. Find the products.

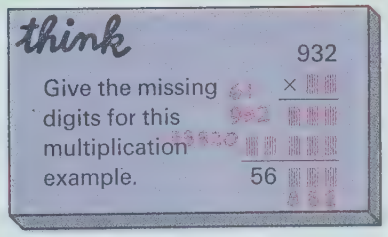
A	347 × 2 = 694	347 × 60 = 20 820	347 × 300 = 104 100	347 × 362 = 125 614
B	641 × 6 = 3846	641 × 20 = 12 820	641 × 400 = 256 400	641 × 426 = 273 066
C	917 × 4 = 3668	917 × 70 = 64 190	917 × 500 = 458 500	917 × 574 = 526 358
D	186 × 7 = 1302		186 × 800 = 148 800	186 × 807 = 150 102
E	342 × 5 = 1710	342 × 30 = 10 260	342 × 900 = 307 800	342 × 935 = 319 770
F	253 × 8 = 2024		253 × 300 = 75 900	253 × 308 = 77 924

4. Find the products.

A 384 × 264 = 101 376	B 462 × 397 = 183 414	C 570 × 642 = 365 940
D 809 × 817 = 660 953	E 6125 × 256 = 1 568 000	F 928 × 407 = 377 696

think

Give the missing digits for this multiplication example.



More practice, page A-5, Set 10



Resources for Active Learning

Mathematics in Modules, WN20, Addison-Wesley

Mathex: Operations and Problem Solving No. 8, "The Russian Peasant's Method," p. 23, Encyclopaedia Britannica Publications Ltd.

Duplicator Masters, page 14
Workbook, page 28
Skill Masters, page 14

Using the Exercises

When you assign the exercises on page 93, call attention to the reasoning in exercises 3 and 4. In exercise 3, children should see quickly that they can find the products by simply adding a certain number of zeros to the first product. In exercise 4, the children should understand quickly that they can do each problem by thinking about only 1-digit multipliers and adding a certain number of zeros according to whether they are multiplying by a 2-digit multiple of ten or a 3-digit multiple of ten.

You might help children get started on the *Think* problem with the following reasoning: "I know the ones' digit in the multiplier has to be 1 because my first partial product is only a 3-digit number. If it were greater than 1, I would have a partial product that is a 4-digit number. If it were zero, I could not get a final product that is in the 56 thousands."

Assignments (page 93)

Minimum: 1, 3. Average: 1–3. Maximum: 1–4.

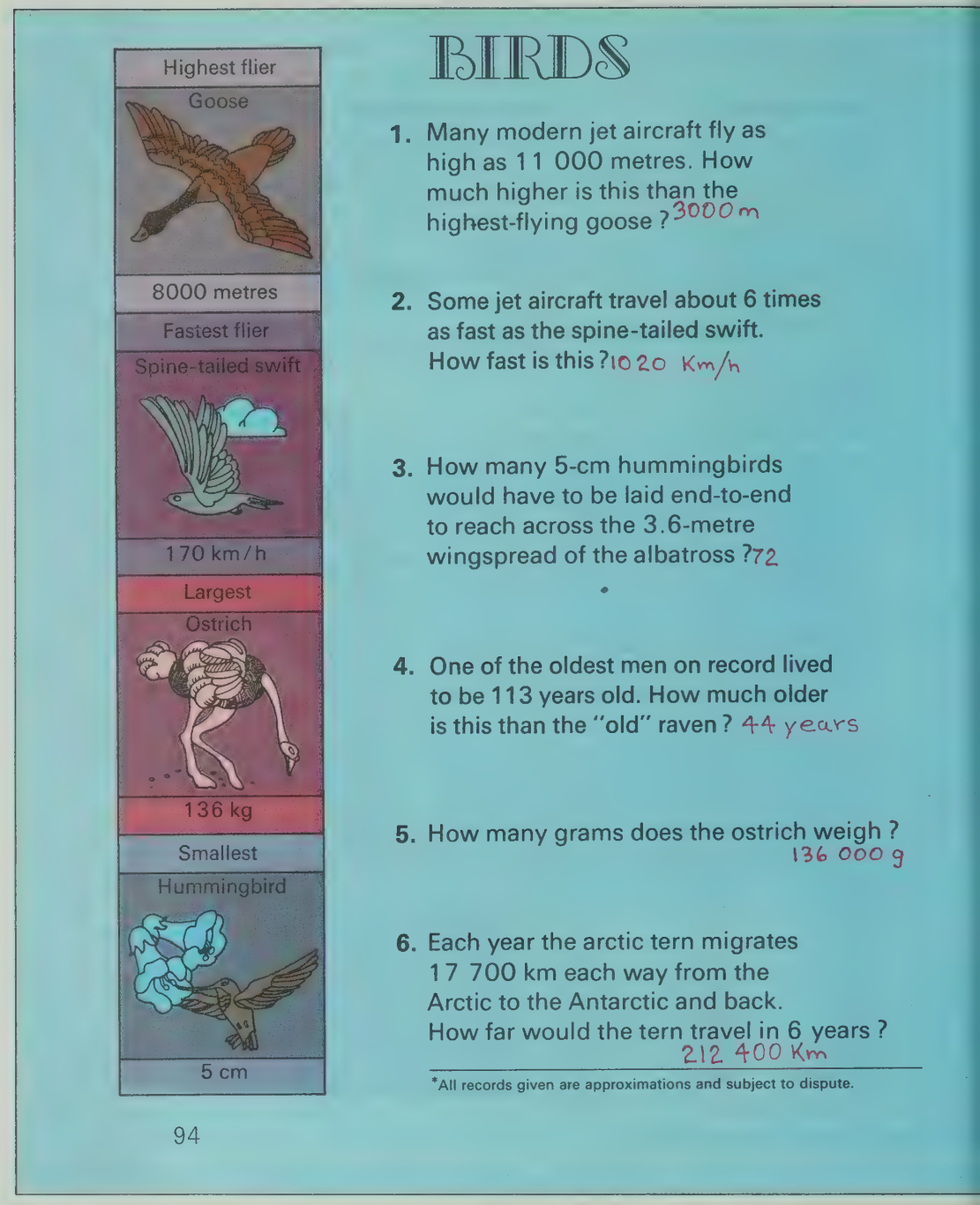
PAGES 94–95

Objective

Given word problems requiring multiplication with a 2-digit factor, the children will be able to solve them by using the multiplication algorithm.

Preparation

To prepare for this lesson, you might give the children oral practice in multiplication with multiples of ten. Or, briefly review any algorithms with which children had difficulty.



Discussion

Study the information accompanying the illustrations of the birds with the children. Keep in mind during this period that one of the chief values of a lesson such as this is the interest it creates among the children. By being exposed to a wide variety of experiences with word problems, the children will gain a greater appreciation of the many uses of arithmetic.

Following a discussion of the factual information, have the children do the exercises.

You should make certain the children see the footnote indicating that the records given are close approximations and that they are not to be regarded as exact figures.

Notice that exercises 11 and 12 are starred, to denote that they are designed primarily as enrichment for the more able children. Although the actual work in these exercises is not too difficult, a chain of reasoning is required which is a little more involved than that required by the other problems. For this reason, you may choose to have all the children attempt the problems, and certainly, you should try to have all the children participate in a discussion of the exercises, after everyone has had a chance to work on them.

7. If an ostrich could run at his top speed for half an hour, how many metres would he travel ?32 500 m

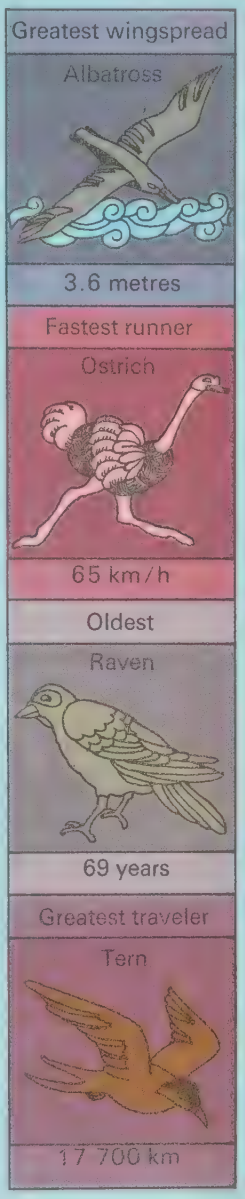
8. The largest flying bird weighs about 18 kg. It would take about 424 hummingbirds to weigh 1 kilogram. How many hummingbirds would it take to weigh 18 kilograms ?
7 632

9. The canvasback duck flies about 48 km/h faster than the ostrich can run. How many metres does the duck fly in an hour ?113 000m

10. An ostrich egg weighs about $1\frac{1}{2}$ kg. A hummingbird egg weighs about $1\frac{1}{2}$ milligrams. How many hummingbird eggs would it take to weigh as much as one ostrich egg ?1 000 000

11. ★ The penguin is a bird that does not fly. Some penguins can swim as fast as 32 km/h. How many metres per second is this ?8.8 mps

12. ★ Sparrow eggs hatch in about 12 days. It takes 5 times that long for penguin eggs. Chicken eggs take 39 days less than penguin eggs. How much time does it take for penguin eggs and chicken eggs to hatch ?60; 21
Penguin eggs, 60 days ; chicken eggs, 21 days



Follow-up

If the interest of some children has been sparked by this lesson on birds, you may wish to extend the lesson by having the children do some research on birds. Graphs and charts that compare weights, wingspans, speed of flight, migration distances, and other quantitative data concerning birds would be excellent extensions of the lesson. Encyclopaedias and bird books will provide some data. Try to find information about the birds that are native to your locality. Your local Audubon society might prove very helpful in this respect.

Resources for Active Learning

Applied Mathematics Cards, "Weight," Group 2/9; Group 3/8, Schofield and Sims. (Available from Mafex Associates)

Math Activity Cards, "Walking," D28, Macmillan.

Nuffield Project: *Computation and Structure* 3, "Multiplication with money . . .," pp. 56–60, Wiley.

Assignments (pages 94–95)

Minimum: 1–9, oral. Average: 1–9.
Maximum: 1–12.

PAGES 96–97

Objectives

Given a number with an exponent (such as 3^4), the child will be able to raise the base to the power given by the exponent.

Given a product with a repeated factor (such as $3 \times 3 \times 3 \times 3$), the child will be able to express the product as a power with the correct base and exponent.

Preparation

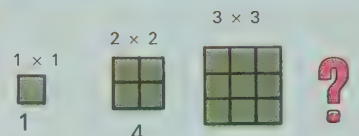
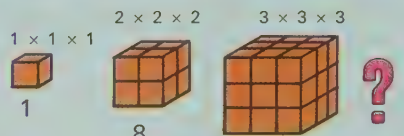
To prepare for this lesson, you might ask the children if they can recall a shortcut for writing 10×10 or $10 \times 10 \times 10$. Remind them that $10 \times 10 = 10^2$, $10 \times 10 \times 10 = 10^3$, and so on. However, use only powers of ten, as the purpose of the investigation is to introduce the children to the use of other bases with exponents.

Investigation

You might want the children to work with a partner as they try to figure out the main questions. Suggest to those who have difficulty that they use graph paper or cubes or both to help them find the squared and cubed numbers. However, many will recognize the sequence established by using 1, 2, and 3 as the base numbers. These children would also benefit from the use of graph paper and cubes so that the geometric expression of these numbers is clear.

● *Let's explore exponents.*

Investigating the Ideas

"Square" Numbers	"Cube" Numbers
 16, 25, 36, ...	 64, 125, 216, ...

? Can you find 3 more "square" numbers and 3 more "cube" numbers? Draw pictures for them.

Answers may vary. Samples are given above.

Discussing the Ideas

1. A How did you find the next three "square" numbers? "cube" numbers? See Discussion.

   B Why do you think it is said that 3 squared is 9, and 2 cubed is 8? See Discussion.

2. Study this chart. Then write and read some symbols like these.

For this symbol	we think	we write	we say
5^3	$5 \times 5 \times 5$	$5^3 = 125$	"5 cubed is 125."
4^2	4×4	$4^2 = 16$	"4 squared is 16."
3^4	$3 \times 3 \times 3 \times 3$	$3^4 = 81$	"3 to the fourth power is 81."

3. When we write 2^3, the "3" is an **exponent.** It tells how many times the **base** 2 is used as a factor. What are the bases and exponents for the examples in the chart above?

   Base 5, exponent 3; base 4, exponent 2; base 3, exponent 4

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Discussion

Use discussion exercise 1 to relate the geometric representations of these numbers to the phrases "square numbers" and "cubed numbers." Carefully explain the chart in exercise 2 and relate it to exercise 3. Emphasize for the children that the exponent tells them how many times to take the base number as a factor. Thus, in 3^4, we use the base number 3 as a factor 4 times: $3 \times 3 \times 3 \times 3$. Present examples on the chalkboard to give children practice in reading them and in identifying both the base number and the exponent. Be sure to point out that 5^3 is read "5 cubed," but 5^4 is read "5 to the fourth power."

Using the Ideas

1. Give the missing words and numbers.

A 3^2 means that 3 is used as a factor __?__ times. 2 3^2 = ▒. 9

B 4^3 means that 4 is used as a factor __?__ times. 3 4^3 = ▒. 64

C 2^5 means that 2 is used as a factor __?__ times. 5 2^5 = ▒. 32

D 10^4 means that 10 is used as a factor __?__ times. 4 10^4 = ▒. 10000

2. Use an exponent to write each of the following.

A 3×3 Answer: 3^2

B $2 \times 2 \times 2 \times 2 \times 2 \times 2$ 2^6

C $5 \times 5 \times 5$ 5^3

D $4 \times 4 \times 4 \times 4$ 4^4

E $10 \times 10 \times 10 \times 10 \times 10$ 10^5

F $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$ 3^7

3. Give the missing numbers. Example: $6^2 = 36$

A 7^2 = ▒ 49 C 4^3 = ▒ 64 E 3^4 = ▒ 81 G 5^2 = ▒ 25 I 7^1 = ▒ 7

B 2^3 = ▒ 8 D 3^2 = ▒ 9 F 2^4 = ▒ 16 H 6^3 = ▒ 216 J 4^4 = ▒ 256

4. Give the correct number for each sentence.

A If $a = 7$, then a^2 = ▒. 49

B If $x = 3$, then x^3 = ▒. 27

C If $m = 4$, then m^2 = ▒. 16

D If $r = 2$, then r^4 = ▒. 16

5. Mark **T** (true) or **F** (false).

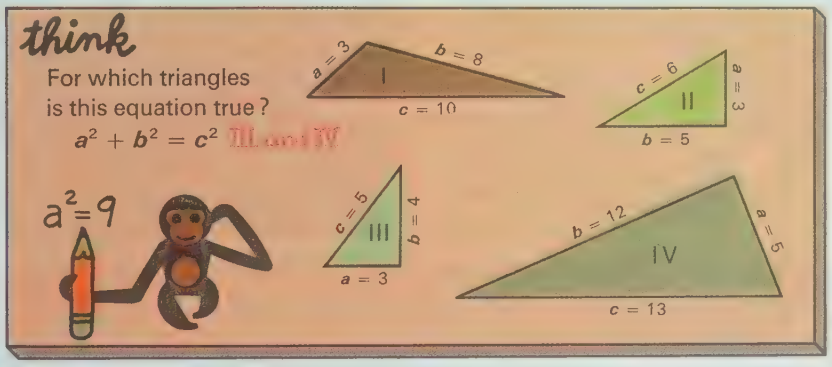
A $5^2 = 2 \times 5$ F

B $4^5 = 4 \times 4 \times 4 \times 4 \times 4$ T

C $3^7 = 3 \times 7$ F

D $5^3 = 125$ T

★ **6.** Is 2^{15} more or less than 15 000? Guess. Then check your guess.
More ($2^{15} = 32\,768$)



Using the Exercises

Give the children an opportunity to do the exercises on their own. Some children might find exercise 4 difficult, but explain that the letter simply represents the base number. Exercise 5 will require some computation; help the children see that they must do the multiplying carefully to arrive at the correct answers. Starred exercise 6 will challenge the more able children to multiply accurately.

Assignments (page 97)
Minimum: 1–3. Average: 1–5.
Maximum: 1–6.

Mathematics

Repeated factors in a product can be expressed concisely by using exponents. The symbol 10^{100} (a googol) means we must multiply the base, 10, by itself 100 times. This would involve writing 1 followed by 100 zeros. Clearly, the symbol 10^{100} is a very convenient way of representing this large number.

Some children may wonder what would be meant by an exponent of 0. Although this exponent does not occur in this lesson, a reasonable explanation for symbols like 3^0, 10^0 is not difficult to make. When we multiply powers that have the same base, we can do this simply by adding the exponents.

Examples: $2^3 \times 2^2 = 2^{3+2} = 2^5 = 32$
$3^2 \times 3^1 = 3^{2+1} = 3^3 = 27$

Now consider 2^0 and suppose we must find the product $2^3 \times 2^0$. Then, using the additive property for exponents illustrated above, we should have

$$2^3 \times 2^0 = 2^{3+0} = 2^3.$$

But, $2^3 \times 1 = 2^3$. Therefore, we must conclude that $2^0 = 1$. Using a similar argument, we can show that any non-zero number raised to the 0 power must equal 1.

Resources for Active Learning

Developmental Math Cards, "Squaring Numbers," L18, Addison-Wesley.

Workbook, page 29

PAGES 98–99

Objective

The child will demonstrate his ability to work with the concepts presented for cumulative review.

Preparation

To prepare for this lesson, you might review any concepts with which the children had previous difficulty. For example, give them practice in multiplying multiples of ten and dividing them by a 1-digit divisor. However, choose your examples so that the basic multiplication facts are reviewed.

Keeping in Touch with

Addition
Multiplication
Division
Place value

1. Find the quotients.

A $24 \div 4$ 6 C $54 \div 9$ 6 E $5400 \div 9$ 600 G $6300 \div 7$ 900 I $420 \div 6$ 70
B $240 \div 4$ 60 D $540 \div 9$ 60 F $63 \div 7$ 9 H $42 \div 6$ 7 J $4200 \div 6$ 700

2. Find the largest possible whole number for n.

A $n \times 40 < 243$ 6
B $n \times 50 < 360$ 7
C $n \times 80 < 331$ 4
D $n \times 20 < 161$ 8
E $n \times 30 < 220$ 7
F $n \times 60 < 375$ 6
G $n \times 40 < 375$ 9
H $n \times 90 < 730$ 8
I $n \times 70 < 400$ 5
J $n \times 30 < 100$ 3
K $n \times 40 < 300$ 7
L $n \times 60 < 500$ 8

3. Find the largest multiple of 10 for n.

A $n \times 20 < 1150$ 50
B $n \times 60 < 2300$ 30
C $n \times 50 < 2400$ 40
D $n \times 30 < 1280$ 40
E $n \times 40 < 3300$ 80
F $n \times 50 < 2700$ 50
G $n \times 80 < 3700$ 40
H $n \times 90 < 5800$ 60
I $n \times 70 < 6100$ 80

4. Give the number for n.

A $3682 = (n \times 100) + 82$ 36
B $5823 = (n \times 10) + 3$ 582
C $9176 = (n \times 100) + 76$ 91

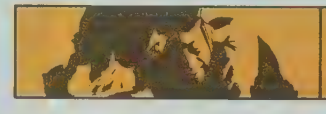
5. For each of the following equations, tell whether the solution is a 1-, 2-, or 3-digit number.

A $n \times 35 = 805$ 2
B $n \times 35 = 8050$ 3
C $n \times 47 = 94$ 1
D $n \times 47 = 940$ 2
E $n \times 47 = 9400$ 3
F $n \times 26 = 182$ 1
G $n \times 26 = 1820$ 2
H $n \times 76 = 684$ 1
I $n \times 76 = 6840$ 2
J $n \times 34 = 2720$ 2

think

Each equation below has two solutions from the set: {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}. Solve the equations.

1. $(n \times n) + 10 = 7 \times n$
   Answer: $n = 2$ or $n = 5$
2. $(n \times n) + 12 = 7 \times n$ 3,4
3. $(n \times n) + 21 = 10 \times n$ 3,7
4. $(n \times n) + 35 = 12 \times n$ 5,7
5. $(n \times n) + 54 = 15 \times n$ 6,9
6. $(n \times n) = 6 \times n$ 0,6



You are invited to explore

ACTIVITY CARD 4
Page 357

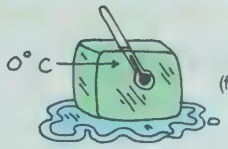
98

Discussion

Page 98 is appropriate for independent work. When the children finish the exercises, help them check their work. In particular, point out the difference of place value in the answers for exercises 2 and 3. Exercise 5 may serve as a basis for discussion since this is important background material for understanding the division algorithm.

Encourage children to solve the *Think* problem by trial and error; no particular technique for solving these equations should be stressed. Instruct the children to check the two answers, 2 and 5, for the first equation in the problem. Performing this check will, perhaps, help them in deciding what numbers are solutions for equations 2 through 6.

TEMPERATURE





1 Iron melts at 1535°C. How much hotter is this than the melting point of ice? 1535°

2 The melting point of lead is 1187C° cooler than that of iron. What is this melting point? 348°

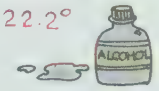
3 The melting point of gold is 472C° less than the melting point of iron. Give this temperature. 1063°

4 The melting point of paraffin is 39C° greater than average room temperature. What is it? 61°

5 Tungsten is used in light bulbs since it has a very high melting point. This melting point is about 153 times the average room temperature. Find it. 3366°



6 The boiling point of alcohol is 77.8°C. How much less is this than the boiling point of water? 22.2°



7 Metals have a boiling point just as they have a melting point. Iron boils at a temperature that is 1465C° hotter than its melting point. What is the boiling point of iron? (See exercise 1.) 3000°

★ **8** Mercury is liquid at room temperature. It has a melting point of ⁻38.8°C. How much does this differ from the melting point of ice? 38.8°

★ **9** Mercury boils at 356.9°C. What is the difference between the melting and boiling points of mercury? 395.7°

99

Using the Exercises

Before assigning page 99, discuss with the children the meaning of melting point and boiling point. Explain that the melting point of a substance is the temperature at which the substance changes from solid to liquid and that the boiling point of a substance is the temperature at which it changes from liquid to gas. Discuss the melting point and boiling point of water.

Following this discussion, direct the children to do the exercises.

Note that exercises 8 and 9 are starred, because they actually concern ideas relating to negative numbers. If the children should bring this up themselves, compliment them, but continue discussing the solutions in terms of whole numbers.

In exercise 8, for example, since mercury melts at 38.8 degrees below zero and ice melts at 0 degrees, the children must add 38.8 and 0. The same thing applies for exercise 9. The point to note is that in other cases the children had to subtract in order to find the difference in temperatures, but in exercises 8 and 9 they must add.

Follow-up

If children are interested in the topic of melting and boiling points, encourage them to do some related research. For example, ask them to find out about the characteristics of water as it changes phases—for instance, what happens to a full jar of water when it freezes. (Since water, unlike most liquids, expands upon freezing, the jar will break.)

Other children might like to find out the melting points of various substances, such as paraffin, or the boiling point of alcohol. However, if children want to experiment with these materials make sure laboratory safety procedures are followed. Unless your science program coincides with such activity, you might limit their research to using reference materials.

Resources for Active Learning

Teaching Aids for Elementary Mathematics, pp. 80, 84, 88, Holt, Rinehart and Winston [Cross-number puzzles].

Applied Mathematics Cards, Group 2/26; Group 3/30, Schofield and Sims [Temperature]. (Available from Mafex Associates)

Measure and Find Out, Book 1, Activities 20, 21, 23; Book 2, 1/6, Scott Foresman [Temperature]. (Available from Gage Educational Publishing Ltd.)

PAGES 100–101

Objective

Given a dividend and a single-digit divisor, the child will be able to find the quotient.

Preparation

It would be helpful to review division as the inverse of multiplication. For example, give two single-digit numbers and have the children give the product and a related division equation. Stress speed as well as accuracy, and keep the preparation brief.

Investigation

Children should work independently on this investigation, solving and then checking their own work. Since children will finish at different rates, you might put the following message in code on the chalkboard for those who finish quickly:

1. $6\overline{)174}$ 2. $3\overline{)255}$ 3. $9\overline{)261}$

4. $5\overline{)1935}$ 5. $7\overline{)301}$

2 = m, 3 = s, 4 = r, 5 = h, 7 = e, 8 = t, 9 = a.

(If the problems are done correctly and in the proper order, the quotients will yield the message *Math Masters.*)

Make sure children decipher the code for the investigation problems correctly so that they can recognize any errors. When all have finished, you might have a child put the numerical answers on the chalkboard.



● *How are your dividing skills?*

Investigating the Ideas

Let's check your dividing skills when the divisor has 1 digit.

1. $9\overline{)63}$ (7) 2. $4\overline{)96}$ (24) 3. $6\overline{)378}$ (63) 4. $9\overline{)7659}$ (851) 5. $8\overline{)5209}$ (651 R1)

Can you find the quotients above without missing more than one? See Investigation.

Use the code on page 84 to check your answers.

Answers 1. h 2. ce 3. gd 4. ifb 5. gfb, R b

Discussing the Ideas

1. This example shows that to find the **quotient** (726), you have to find the greatest number of times the **divisor** (6) can be subtracted from the **dividend** (4359).

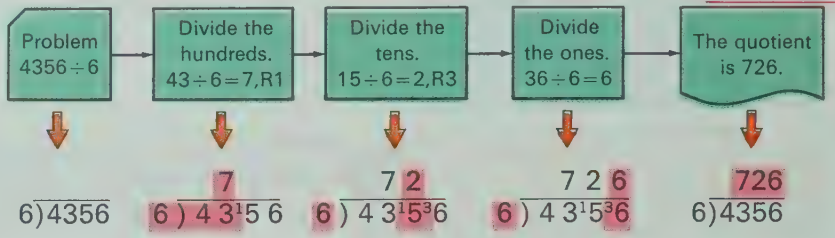
   A How many sixes were subtracted the first time? How many in all? (700; 726)

   B What was the remainder? (3)

   C Can you describe another way this quotient might have been found? See Discussion.

```
      726
6)  4359
  −4200   ← 700 × 6
    159
   −120   ←  20 × 6
     39
    −36   ←   6 × 6
      3
```

2. This flow chart shows a way (called "short division") to find the quotient when the divisor is less than 10. Explain how to use this method to find one of the quotients above. See Discussion.



100

Discussion

Review the meaning of the terms *quotient, divisor,* and *dividend.* As you discuss exercise 1, you might write the example on the chalkboard and work through it step by step. Point out that longer methods of subtracting multiples of a divisor are possible, but stress the efficiency of the method shown in the text.

As you discuss exercise 2, emphasize that for this shortcut method they follow the same steps of estimating, multiplying, and subtracting as used in exercise 1, but they do the computation mentally, writing only the remainders. Have volunteers use this method for the problems in the investigation. Be sure children understand that this method is optional; do not stress its use if it is confusing to them. However, children who are able should try to use the shortcut.

Using the Ideas

Study this example. Then find the quotients and remainders below. Check your work.

Step 1

Think about **hundreds:** $16 \div 8 = 2$

$$8\overline{)1649} \quad 2$$

Step 2

Think about **tens:** There are 0 eights in 4. The remainder is 4.

$$8\overline{)1649} \quad 20$$

Step 3

Think about **ones:** $49 \div 8$ Quotient, 6 Remainder, 1

$$8\overline{)1649} \quad 206\ R1$$

Check

$206 \times 8 = 1648$

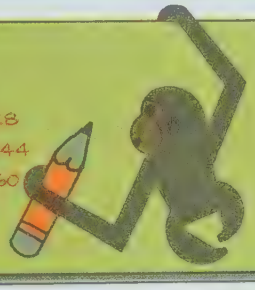
$1648 + 1 = 1649$

1. $2\overline{)568}$ 284
2. $5\overline{)755}$ 151
3. $4\overline{)2964}$ 741
4. $3\overline{)6389}$ 2129 R2
5. $7\overline{)6321}$ 903
6. $8\overline{)5642}$ 705 R2
7. $9\overline{)6584}$ 731 R5
8. $6\overline{)4224}$ 704
9. $4\overline{)3478}$ 869 R2
10. $8\overline{)6666}$ 833 R2
11. $7\overline{)432}$ 61 R5
12. $5\overline{)5305}$ 1061
13. $6\overline{)39\,872}$ 6645 R2
14. $9\overline{)65\,363}$ 7262 R5
15. $3\overline{)18\,654}$ 6218
16. $4\overline{)607}$ 151 R3
17. $7\overline{)9305}$ 1329 R2
18. $8\overline{)42\,603}$ 5325 R3
19. $9\overline{)55\,079}$ 6119 R8
20. $6\overline{)60\,704}$ 10117 R2

think

Some of the digits of this example are covered. Copy this example and give the missing digits.

$$\begin{array}{r} 43 \\ \times\ \blacksquare\blacksquare 8 \\ \hline \blacksquare\blacksquare\blacksquare 44 \\ \blacksquare\blacksquare\blacksquare 60 \\ \hline \blacksquare\blacksquare\blacksquare 4 \\ 120 \end{array}$$





101

Resources for Active Learning

Nuffield Project: *Computation and Structure* 3, "Simple Sharing," pp. 61–64, Wiley.

Duplicator Masters, page 15
Workbook, page 30
Skill Masters, page 15

Using the Exercises

With some groups of children, you will want to use the material at the top of page 101 as a basis for further discussion. If children are to check their answers, they must understand how to treat a remainder. Encourage those who are able, to use the shortcut method, but allow sufficient time for those who find one of the longer methods more suitable.

Children will probably enjoy the *Think* problem.

Assignments (page 101)

Minimum: 1–8. Average: 1–15.
Maximum: 1–20.

PAGES 102–103

Objective

Given a 3-digit dividend and a 2-digit divisor, the child will be able to find the quotient.

Preparation

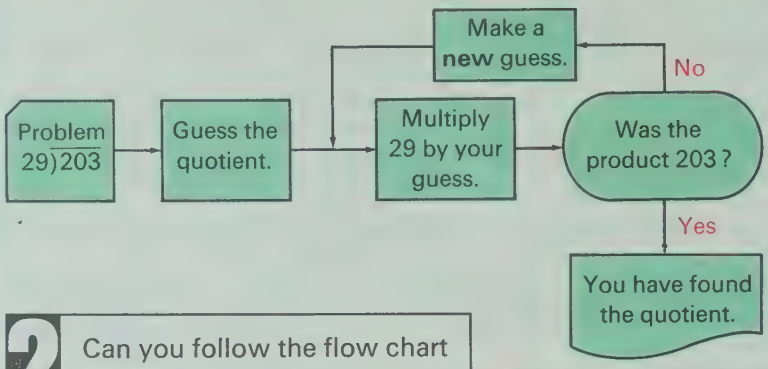
To prepare for this lesson, conduct an oral review of multiplying using multiples of 10. You might include in the review multiples used in the inequalities. For example, say something such as, what multiple of $10 \times 6 \leq 485$, or what multiple of $10 \times 8 \leq 372$. Remind the children that the symbol "$\leq$" is read *is less than or equal to*.

Investigation

It is intended that children work on this investigation independently. Be sure they understand how to follow the flow chart, particularly the step concerning what they should do if their product was not 203. You might suggest to those who finish quickly that they follow the same rules (except the last) for the problem $29\overline{)261}$ or $29\overline{)145}$.

Let's explore divisors less than 100.

Investigating the Ideas



? Can you follow the flow chart and find the quotient? 7

See Investigation.

Discussing the Ideas

1. How many estimates did you make before you found the quotient above?
See Discussion.

$n \times 30 \leq 243$? is the largest number for n.

2. Find the largest number for n in the diagram. Explain why 30 was used in this estimation. What is the remainder? 11

$$29\overline{)\begin{array}{r}?\ 8\\ 243\\ -232\\ \hline 11\end{array}}\ (?)$$

3. $28\overline{)126}$ 4 R14 $\quad 27\overline{)126}$ 4 R18 $\quad 26\overline{)126}$ 4 R22 $\quad 25\overline{)126}$ 5 R1

$31\overline{)267}$ 8 R19 $\quad 32\overline{)267}$ 8 R11 $\quad 33\overline{)267}$ 8 R3 $\quad 34\overline{)267}$ 7 R2

Since **30** is "close" to each of the divisors above, we usually use 30 to help estimate quotients in problems such as these. Explain how 30 is used to estimate the quotients above. Find each quotient and remainder. Explain your work.

4. State a general rule for choosing multiples of 10 to use in estimating quotients. See Discussion.

102

Discussion

Use discussion exercise 1 to elicit the children's explanation of what they did in the investigation. As you discuss exercise 2, point out that in the investigation the guesses they made for the number of twenty-nines in 203 were more or less random guesses; whereas, in exercise 2, by using 30 as a trial divisor and finding that the largest number of thirties in 243 is 8, they can arrive at a good estimate for the number of twenty-nines in 243. (The discussion exercises should show that this method of estimating the quotient, while not absolutely foolproof, is a helpful shortcut.)

When you go on to discussion exercise 3, point out that for a number ending in 5, they should round *up*. Note also that, in $126 \div 25$, rounding up to 30 gives an estimate that is too small. Observe that, in the last example in exercise 2 ($267 \div 34$), the estimate is too large if we round down to 30. The general rule that the children should state for exercise 4 is that, when rounding a 2-digit divisor, we round *up* if the ones' digit is 5, 6, 7, 8, or 9, and we round *down* if it is 1, 2, 3, or 4.

Using the Ideas

1. Find the number for n. Then use this estimate to help you find the quotient.

A 7 $n \times 40 \leq 294$ — $38\overline{)294}$ 7 R28

B 6 $n \times 60 \leq 372$ — $64\overline{)372}$ 5 R52

C 6 $n \times 30 \leq 208$ — $26\overline{)208}$ 8

2. Find the quotients and remainders. Check your work.

A $40\overline{)234}$ 5 R34	B $35\overline{)247}$ 7 R2	C $15\overline{)124}$ 8 R4	D $39\overline{)317}$ 8 R5
E $29\overline{)178}$ 6 R4	F $77\overline{)658}$ 8 R42	G $76\overline{)692}$ 9 R8	H $45\overline{)300}$ 6 R30
I $17\overline{)153}$ 9	J $52\overline{)467}$ 8 R51	K $39\overline{)210}$ 5 R15	L $44\overline{)246}$ 5 R26
M $13\overline{)85}$ 6 R7	N $38\overline{)293}$ 7 R27	O $23\overline{)107}$ 4 R15	P $85\overline{)637}$ 7 R42
Q $50\overline{)371}$ 7 R21	R $60\overline{)500}$ 8 R20	S $94\overline{)758}$ 8 R6	T $12\overline{)113}$ 9 R5

3. Find the divisor for each exercise. The quotient is given.

A 9 $\overline{)648}$	B 6 $\overline{)564}$	C 7 $\overline{)294}$	D 3 $\overline{)177}$	E 1 $\overline{)504}$
72	94	42	59	504

★ **4.** For each exercise, find one number that can serve as both quotient and divisor.

A 10 $\overline{)100}$	B 12 $\overline{)144}$	C 20 $\overline{)400}$	D 30 $\overline{)900}$
E 15 $\overline{)225}$	F 25 $\overline{)625}$		

think

Since it takes the planet Mercury 88 of our days to travel once around the sun, a "year" on Mercury is 88 of our days. If a person were 10 years old, give his age in "Mercury years." Use 365 days for an "Earth year."

Over 41 "Mercury years" old

More practice, page A-6, Set 11



Using the Exercises

With some groups, you will want to work through several of the exercises on page 103 together. With more able children, assign these exercises as independent work, and have them explain exercise 1 when they have finished. In either case, help the children see that the estimate found by using 40 in place of 38 in part A turns out to be correct. In part B, when we round 64 down to 60 to get our estimate, the quotient is too large. And in part C, when we round 26 to to 30 to arrive at the estimate, the estimate turns out to be too small. Emphasize that, even though we arrive at a quotient which is either too small or too large, we are still able to make a fairly good estimate and that we will, more often than not, arrive at the correct quotient.

To solve the *Think* problem, it is necessary first to figure out how many days old a ten-year-old is and then divide by 88 to determine how many "Mercury years" this is. The children will probably be amused by this since a child 10 years old on earth would be 41 "Mercury years" old.

Assignments (page 103)

Minimum: 1–2H. Average: 1–3. Maximum: 1–4.

Follow-up

Reconstruction problems might be used both for those children who had difficulty with this lesson and for faster children.

For those who found this lesson difficult, suggest that they find the missing numbers in these problems.

$15\overline{)84}$	$22\overline{)164}$	$84\overline{)637}$
7	14	8
9	10	49

For those who were able to do starred exercise 4, suggest that they try to find one number that can serve as both quotient and divisor in the following problems. (Answers are in parentheses.)

(13) $\overline{)169}$	(27) $\overline{)729}$	(18) $\overline{)324}$

Resources for Active Learning

Franklin Series: *Making and Using Graphs and Nomographs*, "Nomographs and Division," pp. 79–81, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Duplicator Masters, page 16
Workbook, page 31
Skill Masters, page 16

PAGES 104–105

Objective

Given a division problem which has a 2-digit divisor and a 2-digit quotient, the child will be able to find the 2-digit quotient by estimating and using the long division algorithm.

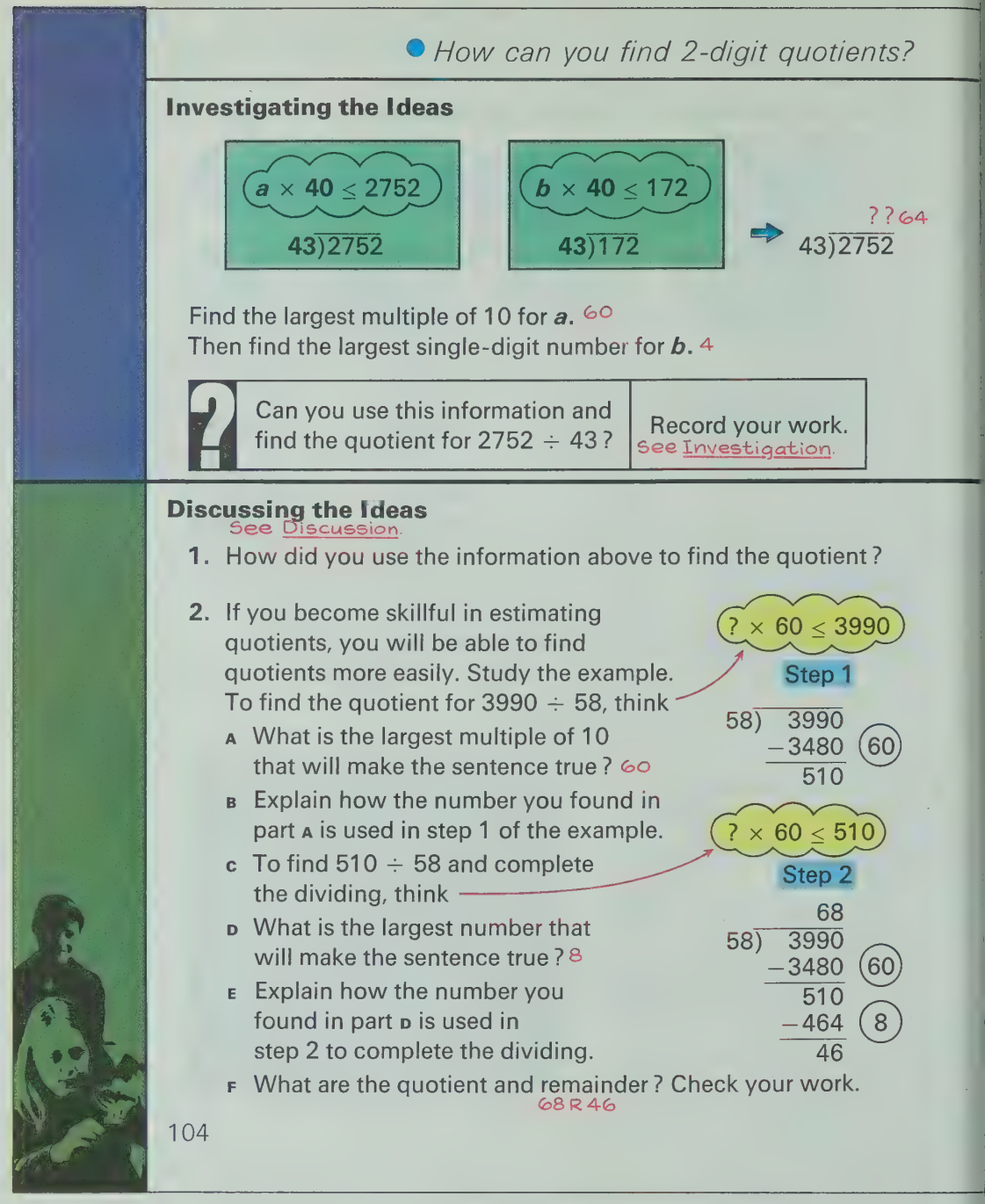
Preparation

To prepare for this lesson, it would be helpful to review inequalities with multiples of 10. For example, write the following on the chalkboard and ask children to give the largest multiple of 10 which could replace n in each:

$n \times 20 \leq 554$
$n \times 40 \leq 3251$
$n \times 50 \leq 3875$
$n \times 30 \leq 1062$
$n \times 70 \leq 5763$

Investigation

Although some children will have difficulty in estimating quotients and shortening their long division work, this investigation attempts to help children realize that proper use of estimation is the most efficient way of reducing the number of steps in solving a division problem. You might want some groups of children to work on this investigation with one or two other classmates. However, if the children have had a good foundation in estimating, you might prefer to have them work independently. As children work on the problem, you might want to remind them that the basic process of subtracting groups of the divisor still applies; the information gained from the inequalities should simply make that method easier.



● *How can you find 2-digit quotients?*

Investigating the Ideas

$a \times 40 \leq 2752$ → $43\overline{)2752}$

$b \times 40 \leq 172$ → $43\overline{)172}$

→ $43\overline{)2752}$??64

Find the largest multiple of 10 for ***a***. 60
Then find the largest single-digit number for ***b***. 4

Can you use this information and find the quotient for 2752 ÷ 43?	Record your work. See Investigation.

Discussing the Ideas

See Discussion.

1. How did you use the information above to find the quotient?

2. If you become skillful in estimating quotients, you will be able to find quotients more easily. Study the example. To find the quotient for 3990 ÷ 58, think

   ? × 60 ≤ 3990 — Step 1

   ```
      58) 3990
        −3480   (60)
          510
   ```

   A What is the largest multiple of 10 that will make the sentence true? 60

   B Explain how the number you found in part A is used in step 1 of the example.

   C To find 510 ÷ 58 and complete the dividing, think

   ? × 60 ≤ 510 — Step 2

   D What is the largest number that will make the sentence true? 8

   ```
        68
      58) 3990
        −3480   (60)
          510
         −464   (8)
           46
   ```

   E Explain how the number you found in part D is used in step 2 to complete the dividing.

   F What are the quotient and remainder? Check your work. 68 R 46

Discussion

Encourage several children to give their explanations for exercise 1. Stress with the children the convenience of making accurate estimates. Note that in Step 1 of the example in exercise 2, the quotient figures are not written in correct place-value positions above the dividend. That is, in Step 1, we did not write 6 above the 9 in the tens' place in the division box. Rather, we estimated that there are as many as 60 fifty-eights in 3990, and wrote 60 out to the side. Then we estimated 8 more multiples of 58 and wrote 8 out to the side. After subtracting, we finally added 60 and 8 to get 68 as the quotient, with a remainder of 46.

The main point here is that we first made an accurate estimate of 60, and then we subtracted 60 fifty-eights. Finally, we estimated 8 more fifty-eights and subtracted to get a remainder which is less than 58. We reemphasize that we were not working with quotient figures but, rather, were thinking of the underlying division concepts. When shortcut methods are introduced, we will write the 6 on top of the division box in our first step.

Use other examples to help children see how to find the quotient after estimating the first multiple.

Using the Ideas

1. When you estimate quotients, you will find that your estimate does not always give the correct number. Often your estimate will be too large, and you will have to choose a smaller number and try again. Other times your estimate will be too small, and you will have to subtract an extra time. Each of these cases is illustrated in the examples below. In each exercise, find the largest multiple of 10 for n. Then use this estimate and find the quotient.

A 20 $n \times 50 \leq 1395$ → 31 → 45)1395

B 90 $n \times 10 \leq 936$ → 72 → 13)936

C 30 $n \times 30 \leq 900$ → 33 R9 → 27)900

2. Find the quotients and remainders. Check your work.

A 87 R17 → 19)1670
B 73 R31 → 35)2586
C 32 R3 → 15)483
D 33 R26 → 77)2567
E 16 R3 → 52)835
F 32 R10 → 17)554
G 69 R33 → 43)3000
H 69 R11 → 56)3875
I 92 R11 → 31)2863
J 67 R5 → 13)876
K 78 R5 → 16)1253
L 10 R20 → 98)1000
M 11 R25 → 79)894
N 13 R2 → 12)158
O 77 R5 → 85)6550
P 50 R7 → 18)907

3. Find the quotients and remainders.

A 800 → 5)4000
B 16 R20 → 30)500
C 833 R5 → 7)5836
D 8888 R8 → 9)80,000
E 7 R60 → 62)494
F 3750 → 8)30,000
G 8 R60 → 80)700
H 50 R8 → 11)558
I 1989 → 3)5967
J 21 R13 → 16)349

think

A rock dropped from the top of a tall building will fall about:
$4.9 \times$ ² m in sec
$4.9 \times$ ² m in sec
$4.9 \times$ ² m in sec
$4.9 \times$ ² m in sec
(use any number for n.)
How many seconds (to the nearest second) would it take a rock to fall from the top of a building 176.4 metres tall?





More practice, page A-6, Set 12

Resources for Active Learning

Franklin Series: *Making and Using Graphs and Nomographs*, pp. 83–86, Lyons and Carnahan [Estimating products and missing factors]. (Available from McGraw-Hill Ryerson)

Duplicator Masters, page 17
Skill Masters, page 17

Using the Exercises

Before assigning the exercises on page 105, use exercise 1 to emphasize the fact that, even though rounding the divisor and finding an estimate of how many times it is contained in the dividend will not always yield the best estimate, it is certainly a tremendous help.

Encourage the children to use a trial-and-error approach to the *Think* problem. The children may want to try six seconds, seven seconds, and so on, in order to arrive at a height that is close to 176.4 metres. (Many inaccuracies occur in a problem such as this because of wind resistance, but they need not be pointed out or discussed unless the children raise the point.)

Assignments (page 105)

Minimum: 1–2H. Average: 1, 2. Maximum: 1–3.

PAGES 106–107

Objective

Given a list of numbers, the child will be able to find their arithmetic mean or average.

Preparation

If you choose to use a preparation for this lesson, you might give the children some mental chain games such as the following:

"Start with 8. . . Add 7. . . Divide by 3. . . Multiply by 6. . . Add 2. . . Divide by 4. . . What's your answer?" (8)

"Start with 9. . . Add 5. . . Divide by 2. . . Multiply by 8. . . Subtract 2. . . Divide by 9. . . What's your answer?" (6)

"Start with 40. . . Divide by 5. . . Add 2. . . Multiply by 13. . . What's your answer?" (130)

"Start with 15. . . Add 5. . . Divide by 5. . . Multiply by 6. . . Divide by 3. . . What's your answer?" (8)

You might also give children a chance to practice adding three and four numbers mentally. For example, say:

"Add: 5 plus 9, plus 7, plus 3. What's your sum?" (24)

"Add: 7 plus 6, plus 5, plus 4. What's your sum?" (22)

"Add: 8 plus 7, plus 9, plus 6. What's your sum?" (30)

Investigation

Before children begin the investigation, be sure they see how the substitution that was made in example A did not change the sum, 21. For those who find the substitutions quickly, you might write a few more lists, such as the following, on the chalkboard:

$$\begin{array}{c} ? \quad ? \quad ? \\ 7 + 5 + 6 = 18 \end{array}$$

$$\begin{array}{c} ? \quad ? \quad ? \\ 9 + 10 + 8 = 27 \end{array}$$

$$\begin{array}{c} ? \quad ? \quad ? \\ 22 + 22 + 24 = 66 \end{array}$$



Can you compute the average?

Investigating the Ideas

A $\begin{array}{c} 7 \quad 7 \quad 7 \\ 4 + 8 + 9 = 21 \end{array}$

B $\begin{array}{c} ? \quad ? \quad ? \\ 5 + 9 + 10 = 24 \end{array}$ 8

C $\begin{array}{c} ? \quad ? \quad ? \quad ? \\ 5 + 12 + 6 + 13 = 36 \end{array}$ 9

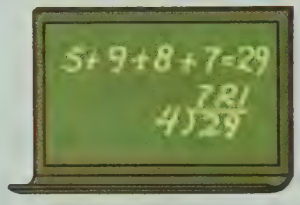
D $\begin{array}{c} ? \quad ? \quad ? \quad ? \\ 20 + 24 + 32 + 36 = 112 \end{array}$ 28

In example A, when 7 is substituted for each addend, the sum is the same as before.

? Can you find a number that will substitute for each addend and give the same sum for each of the other examples? See above.

Discussing the Ideas

1. Since 7 will substitute for each addend and give the same sum in example A, we say that 7 is the **average** (or **arithmetic mean**) of 4, 8, and 9. How did you find the average of 20, 24, 32, and 36? See Discussion.
2. Give the missing words. Then give an example to explain what the following sentence means. To find the arithmetic mean of a set of numbers, divide the __? __ of the numbers sum by the __? __ of addends. number
3. The chalkboard shows Tom's work in finding the arithmetic mean of 4 numbers.



   A If you substitute 7 for each addend, will the sum be the same? No

   B Can any other whole number be substituted for each addend to give a sum closer to 29? We say that the arithmetic mean of 5, 9, 8, and 7 is 7 (to the nearest whole number). No
4. Explain how to find the average of 9, 4, 7, 8, and 5 (to the nearest whole number).
   33÷5=6R3, so average is 7 (to the nearest whole number).

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Discussion

The *average*, or *arithmetic mean*, of a set of numbers may be thought of as a single number that "represents" the numbers in the set. Although children might have found the averages of the sets of numbers in the investigation by examination, use examples to develop the generalization treated in discussion exercise 2. For example, use some addends and sums from the investigation (such as 20 + 24 + 32 + 36 = 112) and have volunteers explain that to find the average they may simply divide 112 by 4, since the number of addends was 4.

In the discussion of the work in exercise 3, be sure to emphasize for the children that we can actually work with arithmetic mean (average) even though the quotient does not have a zero remainder. For a given set of numbers, we can find the arithmetic mean to the nearest whole number.

Sports Stories

1. Jack played 9 holes of golf.
   - A What was his total score? 45
   - B What was his average score for each hole? 5

Score Card					
Hole	1	2	3	4	5
Score	6	4	5	3	5
Hole	6	7	8	9	Total
Score	7	6	5	4	

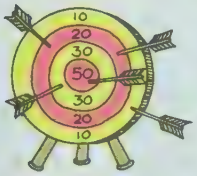
NAME	Score			
	1	2	3	Total
Nancy	108	131	118	
Carol	114	121	98	

2. Nancy and Carol each bowled three games.
   - A Find Nancy's total score. 357
   - B Find Nancy's average score. 119
   - C Find Carol's total score. 333
   - D Find Carol's average score. 111

3. Joe read that a player on the Montreal Expos baseball team averaged one hit for every three times at bat. If he was at bat 294 times, how many hits did he make? 98



4. What is the average score for the arrows on the archery target? 26



5. A Find the average score for the home team. 54

Basketball Scores						
Home	49	50	61	44	68	52
Opponent	52	44	46	45	42	51

   - B Find the opponents' average score to the nearest whole point. 47



Using the Exercises

Before assigning the exercises on page 107, give the children an opportunity to discuss the various score cards. Following this discussion, have the children do the exercises. When they have finished, allow time for further discussion of scores and for checking papers.

As an extension of this set of word problems, you might find it helpful to provide the children with additional practice in finding the average of a given set of numbers to the nearest whole number.

Assignments (page 107)

Minimum: 1–3, oral. Average: 1–5. Maximum: 1–5.

Follow-up

Although the children can practice averaging spelling scores and grades, they might prefer games that offer especially good opportunities for finding averages. A tossing game can be made from the bottom halves of egg cartons. Write the numerals 1 through 12 at random on the back edge of each section, so that they are visible from about 10 m away. Supply each player with 6 bottle caps, wooden beads, or (when they become more adept) table-tennis balls to toss at the carton, and an index card for score-keeping. Ask each player to find his average score per round and then his best round per game.

If you have four games being played simultaneously, the teams can compare team scores and determine high and low individual averages and high and low team scores. If interest is keen, allow the children to play the game several times and keep cumulative averages.

Resources for Active Learning

Mathematics in Modules, S2, Addison-Wesley.

Mathex: Graphing and Probability No. 6, "Statistics," pp. 26–29, Encyclopaedia Britannica Publications Ltd.

Workbook, page 32

PAGES 108–109

Objective

Given problems based on information in accompanying charts, the child will be able to interpret the information on the chart and solve the problems by applying his knowledge of the basic operations.

Preparation

For these pages, no preparation period is necessary. You might refer immediately to the presentation in the text.

Solving Story Problems

The sea contains millions of cubic kilometres of water. This water contains billions of plants and animals and tons of minerals. The sea is one of the great natural resources for water, food, power, and valuable minerals.

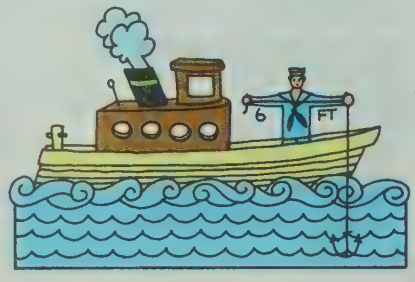
Ocean	Approximate Area (in square kilometres)
Pacific	178 154 000
Atlantic	81 631 000
Indian	73 414 000
Antarctic	14 838 000
Arctic	14 084 000





1. How many square kilometres larger is the Antarctic Ocean than the Arctic Ocean ?
754 000

2. The total area of Earth's surface is 509 903 500 square kilometres. Use the table above to find the total surface area covered by oceans. About how many square kilometres of Earth's surface is covered by land ?147 782 500



3. Sailors used to measure the depth of the ocean by dropping a weighted rope overboard. As they pulled it up, they measured by using their outstretched arms as a unit.
The unit was about 1.8 metres long and was called a **fathom.**
   - A The deepest part of the ocean is 10.8 kilometres. How many fathoms ?6 000
   - B The sea is 4.14 km deep at the North Pole. How many fathoms ?
2 300

108

Discussion

Read and study the material at the top of the page with the children. Encourage them to discuss the various facts presented here. Following this discussion, assign the exercises.

Notice that all the basic operations and the concept of averages are treated on this page. You might write some problem-solving guidelines on the chalkboard for children who find problems difficult:

What do I know? (What information is given in the problem or chart?)

What must I find? (Difference? How many times as . . . ? Average?)

What do I do? (What operation or operations are necessary: add? subtract? multiply? divide?)

Does my answer make sense? (Reread the problem; does this answer really answer the question in the problem?)

Children will benefit from a discussion of their solutions. Also, keep in mind as the children do these exercises that an important part of a lesson like this is the interest which it may stimulate. Encourage the children to discuss the various ideas in each of the problems and, perhaps, to do additional research on their own to find out other facts about the oceans.

Depth (in metres)	Temperature
180	16°C
360	10°C
540	7.2°C
720	5.6°C
900	4.4°C
1080	3.9°C
1260	3.3°C
1440	2.8°C
1520	2.2°C

4. A device called a bathythermograph is used to record underwater temperatures. The chart shows temperatures recorded every 180 metres in a certain spot in the ocean. At greater depths the temperature usually stays about the same, from 0° to 1.7°C. Give the average of the temperatures in the table. 6° (to the nearest whole number)

5. A cubic metre of sea water contains about 20.7 kilograms of salt. Salt is often purchased in 750-gram containers. How many of these containers could be filled with the salt from a cubic metre of sea water? 27.6 or 27½ containers



6. Waves as high as 34.2 metres have been reported on the "high seas." If each floor of a building is 4.2 m tall, the wave would be as tall as a building with how many floors? 8



A cubic kilometre of sea water contains about 12.6 kilograms of gold.

A How many cubic kilometres of water would be needed to have 5292 kilograms of gold? 420 km^3

★ B 1 kg of gold will make a thin wire about 3500 km long. The distance around the earth is 40 068 km. With the gold from 1 cubic kilometre of sea water, how many kilometres of wire would you have left after you wrapped it around the earth once? 4 032 km

Follow-up

A good way to follow up this lesson would be to have the children write some story problems of their own. For example, you might have them write some story problems using the information given in the chart for exercise 4, or you might have some children look up some additional information concerning the oceans and use it as the basis for writing their own story problems.

Duplicator Masters, pages 18, 19
Skill Masters, page 18

Assignments (pages 108–109)
Minimum: 1–4, oral.
Average: 1–7A. Maximum: 1–7.

PAGES 110–111

Objective

Given a division problem which has a 2-digit divisor, the child will be able to find the quotient by using the long division algorithm.

Preparation

To prepare for this lesson, you might work through a few division problems with a single-digit quotient. For example, ask a volunteer to solve a problem such as $42\overline{)148}$. You might also review rounding 2-digit numerals to the nearest multiple of ten.

Investigation

The purpose of this investigation is to help children relate the three problems with single-digit quotients to the longer problem with the 1-digit quotient. Move around the room, observing the children as they work and, if necessary, hint that they should look at the first three digits of the larger dividend, and then at the dividend of problem A. For those who finish quickly, you might suggest another problem, such as $68\overline{)45\,726}$. You might also suggest that they check each of the problems in the investigation.

Let's explore some shortcuts.

Investigating the Ideas

A $39\overline{)225}$ 5R30 B $39\overline{)307}$ 7R34 C $39\overline{)346}$ 8R34

Find each of these single-digit quotients and the remainders.

? Can you use the quotients you found above to help you find the quotient for $39\overline{)22576}$? 578 R34

Discussing the Ideas

1. How did you use the one-digit quotients to find the larger quotient in the Investigation? See Discussion.

2. You can find any quotient, no matter how large, by finding the one-digit quotients for a series of simpler problems. Study this example.

A Dividing hundreds

$$\begin{array}{r} 2 \\ 53\overline{)\,13038} \\ -\,106 \\ \hline 24 \end{array}$$

B Dividing tens

$$\begin{array}{r} 24 \\ 53\overline{)\,13038} \\ -\,106 \\ \hline 243 \\ -\,212 \\ \hline 31 \end{array}$$

C Dividing ones

$$\begin{array}{r} 246 \\ 53\overline{)\,13038} \\ -\,106 \\ \hline 243 \\ -\,212 \\ \hline 318 \\ -\,318 \end{array}$$

A Which three "simpler" division problems were solved in order to find the quotient 13 038 ÷ 53? 130÷53, 243÷53, 318÷53

B Why are the steps in the example labelled "dividing hundreds," "dividing tens," and "dividing ones"? Because in Step A, for example, we are actually dividing 130 hundreds by 53.

3. It is important to be able to find one-digit quotients quickly. Explain how the heavy black numerals can help you estimate the quotient. See Discussion.

60 7R23 $59\overline{)436}$

Discussion

Use exercise 1 to have volunteers explain what they did in the investigation. It would be helpful to use an overhead projector while discussing the problem. Relate each of the single-digit quotient problems to the following steps:

1. Dividing hundreds

$$\begin{array}{r} 5 \\ 39\overline{)\,22576} \\ -\,195 \\ \hline 30 \end{array}$$

2. Dividing tens

$$\begin{array}{r} 57 \\ 39\overline{)\,22576} \\ -\,195 \\ \hline 307 \\ -\,273 \\ \hline 34 \end{array}$$

3. Dividing ones

$$\begin{array}{r} 578 \\ 39\overline{)\,22576} \\ -\,195 \\ \hline 307 \\ -\,273 \\ \hline 346 \\ -\,312 \\ \hline 34 \end{array}$$

It might help to accompany the discussion of this exercise with an exercise using the long method, in which the children write the quotient at the side. Work through exercise 2 similarly. Use exercise 3 to stress the importance of estimating to find single-digit quotients.

Using the Ideas

1. Use the heavy black numerals to help you estimate the quotients.

A $12\overline{)49}$ 4 B 60 $59\overline{)267}$ 4 C $71\overline{)573}$ 8 D 20 $18\overline{)173}$ 9

E 70 $65\overline{)435}$ 6 F $34\overline{)197}$ 6 G 50 $46\overline{)358}$ 7 H $83\overline{)746}$ 9

2. Find the quotients.

A $61\overline{)26\,535}$ 435 B $18\overline{)2232}$ 124 C $49\overline{)25\,780}$ 526 R6 D $93\overline{)22\,150}$ 238 R16

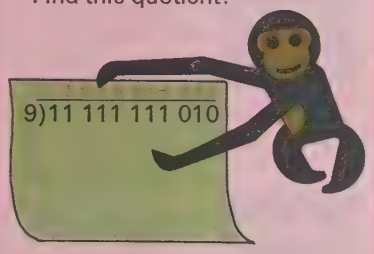
E $17\overline{)14\,235}$ 837 R6 F $85\overline{)9642}$ 113 R37 G $34\overline{)21\,537}$ 633 R15 H $56\overline{)30\,764}$ 549 R20

I $58\overline{)25\,734}$ 443 R40 J $23\overline{)13\,156}$ 572 K $72\overline{)17\,784}$ 247 L $89\overline{)64\,547}$ 725 R22

M $35\overline{)34\,721}$ 992 R1 N $14\overline{)13\,482}$ 963 O $47\overline{)35\,967}$ 765 R12 P $53\overline{)20\,511}$ 387

think

Find this quotient:



What interesting pattern do you see?

3. Find these special quotients. Why are they special?

Each quotient is the same as the divisor.

A $32\overline{)1024}$ 32 B $15\overline{)225}$ 15 C $25\overline{)625}$ 25 D $49\overline{)2401}$ 49

4. Find these quotients. What pattern do you see?

A $32\overline{)736}$ 23 B $74\overline{)3478}$ 47 C $51\overline{)765}$ 15 D $59\overline{)5605}$ 95

Each quotient has the same digits as the divisor but in reverse order.

More practice, page A-7, Set 13



Resources for Active Learning

A Cloudburst, Vol. 1, No. 1421, Midwest Publications [Patterns in division].

Mathematics in Modules, WN21, Addison-Wesley.

Duplicator Masters, page 20
Workbook, page 33
Skill Masters, page 20

Using the Exercises

With some groups of children, you will want to work through parts of exercises 1 and 2 together. Allow others to work independently, but discuss some of the exercises as you have them check their work. For example, in exercise 1, note with the children that we are attempting to work out some shortcuts for making estimates. In example A, we think $4 \div 1$ rather than $49 \div 10$. In example B, we think $26 \div 6$ rather than $267 \div 60$. In example C, we think $57 \div 7$ rather than $573 \div 70$. Be sure to emphasize for the children the fact that this is simply a method of thinking which will help shorten the work in estimation.

Assignments (page 111)

Minimum: 2A–H. Average: 1–2. Maximum: 1–4.

PAGES 112–113

Objective

Given a division problem involving a 2-digit divisor and a quotient with a zero in one place, the child will be able to find the quotient.

Preparation

To prepare for this lesson, you might review estimating single-digit quotients with 2-digit divisors. For example, display problems such as the ones below and review how to round the divisor to make an estimate for the quotient.

$61\overline{)437}$ $19\overline{)128}$ $47\overline{)392}$

Let's work with zeros in the quotient.

Discussing the Ideas

See Discussion.

1. A Explain how to estimate the number of **hundreds** in the quotient. Think 140 ÷ 70 or 14 ÷ 7.
   B Explain the work shown in step 1.
   C Explain how you can tell that there are 0 **tens** in the quotient. 49 < 68
   D Explain how to estimate the number of **ones** in the quotient. Think 490 ÷ 70 or 49 ÷ 7
   E Explain step 3.
   F What is the quotient? 207
   What is the remainder? 20
   G Explain how to check your work. Find the product 207 × 68 and add 20 to see whether the result equals the dividend.

2. A Explain how to estimate the number of **hundreds** in the quotient. Answers will vary.
   B Explain step 1.
   C Explain how to estimate the number of **tens** in the quotient. Answers will vary.
   D Explain step 2. 11 < 14
   E Explain how you can tell that there are 0 **ones** in the quotient.
   F Give the quotient and remainder. 350 R11
   Check the work.

1

Step 1 Dividing hundreds	2 68) 14096 −136 4
Step 2 Dividing tens	20 68) 14096 −136 49
Step 3 Dividing ones	207 68) 14096 −136 496 −476 20

2

Step 1 Dividing hundreds	3 14) 4911 −42 7
Step 2 Dividing tens	35 14) 4911 −42 71 −70 1
Step 3 Dividing ones	350 14) 4911 −42 71 −70 11



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Discussion

As you use these exercises to discuss zeros in the quotient, it may be helpful to have the children work out these exercises using the long method (writing the quotients down the side), so that it will become apparent to them why, in example 1, there are 0 tens and in example 2 there are 0 ones. In other words, for example 1, if the children worked the problem the long way, they would have written at the side 200 and, under that, 7; in example 2, they would have written 300 and, under that, 50. If the children thoroughly understand the ideas presented in the last lesson, they should quickly see that in example 1, when they divide tens, there are 0 tens in the quotient; and in example 2, when they divide ones, there are 0 ones in the quotient.

Using the Ideas

1. Find the quotients and remainders. Check your work.

A $68\overline{)20\,740}$ 305 B $32\overline{)23\,685}$ 740 R5 C $45\overline{)13\,500}$ 300 D $17\overline{)8615}$ 506 R13

E $18\overline{)3726}$ 207 F $31\overline{)8990}$ 290 G $56\overline{)3000}$ 53 R32 H $64\overline{)52\,300}$ 817 R12

I $5\overline{)80\,000}$ 16,000 J $19\overline{)3847}$ 202 R9 K $55\overline{)35\,862}$ 652 R2 L $17\overline{)5246}$ 308 R10

2. Study the example. Then complete the exercises.

Dividing thousands	Dividing hundreds	Dividing tens	Dividing ones
2	24	241	2415
37)89355	37)89355	37)89355	37)89355
74	74	74	74
15	153	153	153
	148	148	148
	5	55	55
		37	37
		18	185
			185

A $29\overline{)64\,932}$ 2239 R1 B $61\overline{)129\,684}$ 2125 R59 C $18\overline{)40\,000}$ 2222 R4 D $42\overline{)326\,572}$ 7775 R22

E $38\overline{)74\,987}$ 1973 R13 F $23\overline{)176\,238}$ 7662 R12 G $57\overline{)64\,593}$ 1133 R12 H $75\overline{)246\,483}$ 3286 R33

3. Study the examples. Then complete the exercises.

A
$$431\overline{)2586}\ 6$$
$$-2586$$

B
$$486\overline{)2670}\ 5 \quad (500)$$
$$-2430$$
$$240$$

C
$$450\overline{)1655}\ 3 \quad (500)$$
$$-1350$$
$$305$$

A $200\overline{)1800}$ 9 B $600\overline{)3000}$ 5 C $205\overline{)1562}$ 7 R127 D $595\overline{)2534}$ 4 R154

E $410\overline{)1640}$ 4 F $823\overline{)7407}$ 9 G $675\overline{)4350}$ 6 R300 H $621\overline{)1972}$ 3 R109

★ I $423\overline{)10\,152}$ 24 ★ J $678\overline{)36\,612}$ 54 ★ K $237\overline{)17\,785}$ 75 R10

More practice, page A-7, Set 14



Using the Exercises

Assign parts of the exercises on page 113 according to the needs of the children. You will not want to assign all of the exercises to children who find division difficult; and those children who have caught on quickly to the method for long division may be wasting their time by working every exercise on this page.

You should note that in exercise 2 the division algorithm is extended to produce a 4-digit quotient and that in exercise 3 it is extended to using a 3-digit divisor. Emphasize for those children who are assigned parts of exercises 2 and 3 and that the basic ideas are the same, but that they are dealing with larger numbers.

When the children have completed the assigned exercises, you will find it helpful to have several presented on the chalkboard and explained. Those exercises which are not used as a part of this lesson can be used later to help the children review the division algorithm from time to time.

Assignments (page 113)

Minimum: 1. Average: 1, 2.
Maximum: 1–3.

Duplicator Masters, page 21
Workbook, page 34
Skill Masters, page 21

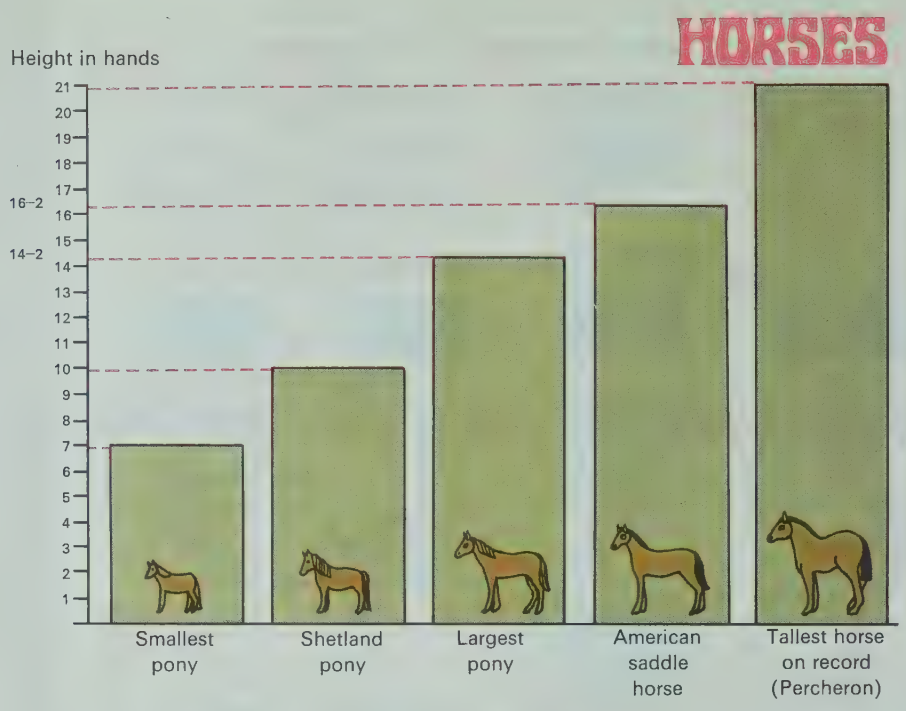
PAGES 114–115

Objective

Given word problems which require use of the basic operations, the child will be able to solve the problems by using the appropriate algorithms.

Preparation

To prepare for this lesson, you might review some basic equivalents or remind the children about the Tables of Measures in the back of the book. They will need a few standard measures for the problems.



Horses are measured by using a unit called a **hand** (10.2 centimetres). A horse's height is measured from the ground to the highest point of the ridge between its shoulder bones. If a horse is 16 hands and 2 centimetres tall, the height can be written 16-2.

1. Give the height (in cm) of each horse mentioned above. 71.4; 102; 144.8; 165.2; 214.2



2. Give the height (in metres) of each horse shown above. 0.714; 1; 1.4; 1.7; 2.1

3. Give the average height (in cm) of the horses mentioned above. Use the numbers you found in exercise 1. 139.5 (to the nearest whole number)

4. The tallest horse on record weighed 1358.5 kilograms. This is 9 times as much as a small Shetland pony. Give the weight of the Shetland pony (to the nearest kilogram). 151 kg

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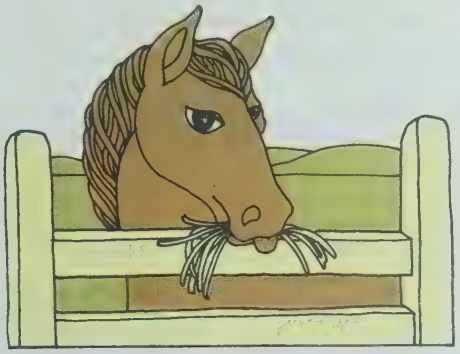
Discussion

Study the chart at the top of the page with the children. Help them see that they can interpret the pictures as they would interpret a bar graph. Discuss the fact that horses are measured according to *hands*, a unit that is equivalent to 10.2 cm. You might have the children measure the width of their hands, as illustrated in the figure, to see how close the width of their hand is to 10.2 cm. Observe with the children that the 14-2 beside the height of the largest pony indicates 14 $\frac{2}{4}$ hands, or 14 hands and 5.1 centimetres.

For some groups, you might want to point out the problem-solving guidelines that were suggested on page 108. Others might need more specific help, particularly with some of the problems on page 115. Note that exercises 10 and 11 are intended primarily for the more able children. When the children finish these problems, allow sufficient time to discuss them and present the solutions to a few on the chalkboard.

Assignments (pages 114–115)

Minimum: 1–9, oral. Average: 1–9. Maximum: 1–11.



Type of horse	Weight (kg)
Pony	225
Saddle horse	480
Light harness horse	516
Heavy harness horse	620
Draft horse	984

This table gives 5 different types of horses and the weight of one horse of each type.

5. Find the average of the weights given in the table. 565 kg (to the nearest whole number)

6. It has been said that a horse should be fed 0.5 kg of grain per day for each 50 kg it weighs. If this rule is followed, about how many kilograms of grain should be given each day to the draft horse whose weight is given in the table above? 9.8 kg

7. The average life span of man, 69 years, is 3 times the average life span of a horse. What is the average life span of a horse? 23

8. A near-record high jump for a horse is 2.5 metres. A near-record high jump for a man is 224 centimetres. How much higher is the horse's jump? 0.26 m

9. A light horseshoe weighs 225 g. Give the total weight in kilograms of the horseshoes needed to shoe a team of 6 horses. 5.4 kg

★ **10.** Fast horses can run about 62 km/h for a short time. If a horse runs at a rate of 48 km/h how far does it run in one minute? 0.8 kg per minute

★ **11.** A strong horse can lift 15 000 kg 30 centimetres high in 1 minute. An engine that can do as much work as one strong horse is called a **1-horsepower** engine. How many kilograms can a 34-h.p. engine lift 30 centimetres in 1 minute? 510 000

Follow-up

A group of children might be interested in looking up information about dogs and comparing that information with the data about horses presented in this lesson.

DOGS*

Heaviest
St. Bernard
116 kg
(86 cm at shoulder)

Tallest
Irish Wolfhound
(100 cm at shoulder)

Fastest
Saluki
69 km/h

Oldest
Labrador, Collie } 27 years each

Smallest
Chihuahua
454 g

"Highest Flyer"
Laika in Sputnik II
1690 km

*Records are approximate and subject to dispute.

PAGES 116–117

Objective

Given multiplication and division problems involving money, the child will be able to solve the problems.

Preparation

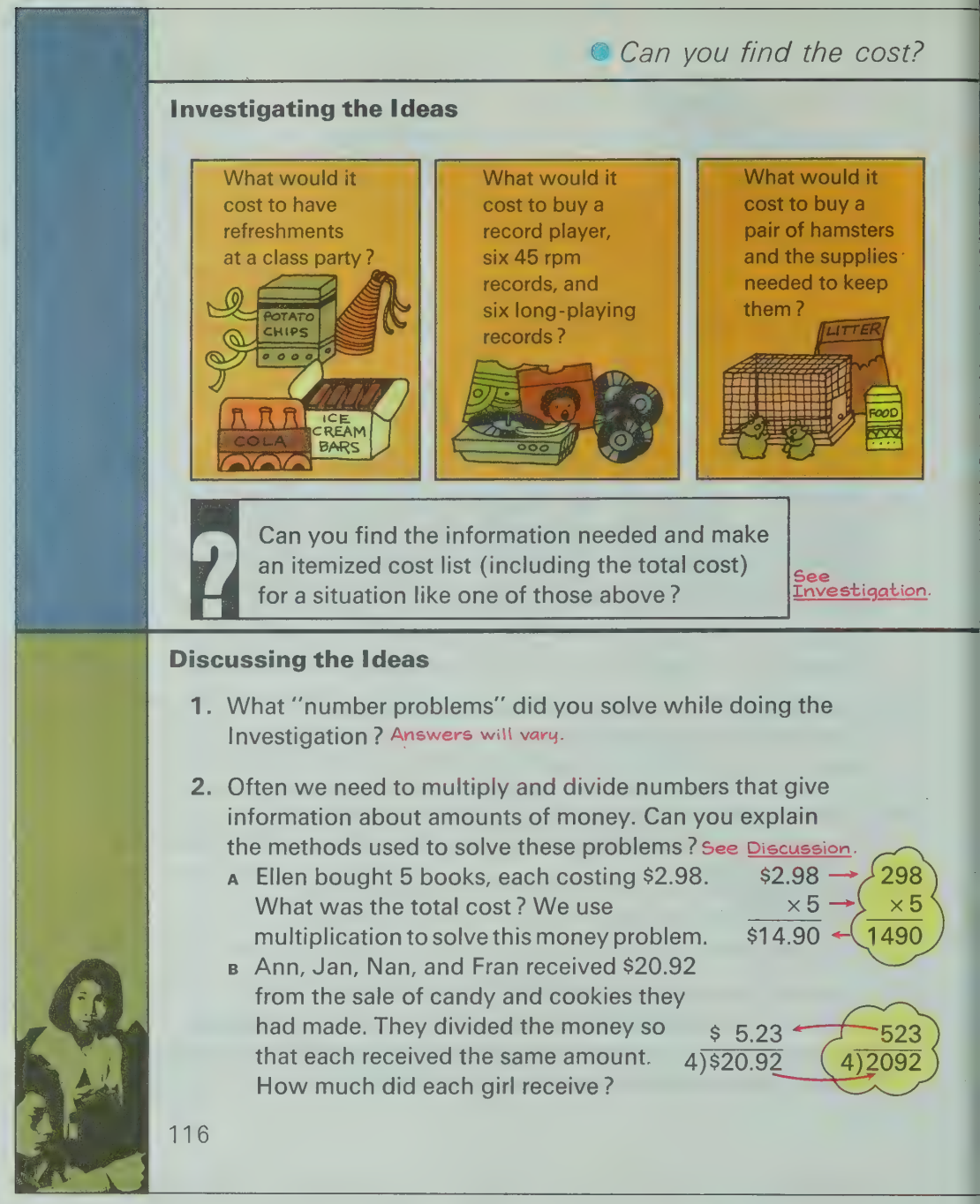
Materials

catalogues; newspaper advertisements

To prepare for this lesson (and with awareness of the children's sensitivities), you might ask them to name some things they would like to buy and to suggest what they think the item would cost.

Investigation

If necessary, adapt the suggested situations to your locality. You might find it suitable to provide catalogues and advertisements for children to use in making a cost list for a situation they make up. With some groups, you might want to supply on the chalkboard some of the information regarding the cost of items, rather than have them try to find it.



Can you find the cost?

Investigating the Ideas

What would it cost to have refreshments at a class party?

What would it cost to buy a record player, six 45 rpm records, and six long-playing records?

What would it cost to buy a pair of hamsters and the supplies needed to keep them?

? Can you find the information needed and make an itemized cost list (including the total cost) for a situation like one of those above?

See Investigation.

Discussing the Ideas

1. What "number problems" did you solve while doing the Investigation? Answers will vary.

2. Often we need to multiply and divide numbers that give information about amounts of money. Can you explain the methods used to solve these problems? See Discussion.

   A Ellen bought 5 books, each costing $2.98. What was the total cost? We use multiplication to solve this money problem.

   $2.98 → 298
   × 5 → × 5
   $14.90 ← 1490

   B Ann, Jan, Nan, and Fran received $20.92 from the sale of candy and cookies they had made. They divided the money so that each received the same amount. How much did each girl receive?

   $ 5.23 ← 523
   4)$20.92 → 4)2092

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Discussion

While you are using the situations presented in the investigation as a basis for discussion, point out the correct money notation. Emphasize that it is proper to work with this money notation just as we have worked with whole-number notation in the previous work in this chapter. Encourage the children to think about the numbers in these problems just as they have thought about the whole numbers in previous algorithms. The only difference here is that when they get their answers they have to reconvert them to money notation. For example, as you discuss exercise 2A, stress to the children that instead of writing the multiplication in a second step without money notation, they may shorten their work simply by thinking about the numbers 298 and 5 when multiplying, and then write money notation for the answer they found. Similarly, in exercise 2B, instead of writing the step in the "cloud" they can write $4\overline{)\$20.92}$ and think about the number 2092 as they divide; finally, they should carefully convert the quotient, 523 back into money notation, $5.23.

Using the Ideas

1. Find the answers to these money problems.

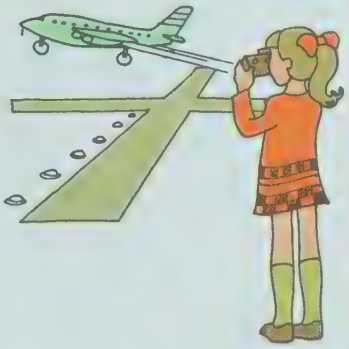
A $\$6.75 \times 3 = \$20.25$ B $\$0.88 \times 6 = \$5.28$ C $\$8.99 \times 8 = \$71.92$ D $\$0.35 \times 23 = \$8.05$ E $\$1.39 \times 365 = \$507.35$

F $5)\$2.95 = \$.59$ G $7)\$44.38 = \$6.34$ H $30)\$87.00 = \$2.90$ I $65)\$22.10 = \$.34$

J $\$0.05)\$1.00 = 20$ K $\$0.50)\$7.50 = 15$ L $\$0.75)\$17.25 = 23$ M $\$0.25)\$20.00 = 80$

2. Solve the problems.

A A jet plane carried 84 passengers from San Francisco to New York. The fare with tax was $160.90. How much money did the airline collect? $13,515.60



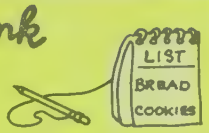
B Jane took 20 pictures. 4 of the pictures were overexposed and were not developed. It cost $3.84 to develop the other pictures. What was the cost of each developed picture? $ 0.24

C Linda works at the school popcorn stand. She needs some coins in order to make change. How many quarters can she get for a $10 bill? 40

D A new bike cost $56.00. Jim received $12.50 for his old bike and paid the remainder in 6 payments. How much was each payment? $7.25

★ **3.** A hamburger and a bag of potato chips cost $0.50. Two hamburgers and a bag of potato chips cost $0.85. What is the cost of a bag of potato chips? $ 0.15

think



A dozen cookies and a loaf of bread cost $0.56. Half a dozen cookies and 2 loaves of bread cost $0.58. What is the cost of a loaf of bread? $0.20



Resources for Active Learning

A Cloudburst, Vol. 2, No. 8393, Midwest Publications.

Duplicator Masters, page 22

Using the Exercises

Assign the exercises on page 117 as independent work, and emphasize again for the children that this work with money notation is essentially the same as their previous work with multiplication and division of whole numbers.

Encourage more able children to try the *Think* problem. They might reason like this:

"Since a dozen cookies and a loaf of bread cost 56 cents, and half a dozen cookies and two loaves of bread cost 58 cents, I know that one loaf of bread costs as much as half a dozen cookies and 2 extra cents.

"I can think of the dozen cookies and loaf of bread which cost 56 cents as 3 half-dozen cookies and 2 cents. So, 3 half-dozen cookies would cost 54 cents, or 18 cents for a half dozen. But I already found that a loaf of bread costs as much as a half a dozen cookies and 2 cents; therefore, the loaf of bread would cost 20 cents."

This is a difficult problem, so children should not be discouraged if they cannot solve it.

Assignments (page 117)

Minimum: 1. Average: 1, 2. Maximum: 1–3.

PAGES 118–119

Objectives

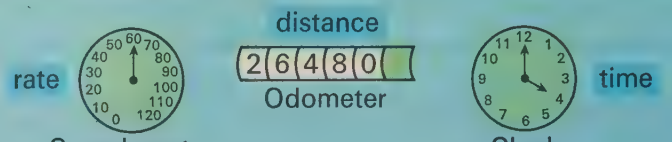
Given word problems in which two of the three measures—rate, time, and distance—are given, the child will be able to find the missing measure by multiplying or dividing.

Given word problems in which n *represents an unknown sum, difference, product, or quotient, the child will have an opportunity to use his reasoning and computational skills to find* n.

Preparation

It would be helpful to review some basic concepts concerning time, rate, and distance. For example, you might give the children an opportunity to discuss some simple time, rate, and distance problems concerning automobile travel. Have the children answer questions such as, "If you travel 40 km per hour for two hours, how far would you go?" In doing such a problem, point out to the children that 40 km per hour is called the rate, two hours is called the time, and the answer, how far, is called the distance; and we speak of this as a time, rate, and distance problem.

Solving Short Story Problems



Speedometer Clock

Give the number for ***r*** (rate), ***d*** (distance), or ***t*** (time).

1. WALK

A Slow: 3km. 3 hours. ***r*** km/h. 1

B Average: 5 km/h. 5 km. Walk for ***t*** hours.

C Fast: 1 hour. 8 km/h. 1 Walk ***d*** kilometres. 8



2. BICYCLE

A Slow: 3 hours. 15 kilometres. ***r*** km/h. 5

B Average: 16 km/h. 32 km. Ride for ***t*** hours. 2

C Fast: 36 km/h. 2 hours. Go ***d*** kilometres. 72



3. CAR

A Slow: 36 km/h. 180 km. Takes ***t*** hours. 5

B Average: 6 hours. 528 km. ***r*** km/h. 88

C Fast: 222 km/h. 3 hours. Must go ***d*** more kilometres to finish the 800 kilometre race. 134

4. SHIP

A Slow: 168 km. 7 hours. ***r*** km/h. 24

B Average: 52 km/h. 468 km. Takes ***t*** hours. 9

C Fast: 65 km/h. 76 hours. Travels ***d*** kilometres. 4940

5. PLANE

A Slow: 912 km. 6 hours. Travels ***r*** km/h. 152

B Average: 620 km/h. 4340 km in ***t*** hours. 7

C Fast: 2440 km/h. 4 hours. Travels ***d*** kilometres. 9760

6. SATELLITE

A Average (in Earth's orbit): 28 232 km/h. 112 928 4 hours. Goes ***d*** kilometres.

B Fast (escaping Earth's orbit): 315 364 km. 8 hours. ***r*** km/h. 39 420.5



★**7.** EARTH

Travels 952 million kilometres in about 365 days. Travels ***r*** km/h (rounded to nearest thousand). 109 000

118

Discussion

Study the chart at the top of the page with the children. Discuss the measuring devices given. You may need to remind the children that rate, given here by the speedometer, indicates km per hour. You also may need to point out that rate need not always be given in km per hour but can be given using any unit of length and any unit of time. For example, we might give the rate in kilometres per minute, and so on. The distance indicated here is km and the time, depending upon the various hands of the clock, can be interpreted in seconds, minutes, or hours. Also be sure they understand that the abbreviations, *r*, *t*, and *d* are used for rate, time, and distance respectively.

Assignments (page 118)

Minimum: 1–3. Average: 1–6. Maximum: 1–7.

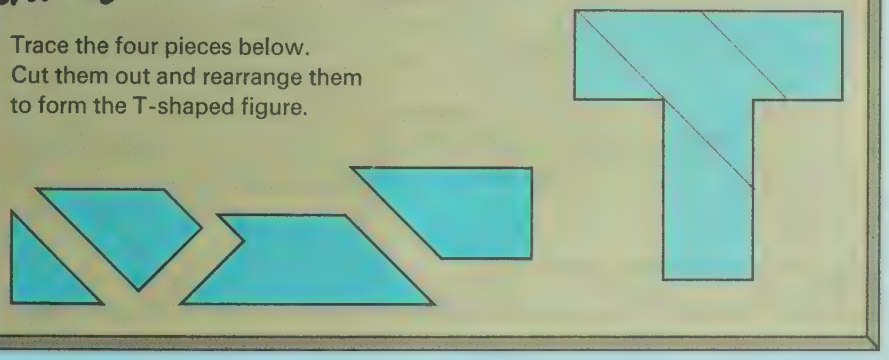
Solving Problems

Find the number for ***n*** in each exercise.

1. ***n*** is 232 more than 649. 881
2. ***n*** is 358 less than 914. 556
3. ***n*** is 913 more than the sum of 432 and 956. 2301
4. ***n*** is 387 less than the sum of 390 and 423. 426
5. ***n*** is 786 more than the difference of 3142 and 787. 3141
6. The sum of 862 and ***n*** is 1465. 603
7. The product of 47 and 12 is 519 less than ***n***. 1083
8. The product of 86 and 35 is 1946 more than ***n***. 1064
9. The product of 37 and 29 is half as large as ***n***. 2146
10. The product of 46 and 54 is twice as large as ***n***. 1242
11. The product of 72 and 24 is 9 times as large as ***n***. 192
12. The product of 15 and 15 is 25 times as large as ***n***. 9
13. The product of 86 and ***n*** is 602. 7
14. The product of 72 and 4 is equal to the product of 32 and ***n***. 9
15. The product of 40 and 7 is equal to the product of 56 and ***n***. 5
16. ★ The product of 28 and 54 is 6 times as much as the product of 36 and ***n***. 7

think

Trace the four pieces below.
Cut them out and rearrange them to form the T-shaped figure.



119

Resources for Active Learning

Applied Mathematics Cards, Group 2/12–13; Group 3/11–13, Schofield and Sims. (Available from Mafex Associates)

Math Activity Cards, E27, Macmillan.

Mathex: Measurement No. 10, pp. 22–24 (pupil pages 60–62), Encyclopaedia Britannica Publications Ltd.

Maths Mini-lab, Cards 83, 84, 94–96, Selective Educational Equipment.

Measure and Find out, Book 1, Activities 15–17; Book 2, Activities 1/3, 8; 2/5–6, Scott Foresman. (Available from Gage Educational)

Workbook, page 35

Using the Exercises

You might want to work some of the exercises on page 119 with the children. Help them to recognize phrases which can be symbolized mathematically. For example, in exercise 10 they can simply replace each phrase with a mathematical expression:

"The product of 46 and 54" (46×54) "is" $(=)$ "twice as large as n" $(2 \times n)$, or $46 \times 54 = 2 \times n$.

Thus, they should be able to find n by applying their understanding of basic operations.

To help children solve the *Think* problem, you might trace the figure exactly and duplicate it for the children. This *Think* is a puzzle which is a classic dissection problem. Do not offer strategies for solution. However, see that the children get a chance to manipulate the pieces before the solution is given.

Assignments (page 119)

Minimum: 1–9. Average: 1–15. Maximum: 1–16.

PAGES 120–121

Objective

The child will demonstrate his ability to work with the concepts presented in this chapter.

Preparation

Review any of the material from this chapter which the children have found particularly difficult. It might be helpful to provide some children with a variety of addition, subtraction, multiplication, and division exercises to be presented on the chalkboard and explained to the entire class.

Reviewing the Ideas

1. Find the sums, products, differences, and quotients.

A $\begin{array}{r} 5764 \\ +3847 \\ \hline 9611 \end{array}$ B $\begin{array}{r} 861 \\ -463 \\ \hline 398 \end{array}$ C $\begin{array}{r} 3947 \\ -1698 \\ \hline 2249 \end{array}$ D $\begin{array}{r} 653 \\ \times 25 \\ \hline 16325 \end{array}$

E $876 + 39 + 5483 + 9$ 6407 F $683 + 4765 + 98 + 17$ 5563

G $7\overline{)6478}$ 925 R3 H $7004 - 39$ 6965 I 37×56 2072 J $50\overline{)3076}$ 61 R26

K $\begin{array}{r} 566 \\ \times 207 \\ \hline 117162 \end{array}$ L $\begin{array}{r} 8070 \\ -4293 \\ \hline 3777 \end{array}$ M $\begin{array}{r} 308 \\ \times 72 \\ \hline 22176 \end{array}$ N $\begin{array}{r} 6003 \\ -5786 \\ \hline 217 \end{array}$

O $62\overline{)434}$ 7 P $79\overline{)635}$ 8 R3 Q $35\overline{)2205}$ 63 R $74\overline{)18\,944}$ 256

S $57\overline{)874}$ 15 R19 T $16\overline{)12\,675}$ 792 R3 U $92\overline{)556\,947}$ 6053 R71 V $486\overline{)3786}$ 7 R384

2. Give the missing numbers.

A $4^2 =$ ▮ 16 B $5^3 =$ ▮ 125 C $6^4 =$ ▮ 1296 D $3^5 =$ ▮ 243

3. Find the arithmetic mean of each set of numbers.

A {562, 385, 419, 646} 503

B {88, 107, 365, 419, 136} 223

4. Find the total amounts.

A $\begin{array}{r} \$18.75 \\ 26.99 \\ \hline \$45.74 \end{array}$ B $\begin{array}{r} \$23.64 \\ 8.98 \\ \hline \$32.62 \end{array}$

5. Find the differences in the amounts.

A $\begin{array}{r} \$65.50 \\ 65.37 \\ \hline \$.13 \end{array}$ B $\begin{array}{r} \$20.00 \\ 9.34 \\ \hline \$10.66 \end{array}$

6. Solve these money problems.

A $\begin{array}{r} \$3.49 \\ \times 4 \\ \hline \$13.96 \end{array}$ B $\begin{array}{r} \$0.87 \\ \times 25 \\ \hline \$21.75 \end{array}$ C $9\overline{)\$21.06}$ $\$2.34$

120

think

1
1 1 ← Level 1 2
1 2 1 ← Level 2 4
1 3 3 1 ← Level 3 8
1 4 6 4 1 ← Level 4 16
1 5 ? ? ? 1 ← Level 5 32
1 6 15 20 15 6 1 ← Level 6 64

The array above is called Pascal's Triangle. The red arrows will help you see how to give the numbers for levels 5 and 6. Make a table of the sums of the numbers, level by level, 1 through 6. Predict the sums of the numbers in levels 7 and 8, and check your prediction.

Level 7: 128
Level 8: 256

Discussion

If you use these two pages as a chapter test or as a group review, you may not want to assign all of the exercises to all the children. Certainly, some children will require a considerable amount of time to do all these exercises, and the faster children, in many cases, will not need this much practice. Allow time for discussion of the exercises when they have had the opportunity to complete the page.

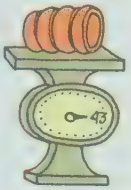
Starred exercises 11 and 12 should challenge some of the more able children. If time permits, you might have them explain their solutions to the class. Encourage faster children to try the *Think* problem when they finish the assigned exercises.

7. A piece of meat weighs 2.3 kg.

A It weighs how many grams less than 3 kilograms? 700 g

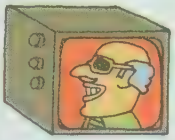
B What is the total cost of 3 kg of meat at $2.79 per kilogram? $8.37

C 2 kg of another type of meat cost $6.16. How much more or less does this meat cost per kilogram than the meat in part B? $0.29 more per kg



8. A plane flew at an average speed of 342 kilometres per hour for 7 hours. How far did it fly? 2394 km

9. Mr. Smith bought a television set. The cost plus interest charges was $340.08. If he pays for it in 12 equal payments, how much will each payment be? $28.34



10. Here is part of Jim's record of the time it takes him to deliver papers each day. He spent a total of 370 minutes delivering papers during this week. How long did it take him to deliver papers on Saturday? 75 min

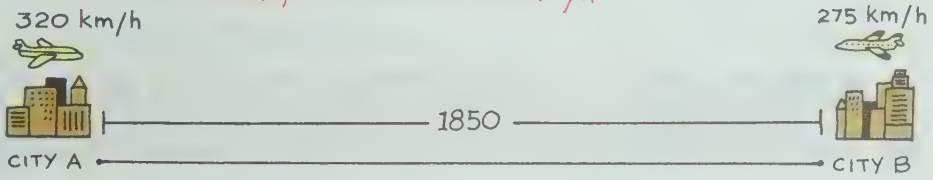
Days	M	T	W	Th	F	S
Minutes	55	65	65	60	50	?

★ **11.** The chart shows the number of minutes Sally practiced the piano during one week. How many minutes must she practice on Saturday in order to have practiced an average of 35 minutes a day? 50

Days	M	T	W	Th	F	S
Minutes	30	25	35	40	30	?

★ **12.** If the planes fly toward each other at the speed given, how far apart will they be after:

A 3 hours? 65 km/h B 4 hours? 530 km/h



121

Workbook, page 36

PAGES 122–123

Objective

The child will demonstrate his ability to work with the concepts presented for cumulative review.

Preparation

Review any of the material from previous parts of the text which the children have found particularly difficult. It may be helpful to place particular emphasis on a review of exponent meaning and function notation. For example, review notation such as that in exercise 5. You might use examples such as the following:

$(4 \times 10^2) + (3 \times 10) + 6 = 436$
$(3 \times 10^3) + (7 \times 10^2) + 9 = 3709$
$(8 \times 10^3) + (5 \times 10^2) + (9 \times 10) + 1 = 8591$

Keeping in Touch with

Addition
Subtraction
Multiplication
Division
Place value
Functions

1. Solve the equations.

A $n \times 6 = 24$ 4
B $q \times 9 = 54$ 6
C $t \times 7 = 63$ 9
D $8 \times t = 0$ 0
E $n \times 11 = 66$ 6
F $15 \times b = 75$ 5
G $8 \times q = 8$ 1
H $7 \times n = 49$ 7

2. Find the products.

A $10^1 \times 10^2$ 1000
B $10^2 \times 10^2$ 10 000
C $10^1 \times 10^3$ 10 000
D 50×80 4000
E $5 \times 10 \times 4 \times 10$ 2000
F $6 \times 10^2 \times 3 \times 10$ 18 000
G 90×600 54 000
H 800×400 320 000

3. Give the missing words and numbers.

A 438 614: The 8 in the __?__ place means 8 × ▮. thousands'; 1000
B 73 602 549: The 7 in the __?__ place means 7 × ▮. ten millions'; 10 000 000

4. Write the number that is

A 10 000 more than one million. 1 010 000
B 1 less than one million. 999 999

5. Write the ordinary base-ten numeral for each exercise.

A 8×10^2
B 6×10^3
C 70×10^1
D 80×10^2
E $(8 \times 10^3) + (3 \times 10^2) + (4 \times 10) + 6$
F $(7 \times 10^5) + (4 \times 10^2) + 3$

A 800 B 6000 C 700 D 8000 E 8346 F 700 403

6. Give the missing numbers for the table.

	Function Rule $n^2 + 5$	
	n	$f(n)$
A	6	▮ 41
B	10	▮ 105
C	0	▮ 5
D	7 ▮	54

★ **7.** Give the missing numbers in each row.

Sums +
Quotients ÷

A	12	▮ 9	3 ▮	3
B	12	▮ 10	2 ▮	5
C	18	▮ 9	9 ▮	1
D	5	▮ 0	5 ▮	0



You are invited to explore

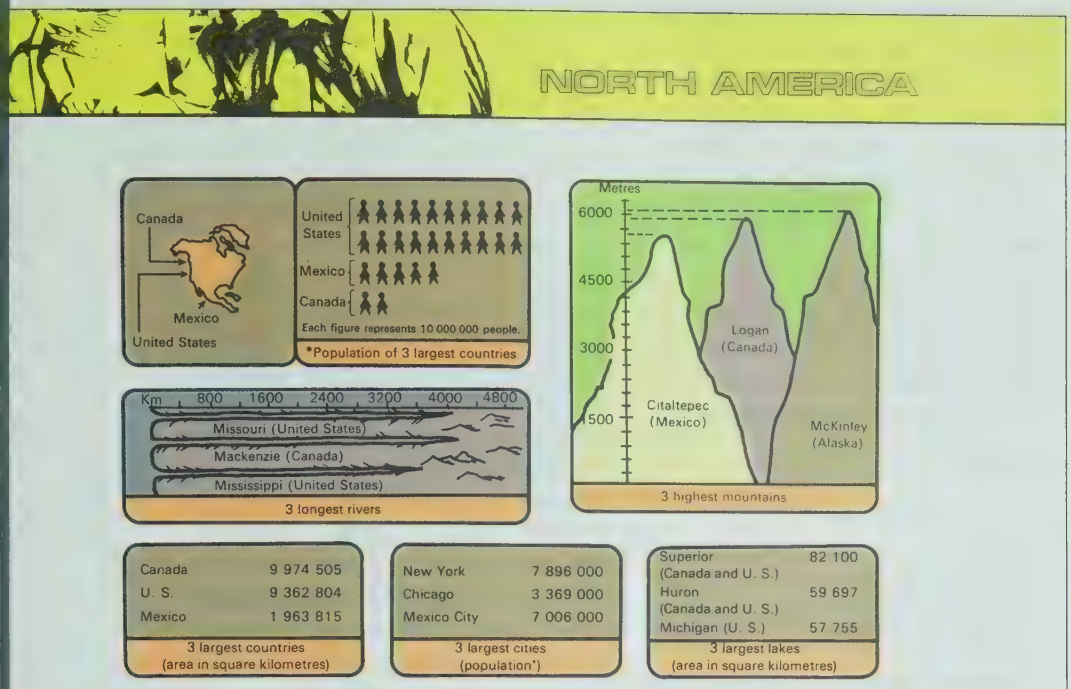
ACTIVITY CARD 5
Page 357

122

Discussion

The exercises on page 122 are suitable for use as independent work. However, you might prefer to work through several of the exercises as a class review. In either case, allow time for discussion and checking papers.

Exercise 7 should prove unusually stimulating for the more able children. As soon as the correct answers are given, even the slower children can understand that the sum $9 + 3$ is 12 and that the quotient $9 \div 3$ is 3. However, finding this number pair is certainly not as simple as understanding the idea when the pair has been found.



1. Use the graphs above to give See Answers, T.E. page 123.
   - A the populations of the 3 largest countries in North America (to the nearest 5 million people).
   - B the lengths of the 3 longest rivers (to the nearest hundred kilometres).
   - C the heights of the 3 highest mountains (to the nearest 50 metres).
   - D an estimate of the difference in height between Mt. McKinley and Mt. Logan.

2. Use the tables above to give See Answers, T.E. page 123.
   - A the areas of the 3 largest countries (rounded to the nearest 500 000).
   - B the populations of the 3 largest cities (rounded to the nearest million).
   - C the average area of the 3 largest lakes (to the nearest whole number).

3. The average number of persons per square kilometre in North America is about 12. The area of North America is 24 400 900 square kilometres. About what is the population of North America? 292 810 800

*Population figures based on 1970 census.

Using the Exercises

The charts at the top of page 123 provide material for an introductory discussion. Be sure to give adequate attention to the interpretation of the various graphs and charts given here. For example, in the chart showing populations of the three largest countries, help the children see that the stick figures shown beside each country can be counted to determine its population.

Ask the children to attempt to read the values from the two bar graphs showing the longest rivers and the highest mountains. Following this discussion, have the children do the exercises. When they have finished, allow time for further discussion and checking papers.

Follow-up

Encourage the children to do research on the latest census figures on population for the three largest cities, the three largest countries, and the density of population for North America, in current almanacs, encyclopaedias, or government publications. They can then compare these figures with those given on text page 123. Translating current data into graphs of various kinds will also give the class valuable experience.

Answers, exercises 1 and 2, page 123

1. A U.S. 200 000 000
Mex. 50 000 000
Can. 20 000 000

B Missouri 4100
MacKenzie 4200
Mississippi 3600

C Citlaltepec 5600
Logan 5900
McKinley 6100

D Any estimate between 200 m and 350 m would be acceptable.

2. A Can. 10 000 000
U.S. 9 000 000
Mex. 2 000 000

B New York 8 000 000
Chicago 3 000 000
Mexico City 7 000 000

C 66 517 sq. km. (to the nearest sq. km)

CHAPTER 6 | Number Theory

Pages 124–141

General Objectives

To provide experiences with factors and multiples
To introduce "least common" multiples
To introduce "greatest common" factors
To provide work with prime numbers
To provide experiences with union and intersection of sets
To provide work with composite numbers
To develop skills in finding the prime factorization of a composite number
To provide an opportunity to explore clock (modular) arithmetic

The first lesson of Chapter 6 deals with finding the factors of a number and develops simple rules for determining when 2, 3, or 5 are factors of a number. The second lesson introduces factor trees. This is followed by work with prime numbers and union and intersection of sets, in preparation for the material on greatest common factor and least common multiple. The next pages concentrate on developing skill in finding greatest common factors and least common multiples. After this, operations and some basic principles for clock numbers are explored. Story problems and cumulative and chapter reviews complete the chapter.

Mathematics

Consider the following definition.

> If a, b, and c are whole numbers such that $a \times c = b$, then a and c are whole-number factors of b.

Prime numbers may be defined as follows:

> The set of prime numbers is the set of all whole numbers x, such that x has exactly two factors.

Thus:

2 is a prime number (factors: 2, 1).
3 is a prime number (factors: 3, 1).
5 is a prime number (factors: 5, 1).
7 is a prime number (factors: 7, 1).
11 is a prime number (factors: 11, 1).

Note that:

0 is not prime (Every number is a factor of zero).
1 is not prime (factor: 1).
6 is not prime (factors: 1, 2, 3, 6).
91 is not prime (factors: 1, 7, 13, 91).

Every number greater than 1 that is not prime is a product of one and only one collection of prime numbers. Such numbers are called *composite numbers*. For example, $6 = 2 \times 3$; $8 = 2 \times 2 \times 2$; $9 = 3 \times 3$; $100 = 2 \times 2 \times 5 \times 5$; and so on. Expressing a composite number as a product of primes is called the prime factorization of a number. The fact that each composite number can be expressed as a unique product of primes (without regard to order) is called the Fundamental Theorem of Arithmetic. It is also referred to as the Unique Factorization Theorem.

Closely related to the idea of the factors of a number is the idea of the common factors of two numbers and the greatest common factor of two numbers. The greatest common factor of two numbers is the largest number in the set of common factors of the two numbers. In arithmetic, the most important application of greatest common factor is in changing fractions to lowest terms; that is, if a fraction is not in lowest terms, it can be changed to lowest terms by dividing the numerator and the denominator by the greatest common factor of the two numbers.

Consider the following definition of the multiples of a number.

> If a and b are whole numbers and c is the product of a and b, then c is said to be a multiple of a (and of b).

Although zero is a multiple of every whole number, we frequently fail to include the number zero in listing multiples of a given number. When we speak of the least common multiple of two numbers, we mean the smallest number other than zero that is a multiple of both numbers. The concept of least common multiple is valuable to the study of fractional numbers, since the least common multiple of the denominators of two fractions is the least common denominator for the fractions. Understanding this is useful in finding the sum of two fractional numbers.

The following examples illustrate the way in which greatest common factor and least common multiple are used. Consider the fraction $\frac{20}{30}$. Ten is the greatest common factor of the two numbers 20 and 30, and by dividing both 20 and 30 by 10, we change the fraction $\frac{20}{30}$ to $\frac{2}{3}$. Now, suppose we wish to add the numbers $\frac{3}{4}$ and $\frac{1}{6}$. We see that 12 is the least common multiple of the numbers 4 and 6. Hence, we select 12 as our common denominator, writing $\frac{3}{4}$ as $\frac{9}{12}$, and $\frac{1}{6}$ as $\frac{2}{12}$. Then we can add the fractions $\frac{9}{12}$ and $\frac{2}{12}$ to find $\frac{11}{12}$, the fractional number which is the sum of $\frac{3}{4}$ and $\frac{1}{6}$.

Number systems with a limited number of elements, such as the one introduced on pages 136 and 137 in this chapter, are called modular systems, or, sometimes, modular arithmetic.

The children's experience with modular systems developed in this chapter differs from their previous experiences because the sets of objects used here contain a limited rather than an unlimited number of members. For example, the set of numbers for the "twelve-clock" contains exactly 12 elements, while

the set of whole numbers contains an unlimited number of elements. Likewise, the "five-clock" contains only 5 elements rather than an infinite number.

Basically, the same operations (with modifications) that are employed with whole numbers are used with modular systems, and the children are led to discover that the same basic principles also hold for this new set of numbers. As illustrated by the "five-clock," the number of elements in modular systems may be greater or less than 12.

Teaching the Chapter

Materials

Counters or other suitable objects for counting

Display grid, 10 rows by 10 columns, approximately three metres square on acetate for overhead projector (for Sieve of Eratosthenes)

Overhead projector (if available) and transparencies

Vocabulary

clock (modular) arithmetic
clock numbers
common factor
common multiple
composite number
factor
factor tree
greatest common factor
intersection of sets
least common multiple
prime factor
prime number
union of two sets

The majority of the words in the vocabulary list are vital to the communication necessary for development of the topics in this chapter. However, most of these words will be quite familiar to those children who studied Book 5 of this series. You need not stress the mastery of the terms union and intersection of sets, since these words can be relearned when they are used later in the development of other topics. These terms are included in the vocabulary list because the ideas are used in finding common multiples and common factors of two numbers.

Lesson Schedule

A week and a half to two weeks should be adequate time for this chapter, although you will want to adjust your schedule to the needs and abilities of your children. If you have an above-average group of children, you may be able to cover the chapter in less time. However, since an above-average group of children often spends more time on stimulating topics, such as number theory, it is difficult to set a definite limit for them. If the children become extremely interested in these topics, you should by all means allow some extra time for them to spend on the ideas. On the other hand, if you have an especially slow class, you may find it necessary to spend extra time on the chapter. We would urge you, however, to attempt to limit class time on the chapter to two weeks.

Evaluation of Progress

The children's mastery of the topics in the chapter can be evaluated partly from their achievement on tests for skills in the following: determining whether a given number is a factor of another number, determining whether a number is a prime, constructing a factor tree, and finding the greatest common factor of two numbers or the least common multiple of two numbers. We urge you not to base your entire evaluation on these somewhat mechanical skills. Through day-to-day observation, you should give particular attention to the children's overall understanding and ability to reason through the ideas when they are introduced.

Pages 138 through 141 supply review material to help you evaluate the children's retention of concepts and, also, their progress in understanding the new material introduced in this chapter.

Resources for Active Learning

General Activities

Franklin Series: *Making and Using Graphs and Nomographs*, "Nomographs," pp. 19–24, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Nuffield Project: *Computation and Structure* 3, "Factors and Primes," pp. 67–69, Wiley

Commercial Games

Domino Number Games, Factors and Multiples (Heath)

On-Sets (Hammett; Nasco; Wff 'N Proof)

Operational Systems Games—Union and Intersection, Multiples and Divisors, Clock Arithmetic (Available from McGraw-Hill Ryerson)

Ranko—white cards (Midwest Publications)

PAGES 124–125

Objective

Given a number which is a product of one of the basic multiplication facts, the child will be able to find its factors.

Preparation

Materials

countable objects—toothpicks, counters, slips of paper, or the like (30 per child)

To prepare for this lesson, it would be helpful to review the meaning of equivalent sets. For example, you might draw the following sets on the chalkboard.

× × × × × + + + +

○ ○ ○ ○ – – –

1 1 1 1 1 * * * * *

Ask children which of these sets are equivalent. Recall with them that equivalent sets are sets which have the same cardinal number; in other words, if both sets have the same number of elements, the sets are equivalent. (Equal sets are sets which have identical elements.) Thus, for the sets illustrated above, the set of plus signs is equivalent to the set of circles and the set of ×'s is equivalent to both the set of ones and the set of asterisks.

Investigation

If you do not have 30 objects for for each child, you might have the children work in small groups. Be sure to have the children record their findings. You might suggest that the example in the book would be recorded as 5 sets of 6. The other possibilities are:

1 set of 30	3 sets of 10
30 sets of 1	10 sets of 3
2 sets of 15	6 sets of 5
15 sets of 2	

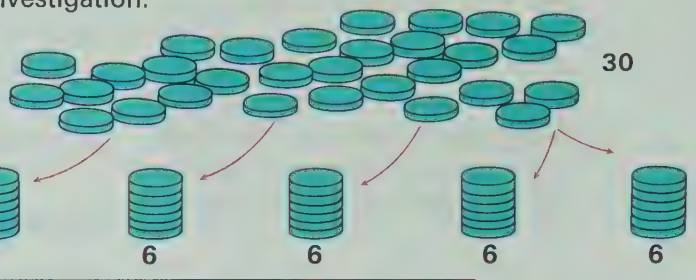
If some children finish very quickly, suggest that they try the same activity with 12 objects.

6 Number Theory

When is one number a factor of another?

Investigating the Ideas

Use a set of 30 objects (counters, toothpicks, slips of paper) for this Investigation.



? How many ways can you find to divide the set into equivalent sets?

See Investigation.

Discussing the Ideas

1. The numbers of the equivalent sets you found in the Investigations are **factors** of 30. How many of the factors of 30 can you give?

2. Solve the equations in the table to find the factors of 36.

	Factor		Factor		Product
A	36*n*	×	1	=	36
B	18*n*	×	2	=	36
C	12*n*	×	3	=	36
D	9*n*	×	4	=	36
E	6*n*	×	6	=	36

3. Find the quotient and remainder.

A $2\overline{)36}$ 18 B $3\overline{)36}$ 12 C $4\overline{)36}$ 9

D $5\overline{)36}$ 7 R1 E $6\overline{)36}$ 6 F $7\overline{)36}$ 5 R1

4. If a number is divided by one of its factors, what can you say about the quotient and remainder? The quotient will be another one of its factors and the remainder will be zero.



Discussion

As you discuss exercise 1, help the children realize the meaning of the term *factor*. The numbers 30, 1, 15, 2, 10, 3, 5, and 6 are all factors of 30. As is brought out in exercise 4, these numbers can also be described by saying that 30 can be divided by any one of its factors and the quotient will be a whole number; that is, the remainder will be zero. As you discuss exercises 2 and 3, you might explain that, when the product of two whole numbers is 36, we say that each of the numbers is a factor of 36. We have learned that the product divided by one factor gives the other factor. For example, since

$$\underset{\text{P}}{36} \div \underset{\text{F}}{2} = \underset{\text{F}}{18},$$

we say that 2 is a factor of 36, and 18 is a factor of 36.

Use pairs of factors to develop correct understanding of the terms factor and product. Encourage the children to express the fact that, in the equation $9 \times 4 = 36$, 9 and 4 are factors of 36, and 36 is the product of 9 and 4.

As you discuss exercises 3 and 4, help the children realize that they can find factors of a number by division. Thus, if they know that 7 is a factor of 42, they can

Using the Ideas

1. Solve the equations that have whole-number solutions.
Then list all the factors of 96.

A $n \times 1 = 96$ 96 F $n \times 6 = 96$ 16
B $n \times 2 = 96$ 48 G $n \times 7 = 96$ ✳
C $n \times 3 = 96$ 32 H $n \times 8 = 96$ 12
D $n \times 4 = 96$ 24 I $n \times 9 = 96$ ✳
E $n \times 5 = 96$ ✳ J $n \times 10 = 96$ ✳

✳ No whole-number solution
Factors of 96 : 1,2,3,4,6,8,12,16,24,32,48,96

think

Give the smallest number that has 2, 3, and 5 as factors. 30

Give the smallest 3-digit number that has 2, 3, and 5 as factors. 120

Give the smallest 4-digit number that has 2, 3, and 5 as factors. 1020

2. Solve the equations that have whole-number solutions.
Then list all the factors of 64.

A $64 \div 1 = n$ 64 C $64 \div 3 = n$ ✳ E $64 \div 5 = n$ ✳ G $64 \div 7 = n$ ✳
B $64 \div 2 = n$ 32 D $64 \div 4 = n$ 16 F $64 \div 6 = n$ ✳ H $64 \div 8 = n$ 8

✳ No whole-number solution Factors of 64 : 1,2,4,8,16,32,64

3. Is the first number a factor of the second?

A 3, 51 Yes B 5, 63 No C 8, 96 Yes D 7, 98 Yes E 3, 160 No F 9, 9 Yes

4. List all the factors of each number.

A 16 1,2,4,8,16 B 30 1,2,3,5,6,10,15,30 C 13 1,13 D 24 1,2,3,4,6,8,12,24 E 40 1,2,4,5,8,10,20,40 F 23 1,23 G 72 1,2,3,4,6,8,9,12,18,24,36,72

5. For each set, give the number (other than 1) that is a factor of each number in the set.

A {0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, . . .} 2
B {0, 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, . . .} 5

6. Study the examples below.

6 + 5 + 4 = 15 ← If 3 is a factor of this sum, then
6 5 4 ← 3 is a factor of this number.

2 + 1 + 5 + 6 = 14 ← If 3 is **not** a factor of this sum, then
2 1 5 6 ← 3 is **not** a factor of this number.

Which numbers in the set {282, 540, 4394, 555, 46 338, 5700, 369 050, 144, 7655, 549} have a factor of

A 2? B 5? C 3?

A 282; 540; 4394; 46 338; 5700; 369 050; 144
B 540; 555; 5700; 369 050; 7655
C 282; 540; 555; 46 338; 5700; 144; 549

125

find at least one other factor by dividing: $42 \div 7 = 6$. Also, if they want to find out if one number is a factor of another number, they again divide. For example, to find out if 6 is a factor of 72, they must divide: $72 \div 6$. Since $72 \div 6 = 12$, with remainder zero, 6 is a factor of 72.

Using the Exercises

Before assigning the exercises on page 125, be sure children realize that the phrase "equations that have whole-number solutions," refers to division in which the remainder is zero. In exercise 5, help children realize that any even number has 2 as a factor, and any number ending in 0 or 5 has 5 as a factor.

The *Think* problem allows children to use the generalizations developed in exercises 5 and 6.

Assignments (page 125)
Minimum: 1–4. Average: 1–6.
Maximum: 1–6.

Mathematics

If a and b are non-zero whole numbers and c is a whole number such that $a \times b = c$, then both a and b are said to be factors of the number c.

One significant point related to this definition of factors of a number is that, since neither a nor b is zero, we can apply the definition to division. Observe that a quotient with a remainder of zero must be possible if the divisor is a factor of the dividend. Therefore, division is probably the technique most commonly used to determine whether or not one number is a factor of another (for example, simply divide one by the other to see whether or not the remainder is zero).

Follow-up

Children might be interested in developing further generalizations for testing whether one number is a factor of another. Some other tests for divisibility are given below.

If a number is even and if 3 is a factor of the sum of its digits, then 6 is a factor of the number. [3126 is even, and 3 is a factor of the sum of the digits ($3 + 1 + 2 + 6 = 12$), so 6 is a factor of 3126.]

If 9 is a factor of the sum of the digits of a number, then 9 is a factor of that number. [711 054: 9 is a factor of the sum of the digits ($7 + 1 + 1 + 0 + 5 + 4 = 18$), so 9 is a factor of 711 054.]

If 4 is a factor of the number formed by the digits in the tens' and ones' places, then 4 is a factor of the number. [724: 4 is a factor of 24, so 4 is a factor of 724.]

If 8 is a factor of the number formed by the last three digits of a number, then 8 is a factor of that number. [15 288: 8 is a factor of 288, so 8 is a factor of 15 288.]

(See also, *Topics in Mathematics for Elementary School Teachers*, Booklet 5, "Numbers and Their Factors," 1965. NCTM, 1201 Sixteenth St., N. W., Washington, D.C. 20030.)

Resources for Active Learning

A Cloudburst, Vol. 3, No. 2435, Midwest Publications.

PAGES 126–127

Objective

Given a number which has at least 2 factors other than 1 and itself, the child will be able to build a factor tree for it.

Preparation

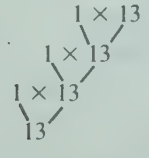
You might begin this lesson by reviewing finding factors of numbers on the chalkboard and by asking the children if one number is a factor of the other; for example:

7, 32 (No, because 32 ÷ 7 has a remainder.)
8, 40
6, 18
2, 23
3, 1239
2, 4056

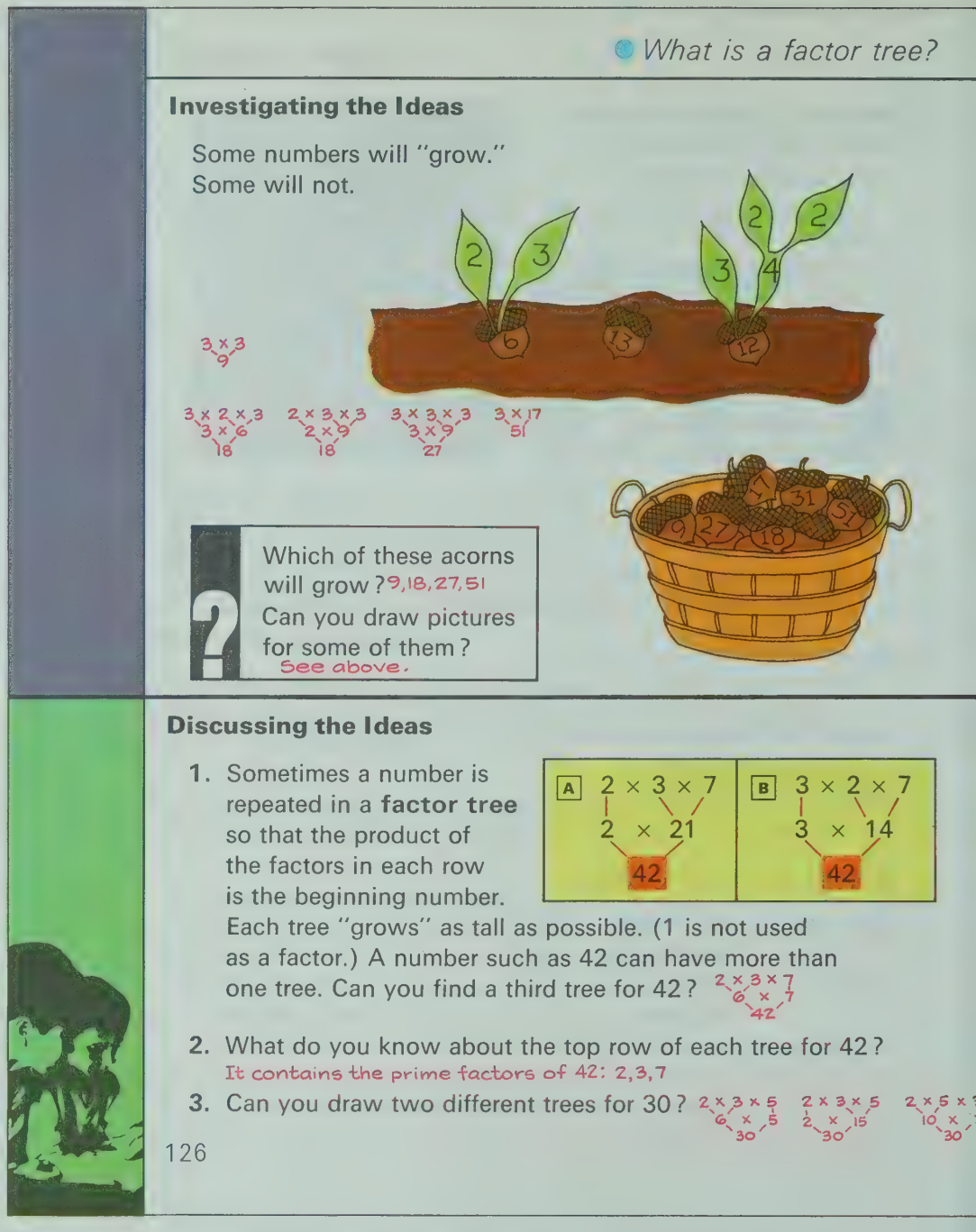
When possible, refer to the rules developed in the previous lesson, or simply apply the basic test of division to see whether there is a remainder.

Investigation

Discuss the illustrated examples to be sure the children understand that the term "grow" is used in a descriptive manner; it is not a mathematical term. You might explain that 13 has no tree because it cannot "grow." Point out that, since 1 is a factor of every number, we do not include it on the tree. Thus, even though $1 \times 13 = 13$ is a true equation, we do not use 1 on the factor tree. If we did, the tree could grow forever!



For those who finish the investigation activity quickly, you might want to suggest other numbers from which they can build a tree, such as 16 or 24.



Discussion

The term *factor tree* ordinarily refers to the kind of expressions pictured in exercise 1. Use discussion exercise 1 to extend the type of tree the children drew in the investigation to the kind usually used in the factoring of numbers. Stress the fact that each row contains factors whose product is the beginning number. Use several examples to emphasize the fact that different factor trees for the same number will end so that the factors in the top row are the same in each tree for that number, as illustrated in the next column.



Also, point out that the order of the factors in the top row is of no significance. During the work with these and other examples, bring out the fact that the numbers in the top row of each tree have no factor tree. That is, they are not the product of two smaller factors.

Using the Ideas

1. Copy each factor tree and give the missing factors.

A $5 \times 3 = 15$ B $7 \times 7 = 49$ C $2 \times 29 = 58$ D $3 \times 37 = 111$

Order of prime factors may vary.

E $2 \times 2 \times 7 = 28$ F $5 \times 2 \times 7 = 70$ G $7 \times 3 \times 5 = 105$

H $3 \times 2 \times 13 = 78$ (or 2×39) I $3 \times 2 \times 97 = 582$ J $5 \times 3 \times 89 = 1335$ (or 3×445 or 89×15)

2. Find at least two trees for each number.

A 84 B 126
C 60 D 64

See Answers, T.E. page 127.

3. Below is a tree for 180. Can you find four more?

$2 \times 2 \times 3 \times 3 \times 5$
$4 \times 3 \times 3 \times 5$
12×15
180

See Answers, T.E. page 127.
More practice, page A-8, Set 15

think

PERFECT NUMBERS

Factors of 6: 1, 2, 3, 6
Notice that
$2 \times 6 = 1 + 2 + 3 + 6$.
When the sum of the factors of a number is twice the number, the number is called a [illegible]. Show that 28 and 496 are perfect numbers. The only perfect numbers less than 500 are 6, 28, and 496.

$2 \times 28 = 1+2+4+7+14+28$
$2 \times 496 = 1+2+4+8+16+31+62+124+248+496$



Using the Exercises

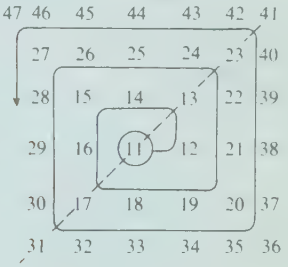
Assign the exercises on page 127 as independent work. Remind the children that there are various ways of determining whether or not a given number is a factor of another number, and that, as soon as they have figured out one factor, they can simply divide to find another. Stress to the children that, in exercise 2, they should carefully check the top row of each tree to make sure that none of the numbers in it has smaller factors.

It may be particularly instructive to have this *Think* problem presented to the class, even though some children may not be able to do it on their own. The last statement in the problem, which calls attention to the fact that 6, 28, and 496 are the only perfect numbers less than 500, is given so that the children will not waste time looking for any other perfect numbers. (Since other perfect numbers are extremely large, it would be impractical for the children to spend time trying to find them.)

Assignments (page 127)
Minimum: 1A–H. Average: 1–3. Maximum: 1–3.

Follow-up

Children might enjoy observing the pattern of primes which appears in a spiral formation of consecutive whole numbers beginning with 11:



You might challenge some children to see if they can figure out how far this pattern continues, or to try to find other patterns.

Answers, exercises 2 and 3, page 127

2. Sample trees are shown below; variations are possible.

A $7 \times 2 \times 2 \times 3$; $7 \times 4 \times 3$; 7×12; 84 — $2 \times 2 \times 3 \times 7$; 4×21; 84

B $2 \times 3 \times 3 \times 7$; 6×21; 126 — $3 \times 3 \times 2 \times 7$; 9×14; 126

C $2 \times 2 \times 3 \times 5$; 4×15; 60 — $5 \times 3 \times 2 \times 2$; $5 \times 3 \times 4$; 5×12; 60

D $2 \times 2 \times 2 \times 2 \times 2 \times 2$; $4 \times 4 \times 2 \times 2$; 16×4; 64 — $2 \times 2 \times 2 \times 2 \times 2 \times 2$; $2 \times 4 \times 4 \times 2$; 8×8; 64

3. Other trees may be developed from "first-branch" factors of 2 and 90, 3 and 60, 4 and 45, 5 and 36, 6 and 30, 9 and 20, or 10 and 18. Many variations are possible for intermediate branches, but of course all top rows should consist of the factors 2 (twice), 3 (twice), and 5.

Resources for Active Learning

A Cloudburst, Vol. 3, No. 2445, Midwest Publications.

Workbook, page 37

PAGES 128–129

Objective

Given a composite number less than 500, the child will be able to find its prime factorization by building a factor tree and recognizing prime factors.

Preparation

To prepare for this lesson, review the building of factor trees. For example, work with the children to build on the chalkboard a factor tree for a familiar number such as 12, 24, 30, or 36. If children understand how to build factor trees quite well, you might prefer to have them begin immediately with the investigation.

Investigation

Although this investigation is suitable for use as independent work, some children might benefit from an opportunity to share ideas with one or two of their classmates. Therefore, you might have them work in groups of two or three and encourage them to share things which they notice about each set. As suggested in the text, children should try to find other numbers in the special set. You might also suggest to some that they list other numbers which are not in the special set.

● *Let's explore prime numbers.*

Investigating the Ideas

Terry is thinking about a set of numbers.

?	Can you use the clues below to answer the question?	List some other numbers in Terry's set to show you know. 2,7,11,37,etc.

See Investigation.

Clues		Question
All of these are in Terry's set.	None of these are in Terry's set.	Which of these are in Terry's set?
17 83 31	24 8 49	93 13 29
5 29 19	35 27 51	15 13, 29
		16 53 91 53

Discussing the Ideas

1. The numbers in Terry's set are called **prime numbers**. How would you describe prime numbers?
Descriptions will vary. See discussion.

2. Prime numbers cannot be shown as a rectangular array of dots. Use dots to show that 7 is a prime and that 9 is not a prime.



6 is not a prime.



5 is a prime.

Array of 7 will not form rectangle. Array of 9: 3x3 rectangle.

3. Prime numbers have exactly two factors. Is 1 a prime number? No
How many factors does 21 have? 4
Is it prime? No

Number	Factors
1	1
2	1, 2
3	1, 3
4	1, 2, 4

4. Try to build a factor tree for each of the numbers 38, 57, and 73. Which of these numbers is prime? 73
2 × 19 (38) 3 × 19 (57)

5. A Which is the smallest prime number? 2
B Do you think there is a largest prime number? No

128

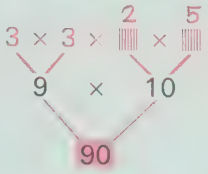


Discussion

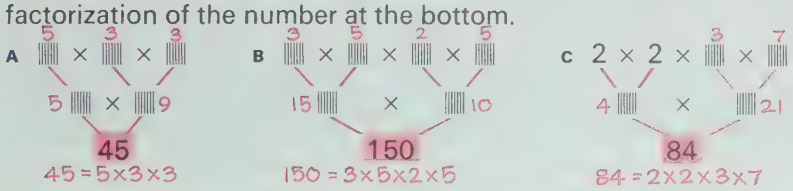
Ask several children to name some numbers which are in the special set. Ask them to explain why they think the numbers they chose are in Terry's set. Relate their explanations to the points stressed in the discussion exercises. Note, first, that *prime numbers give the number of a set of dots that cannot be arranged in a rectangular array.* (It would be helpful to have children use some of the prime numbers from the investigation to show this.) Second, *prime numbers have no factor tree.* We agreed that 1 could not be used on a factor tree, so numbers such as 13 and 17 have no factor tree. Finally, *prime numbers have exactly two factors.* Point out that every number has 1 and itself as factors. A prime number is a number greater than 1 which has only the factors 1 and itself. Explain that numbers greater than 1 that are not prime are called *composite numbers.* Notice that these explanations mean that 1 is neither prime nor composite. Zero is not usually included in discussion of prime numbers, but you might note that zero is neither prime nor composite. Zero has infinitely many factors; thus zero is not prime. Since zero is not greater than 1, zero is not composite.

Using the Ideas

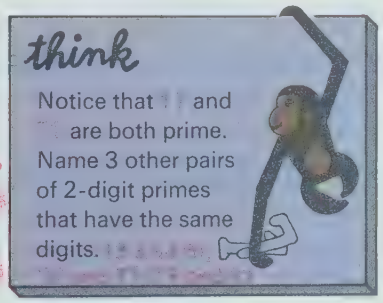
1. List all the prime numbers
   A through 50. B between 60 and 70. C through 100.
   A 2,3,5,7,11,13,17,19,23,29,31,37,41,43,47 B 61,67 C (All of A) 53,59,61,67,71, 73,79,83,89,97
2. A Express 30 as the sum of two prime numbers.
   B Express 30 as the product of prime numbers.
   A 7+23 or 11+19 or 13+17 B 2×3×5
3. Find two prime numbers whose product is an even number.
   (2 times any other prime number)
4. When you express a number as a product of **prime** factors, you have found the **prime factorization** of the number. Give the missing numbers. The prime factorization of 90 is 3 × 3 × ▒ × ▒



5. Copy and complete each factor tree. Then give the prime factorization of the number at the bottom.



6. The red lines should remind you of "branches" of an "upside-down" factor tree. Copy each exercise without the red lines and give the missing numbers.
   A $66 = 11 \times \blacksquare$ ($\blacksquare = 6$; $6 = 2 \times 3$)
   $66 = 11 \times \blacksquare \times \blacksquare$
   B $130 = 26 \times 5$ ($26 = 2 \times 13$)
   $130 = \blacksquare \times \blacksquare \times 5$
   C $210 = 21 \times \blacksquare$ ($21 = 3 \times 7$; $\blacksquare = 10$; $10 = 2 \times 5$)
   $210 = \blacksquare \times \blacksquare \times \blacksquare \times \blacksquare$



7. Give the prime factorization of each number.

A	20 $2\times2\times5$	E	126 $2\times3\times3\times7$	I	45 $3\times3\times5$
B	30 $3\times2\times5$	F	51 3×17	J	250 $5\times5\times2\times5$
C	72 $2\times2\times2\times3\times3$	G	98 $2\times7\times7$	K	231 $7\times3\times11$
D	81 $3\times3\times3\times3$	H	100 $2\times2\times5\times5$	L	330 $3\times11\times2\times5$

Using the Exercises

Before assigning the exercises on page 129, point out the phrase in exercise 4 "prime factorization." Note with the children that the top row of a completed factor tree shows the prime factorization of the number. That is, the top row should contain only prime factors, and the product of these factors is the number with which we started.

Direct the children to do the exercises. During the discussion of this assignment, you might note again that the order of the factors in the prime factorization of a number is unimportant.

Assignments (page 129)
Minimum: 1–5, 7. Average: 1–7.
Maximum: 1–7.

Follow-up

You may help children find the primes through 100 by using the technique of the ancient Greek, Eratosthenes, called the "Sieve of Eratosthenes."

List the numerals 1 through 100 on a grid chart on the chalkboard, or show them on an overhead projector. Cross out 1; circle 2, then cross out the multiples of 2; circle 3, and cross out the multiples of 3. Continue this process for 5 and 7. The numbers remaining are primes.

The *Think* problem on this page suggested one special grouping of pairs of primes. The children might also be interested in the pairs of primes which are called *twin primes:* two primes whose difference is two. For example, 3 and 5 are twin primes. Suggest to children that they try to find other twin primes.

Also, you might introduce the children to the interesting theory known as "Goldbach's conjecture." Goldbach theorized that every even number greater than 2 can be written as the sum of two primes. No one has ever succeeded in providing a mathematical proof for this, but no one has been able to disprove it by finding an even number which is not the sum of two primes.

Resources for Active Learning

Discovery, Section I, Activities 18–19, pp. 21–22, Encyclopaedia Britannica Educational Corp.

Madison Project: *Discovery in Mathematics* (Teachers' Text), "Primes," pp. 260–263, Addison-Wesley.

Mathex: Numeration No. 7, "Prime Numbers," pp. 6–9; "The Sieve of Eratosthenes," pp. 9–12, Encyclopaedia Britannica Publications Ltd.

Modern Math Games, "Two for the Primes" (a game), p. 12; "Puzzles," pp. 49, 50, Fearon. (Available from Clarke, Irwin)

Nuffield Project: *Problems*—Red Set, No. 4–4A, Wiley.

Duplicator Masters, page 23
Workbook, page 38
Skill Masters, page 23

PAGES 130–131

Objective

Given two sets, the child will be able to find their union and their intersection.

Preparation

To prepare for this lesson, you might review the concept of set. For example, ask children to think of the set of all wooden objects in the classroom, the set of all brown automobiles, or the like. Point out that a set must be defined in such a way that, given a specific object, one can determine whether or not the object is a member of that set.

Investigation

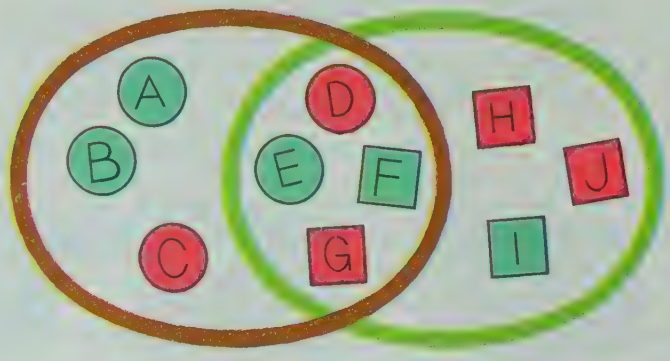
It is intended that children use the figures in the text as the elements of the set they choose to think about. Have the children play the game in pairs. One child thinks of some elements in the text, and the other child tries to guess his set by asking no more than 5 questions. They may find a need to invent some rules of their own, depending on the kinds of sets they choose. For example, a child may think of the set of blue squares, or the set of red circles, or the set of objects in the brown ring. But another child may think of the set whose members are A and I; this set may be so challenging that children cannot definitely identify it in 5 questions.

With many groups, you may find it necessary to read the sample game together to be sure the children know what to do.

Let's explore union and intersection of sets.

Investigating the Ideas

Let's play "GUESS MY SET."



Bill is thinking of a special set of these figures.
Jill is trying to guess his set with five or fewer questions.

1. Jill: Are they all in the brown ring? Bill: Yes.
2. Jill: Are any of them outside the green ring? Bill: No.
3. Jill: Are any of them circles? Bill: No.
4. Jill: Are there more than one figure in your set? Bill: Yes.

What are the figures in Bill's set? Squares F and G

? Can you choose a special set of the objects shown above and play "Guess My Set" with a classmate?

See Investigation



Discussing the Ideas

1. The **union** of the figures in the brown set with the figures in the green set is the set of all the figures in the two sets. What are these figures? A, B, C, D, E, F, G, H, I, J
2. The **intersection** of the two sets (brown and green) is the set of figures that are inside the green set and also inside the brown set. What figures are in the intersection of the two sets? D, E, F, G

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Discussion

The goal of this lesson is for children to understand what is meant by the union and intersection of sets. Use examples other than the sets of figures in the text to clarify these concepts and to teach the symbols. The union of two sets is that set consisting of those objects which are in *one or the other or both* of the two sets. In reference to the text, if we call the set of figures in the brown ring set S and those in the green ring set T, we write the union of these sets as

$$S \cup T = \{A, B, C, D, E, F, G, H, I, J\}.$$

$S \cup T$ is read: "S union T."

The *intersection* of two sets is the set containing those objects and only those objects that are in both of two sets. In reference to the sets in the text, we write

$$S \cap T = \{D, E, F, G\}$$

We read the symbols $S \cap T$, "S intersect T."

It would be helpful to list sets on the chalkboard and ask children to find the union and intersection of them. For example, list the factors of 12, and then list the factors of 16. List their union: {1, 2, 3, 4, 6, 8, 12, 16}. List their intersection: {1, 2, 4}.

Using the Ideas

1. The table below will help you better understand union and intersection of sets. Give the missing numbers in the sets.

Set P = {2, 3, 5, 7}	The set of **prime** numbers less than 10
Set O = {1, 3, 5, 7, 9}	The set of **odd** numbers less than 10
Set E = {0, 2, 4, 6, 8}	The set of **even** numbers less than 10

The union of sets O and E	contains	the numbers that are in one or the other or both sets (all the whole numbers less than 10).
O ∪ E	=	{0, 1, 2, 3, 4, 5, 6, 7, 8, 9}
The intersection of sets P and O	contains	only the numbers that are in both set P and set O (the odd prime numbers less than 10).
P ∩ O	=	{3, 5, 7}

2. What numbers are in the intersection of sets O and E? None
We call this set the **empty** set. We write: O ∩ E = { }

3. For each exercise, give the set of numbers.
   A P ∪ E {0,2,3,4,5,6,7,8} B E ∩ P {2} C O ∪ P {1,2,3,5,7,9} D E ∩ O { }

4. A Give the set of numbers less than 25 that have 2 as a factor. Call this set T. Set T={0,2,4,6,8,10,12,14,16,18,20,22,24}
   B Give the set of numbers less than 25 that have 3 as a factor. Call this set R. Set R={0,3,6,9,12,15,18,21,24}
   C Give the set of numbers for T ∩ R. T∩R={0,6,12,18,24}
   D Give the set of numbers less than 25 that have 6 as a factor. {0,6,12,18,24}

★5. How many numbers less than 100 have
   A 2 as a factor? 50 C 2 and 5 as factors? 10
   B 5 as a factor? 20 D 10 as a factor? 10

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Using the Exercises

Before assigning the exercises on page 131, you might explain the concept of the empty set as presented in exercise 2. Point out that the empty set is simply the set that has no members. You might provide amusement by suggesting some non-mathematical descriptions of the empty set, such as the set of all children in the classroom who are 4 metres tall. Point out that the empty set also may be symbolized as ∅.

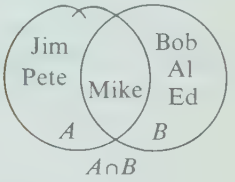
When children finish the exercises and you help them check their work, stress the idea that union is defined as the objects that are in *either* of the two sets and that intersection is defined as only those objects which are in both sets. Note with them that sets P, O, and E contain, respectively, certain prime numbers, odd numbers, and even numbers. Call their attention to the union and intersection of pairs of these sets. The union of sets O and E is the set of all whole numbers less than 10, while the intersection of sets P and O is the set of odd prime numbers less than 10.

Assignments (page 131)*
Minimum: 1–4. Average: 1–4.
Maximum: 1–5.

Follow-up

Venn diagrams provide an enjoyable way of showing intersecting sets. Explain an example such as the following and encourage children to make Venn diagrams for the other sets listed below.

Set *A* Baseball Team: Jim, Mike, Pete
Set *B* Football Team: Mike, Ed, Bob, Al



A ∩ *B*

Other examples:
Use a Venn diagram to show:
1) *A* ∩ *B*, when Set *A* = {2, 4, 6, 8} and Set *B* = {1, 2, 3, 4, 5}
2) *A* ∩ *B*, when Set *A* = {10, 20, 30, . . .} and Set *B* = {2, 4, 6, 8, 10, 12, 14, 16, 18, 20}
3) *A* ∩ *B*, when Set *A* is the set of all girls taking sewing and music and Set *B* is the set of all girls taking weaving and sewing.

Resources for Active Learning

Franklin Series: *Probability*, "Sets," pp. 5–16, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Nuffield Project: *Problems*—Green Set, No. 25, 37; Red Set, No. 1, 2, Wiley.

Teaching Aids for Elementary Mathematics, "Venn Diagrams," pp. 120–125, Holt, Rinehart and Winston.

PAGES 132–133

Objective

Given two numbers, the child will be able to find their greatest common factor.

Preparation

To prepare for this lesson, review how to determine whether or not one number is a factor of another. For example, 4 is a factor of 12 because $12 \div 4 = 3$, with zero remainder. You might also write some numbers on the chalkboard and ask children to list the factors of each.

Investigation

You might want the children to work in groups of two or three and share ideas on how to find the 5 other numbers. As children work, you might ask questions to guide their thinking. For example, you might say: "Think of the basic multiplication facts. Do you recognize familiar products?" Or: "Add the digits in each numeral. Does this give you a clue?" "Have you divided each number by the factors you know?" "Can you make a factor tree for each of the two numbers?" Use questions such as these only for children who have difficulty in thinking of ways of finding factors for the numbers.

What is the greatest common factor of two numbers?

Investigating the Ideas

2 is a factor of both of these numbers. → 72 450

Can you use division to show this fact? $72 \div 2 = 36$ R0 $450 \div 2 = 225$ R0

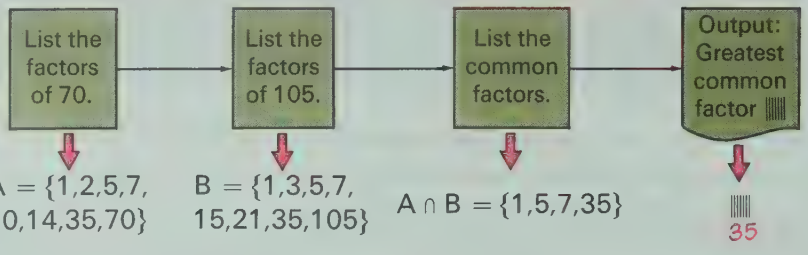
? There are five other numbers that are factors of both 72 and 450. How many of them can you find?

1,3,6,9,18

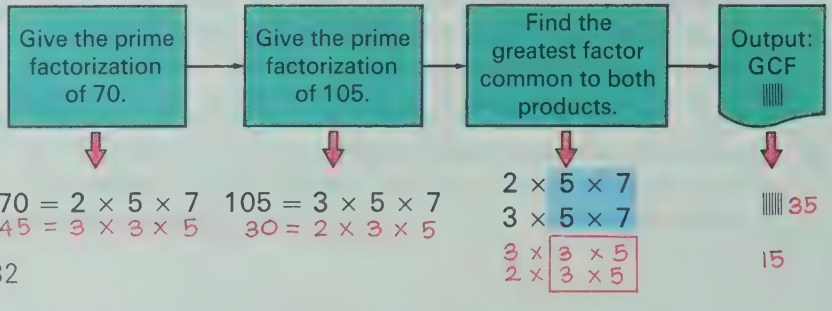
See Investigation

Discussing the Ideas

1. Study the flow chart below for finding the **greatest common factor** (GCF) of 70 and 105. Explain each step and then give the output number. See Discussion.



2. The flow chart below shows a second method for finding the GCF of 70 and 105. Use the method in this flow chart to find the GCF of 45 and 30.





Discussion

Ask the children to list the factors they found in the investigation, and encourage them to explain how they found them. Point out that the 6 numbers which are factors of both 72 and 450 (1, 2, 3, 6, 9, 18) are called *common factors*. You might also point out that when they are looking for common factors they need not find the factors of 450 which are greater than 72, because these numbers clearly will not be factors of 72. When you are sure children understand the idea of common factors, introduce the concept of *greatest common factor*. For example, have them identify the greatest common factor of 72 and 450. Use the discussion exercises to develop this concept. Carefully work through the flow chart steps of the two methods for finding the greatest common factor.

Make certain that the children see that one method consists primarily of choosing the largest factor from the set of common factors and that the other method involves simply choosing the common prime factors of the two numbers. In the illustration of the second method, for example, 5 and 7 appear in both prime factorizations; hence, by choosing 5×7, we get the greatest common factor, 35.

Using the Ideas

1. A List the factors of 12. 1,2,3,4,6,12
B List the factors of 30. 1,2,3,5,6,10,15,30
C List the common factors of 12 and 30. 1,2,3,6
D What is the GCF of 12 and 30? 6

2. A List the factors of 50. 1,2,5,10,25,50
B List the factors of 30. 1,2,3,5,6,10,15,30
C List the common factors of 50 and 30. 1,2,5,10
D What is the GCF of 50 and 30? 10

3. Use the method shown in the first flow chart on page 132 to give the GCF of each pair of numbers.
A 18, 24 6 B 20, 27 1 C 12, 60 12

4. The prime factorization of each number is given. Give the GCF of the two numbers.
A 70: $2\times5\times7$ 20: $2\times2\times5$ Answer: 10
C 66: $2\times11\times3$ 165: $11\times3\times5$ 33
E 135: $3\times3\times3\times5$ 36: $2\times2\times3\times3$ 9
B 126: $3\times3\times2\times7$ 24: $3\times2\times2\times2$ 6
D 525: $3\times5\times5\times7$ 330: $2\times3\times5\times11$ 15
F 210: $3\times2\times7\times5$ 60: $2\times5\times2\times3$ 30

5. Give the GCF for each pair of numbers. Use the flow chart in exercise 2, page 132.
A 36, 42 6
B 20, 36 4
C 90, 315 45
D 72, 450 18
E 175, 105 35
F 525, 441 21

★ **6.** Give the GCF of 36, 42, and 54. 6

★ **7.** Give the GCF of 70, 28, 42, and 56. 14

think

Let's agree that ▫ between two numbers means:

Multiply the first number by itself and add the second to this product.

So, 2 ▫ 1 = 5, 3 ▫ 2 = 11, and 4 ▫ 2 = 18

1. Solve the equations.
A 2 ▫ 4 = *n* C 6 ▫ 6 = *n*
B 5 ▫ 1 = *n* D 1 ▫ 1 = *n*

2. Answer T (true) or F (false).
A 2 ▫ 5 = 5 ▫ 2 D 4 ▫ 0 = 0
B 4 ▫ 3 = 3 ▫ 4 E 0 ▫ 1 = 1
C 0 ▫ 4 = 4 F 0 ▫ 2 = 2



More practice, page A-8, Set 16

Using the Exercises

Note with the children that the steps in exercises 1 and 2 on page 133 lead them through the steps outlined in the first flow chart. Exercises 4 and 5 relate to the method shown in the second chart. Starred exercises 6 and 7 are intended for children who want to choose one of the two methods and work through it on their own. You might want those who work these exercises to explain their solutions to the class.

Some children will enjoy the *Think* problem. You might encourage them not only to solve the equations but also to write some of their own. If time permits, they might explain the equations and answers to others in the class.

Assignments (page 133)*
Minimum: 1–5. Average: 1–5.
Maximum: 1–7.

Mathematics

Two whole numbers are said to be *relatively prime* to each other if their greatest common factor is 1.

The numbers 8 and 15 are relatively prime. The factors of 8 are 1, 2, 4, and 8. The factors of 15 are 1, 3, 5, and 15. Since 1 is their greatest common factor, 8 and 15 are relatively prime. Every pair of *consecutive whole numbers* is relatively prime. Any number larger than 1 which is the factor of the smaller number of the consecutive pair cannot be a factor of the larger number, for division of the larger number by the factor will always give a remainder of 1.

A fraction is in lowest terms if the numerator and denominator are a pair of relatively prime whole numbers.

Follow-up

A suitable follow-up for children who clearly understood the concepts and techniques of this lesson would be a study of the Euclidean algorithm for finding the greatest common factor (GCF) of two large numbers. The following steps may be given to these children to study on their own.

1. Compare two numbers.
2. Divide the greater number by the smaller.
3. Is the remainder zero?
4. If it is, recall the last divisor—this is the greatest common factor of the two numbers.
5. If the remainder is not zero, divide the last divisor by that remainder.
6. Is the new remainder zero?
7. Repeat step 4 or 5.

Resources for Active Learning

A Cloudburst, Vol. 2, No. 2613, 2623; Vol. 3, 2615–35, Midwest Publications.

SMSG: *Puzzle Problems and Games Project*, ". . . Greatest Common Divisor," pp. 69–77, Stanford University.

Duplicator Masters, page 24
Workbook, page 39
Skill Masters, page 24

PAGES 134–135

Objective

Given two numbers, the child will be able to find the least common multiple of the pair.

Preparation

To prepare for this lesson, you might review the term "multiple." A first number is a multiple of a second number if there is a whole number that multiplies by the second to give the first. For example, 32 is a multiple of 4 because $8 \times 4 = 32$. You might list the following numbers on the chalkboard and ask children to identify the multiples of 5, the multiples of 6, and so on.

15, 21, 30, 42, 50, 12, 28, 45

Children might respond by saying something such as, "15 is a multiple of 5 because $3 \times 5 = 15$."

What is the least common multiple of two numbers?

Discussing the Ideas

1. The **least common multiple** (LCM) of two numbers is the smallest number (other than zero) that is a multiple of both numbers. Study method 1. Then use it to find the LCM of 3 and 4. 12

Method 1 — Find the LCM of 6 and 8.

1. List some multiples of 6. → {0,6,12,18,24,30,36,42,48, . . .} = S
2. List some multiples of 8. → {0,8,16,24,32,40,48, . . .} = E
3. List **common multiples** of 6 and 8. → {0,24,48, . . .} = S ∩ E
4. Choose the **least common multiple.**

2. Study method 2. Then use it to find the LCM of 8 and 12. 24

Method 2 — Find the LCM of 9 and 12.

1. Choose the greater of the two numbers: 12
2. Find the least multiple of 12 (other than 0) that has 9 as a factor.
   12; 9 **is not** a factor of 12.
   24; 9 **is not** a factor of 24.
   36; 9 **is** a factor of 36.
3. 36 is the **least common multiple** of 9 and 12.

3. Study method 3. See if you can explain why it gives the LCM of two numbers. See Discussion.

Method 3 — Find the LCM of 30 and 42.

$2 \times 3 \times 5$ ← The prime factorization of 30.

$2 \times 3 \times 7$ ← The prime factorization of 42.

$2 \times 3 \times 5 \times 7$ ← This product is a multiple of both 30 and 42.

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Discussion

Before developing the idea of *least common multiple*, as in discussion exercise 1, help children realize what a *common multiple* is. For example, list certain sets of multiples and ask children to give the intersection of them.

Multiples of 4: 0, 4, 8, 12, 16, 20, 24, 28, . . .
Multiples of 6: 0, 6, 12, 18, 24, 30, . . .

Explain that 0, 12, 24, . . . are common multiples of 4 and 6. Point out that, other than 0, 12 is the least common multiple of 4 and 6. Carefully work through exercise 1, giving the children other suitable examples to try. With some children, you might want to devote the entire time to this method; with others, you might present both methods 1 and 2. It would be best to use method 3 only with more able children. Note that method 3 works because only those factors needed to make the resulting product a multiple of both numbers are chosen.

Using the Ideas

1. A List the multiples (to 60) of 6. 0,6,12,18,24,30,36,42,48,54,60
B List the multiples (to 60) of 4. 0,4,8,12,16,20,24,28,32,36,40,44,48,52,56,60
C List the common multiples (to 60) of 6 and 4. 0,12,24,36,48,60
D Give the least common multiple of 6 and 4. 12

2. A List the multiples (to 30) of 5. 0,5,10,15,20,25,30
B List the multiples (to 30) of 2. 0,2,4,6,8,10,12,14,16,18,20,22,24,26,28,30
C List the common multiples (to 30) of 5 and 2. 0,10,20,30
D Give the LCM of 5 and 2. 10

3. A List the multiples (to 72) of 12. 0,12,24,36,48,60,72
B Give the least multiple of 12 (other than 0) that has 8 as a factor. 24
C What is the LCM of 8 and 12? 24

4. Use method 2 to find the LCM for each pair of numbers.
A 3, 4 12 C 10, 8 40 E 9, 15 45 G 6, 9 18 I 10, 15 30 K 24, 18 72
B 12, 16 48 D 7, 14 14 F 8, 20 40 H 5, 9 45 J 10, 100 100 L 24, 40 120

5. The prime factorization of each number is given. Give the LCM of the two numbers.

A $10 = 2 \times 5$
$15 = 5 \times 3$
30

C $6 = 2 \times 3$
$15 = 3 \times 5$
30

E $75 = 3 \times 5 \times 5$
$50 = 5 \times 5 \times 2$
150

B $30 = 2 \times 3 \times 5$
$70 = 2 \times 5 \times 7$
210

D $12 = 2 \times 2 \times 3$
$18 = 2 \times 3 \times 3$
36

F $6 = 2 \times 3$
$35 = 5 \times 7$
210

6. What is the LCM for each pair of numbers?
A 2, 5 10
B 6, 5 30
C 10, 12 60
D 10, 6 30
E 20, 10 20
F 4, 12 12
G 21, 6 42
H 17, 2 34
I 20, 24 120
J 90, 63 630

7. What is the LCM of
A 2, 6, and 10? 30
B 15, 6, and 4? 60
C 18, 15, and 8? 360
D 8, 6, and 4? 24
E 5, 10, and 3? 30
F 10, 12, and 15? 60

More practice, page A-9, Set 17



Using the Exercises

Assign the exercises on page 135 as independent work, according to which method for finding the LCM the children studied. Encourage children to choose the method they like best and to develop it by practice in exercises 6 and 7. When they have finished, allow time for further discussion and for checking answers.

Assignments (page 135)*
Minimum: 1–3, 6. Average: 1–3, 5–7. Maximum: 1–7.

Follow-up

Children might enjoy an oral game for identifying multiples. For example, explain that for all the multiples of 4, they say *Buzz;* for those of 5, they say *Clang;* and for those of 6, they say *Bam.* Then have the group begin to count, remembering to use the *Buzz, Clang,* and *Bam* substitutes at the proper time. The first 20 numbers would sound as follows:

1, 2, 3, *Buzz, Clang, Bam,* 7, *Buzz,* 9, *Clang,* 11, *Buzz-Bam,* 13, 14, *Clang, Buzz,* 17, *Bam,* 19, *Buzz-Clang* . . .

Resources for Active Learning

A Cloudburst, Vol. 2, No. 2713, 2723; Vol. 3, 2715, Midwest Publications.

Duplicator Masters, page 25
Workbook, page 40
Skill Masters, page 25

PAGES 136–137

Objective

Given addition, subtraction, and multiplication operations in clock arithmetic, the child will be able to write and solve an equation for each.

Preparation

Although the topic of clock arithmetic is developed in Book 5 of this series, this presentation may be the first for many of the children. If you choose, extend this lesson over more than one period.

As long as children are familiar with the use of a number line, there is no need for a specific preparation for this lesson.

Investigation

If children have not had previous experience with clock arithmetic read the paragraph at the top of the page and discuss the one-handed clock. Then encourage them to try the investigation equation. Move around the room as the children work and help any who need guidance.

Let's explore some clock arithmetic.

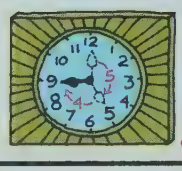
Investigating the Ideas

The figure shows how you can use a one-handed clock to think of addition in "clock arithmetic." You used the number line to show operations in ordinary arithmetic. A clock may be used in much the same way to show operations in clock arithmetic.



Equation: $3 + 4 = 7$

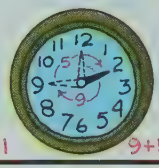
? Can you write a "clock arithmetic" equation for each clock below?



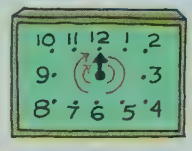
5+4=9



7+6=1



9+5=2



12+12=12

Discussing the Ideas

1. Use a clock to explain each of the following sums.

   A $5 + 6 = 11$ B $5 + 7 = 12$ C $5 + 8 = 1$ D $5 + 9 = 2$

   See Discussion.

2. Use a clock to explain each of these differences.

   A $10 - 8 = 2$ B $10 - 9 = 1$ C $10 - 10 = 12$ D $10 - 11 = 11$

   See Discussion.

3. Suppose you have a special one-handed clock with just 5 numbers on it. Also, suppose you use 0 instead of 5 on your clock. You can do some interesting arithmetic with this clock, much as you did with an ordinary clock. Choose some pairs of "five-clock numbers" and explain how you can find their sums.



   Choices will vary. Samples: 2+3=0; 4+3=2; 3+3=1

4. Use the five-clock to explain each equation.

   A $4 + 2 = 1$ B $1 - 2 = 4$ C $2 \times 4 = 3$ D $3 \times 2 = 1$

   See Discussion.

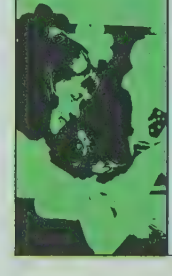
5. Is it true that for any pair of five-clock numbers a and b

   $a + b = b + a$? Yes

   Choose a pair of five-clock numbers and illustrate this principle.

   Sample: 3+4=2 and 4+3=2, so 3+4=4+3

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Discussion

Before discussing the exercises, check the equations the children wrote for the clocks in the investigation. As you discuss these equations, and the first two discussion exercises, stress that the clock being used is a "twelve-clock"; these equations make sense only if it is understood that they refer to twelve-clock arithmetic.

Use exercise 3 to introduce clock arithmetic other than 12. Point out that five-clock arithmetic is but one of many examples which could have been chosen. When children discuss exercises 4 and 5, be sure they realize that they must refer to the five-clock which is illustrated in exercise 3.

Using the Ideas

1. Answer the question. Then solve the equation.

A What time will it be 5 hours after 8 o'clock? 1 o'clock
$8 + 5 = n$ 1

B What time was it 6 hours before 2 o'clock? 8 o'clock
$2 - 6 = n$ 8

2. Use twelve-clock numbers to find the sums.

A $7 + 7$ 2 B $7 + 7 + 7$ 9 C $8 + 9$ 5 D $11 + 11$ 10 E $12 + 12$ 12

3. Use twelve-clock numbers to find the differences.

A $4 - 6$ 10 B $4 - 4$ 12 C $8 - 7$ 1 D $7 - 8$ 11 E $12 - 12$ 12

4. Find the sum. Then find the product. Use twelve-clock numbers.

A $2+2+2+2=n$ 8
$4 \times 2 = n$ 8

B $5+5+5=n$ 3
$3 \times 5 = n$ 3

C $7+7+7+7=n$ 4
$4 \times 7 = n$ 4

Use the **five-clock** for all of these exercises.

5. Answer the question. Then solve the equation.

A Start at 0. Move forward 3. Move forward 4 more. Where are you?
$3 + 4 = n$ 2

B Start at 0. Move forward 2. Move backward 4. Where are you?
$2 - 4 = n$ 3

C Start at 0. Move forward 4. Move forward 4 more. Where are you?
$2 \times 4 = n$ 3

6. Solve the equations.

A $3 + 0 = n$ 3
B $3 + 1 = a$ 4
C $3 + 2 = b$ 0
D $3 + 3 = s$ 1
E $2 - 1 = t$ 1
F $2 - 2 = c$ 0
G $2 - 3 = n$ 4
H $2 - 4 = d$ 3
I $2 \times 3 = y$ 1
J $3 \times 3 = p$ 4
K $3 \times 4 = q$ 2
L $4 \times 0 = r$ 0

7. Copy and complete the addition and multiplication tables.

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

×	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1



Using the Exercises

Assign the exercises on page 137 as independent work unless the children had particular difficulty with the discussion section. Point out that exercises 1 through 4 refer to twelve-clock arithmetic; exercises 5, 6, and 7 refer to five-clock arithmetic. Allow time for checking answers when the children have completed this page.

Assignments (page 137)
Minimum: 1–4. Average: 1–7.
Maximum: 1–7.

Follow-up

Suggest to the children that they make both an addition and a multiplication table for some clock arithmetic of their choice. These may then be displayed around the classroom. They might also write equations under these tables and challenge others to find the solutions.

Resources for Active Learning

[The following are clock arithmetic activities.]

Developmental Math Cards, L¹14, Addison-Wesley.

Franklin Series: *Mathematics Around the Clock*, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Math Activity Cards, E2, Macmillan.

Nuffield Project: *Computation and Structure* 4, "Modular Arithmetic," pp. 13–21, Wiley.

Nuffield Project: *Problems*—Green Set, No. 15, Wiley.

Teaching Aids for Elementary Mathematics, pp. 62–67, Holt, Rinehart and Winston.

Nuffield Project: *Shape and Size* 3, "Patterns on Circles," pp. 11–18, Wiley.

Workbook, page 41

PAGES 138–139

Objective

The child will demonstrate his ability to work with the concepts presented in this chapter.

Preparation

If you choose to use these pages as an evaluation instrument, you will probably want to spend some time in preparation. It would be beneficial for children to build several factor trees, list all the factors of several numbers, and find some multiples of small numbers.

Reviewing the Ideas

1. Solve the equations that have whole-number solutions.

	Factor		Factor		Product
A	f 32	×	1	=	32
B	f 16	×	2	=	32
C	f	×	3	=	32
D	f 8	×	4	=	32
E	f	×	5	=	32
F	f	×	6	=	32

	Factor		Factor		Product
G	f 28	×	1	=	28
H	f 14	×	2	=	28
I	f	×	3	=	28
J	f 7	×	4	=	28
K	f	×	5	=	28
L	f	×	6	=	28

2. A List the factors of 32. 1, 2, 4, 8, 16, 32 B List the factors of 28. 1, 2, 4, 7, 14, 28

3. Is the first number a factor of the second?

A 7, 42 Yes B 10, 55 No C 6, 54 Yes D 7, 91 Yes E 1, 85 Yes

4. List the factors of: A 12 1,2,3,4,6,12 B 42 1,2,3,6,7,14,21,42 C 56 1,2,4,7,8,14,28,56 D 100 1,2,4,5,10,20,25,50,100

5. Find each product. Then give all the factors of that number.

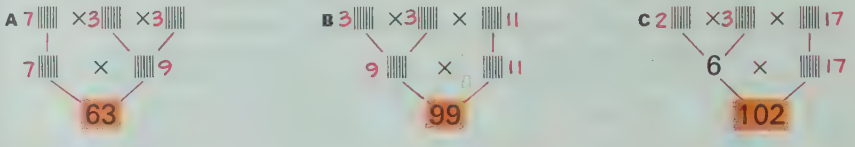
A 3×5 B 2×7 C 5×11 D 7×7 E $3 \times 5 \times 7$ F $2 \times 3 \times 3$ G $2 \times 2 \times 2$ H $2 \times 5 \times 7$

See Answers, T.E. page 138.

6. Give all the numbers in the set {82, 75, 102, 300, 537, 870, 377, 468, 1266, 4575} that have

A 2 as a factor. 82, 102, 300, 870, 468, 1266 B 5 as a factor. 75, 300, 870, 4575 C 3 as a factor. 75, 102, 300, 537, 870, 468, 1266, 4575

7. Copy each factor tree and give the missing factors.



8. Which of these numbers are prime? 37, 47, 53, 61

{21, 27, 33, 37, 47, 49, 51, 53, 61, 63}

9. Give two prime numbers whose difference is 1. 2, 3

Discussion

Because of the number of exercises on these pages, you might assign only selected exercises for the children to do. Or, you might extend the length of time you allow for the children to complete them.

Remind the children that some of the equations in exercise 1 do not have whole-number solutions; they are asked only to solve those that do. If children have difficulty remembering a certain topic, direct them to the appropriate pages in the chapter to help them refresh their memory. Give help to those children who seem to be finding the exercises difficult.

Answers, exercise 5, page 138

A 15; 1, 3, 5, 15
B 14; 1, 2, 7, 14
C 55; 1, 5, 11, 55
D 49; 1, 7, 49
E 105; 1, 3, 5, 7, 15, 21, 35, 105
F 18; 1, 2, 3, 6, 9, 18
G 8; 1, 2, 4, 8
H 70; 1, 2, 5, 7, 10, 14, 35, 70

10. Give the prime factorization of the numbers at the bottom of the factor trees in exercise 7. A $63 = 3 \times 3 \times 7$ B $99 = 3 \times 3 \times 11$ C $102 = 2 \times 3 \times 17$

11. The red lines should remind you of the branches of an "upside-down" factor tree. Copy each exercise without the red lines and give the missing numbers.

A $78 = 13 \times 6$, $78 = 13 \times 2 \times 3$ B $154 = 11 \times 14$, $154 = 11 \times 2 \times 7$ C $170 = 10 \times 17$, $170 = 2 \times 5 \times 17$

12. Give the prime factorization of each number.

A 45 $3 \times 3 \times 5$ B 28 $2 \times 2 \times 7$ C 75 $5 \times 5 \times 3$ D 120 $2 \times 5 \times 2 \times 2 \times 3$ E 72 $3 \times 3 \times 2 \times 2 \times 2$ F 180 $3 \times 2 \times 3 \times 2 \times 5$

13. Set T = {0, 3, 6, 9, 12, 15} Set E = {0, 2, 4, 6, 8, 10, 12, 14}

Give the set of numbers for: A $T \cap E$ {0, 6, 12} B $T \cup E$ {0, 2, 3, 4, 6, 8, 9, 10, 12, 14, 15}

14. Give the **greatest common factor** for each pair of numbers.

A 6, 9 3 B 12, 18 6 C 32, 36 4 D 60, 48 12 E 36, 42 6 F 42, 55 1

15. Give the **least common multiple** for each pair of numbers.

A 4, 6 12 B 12, 16 48 C 14, 10 70 D 8, 12 24 E 30, 66 330 F 30, 45 90

★**16.** Give two numbers whose least common multiple Answers will vary.

A is their product. Sample answers: 2, 3; 7, 15; 8, 15

B is one of the numbers. Sample answers: 4, 8; 2, 6; 1, 5

think

Write symbols for the numbers 1 to 40, but use only 1, 3, 9, 27 and + or −. Use no number more than once in a given set of symbols. The examples will help you get started. See Solutions, T.E. page 139



Symbols	Numbers
1	1
3 − 1	2
3	3
3 + 1	4
(9 − 3) − 1	5
9 − 3	6
9 − (3 − 1)	7
⋮	⋮

Solutions, Think, page 139

The *Think* problem is quite challenging. Several solutions are possible for some of the numbers. For example, the symbol for 7, given on page 139, is $9 - (3 - 1)$. Another symbol for 7 is $(9 - 3) + 1$. Sample solutions for 8 through 40 follow:

Symbols	*Numbers*
$9 - 1$	8
9	9
$9 + 1$	10
$9 + (3 - 1)$	11
$9 + 3$	12
$9 + (3 + 1)$	13
$27 - [9 + (3 + 1)]$	14
$27 - (9 + 3)$	15
$27 - [9 + (3 - 1)]$	16
$27 - (9 + 1)$	17
$27 - 9$	18
$27 - (9 - 1)$	19
$27 - [(9 - 3) + 1]$	20
$27 - (9 - 3)$	21
$27 - [(9 - 3) - 1]$	22
$27 - (3 + 1)$	23
$27 - 3$	24
$27 - (3 - 1)$	25
$27 - 1$	26
27	27
$27 + 1$	28
$27 + (3 - 1)$	29
$27 + 3$	30
$27 + (3 + 1)$	31
$27 + [(9 - 3) - 1]$	32
$27 + (9 - 3)$	33
$27 + [9 - (3 - 1)]$	34
$27 + (9 - 1)$	35
$27 + 9$	36
$27 + (9 + 1)$	37
$27 + [9 + (3 - 1)]$	38
$27 + (9 + 3)$	39
$27 + [9 + (3 + 1)]$	40

Workbook, page 42

PAGES 140–141

Objective

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

Review any topics covered thus far in the text which may have been difficult for the children. If no difficulty comes readily to mind, it will probably be most helpful to spend this time reviewing the division algorithm.

Keeping in Touch with Addition Multiplication Division Place value

1. Find the products.

A 10×10 100 C 100×10 1000 E 10×1000 10000 G 100×1000 100000

B 10×100 1000 D 100×100 10000 F 1000×10 10000 H 1000×1000 1000000

2. Give the smallest number that has 4 thousands, 6 tens, 5 hundreds, 3 ten thousands, 2 ones, 6 hundred thousands, 9 millions, 6 hundred millions, and 0 ten millions. 609,634,562

3. Write each number as a power of ten or as a number less than 10 times a power of ten.

A 500 Answer: 5×10^2 B 600 6×10^2 C 1000 10^3 D 5000 5×10^3 E 10 000 10^4

4. Give the ordinary base-ten numeral for each ▥.

A $4^2 =$ ▥ 16 C $3^3 =$ ▥ 27 E $6^3 =$ ▥ 216 G $5 \times 10^3 =$ ▥ 5000

B $10^2 =$ ▥ 100 D $2^5 =$ ▥ 32 F $13^3 =$ ▥ 2197 H $8 \times 10^4 =$ ▥ 80000

5. Find the products.

A	B	C	D	E	F
300	237	39	54	387	465
$\times 9$	$\times 6$	$\times 20$	$\times 47$	$\times 29$	$\times 174$
2700	1422	780	2538	11223	80910

6. Find the quotients and remainders.

A $8\overline{)3642}$ 455 R2 B $40\overline{)2963}$ 74 R3

C $78\overline{)457}$ 5 R67 D $39\overline{)2164}$ 55 R19

E $65\overline{)4762}$ 73 R17 F $84\overline{)5967}$ 71 R3

7. Find the arithmetic mean for each set of numbers.

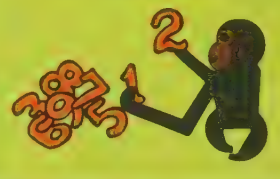
A {613, 786, 494} 631

B {86, 97, 81, 76} 85

think

Copy the problem.
Give the missing digits.

▥▥ 11
73) ▥▥▥ 803
▥▥▥ 730
▥▥ 73
▥ 3 7
0





You are invited to explore

ACTIVITY CARD 6 Page 358

140

Discussion

You might choose to work through several exercises on page 140 together with the class.

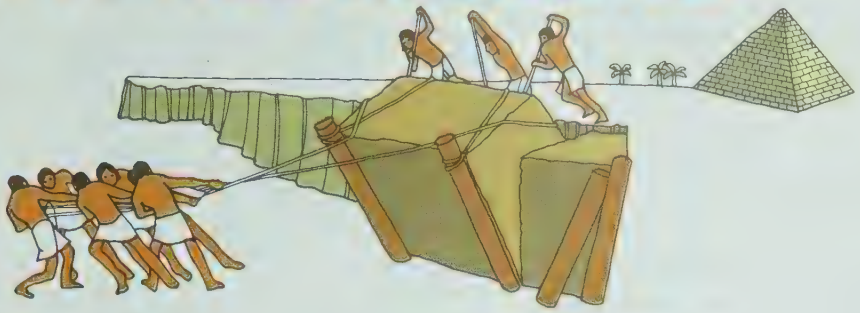
It may help to have several parts of exercise 6 written on the board and explained to the class. It may also help to discuss several parts of exercises 5 and 7 in some detail. For exercise 7, you may need to help some of the children review the definition of an arithmetic mean.

For the *Think* problem, encourage reasoning such as: "I know that the ones' digit of the quotient is 1 since my last product is a 2-digit number ending in 3; hence, it would have to be 1×73."

Following this, the child should reason that the tens' digit in the quotient cannot be as great as 2 since 20×73 would be a number greater than 1000, and the dividend is only a 3-digit number. If the tens' digit of the quotient cannot be greater than 1, it must be 1. As soon as the quotient of 11 is obtained, the rest of the digits can easily be supplied simply by multiplying 11 times 73 to find the dividend.



EGYPTIAN PYRAMIDS



The Great Pyramid at El Gizeh is the largest of the approximately 80 pyramids that still stand in Egypt. The Pharaoh Cheops had it built as a tomb for himself around 2600 B.C. One hundred thousand slaves worked for 20 years to complete the project, which required 2 300 000 stone blocks, weighing about 2275 kg each. The Great Pyramid is considered one of the seven wonders of the ancient world.

1. The Great Pyramid was originally 144 m tall. The Great Pyramid was as tall as a building of how many stories, if you use 4 metres per story ? 36

2. When the capstone was removed from the top, the Great Pyramid was 137 m tall and the top was a square surface about 10 m on each side. If a car takes up a space of 4.5 by 2 m, about how many cars could be placed on top of the pyramid ? About 11

3. The bottom of the pyramid is a square about 228 metres on a side. A hectare is 10 000 square metres. Does the pyramid cover an area closer to 5 or to 6 hectares ? 5

4. Estimate the number of hockey rinks (61 m long and 26 m wide) it would take to provide room for the pyramid. 33

5. How many years ago (to the nearest 100 years) was the Great Pyramid built ? 4600

141

Using the Exercises

You may prefer to assign page 140 as independent work and discuss the introductory material for the problems on page 141. Give children an opportunity to discuss the illustration and perhaps encourage them to consult an encyclopaedia for additional information about the Egyptian pyramids.

Follow-up

Children might enjoy a version of "I Know Your Answer." They might play it with a fellow classmate or friends and relatives outside of class.

1. Choose and write down any number.
2. Multiply your number by 2.
3. Add 16 to this product.
4. Divide the sum by 2.
5. Now subtract the number that you started with.

[The answer is always 8.]

You might challenge some children to try to make up other rules patterned on these.

Resources for Active Learning

Math Activity Cards, "Cheops' Pyramid," D36, Macmillan.

General Objectives

To provide experiences with fraction concepts
To provide work with equivalent fractions
To stress building sets of equivalent fractions
To make a transition from sets of equivalent fractions to fractional numbers
To introduce fractional numbers
To introduce the fractional-number line
To establish the fact that each whole number is also a fractional number
To introduce equality and inequality for fractional numbers
To review skills for finding lowest-terms fractions

The first lesson of the chapter deals with the general concept of fractions, followed by development of the concepts of equivalent fractions and building sets of equivalent fractions. Then material on lowest-terms fractions and finding the lowest-terms fraction for a given fraction is provided.

After this introduction, material is presented that should help children move smoothly from sets of equivalent fractions to sets of fractional numbers. The last part of the chapter deals with equality and inequality for fractional numbers, including fractional numbers greater than one, and whole numbers as fractional numbers.

Mathematics

Fractions are different in concept from the numbers they represent. Initially, we take the point of view that fractions are symbols for number pairs; later, we also use fractions as symbols for fractional numbers.

The work with equivalent fractions is very important to the early part of the chapter. This work prepares children for fractional-number concepts. A formal definition of equivalent fractions states:

> Two fractions
> $$\frac{a}{b} \text{ and } \frac{c}{d}$$
> are equivalent to each other if and only if $a \times d = b \times c$.

After providing children with experiences using equivalent fractions, we lead them to build sets of equivalent fractions, starting with lowest-terms fractions such as $\frac{1}{2}$ and $\frac{2}{3}$. For example:

$A = \{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}, \frac{7}{14}, \frac{8}{16}, \ldots\}$

$B = \{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \ldots\}$

$C = \{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \ldots\}$

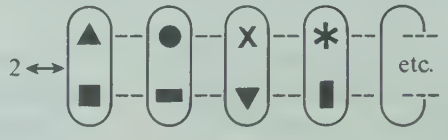
These patterns could be continued indefinitely, for each set contains an unlimited number of fractions. It is important to note that we can make this assumption: If these sets are endless and if any fraction that is not equivalent to $\frac{1}{2}$ is not in set A, then any fraction equivalent to $\frac{1}{2}$ is in set A. Similar statements are true for sets B and C, as well as for any other set of fractions constructed in this way. These points are essential to the general concepts of this chapter:

1. The definition of equivalent fractions divides the fractions into classes.
2. Any two fractions in one class are equivalent.
3. A fraction from one class and a fraction from a different class are not equivalent.
4. In any class, there is an unlimited number of fractions.
5. Every fraction is in exactly one class.

Building the fractional-number concept from fractions (number pairs) is very much like building the cardinal-number concept from sets. In the earlier *Investigating School Mathematics* texts, we began to develop the concept of cardinal numbers by establishing an understanding of equivalent sets. Next, children were led to think about classes of equivalent sets and the cardinal number associated with each class. For example, they learned to associate cardinal number 2 with the class of sets that are equivalent to:



The equivalence of the sets in this class is illustrated as follows:



Just as we abstracted cardinal number 2 from the class of sets illustrated above, so we can abstract the fractional number one half from the class of equivalent fractions

$$\{\tfrac{1}{2}, \tfrac{2}{4}, \tfrac{4}{8}, \tfrac{5}{10}, \tfrac{6}{12}, \tfrac{7}{14}, \ldots\}.$$

We can start with any lowest-terms fraction and imagine an infinite set of equivalent fractions. With each of the sets, we associate exactly one fractional number. The number line is very effective for illustrating this idea. For example, the number-line exercises on pages 148 and 149 of the text demonstrate that for each set of equivalent fractions, there is one and only one fractional number.

As shown in exercise 5, page 151, the number line also helps illustrate that each whole number can be considered as a fractional number and that fractional numbers greater than one have corresponding fractions.



Although we think of exactly one

number for a given point on the number line (and of exactly one point for a given fractional number), we may label the point with any fraction from the set of equivalent fractions for that number. Thus, the point associated with one half could be labelled with any fraction from the set $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{5}{10}, \frac{6}{12}, \ldots$.

The fact that each fractional number has many different names leads us to consider the concept of equality for fractional numbers. When we write $\frac{1}{2} = \frac{4}{8}$, we are indicating that $\frac{1}{2}$ and $\frac{4}{8}$ represent the same fractional number. This also indicates that the fractions are equivalent. Keep in mind that $\frac{1}{2} = \frac{4}{8}$ is a statement about numbers, not about fractions.

In general, we would say that $\frac{a}{b} = \frac{c}{d}$ means that $\frac{a}{b}$ and $\frac{c}{d}$ name the same fractional number. Again we observe that two fractions name the same fractional number if and only if they are equivalent.
Therefore:

$\frac{a}{b} = \frac{c}{d}$ if and only if $a \times d = b \times c$.

If we wish to compare two fractional numbers such as $\frac{3}{8}$ and $\frac{5}{8}$, we could consider the physical interpretations from which the fractions $\frac{3}{8}$ and $\frac{5}{8}$ arise.

Consider these examples.

(1) $\frac{3}{8} \neq \frac{5}{14}$ since $3 \times 14 \neq 8 \times 5$
(2) $\frac{9}{15} \neq \frac{4}{6}$ since $9 \times 6 \neq 15 \times 4$

Using the number line, we could clearly illustrate the preceding examples. Also, we could find fractions with common denominators for the two fractional numbers. Consider example 2. Since $\frac{9}{15} = \frac{18}{30}$ and $\frac{4}{6} = \frac{20}{30}$, then $\frac{9}{15} < \frac{4}{6}$ because $\frac{18}{30} < \frac{20}{30}$.

Although it is inconvenient to use physical objects or the number line to consider the inequality in the preceding example, such interpretations for certain simple pairs of fractions will aid the children. For example, to demonstrate that $\frac{1}{2} > \frac{1}{3}$, we could examine the scale on a measuring cup. To see that $\frac{1}{5} < \frac{1}{4}$, we could use a number line or some physical object.

Teaching the Chapter

Materials

Colored strips (1 set per child)
Demonstration number line
Overhead projector (if available) and transparencies
Regions shaded in fractional parts

Vocabulary

denominator	higher terms
equivalent fraction	improper fraction
fraction	lower terms
fractional number	lowest terms
	number pair
	numerator

You may find it helpful, and even necessary, to show some objects being cut into parts, or at least draw some pictures of cut-up objects to help slower children develop a feeling for fraction and fractional-number concepts. Continue to guide the children toward more abstract thinking as their experiences with fractions and fractional numbers intensify throughout this chapter and those to come. In the early part of the chapter, sufficient material of a concrete nature is provided for most of the children to gain a feeling for fraction concepts. As fractional-number ideas are developed later in the chapter, encourage the children to abstract the fraction and fractional-number concepts from the physical world.

Most of the words in the vocabulary list are familiar to the children. Minimize use of the expression *improper fraction*, and try to avoid having the children think that there is anything wrong with a fraction such as $\frac{8}{5}$. The children should be exposed to the expression *improper fraction* but, when possible, just use the word *fraction* for the improper fraction.

Lesson Schedule

Plan to cover this material in about a week and a half to two weeks. As with all suggested schedules, you should adjust this one to the needs and abilities of your children.

Evaluation of Progress

Evaluating children's achievement for this chapter is not simple. While there are several important skills presented, do not underrate the importance of having children relate a single quantitative idea to each given set of equivalent fractions. This is a key concept of the chapter, but it is difficult to test. We suggest that you base much of your evaluation of the children's achievement on their day-to-day experiences and participation in class discussions.

Pages 154 and 155 provide chapter and cumulative reviews.

Resources for Active Learning

General Activities

Franklin Series: *Patterns and Puzzles*, "Fraction Fun," pp. 64–66. Lyons and Carnahan. (Available from McGraw-Hill Ryerson)
It's a Tangram World, "Fractions," pp. 39–52, Educational Science Consultants
Notes on Mathematics in Primary Schools, "Fractions," pp. 190–193, Cambridge University Press. (Available from Macmillan)
Nuffield Project: *Computation and Structure* 3, "Fractions," pp. 70–78, Wiley

Manipulative Devices

Cuisenaire Cubes, Squares, and Rods (Cuisenaire Co.)
Discovery Blocks (Educational Teaching Aids)
Fraction Blocks (Creative Publications)
"Invicta" Math Balance (Math Media; Selective Educational Equipment)
Modulated Cutouts (Encyclopaedia Britannica Publications Ltd.)
Unifix Fractions Kit (Educational Teaching Aids)

PAGES 142–143

Objective

Given appropriate parts of regions or sets, the child will be able to write fractions to compare the part to the whole region or set.

Preparation

Materials

colored strips

To prepare for this lesson, you might refer to the strips and ask children to mention some of the ways in which they have been used. For example, they have used them in work with different bases. If they have used previous books of this series, they might recall that they used them in measurement and with fractions, as they will in this chapter.

Investigation

Read the introductory material with the children, helping them see that each strip is being considered in parts. Have them note the side of the strips which have the centimetre divisions marked. They might find this marked side of the strips easier to work with. Remind the children to record both the colored strips and the fraction.

Some may make a table such as the following:

Strips	Fractions
yellow / black	$\frac{5}{7}$
orange / blue	$\frac{10}{9}$
white / red	$\frac{1}{2}$

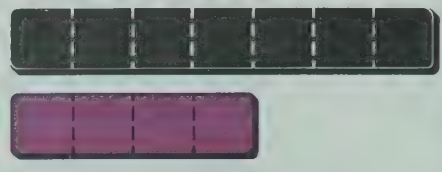
7 Fractional Numbers

Let's explore fractions.

Investigating the Ideas



The light green strip has 3 parts, compared with 5 parts for the yellow strip. The **fraction** $\frac{3}{5}$ compares the light green strip with the yellow strip.



The black strip has 7 parts, compared with 4 parts for the purple strip. The **fraction** $\frac{7}{4}$ compares the black strip with the purple strip.

? Choose four of your strips. Can you write fractions that compare each strip with each of the other strips? Answers will vary. See Investigation.

Record each pair of strips and the corresponding fractions.

Discussing the Ideas

1. You can say that the light green strip above is $\frac{3}{5}$ the size of the yellow strip. How can you use a fraction to describe this pair?



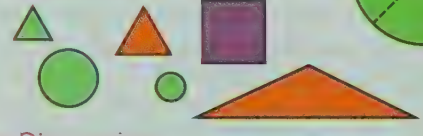
The red strip is $\frac{3}{5}$ the size of the green strip.

2. How can you use fractions to describe what you see in this picture of a circular region?



Answers will vary. See Discussion.

3. How can you use fractions to describe what you see in this picture?



Answers will vary. See Discussion.



142

Discussion

Give children an opportunity to share the results of the investigation. When appropriate, point out that some of their comparisons were expressed as improper fractions and review the idea that such a fraction is a fraction in which the numerator is greater than or equal to the denominator.

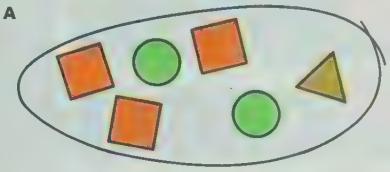
As you discuss the exercises in the text, you will want to bring out one important point: When we speak of a fractional part of a region, we consider that the region has been divided into parts of the same size. For example, in exercise 1, refer to the centimetre parts of the strips. Since the light green strip is the strip with which the red strip is being compared, and since it is divided into 3 parts of the same size, the correct fraction is $\frac{3}{5}$. Exercise 2 treats the concept of fraction in relation to fractional parts of a region. Encourage the children to use as many different fractions as possible: $\frac{1}{4}$ and $\frac{2}{8}$ of the region is red, $\frac{1}{2}$ and $\frac{4}{8}$ of the region is green, $\frac{3}{4}$ of the region is not white, and so on.

Exercise 3 introduces the use of fractions to describe fractional parts of a set rather than of a region. Notice that, in this case, we are not concerned with the size of

Using the Ideas

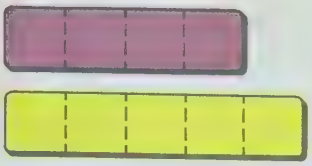
1. Find the numbers for ***a*** and ***b***.
Then write the fraction for the ▥.

A



3
a of the regions are squares.
6
b regions in all.
3/6 ▥ of the regions are squares.

B



The purple strip has ***a*** parts. 4
The yellow strip has ***b*** parts. 5
The purple strip is ▥ of the 4/5 yellow strip.

C



2 6
a of the ***b*** parts of the dark green strip are covered.
2/6 ▥ of the dark green strip is covered.

D

3 4
a of ***b*** parts are colored.



3/4 ▥ of the square is colored.

2. A Give two fractions to tell what part of the set of children are girls. 6/12, 1/2

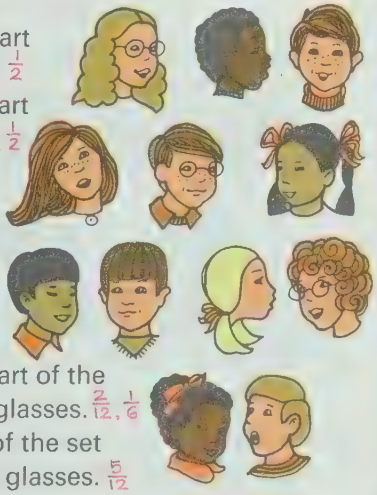
B Give two fractions to tell what part of the set of children are boys. 6/12, 1/2

C Give two fractions to tell what part of the set of children are wearing glasses. 3/12, 1/4

D Give two fractions to tell what part of the set of girls are wearing glasses. 2/6, 1/3

E Give two fractions to tell what part of the set of children are girls wearing glasses. 2/12, 1/6

F Give a fraction to tell what part of the set of children are boys not wearing glasses. 5/12



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the objects, but rather with the number of each kind of object, such as triangles, squares, circles, or the color of the regions. Again encourage variety in answers: $\frac{1}{2}$ of the figures are green, $\frac{1}{6}$ of the figures are blue, $\frac{2}{6}$ of the figures are circles, $\frac{1}{2}$ of the figures are triangles, and so on. You might want to use other examples of fractional parts of a set.

Using the Exercises

Assign the exercises on page 143 as independent work. When children finish, carefully check their work and discuss various exercises. Use parts of exercise 2 to point out how important it is to realize which set is the one to which the second set is being compared. Here, we are not concerned with the size of the objects in the set; we are concerned only with the number of objects in the set.

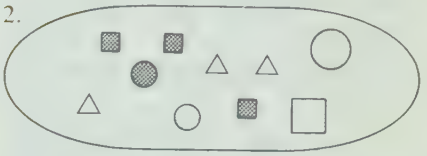
Assignments (page 143)
Minimum: 1, 2. Average: 1, 2.
Maximum: 1, 2.

Follow-up

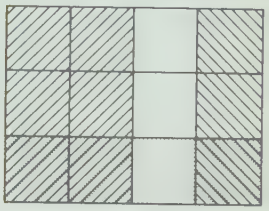
Suggest to the children that they choose one of the following situations and write as many fractions related to it as they can.

1. 8 children; 10 candy bars; 6 milkshakes; 4 boys; 4 girls; 2 tables; 8 hamburgers.

2.



3.



Resources for Active Learning

A Cloudburst, Vol. 2, No. 3113–3153, Midwest Publications.

Math Activity Cards, E36–37, Macmillan [Fraction sequences].

Mathex: Numeration No. 7 "The Meaning of Fractions," pp. 34–40 (pupil pages 27–28), Encyclopaedia Britannica Publications Ltd.

PAGES 144–145

Objective

Given fractional parts of a region or fractional parts of a set, the child will be able to give a fraction to describe the parts and be able to build a set of equivalent fractions.

Preparation

Materials

rolls of adding machine tape (optional)

If you wish to have a preparation for this lesson, display various regions with fractional parts shaded and ask children to describe each with a fraction. You might also use pairs of the strips for this same purpose.

Investigation

With most groups of children, you will want to read the introductory material together and make sure children realize that the fractions $\frac{1}{3}$, $\frac{2}{6}$, and $\frac{3}{9}$ all compare the red strip to the dark green strip. Some children may need specific guidance in making other "tapes." Using rolls of adding machine tape may stimulate interest in this activity. You might help the children get started by asking them to compare the yellow strip with the orange strip, the red strip with the black strip, or the light green strip with the yellow strip.

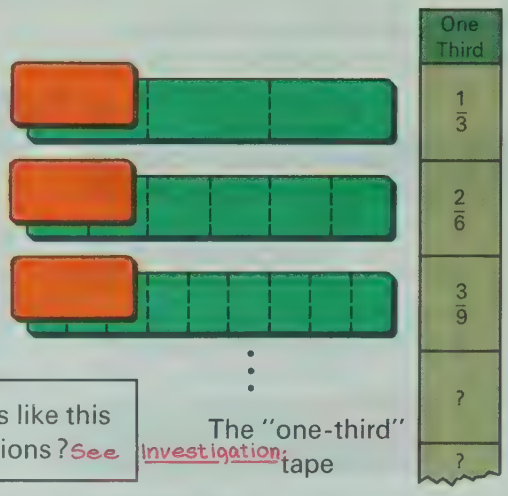


Can you build sets of equivalent fractions?

Investigating the Ideas

Imagine the dark green strip divided equally into different numbers of parts. Give the next four fractions on the "one-third" tape.

$\frac{4}{12}, \frac{5}{15}, \frac{6}{18}, \frac{7}{21}$



The "one-third" tape

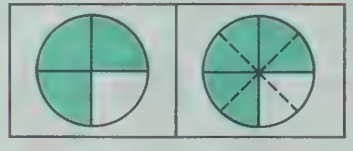
Can you make tapes like this for some other fractions? See Investigation.

Discussing the Ideas

A pair of fractions that suggest the same number of objects in a set or the same part of an object are called **equivalent fractions.**

1. Use the strips pictured above to explain why $\frac{2}{6}$ is equivalent to $\frac{3}{9}$. Sample answer: The red strip covers the same amount of the green strip in both cases.
2. What pair of equivalent fractions is suggested by each pair of pictures? Explain. See Discussion.

A



$\frac{3}{4}, \frac{6}{8}$ (or $\frac{1}{4}, \frac{2}{8}$)

B



$\frac{6}{9}, \frac{2}{3}$ (or $\frac{3}{9}, \frac{1}{3}$)

3. Can you explain an easy way to make the "equivalent-fraction" tapes? See Discussion.
4. Explain why any two of the fractions from the one-third tape are equivalent. See Discussion.

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Discussion

As you explain equivalent fractions, use both the example of strips in the text and examples from the children's investigation results. Stress that we can imagine each pair of strips as being divided into different parts. In exercise 2 point out that in example A, if the children consider the dotted lines in the second figure, they would think of $\frac{6}{8}$ rather than $\frac{3}{4}$, but they should observe that the same amount of both regions is shaded. Hence, we can say that $\frac{3}{4}$ is equivalent to $\frac{6}{8}$. In example B, using the dotted lines will help the children to think of $\frac{2}{3}$ rather than $\frac{6}{9}$. Since the same number of squares is shaded, $\frac{6}{9}$ is equivalent to $\frac{2}{3}$.

Use several examples as you discuss exercise 3, to demonstrate the process of building equivalent sets in the manner suggested by exercise 4 on page 145.

In discussing exercise 4, relate the idea to other sets of equivalent fractions. Stress that each fraction in such a set names the same region or describes the same part of a set, no matter what equal parts may be used to describe it. Help children realize that the "torn-off" end on the tapes indicates that there is an unlimited number of equivalent fractions for each tape.

Using the Ideas

Note that answers in exercises 1 and 2 will vary if children choose to describe the figures in terms of the white rather than the shaded parts.

1. Each exercise below suggests a pair of equivalent fractions. For example, figure A suggests the fractions $\frac{1}{2}$ and $\frac{2}{4}$. Give a pair of equivalent fractions for each figure.

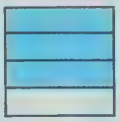
A 
$\frac{1}{2}, \frac{2}{4}$

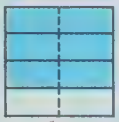
B 
$\frac{1}{4}, \frac{3}{12}$ (or $\frac{3}{4}, \frac{9}{12}$)

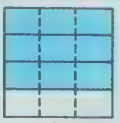
C 
$\frac{2}{3}, \frac{4}{6}$ (or $\frac{1}{3}, \frac{2}{6}$)

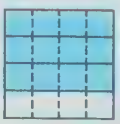
D 
$\frac{1}{4}, \frac{4}{16}$ (or $\frac{3}{4}, \frac{12}{16}$)

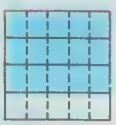
2. Give the fraction suggested by each figure.
Are any two of the fractions equivalent? All are equivalent.

A 
$\frac{3}{4}$ (or $\frac{1}{4}$)

B 
$\frac{6}{8}$ (or $\frac{2}{8}$)

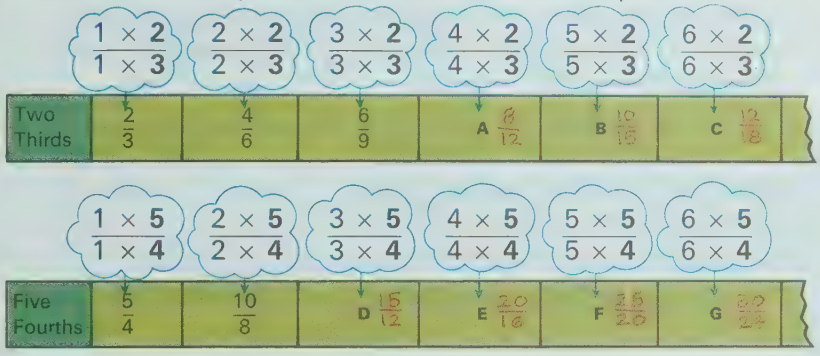
C 
$\frac{9}{12}$ (or $\frac{3}{12}$)

D 
$\frac{12}{16}$ (or $\frac{4}{16}$)

E 
$\frac{15}{20}$ (or $\frac{5}{20}$)

3. Give the next 3 fractions for each set of equivalent fractions.
A $\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \frac{15}{20}, \ldots\}$ $\frac{18}{24}, \frac{21}{28}, \frac{24}{32}$
B $\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \frac{5}{20}, \ldots\}$ $\frac{6}{24}, \frac{7}{28}, \frac{8}{32}$

4. Give the missing fractions for each of these "tapes."



5. Copy each set of fractions. Give 3 more fractions for each set.
A $\{\frac{1}{5}, \frac{2}{10}, \frac{3}{15}, \ldots\}$ $\frac{4}{20}, \frac{5}{25}, \frac{6}{30}$ C $\{\frac{1}{2}, \frac{2}{4}, \ldots\}$ $\frac{4}{8}, \frac{5}{10}, \frac{6}{12}$ E $\{\frac{1}{3}, \ldots\}$ $\frac{2}{6}, \frac{3}{9}, \frac{4}{12}$ G $\{\frac{9}{10}, \ldots\}$ $\frac{18}{20}, \frac{27}{30}$,
B $\{\frac{6}{5}, \frac{12}{10}, \frac{18}{15}, \ldots\}$ $\frac{24}{20}, \frac{30}{25}, \frac{36}{30}$ D $\{\frac{3}{10}, \frac{6}{20}, \ldots\}$ $\frac{9}{30}, \frac{12}{40}, \frac{15}{60}$ F $\{\frac{8}{5}, \ldots\}$ $\frac{16}{10}, \frac{24}{15}, \frac{32}{20}$ H $\{\frac{1}{20}, \ldots\}$ $\frac{36}{40}$ $\frac{2}{40}, \frac{3}{60}, \frac{4}{80}$

Using the Exercises

Exercises 1, 2, and 3 on page 145 stress the basic meaning of equivalent fractions and the ability to recognize equivalent fractions. Exercises 4 and 5 are intended to develop the skill of building sets of equivalent fractions. If you did not already do so during the discussion, emphasize the method used to build equivalent sets in exercise 4. Have the children practice using this method for at least some of the sets in exercise 5. When children have completed the work, help them carefully check their answers, particularly for exercise 5.

Assignments (page 145)
Minimum: 1–5. Average: 1–5.
Maximum: 1–5.

Mathematics

In this lesson, instead of providing a formal definition for the equivalence of two fractions, we introduce the idea in terms of sets and parts of a region, in such a way that children perceive intuitively the general idea of equivalent fractions.

The standard definition of equivalent fractions follows.

> The fraction $\frac{a}{b}$ is equivalent to $\frac{c}{d}$ if, and only if, $a \times d = b \times c$.

Consider this example: The fraction $\frac{2}{3}$ is equivalent to $\frac{2}{3}$ because $2 \times 3 = 3 \times 2$. Now let us examine the two fractions $\frac{2}{3}$ and $\frac{4}{6}$. If we write the numerator and denominator of $\frac{4}{6}$ in factored form, that is, as $\frac{2 \times 2}{2 \times 3}$, it is easy to see that $\frac{4}{6}$ differs from $\frac{2}{3}$ only in having an additional factor of two in both the numerator and the denominator. Hence, $2 \times (2 \times 3) = 3 \times (2 \times 2)$.

The equivalence relation divides the set of all fractions into equivalence classes. In other words, any two fractions within the same class of fractions are equivalent. Also, we see that if two fractions are not equivalent they are in different classes, and if they are equivalent they are in the same class.

Also, note that there is an unlimited number of fractions in each of these equivalence classes. This idea of dividing the fractions into equivalence classes leads us to the concept of fractional number.

Follow-up

Most children would benefit from additional experiences in building sets of equivalent fractions. You might ask them to fill in these sets:

$\left\{\frac{2}{5}, \frac{4}{10}, \frac{?}{15}, \frac{?}{?}, \frac{?}{?}, \frac{?}{?}, \frac{?}{?}, \frac{?}{?}, \ldots\right\}$.

$\left\{\frac{4}{3}, \frac{?}{?}, \frac{?}{?}, \frac{16}{12}, \frac{?}{15}, \frac{24}{?}, \frac{?}{?}, \frac{?}{?}, \frac{?}{?}, \ldots\right\}$.

Resources for Active Learning

Mathex: Numeration No. 7, "Equivalent Fractions," pp. 41–45 (pupil pages 39–41), Encyclopaedia Britannica Publications Ltd.

Workbook, page 43

PAGES 146–147

Objective

Given a fraction, the child will be able to express it in lowest terms.

Preparation

It would be helpful to review how to build sets of equivalent fractions. Write a fraction on the chalkboard and ask the children to describe how to find other fractions in this set. You might also use these fractions to review the terms *numerator* and *denominator*.

Investigation

Children might benefit from working on this investigation in groups of two or three, so that they can share ideas and explain their reasons for a particular choice to each other. Move around the room as children work on the investigation, and give indirect guidance when necessary. For example, ask a child who is having difficulty with a particular fraction whether he can express the numbers in the fraction in terms of their factors. Thus, $\frac{12}{16}$ might be expressed as

$$\frac{3 \times 4}{4 \times 4}, \text{ or } \frac{2 \times 6}{2 \times 8}.$$

Such an approach should help the child break down the terms of the fractions until he eventually finds the lowest-terms fraction. You might also point out that $\frac{9}{15}$ and $\frac{12}{20}$ are equivalent fractions.

How do you find lowest-terms fractions?

Investigating the Ideas

When you build a set of equivalent fractions, you start with a **lowest-terms** fraction.

Lowest-Terms Fraction	$\frac{2 \times 2}{2 \times 5}$	$\frac{3 \times 2}{3 \times 5}$	$\frac{4 \times 2}{4 \times 5}$	$\frac{5 \times 2}{5 \times 5}$	$\frac{6 \times 2}{6 \times 5}$	$\frac{7 \times 2}{7 \times 5}$
Two Fifths	$\frac{2}{5}$	$\frac{4}{10}$	$\frac{6}{15}$	$\frac{8}{20}$	$\frac{10}{25}$	$\frac{12}{30}$	$\frac{14}{35}$

? Can you find the lowest-terms fraction that was used to "build" each of these fractions? See Investigation.

A	B	C	D	E
$\frac{3}{5}$ $\frac{9}{15}$ $\frac{12}{20}$	$\frac{1}{2}$ $\frac{3}{6}$	$\frac{3}{8}$ $\frac{9}{24}$	$\frac{5}{7}$ $\frac{10}{14}$	$\frac{3}{4}$ $\frac{12}{16}$

Discussing the Ideas

1. When the numerator and the denominator of a fraction have no common factor other than 1, the fraction is a lowest-terms fraction. In the examples below, the factors of the numerator and the denominator are given.

   The fraction $\frac{12}{15}$ is not a lowest-terms fraction. Why?

   $\frac{12}{15}$ {1, 2, 3, 4, 6, 12} / {1, 3, 5, 15}

   They have common factor other than 1.

   The fraction $\frac{8}{9}$ is a lowest-terms fraction. Why?

   $\frac{8}{9}$ {1, 2, 4, 8} / {1, 3, 9}

   They have no common factor other than 1.

2. To find the lowest-terms fraction you can "divide out" common factors. Explain examples A and B and then use this method to find the lowest-terms fraction for $\frac{24}{30}$.

   A $\frac{60}{90} \rightarrow \frac{60 \div 10}{90 \div 10} \rightarrow \frac{6}{9} \rightarrow \frac{6 \div 3}{9 \div 3} \rightarrow \frac{2}{3}$

   B $\frac{24}{18} \rightarrow \frac{24 \div 2}{18 \div 2} \rightarrow \frac{12}{9} \rightarrow \frac{12 \div 3}{9 \div 3} \rightarrow \frac{4}{3}$

   $\frac{24}{30} \rightarrow \frac{24 \div 2}{30 \div 2} \rightarrow \frac{12}{15} \rightarrow \frac{12 \div 3}{15 \div 3} \rightarrow \frac{4}{5}$

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Discussion

Have the children name the lowest-terms fractions they found in the investigation. Ask them to explain why they decided these were lowest-terms fractions. On the basis of their experience in the investigation, some children will probably say that all the other fractions in a given set can be built from its lowest-terms fraction. The discussion exercises are intended to help children refine their concept of lowest-terms fraction.

As you work through exercises 1 and 2, emphasize the fact that we can "divide out" common factors in order to find the lowest-terms fraction for a given fraction, as illustrated in examples A and B of exercise 2.

At this point you might also stress that if we are able to determine the greatest common factor of the numerator and denominator, we can arrive at the lowest-terms fraction in one step rather than in several. For example, B, note that $\frac{24}{18}$ could have been reduced to $\frac{4}{3}$ in one step by dividing by 6 the greatest common factor. Do not insist, however, that children find the greatest common factor in one step so long as they eventually succeed in finding lowest-terms fractions.

Using the Ideas

1. Build a set of six equivalent fractions from each of these lowest-terms fractions.
Example: $\frac{3}{4} \rightarrow \{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \frac{15}{20}, \frac{18}{24}\}$

A $\frac{1}{2}$ B $\frac{1}{4}$ C $\frac{2}{3}$ D $\frac{5}{8}$ E $\frac{1}{10}$ F $\frac{10}{7}$ G $\frac{5}{6}$ H $\frac{3}{5}$

See Answers, T.E. page 147.

2. In each of these sets of fractions, the lowest-terms fraction is covered. Give the lowest-terms fraction for each set.

A $\{\blacksquare, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \ldots\}$ $\frac{1}{3}$
B $\{\blacksquare, \frac{6}{20}, \frac{9}{30}, \frac{12}{40}, \ldots\}$ $\frac{3}{10}$
C $\{\blacksquare, \frac{10}{14}, \frac{15}{21}, \frac{20}{28}, \ldots\}$ $\frac{5}{7}$
D $\{\blacksquare, \frac{2}{12}, \frac{3}{18}, \frac{4}{24}, \ldots\}$ $\frac{1}{6}$
E $\{\blacksquare, \frac{14}{16}, \frac{21}{24}, \frac{28}{32}, \ldots\}$ $\frac{7}{8}$
F $\{\blacksquare, \frac{18}{20}, \frac{27}{30}, \frac{36}{40}, \ldots\}$ $\frac{9}{10}$

3. Tell whether or not the fraction is a lowest-terms fraction. The factors of the numerator and denominator are given in red.

A $\frac{14}{15}$ {1, 2, 7, 14} {1, 3, 5, 15} — Yes
B $\frac{16}{18}$ {1, 2, 4, 8, 16} {1, 2, 3, 6, 9, 18} — No
C $\frac{8}{12}$ {1, 2, 4, 8} {1, 2, 3, 4, 6, 12} — No
D $\frac{9}{16}$ {1, 3, 9} {1, 2, 4, 8, 16} — Yes
E $\frac{12}{20}$ {1, 2, 3, 4, 6, 12} {1, 2, 4, 5, 10, 20} — No
F $\frac{15}{28}$ {1, 3, 5, 15} {1, 2, 4, 7, 14, 28} — Yes

4. Tell whether or not these fractions are in lowest terms.

A $\frac{6}{10}$ No B $\frac{9}{15}$ No C $\frac{8}{15}$ Yes D $\frac{10}{18}$ No E $\frac{24}{9}$ No F $\frac{15}{20}$ No G $\frac{21}{5}$ Yes H $\frac{30}{25}$ No

5. Give the lowest-terms fraction for each fraction.

A $\frac{8}{12}$ $\frac{2}{3}$
B $\frac{16}{20}$ $\frac{4}{5}$
C $\frac{8}{20}$ $\frac{2}{5}$
D $\frac{20}{60}$ $\frac{1}{3}$
E $\frac{25}{100}$ $\frac{1}{4}$
F $\frac{25}{10}$ $\frac{5}{2}$
G $\frac{4}{24}$ $\frac{1}{6}$
H $\frac{15}{50}$ $\frac{3}{10}$
I $\frac{6}{50}$ $\frac{3}{25}$
J $\frac{75}{100}$ $\frac{3}{4}$
K $\frac{9}{21}$ $\frac{3}{7}$
L $\frac{8}{10}$ $\frac{4}{5}$
M $\frac{28}{14}$ $\frac{2}{1}$
N $\frac{50}{75}$ $\frac{2}{3}$
O $\frac{16}{30}$ $\frac{8}{15}$
P $\frac{50}{100}$ $\frac{1}{2}$

think

Susan has some coins.
Only 2 of them are alike.
What coins does she have if she has 96¢ in all?

2 possible solutions:
1-50¢ 1-25¢ 1-10¢ 2-5¢ 1-1¢
or
1-50¢ 1-25¢ 2-10¢ 1-1¢



More practice, page A-9, Set 18

Follow-up

Various card games can be used with children who have difficulty recognizing even the more common lowest-terms fractions. For example, a game similar to "Fish" may be played. Each player receives seven cards. His object is to find among his cards pairs of equivalent fractions. The first player may ask another player for a particular card which he needs in order to make a fraction. If that player does not have it, the asker must "fish" from the deck; if that player does have it, the asker takes it and may then ask another player for another card. When a player can find a pair of equivalent fractions, he puts the four cards in a "book" in front of him. The object of the game is to get the most books. The game ends when the centre "Fish" pile has been used up or when one player has succeeded in making all of his cards into books.

Answers, exercise 1, page 147

A $\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}\}$
B $\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \frac{5}{20}, \frac{6}{24}\}$
C $\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15}, \frac{12}{18}\}$
D $\{\frac{5}{8}, \frac{10}{16}, \frac{15}{24}, \frac{20}{32}, \frac{25}{40}, \frac{30}{48}\}$
E $\{\frac{1}{10}, \frac{2}{20}, \frac{3}{30}, \frac{4}{40}, \frac{5}{50}, \frac{6}{60}\}$
F $\{\frac{10}{7}, \frac{20}{14}, \frac{30}{21}, \frac{40}{28}, \frac{50}{35}, \frac{60}{42}\}$
G $\{\frac{5}{6}, \frac{10}{12}, \frac{15}{18}, \frac{20}{24}, \frac{25}{30}, \frac{30}{36}\}$
H $\{\frac{4}{7}, \frac{8}{14}, \frac{12}{21}, \frac{16}{28}, \frac{20}{35}, \frac{24}{42}\}$

Duplicator Masters, page 26
Workbook, page 44
Skill Masters, page 26

Using the Exercises

Some groups of children would benefit from working through parts of the exercises on page 147 with you. However, encourage most to do the exercises independently. Use exercise 3 to stress the importance of being able to recognize common factors and of understanding that if the numerator and denominator have a common factor greater than 1, the fraction is not in lowest terms. Some children will be able to recognize and find lowest-terms fractions quickly; others may need to list the factors of the numerator and denominator to find and "divide out" common factors.

The *Think* problem may challenge some children. Note that we do not have to be told how many coins Susan has. The stipulation that only two of them are alike determines the answer.

Assignments (page 147)
Minimum: 1, 2, 5. Average: 1–5.
Maximum: 1–5.

PAGES 148–149

Objective

Given a set of equivalent fractions, the child will be able to match the set with one fractional number and one point on a number line.

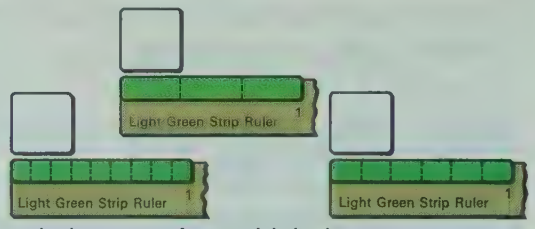
Preparation

A good way to prepare for this lesson is to be sure that the children are quite familiar with sets of equivalent fractions. You might, therefore, find it helpful to give the children an opportunity to build several sets of equivalent fractions, starting with given lowest-terms fractions.

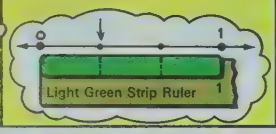
● *Let's explore fractional numbers.*

Discussing the Ideas

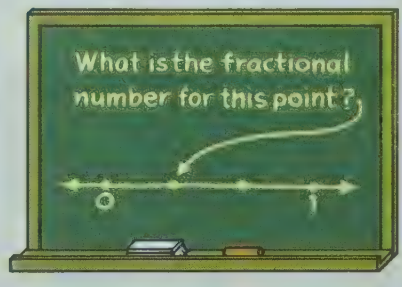
1. What is the **length** of the white strip when the light green strip is the unit? After choosing the unit, the length of the strip is a **number** which does not change, but different fractions can be used to represent it. What are some of these fractions? $\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \cdots\}$

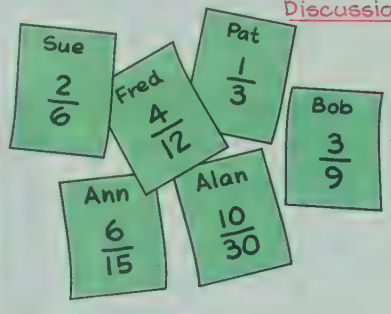


2.

For each set of equivalent fractions	we think of one fractional number	and one point on the number line.
One Third: $\frac{1}{3}$ $\frac{2}{6}$ $\frac{3}{9}$ $\frac{4}{12}$		

Don is thinking of a fractional number. How do you think he found the point for this number on the number line? See Discussion.

3. 



Any fraction from a set of equivalent fractions can be used to name the fractional number for that set. Can you find the one paper that is incorrect? Give a name for this fractional number that is different from any of the names above.
Ann's paper is incorrect. Other names for $\frac{1}{3}$: $\{\frac{5}{15}, \frac{6}{18}, \frac{7}{21}, \cdots\}$

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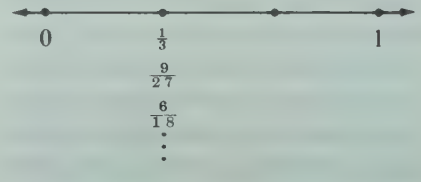


Discussion

The most important point of this lesson is to help children realize that any fraction they have studied is one of many names for a number-idea, that is, for a fractional number. As you discuss exercise 1, recall with the children that, once a unit of measure has been selected, the length of an object is fixed. The different subdivisions of the unit do not change the length of the object being measured; they only give us different ways of expressing the length. Thus, $\frac{1}{3}$, $\frac{2}{6}$, and $\frac{3}{9}$ are all symbols for the number that gives the length of the white strip when the light green strip is the unit.

In exercise 2, the first and second column show that, for a set of equivalent fractions, we can think of one fractional number. Columns two and three show that this one fractional number can be matched with one point on the number line. Thus, any of the fractions in the first column can be used to name the number which matches that particular point on the number line.

Use several examples to develop this idea. For example, use a number line on the chalkboard or overhead projector and mark some obvious divisions (halves, fourths, or thirds). Point to a specific point and ask the children to give several fractions which name the fractional number for this point.



Using the Ideas

1. Each set of equivalent fractions indicates one fractional number. Give the correct point (*X*, *Y*, or *Z*) for that fractional number.

A $\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \ldots\}$ Z

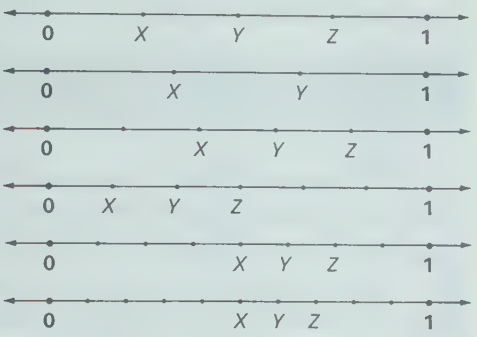
B $\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \ldots\}$ Y

C $\{\frac{4}{5}, \frac{8}{10}, \frac{12}{15}, \frac{16}{20}, \ldots\}$ Z

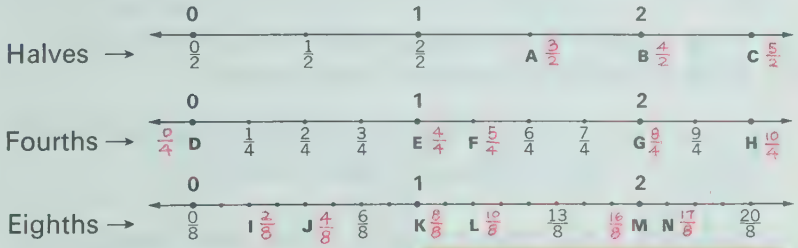
D $\{\frac{1}{6}, \frac{2}{12}, \frac{3}{18}, \frac{4}{24}, \ldots\}$ X

E $\{\frac{5}{8}, \frac{10}{16}, \frac{15}{24}, \frac{20}{32}, \ldots\}$ Y

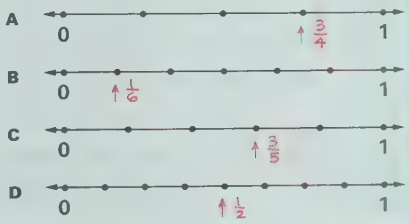
F $\{\frac{7}{10}, \frac{14}{20}, \frac{21}{30}, \frac{28}{40}, \ldots\}$ Z



2. Using the denominator suggested at the left of each number line, give the fraction for the point above each letter.



3. Give the lowest-terms fraction that names the fractional number for the point over the red arrow.



think

Janet had 69¢. Sally asked her for change for a 50-cent piece. Janet tried to make change but found she could not. What coins did she have if they were all less than 50¢?

One quarter, four dimes, four pennies



Mathematics

This lesson contains the heart of the mathematical development of the fractional-number concepts: for each set of equivalent fractions, there is exactly one fractional number and one point on the number line. Exercise 2 on page 148 illustrates this fact. Notice that at the left is a set of equivalent fractions. In the centre is a child thinking about exactly one number for this set of equivalent fractions. At the right, we see that for this one number there is exactly one point on the number line. Thus, there are three distinct things to be considered: the set of equivalent fractions, the fractional number (which is undefined), and the point on the number line for that fractional number.

It is not unusual to leave the idea of number undefined. We did not attempt to define precisely what is meant by cardinal number. Similarly, we never attempt to say precisely what a fractional number is. Rather, we simply point out that, for each set of equivalent fractions, we think of exactly one number and one point on the number line. It is important for the children to understand that there is a certain quantitative sameness associated with all the fractions in a given set of equivalent fractions.

Children need not know precise mathematical definitions of whole numbers or fractional numbers in order to work with them. They should have a general intuitive feeling for the number ideas, the symbols used to represent the numbers, the quantitative size of each number (location on the number line), and rules, or basic principles, to guide them in computing.

Each fractional number is also a rational number, but the term fractional number is preferred at this level. Rational numbers include the set of all fractional numbers and their negatives. Fractional numbers include only the set of positive rational numbers and zero.

Using the Exercises

Assign the exercises on page 149 as independent work. Note that in exercise 1 children must study the divisions in the number lines carefully, in order to identify the correct point.

When children finish, help them check their work. Stress that they can use any fraction from the set of equivalent fractions to name a given fractional number, and point out also that the lowest-terms fraction is the fraction most often chosen to name the fractional number represented by a set of equivalent fractions.

You may wish to give some children play coins to manipulate as they try to work out the *Think* problem. Capable children will probably be able to decide, without using coins, that Janet does not have any nickels.

Assignments (page 149)

Minimum: 1. Average: 1–3.
Maximum: 1–3.

Workbook, page 45

PAGES 150–151

Objective

Given a pair of fractions, the child will be able to determine whether or not they are equivalent fractions by using the cross-product method.

Preparation

Materials

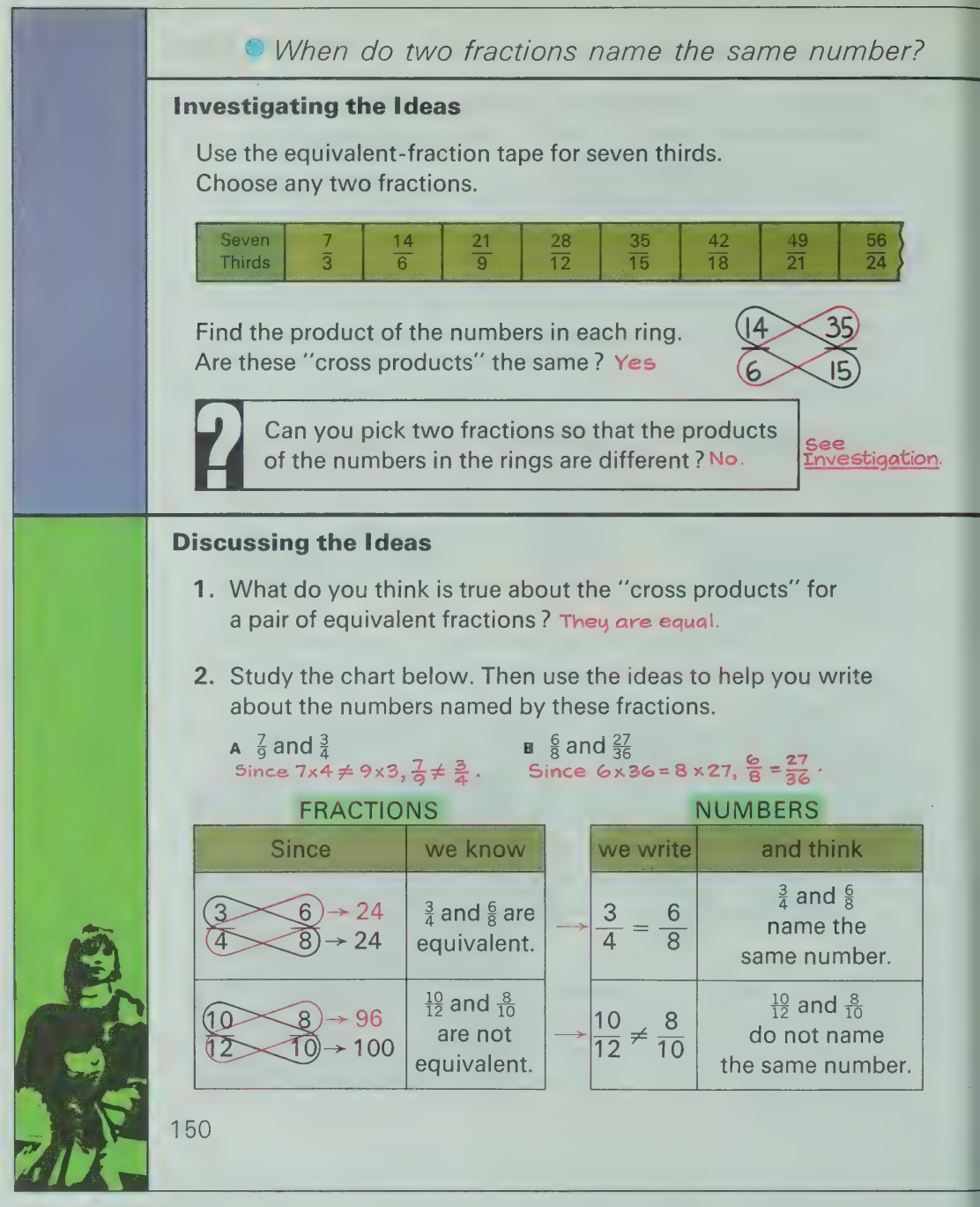
demonstration number line or overhead projector (*optional*)

To prepare for this lesson, you might review the main concept of the previous lesson, namely, that any fraction from a set of equivalent fractions can be used to name the fractional number represented by that set. For this purpose, use a number line on the chalkboard or overhead projector and ask children to give several fractions which can be used to name the fractional number matched with the point.



Investigation

Encourage the children to work on this investigation independently. Some of them may recall the use of cross products from grade five. Others should discover that the cross products they find for the pairs chosen from the equivalent-fraction tape for seven thirds are equal regardless of whether or not they equal 210. In the process of looking for two fractions whose cross products are different, they should observe that the cross products of any pair of equivalent fractions are equal.



When do two fractions name the same number?

Investigating the Ideas

Use the equivalent-fraction tape for seven thirds.
Choose any two fractions.

Seven Thirds	$\frac{7}{3}$	$\frac{14}{6}$	$\frac{21}{9}$	$\frac{28}{12}$	$\frac{35}{15}$	$\frac{42}{18}$	$\frac{49}{21}$	$\frac{56}{24}$

Find the product of the numbers in each ring.
Are these "cross products" the same? Yes

? Can you pick two fractions so that the products of the numbers in the rings are different? No.

See Investigation.

Discussing the Ideas

1. What do you think is true about the "cross products" for a pair of equivalent fractions? They are equal.

2. Study the chart below. Then use the ideas to help you write about the numbers named by these fractions.

A $\frac{7}{9}$ and $\frac{3}{4}$ Since $7\times4 \neq 9\times3$, $\frac{7}{9} \neq \frac{3}{4}$.

B $\frac{6}{8}$ and $\frac{27}{36}$ Since $6\times36 = 8\times27$, $\frac{6}{8} = \frac{27}{36}$.

FRACTIONS		NUMBERS	
Since	we know	we write	and think
$3\times8 = 24$, $4\times6 = 24$	$\frac{3}{4}$ and $\frac{6}{8}$ are equivalent.	$\frac{3}{4} = \frac{6}{8}$	$\frac{3}{4}$ and $\frac{6}{8}$ name the same number.
$10\times10 = 100$, $12\times8 = 96$	$\frac{10}{12}$ and $\frac{8}{10}$ are not equivalent.	$\frac{10}{12} \neq \frac{8}{10}$	$\frac{10}{12}$ and $\frac{8}{10}$ do not name the same number.

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Discussion

Use discussion exercise 1 to clarify the rule children discovered in the investigation. Be sure they realize that, when the cross products of any two fractions are equal, the fractions are equivalent.

Exercise 2 stresses the concept of equality for fractional numbers. Help the children realize that when we use the equal sign between two fractions we are thinking that both of these fractions name the same fractional number. Since the only fractions which name the same fractional number are equivalent fractions, the check for equivalent fractions can be used to test equality of fractions. During this discussion, stress the relationship between the two charts. Notice with the children that the chart on the left deals with fraction concepts and equivalent fractions and that the chart on the right deals with fractional numbers.

Using the Ideas

1. Copy each exercise and give the correct sign ($=$ or $\neq$). The red and black rings are shown to remind you of the cross-product check for equivalent fractions.

A $\frac{4}{8} \neq \frac{6}{9}$ B $\frac{10}{18} = \frac{5}{9}$ C $\frac{16}{12} = \frac{8}{6}$ D $\frac{8}{20} = \frac{4}{10}$

E $\frac{3}{6} \neq \frac{4}{10}$ F $\frac{8}{7} \neq \frac{22}{20}$ G $\frac{3}{5} = \frac{60}{100}$ H $\frac{20}{30} = \frac{6}{9}$

I $\frac{4}{12} \neq \frac{8}{20}$ J $\frac{9}{21} = \frac{6}{14}$ K $\frac{12}{8} = \frac{6}{4}$ L $\frac{15}{10} = \frac{30}{20}$

2. Use the set of equivalent fractions to help you give the missing numerator or denominator. $\{\frac{5}{6}, \frac{10}{12}, \frac{15}{18}, \frac{20}{24}, \frac{25}{30}, \frac{30}{36}, \frac{35}{42}, \ldots\}$

A $\frac{5}{6} = \frac{\square}{18}$ 15 B $\frac{20}{24} = \frac{\square}{12}$ 10 C $\frac{10}{12} = \frac{15}{\square}$ 18 D $\frac{25}{\square} = \frac{5}{6}$ 25 E $\frac{35}{42} = \frac{\square}{12}$ 10 F $\frac{25}{\square} = \frac{30}{36}$ 30

3. Give the number for ***a***. Then give the number for ***b***.

A $1 \times 6 = 2 \times a \rightarrow \frac{1}{2} = \frac{b}{6}$ 3,3

B $2 \times a = 3 \times 6 \rightarrow \frac{2}{3} = \frac{6}{b}$ 9,9

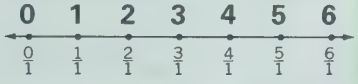
C $a \times 8 = 4 \times 6 \rightarrow \frac{b}{4} = \frac{6}{8}$ 3,3

D $1 \times a = 6 \times 4 \rightarrow \frac{1}{6} = \frac{4}{b}$ 24,24

4. Give the missing numerator or denominator.

A $\frac{3}{2} = \frac{6}{\square}$ 4 B $\frac{3}{4} = \frac{6}{\square}$ 8 C $\frac{7}{8} = \frac{14}{\square}$ 16 D $\frac{6}{8} = \frac{3}{\square}$ 4 E $\frac{4}{10} = \frac{\square}{20}$ 8 F $\frac{12}{6} = \frac{\square}{4}$ 8 G $\frac{10}{16} = \frac{15}{\square}$ 24 H $\frac{18}{3\square} = \frac{24}{4}$ 3 I $\frac{8\square}{10} = \frac{80}{100}$ 8 J $\frac{75}{100} = \frac{\square}{4}$ 3

5. This number-line picture shows two ways to represent whole numbers.



Which whole numbers are named by these fractions?

A $\frac{8}{2}$ 4 B $\frac{2}{1}$ 2 C $\frac{12}{4}$ 3 D $\frac{12}{3}$ 4 E $\frac{7}{7}$ 1 F $\frac{7}{1}$ 7 G $\frac{8}{4}$ 2 H $\frac{20}{5}$ 4 I $\frac{20}{4}$ 5

think

Trace and cut out four of these "birds." Can you put four of these birds together to form a square?



See Solution, T.E. page 151.

More practice, page A-10, Set 19

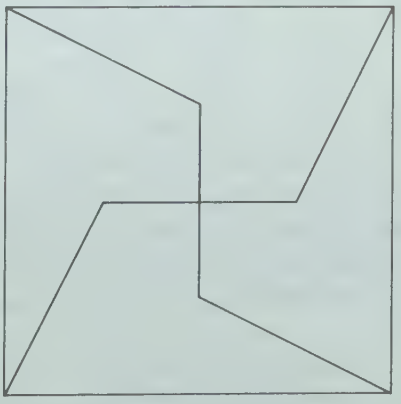


Using the Exercises

Assign the exercises on page 151 as independent work. In exercises 2, 3, and 4, be sure children realize that they can use the cross-product rule to find the missing numerator or denominator. When you discuss exercise 5, stress that whole numbers may be expressed as fractions.

As the children work through the exercises, remember that the development of skill and technique is not the primary objective of this lesson. The primary objective is to help the children understand equality for fractional numbers.

Solution, Think, page 151



Assignments (page 151)

Minimum: 1. Average: 1–5.
Maximum: 1–5.

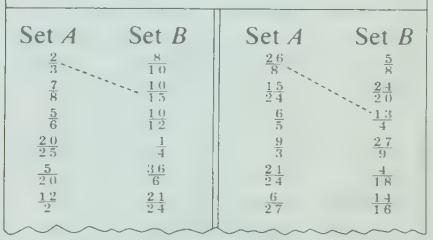
Mathematics

The concept of equality for fractional numbers is basically the same thing as speaking about different names for one fractional number, because when we speak of choosing any fraction from a set of equivalent fractions to name the fractional number for that set, we are essentially working with the idea of equality for fractional numbers. When we write an equals sign between two fractions, we are saying that these two fractions name the same fractional number (they belong to the same set of equivalent fractions). Hence, we can determine whether or not a given statement concerning equality of fractional numbers is true by testing to see whether the fractions are equivalent.

Follow-up

A worksheet similar to the one illustrated below would provide practice in identifying equivalent fractions for any children who might need it.

Match a fraction from Set *A* with an equivalent fraction from Set *B*. Use the cross-product rule to check your matchings.



You might encourage children who find this worksheet easy to write a third fraction for each set of equivalent fractions.

Resources for Active Learning

Developmental Math Cards, "Number Line Fractions," J-19, Addison-Wesley.

Workbook, page 46

PAGES 152–153

Objective

Given two fractional numbers, the child will be able to compare them by using the signs <, =, or > to form a correct number sentence.

Preparation

Materials

colored strips

To prepare for this lesson, recall with the children how strips can be used to represent different fractions according to the unit. For example, ask, "If the red strip is the unit, what do we call the white strip?" Use only one or two examples and then devote time to the introductory material for the investigation.

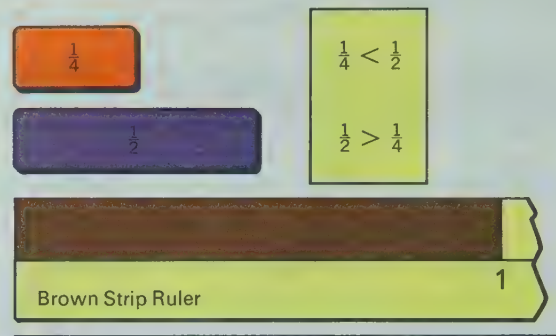
Investigation

With most groups of children it would be helpful to work through the three introductory steps together. Children should see that $\frac{1}{2} > \frac{1}{4}$ because the purple strip is longer than the red. You might suggest two other strips such as the yellow and the black strips in relation to the brown strip as the unit. Then, after comparing the lengths of the strips, the children should write $\frac{5}{8} < \frac{7}{8}$. As children continue with the challenge question on their own, you might suggest the following form for recording their results.

Unit Strip	Strips	Statement
brown	red / light green	$\frac{1}{4} < \frac{3}{8}$
brown	purple / light green	$\frac{1}{2} > \frac{3}{8}$

● *Let's explore fractional-number inequalities.*

Investigating the Ideas



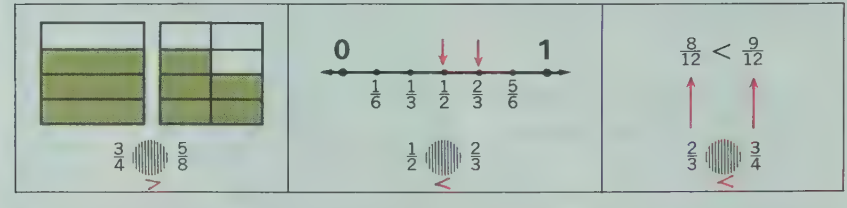
1. Use the brown strip as the unit.
2. Compare two other strips.
3. Use the lengths of the two strips to write inequality statements. Possibilities: $\frac{1}{4} < \frac{3}{8}$, $\frac{3}{8} > \frac{1}{4}$, $\frac{3}{8} < \frac{1}{2}$, $\frac{1}{2} > \frac{3}{8}$

? With the brown strip as the unit, how many different inequality statements can you write by using the red, light green, and purple strips? See Investigation.

Discussing the Ideas

1. How would you use strips to help you compare $\frac{2}{3}$ and $\frac{5}{6}$? See Discussion.
2. Three other ways to compare fractional numbers are shown below.

   A Give the correct symbol (<, >) for each ◯.



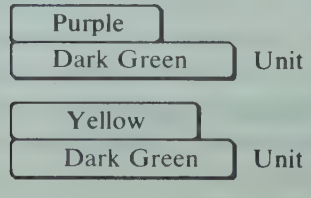
   B Which of these methods would you use to compare $\frac{2}{3}$ with $\frac{3}{5}$? Write the inequality statement. $\frac{2}{3} > \frac{3}{5}$ Answers will vary.

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Discussion

As children explain the results of their investigation, stress the use of the brown strip as the unit.

In order to compare $\frac{2}{3}$ and $\frac{5}{6}$ using the strips as suggested in exercise 1, it is necessary to choose another strip as the unit. The natural choice for the unit in this case is the dark green strip. The purple strip, then, will represent $\frac{2}{3}$; the yellow, $\frac{5}{6}$.



Comparing the lengths of the purple and yellow strips, will enable the child to see that $\frac{2}{3} < \frac{5}{6}$. Some children may find it easier to substitute 2 red strips for the purple strip to visualize $\frac{2}{3}$, when the green strip is the unit. As you discuss exercise 2, observe with the children that, with the regions given, it is easy to observe that $\frac{3}{4} > \frac{5}{8}$. Then observe that, if one number is indicated on the number line to the left of another, it is smaller. Also, remind the children that, if they write two fractional numbers using fractions that have the same denominator, they can easily tell which of the two is greater or small-

Using the Ideas

1. Give the numerators for ***a*** and ***b***. Then give the correct sign (> or <) for the ◯.

A $\frac{1}{2} = \frac{a}{6}$ (3) $\quad \frac{1}{3} = \frac{b}{6}$ (2) → $\frac{1}{2}$ > $\frac{1}{3}$

C $\frac{1}{4} = \frac{a}{20}$ (5) $\quad \frac{1}{5} = \frac{b}{20}$ (4) → $\frac{1}{4}$ > $\frac{1}{5}$

E $\frac{4}{5} = \frac{a}{20}$ (16) $\quad \frac{3}{4} = \frac{b}{20}$ (15) → $\frac{4}{5}$ > $\frac{3}{4}$

B $\frac{3}{4} = \frac{a}{12}$ (9) $\quad \frac{5}{6} = \frac{b}{12}$ (10) → $\frac{3}{4}$ < $\frac{5}{6}$

D $\frac{7}{8} = \frac{a}{24}$ (21) $\quad \frac{2}{3} = \frac{b}{24}$ (16) → $\frac{7}{8}$ > $\frac{2}{3}$

F $\frac{5}{6} = \frac{a}{12}$ (10) $\quad \frac{2}{3} = \frac{b}{12}$ (8) → $\frac{5}{6}$ > $\frac{2}{3}$

2. Give the correct sign for each ◯.

A $\frac{1}{2}$ > $\frac{3}{8}$ $\quad$ C $\frac{1}{2}$ > $\frac{1}{3}$ $\quad$ E $\frac{6}{8}$ > $\frac{11}{16}$ $\quad$ G $\frac{7}{2}$ < 4 $\quad$ I $\frac{14}{3}$ < 5

B 6 < $\frac{7}{1}$ $\quad$ D 9 = $\frac{9}{1}$ $\quad$ F $\frac{2}{3}$ < $\frac{7}{9}$ $\quad$ H $\frac{5}{8}$ < $\frac{3}{4}$ $\quad$ J $\frac{4}{5}$ > $\frac{2}{3}$

Short Stories

1. Miss Brown's class: $\frac{1}{2}$ are girls. Miss White's class: 3 out of each 5 are girls. Same number of children in each class. Which class has more girls?
Miss White's class

2. 20 minutes. $\frac{1}{4}$ hour. Which is longer?
20 minutes

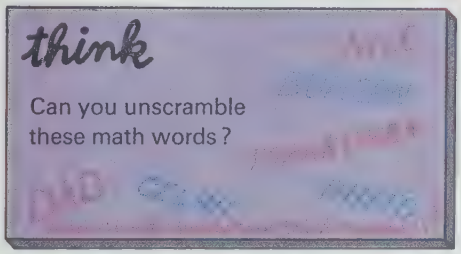


3. 3500 ml. 4 litres. Which is more?
4 litres



4. 130 cm. 1.5 m. Which is longer? 1.5 m

5. Orchard: $\frac{2}{10}$ apple trees, $\frac{1}{2}$ pear trees, $\frac{5}{50}$ peach trees, $\frac{5}{25}$ apricot trees.
   - A The orchard has most of which kind of tree? Pear
   - B The orchard has fewest of which kind of tree? Peach
   - C The orchard has the same number of which two kinds of trees? Apple and apricot



More practice, page A-10, Set 20

153



er simply by comparing their numerators.

Point out that, while examples A and B are quite simple, it is not always convenient to draw regions to represent the two fractions, nor is it always convenient to find where the numbers are located on the number line. Therefore, the method most frequently used for determining which of two fractional numbers is greater is the common-denominator method. Use several examples or some of the exercises on page 153 to ensure that the children understand this method.

Using the Exercises

Assign the exercises on page 153 as independent work, unless you see a need for some children to work through them together. In exercise 2, children should feel free to use any method, such as drawing regions, marking number lines, or finding common denominators.

Some children may find the short stories challenging. All would benefit from having those who solved them explain their solutions.

Assignments (page 153)

Minimum: 1. Average: 1, 2, Story problems. Maximum: 1, 2, Story problems.

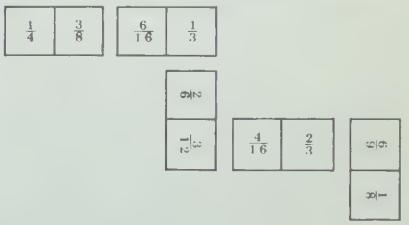
Mathematics

In this lesson, we present the idea of inequalities for fractional numbers through the use of the colored strips, regions, the number line, and common denominators. If fractions representing two fractional numbers are not equivalent, then the fractional numbers they represent are different. Such a concept actually involves a precise mathematical definition, but it is not presented to the children at this time. Based on the definition for equivalent fractions, inequality for fractional numbers is defined as:

$\frac{a}{b} \neq \frac{c}{d}$ if and only if $a \times d \neq b \times c$.

Follow-up

Matching equivalent fractions in a game using special dominoes can provide more practice for less capable children. Cut tagboard into 4-by-8-cm cards. Rule each card and write non-equivalent fractions on either half, as shown below. Design the cards so that there will be six fractions in each equivalent set. Two to four players can draw four cards from the "fraction dominoes" that are face-down on the table. To play, each child must match a domino with one of those face-up on the table or draw another domino. The player who plays all his domino cards first is the winner.



Resources for Active Learning

A Cloudburst, Vol 3, No. 3115, "Farey Series," Midwest Publications.

Duplicator Masters, page 27
Workbook, page 47
Skill Masters, page 27

PAGES 154–155

Objectives

The child will demonstrate his ability to work with the concepts presented in this chapter.

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

Review any topics developed in the chapter which the children found particularly troublesome. For example, with some children you might want to review the method of finding lowest-terms fractions.

$$\frac{4}{16} = \frac{4 \times 1}{4 \times 4} = \frac{1}{4}$$

$$\frac{6}{15} = \frac{3 \times 2}{3 \times 5} = \frac{2}{5}$$

With others, you might want to review the cross-product method for checking equivalence of fractions and for use in finding missing terms.

$$\frac{2}{5} = \frac{?}{10}$$

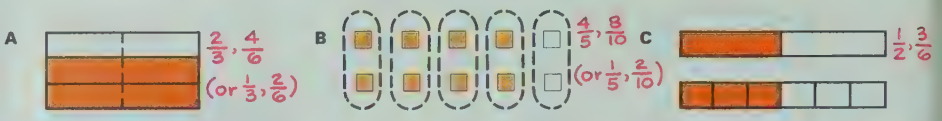
$$2 \times 10 = 5 \times ?$$

$$2 \times 10 = 5 \times 4$$

$$\frac{2}{5} = \frac{4}{10}$$

Reviewing the Ideas

1. Give the pair of equivalent fractions suggested by each figure.



2. Copy each set of fractions and give three more fractions for the set.

A $\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \ldots\}$ $\frac{12}{16}, \frac{15}{20}, \frac{18}{24}$ B $\{\frac{7}{5}, \frac{14}{10}, \frac{21}{15}, \ldots\}$ $\frac{28}{20}, \frac{35}{25}, \frac{42}{30}$ C $\{\frac{9}{10}, \frac{18}{20}, \frac{27}{30}, \ldots\}$ $\frac{36}{40}, \frac{45}{50}, \frac{54}{60}$

Note: In exercises 2 and 3, the equivalents children list may vary.

3. Give a set of six equivalent fractions for each lowest-terms fraction.

A $\frac{1}{8}$ $\{\frac{1}{8}, \frac{2}{16}, \frac{3}{24}, \frac{4}{32}, \frac{5}{40}, \frac{6}{48}\}$ B $\frac{5}{6}$ $\{\frac{5}{6}, \frac{10}{12}, \frac{15}{18}, \frac{20}{24}, \frac{25}{30}, \frac{30}{36}\}$ C $\frac{3}{10}$ $\{\frac{3}{10}, \frac{6}{20}, \frac{9}{30}, \frac{12}{40}, \frac{15}{50}, \frac{18}{60}\}$ D $\frac{2}{5}$ $\{\frac{2}{5}, \frac{4}{10}, \frac{6}{15}, \frac{8}{20}, \frac{10}{25}, \frac{12}{30}\}$ E $\frac{5}{2}$ $\{\frac{5}{2}, \frac{10}{4}, \frac{15}{6}, \frac{20}{8}, \frac{25}{10}, \frac{30}{12}\}$ F $\frac{7}{3}$ $\{\frac{7}{3}, \frac{14}{6}, \frac{21}{9}, \frac{28}{12}, \frac{35}{15}, \frac{42}{18}\}$

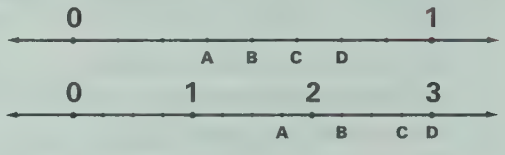
4. Give the lowest-terms fraction for each fraction.

A $\frac{7}{14}$ $\frac{1}{2}$ B $\frac{6}{24}$ $\frac{1}{4}$ C $\frac{40}{60}$ $\frac{2}{3}$ D $\frac{15}{24}$ $\frac{5}{8}$ E $\frac{7}{70}$ $\frac{1}{10}$ F $\frac{70}{100}$ $\frac{7}{10}$ G $\frac{6}{10}$ $\frac{3}{5}$ H $\frac{35}{50}$ $\frac{7}{10}$

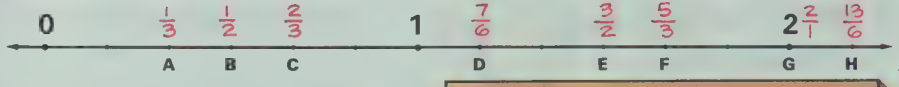
5. Give the point for the fractional number indicated by the set of equivalent fractions.

A $\{\frac{5}{8}, \frac{10}{16}, \frac{15}{24}, \frac{20}{32}, \ldots\}$ C

B $\{\frac{9}{4}, \frac{18}{8}, \frac{27}{12}, \frac{36}{16}, \ldots\}$ B



6. Give the lowest-terms fraction for each letter.



7. Give the number for n.

A $\frac{1}{2} = \frac{n}{10}$ 5 C $\frac{5}{8} = \frac{n}{24}$ 15 E $\frac{n}{10} = \frac{4}{5}$ 8

B $\frac{2}{3} = \frac{8}{n}$ 12 D $\frac{6}{n} = \frac{9}{12}$ 8 F $\frac{2}{8} = \frac{n}{36}$ 9

8. Give the correct sign (<, =, >).

A $\frac{11}{2}$ > 5 C $\frac{6}{12}$ = $\frac{5}{10}$ E $\frac{3}{4}$ > $\frac{4}{6}$

B $\frac{3}{8}$ > $\frac{1}{4}$ D $\frac{7}{8}$ > $\frac{3}{4}$ F $\frac{8}{6}$ > $\frac{10}{8}$

think

Study the example.
Then give the lowest-terms fraction for A, B, and C.

Example: $\frac{3 \times 5}{4 \times 5^2}$ $\frac{3}{4 \times 5}$ $\frac{3}{20}$

A $\frac{2 \times 5^2}{7 \times 5}$ B $\frac{3 \times 5^2}{7 \times 5^3}$ C $\frac{3 \times 5^3}{6 \times 5^5}$

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Discussion

Page 154 may be used as a class review in which you work through several problems together, or as an evaluation page. In either case, stress the points that children found most difficult. Also, emphasize again the concept developed in exercise 5. Help the children realize that each fraction in a set of equivalent fractions represents one fractional number which can be matched to one point on the number line.

The *Think* problem may be challenging even to the more able children. Let those who see the pattern explain it to the others.

Keeping in Touch with

Addition
Subtraction
Multiplication
Division
Place value
Functions

1. Give the ordinary base-ten numeral for each number.

A 10^3 1000
B 5×10^3 5000
C 37×10^3 37000
D 6×10^4 60000
E 8×10^7 80 000 000
F 35×10^6 35 000 000
G 427×10^2 42700
H $10^6 \times 10^3$ 1 000 000 000

2. In a recent year, the combined population of North and South America was 475 387 000.

A Give this population figure to the nearest ten thousand. 475 390 000
B Give this population figure to the nearest million. 475 000 000

3. Solve the equations.

A $327 + 8465 + 29 + 7 = n$ 8828
B $84\,206 - 7349 = n$ 76 857
C $82\,654 + n = 97\,612$ 14 958
D $74\,261 + 93\,483 + 68 = n$ 167 812
E $n - 820\,064 = 132\,576$ 952 640
F $843 + 7 + 6929 + 78 = n$ 7857

4. Give the missing numbers.

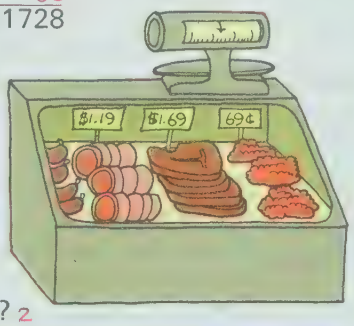
Function Rule: $n \times 37$

	n	$f(n)$
A	6	222
B	48	1776
C	465	17 205

5. Find the products and quotients.

A $286 \times 53 = 15158$
B $4681 \times 24 = 112344$
C $31\overline{)1829}$ 59
D $48\overline{)1728}$ 36

6. A If a roast is $1.19 for 500 g, how much will a 3.5 kg roast cost? $8.33
B If steak is $1.69 for 500 g, what will a 1.5 kg steak cost? $5.07
C If hamburger costs 69¢ for 500 g, how many kilograms can you buy for $2.76? 2





You are invited to explore

ACTIVITY CARD 7
Page 358

Using the Exercises

Page 155 may be assigned as independent work, although you may want to use some of the exercises to review some concepts such as exponential notation and function rules. You might also give some children an opportunity to present several parts of exercises 3 and 5 on the chalkboard and explain the appropriate algorithms to the rest of the class.

Follow-up

A worksheet like the following may challenge more capable children and still provide a great deal of review in finding and ordering equivalent fractions.

In each row circle the largest quantity; put a square around the smallest quantity.

1.	$\frac{3}{8}$,	$\frac{1}{4}$,	$\frac{5}{16}$,	$\frac{4}{2}$,	$\frac{7}{8}$,	$\frac{15}{16}$.
2.	$\frac{3}{6}$,	$\frac{1}{3}$,	$\frac{5}{6}$,	$\frac{6}{1}$,	$\frac{4}{2}$,	$\frac{2}{3}$.
3.	$\frac{6}{10}$,	$\frac{3}{5}$,	$\frac{1}{30}$,	$\frac{2}{15}$,	$\frac{10}{2}$,	$\frac{2}{10}$.
4.	$\frac{12}{12}$,	$\frac{1}{4}$,	$\frac{8}{12}$,	$\frac{3}{4}$,	$\frac{1}{3}$,	$\frac{1}{2}$.
5.	$\frac{1}{2}$,	$\frac{12}{20}$,	$\frac{2}{5}$,	$\frac{9}{10}$,	$\frac{3}{2}$,	$\frac{5}{20}$.

Workbook, page 48

CHAPTER 8 | Addition and Subtraction of Fractional Numbers

Pages 156–181

General Objectives

To provide additional experiences with fractional numbers
To introduce addition of fractional numbers
To introduce subtraction of fractional numbers
To develop skill in finding sums and differences of fractional numbers
To introduce mixed-numeral notation
To provide experience with word problems requiring addition and subtraction of fractional numbers

The first investigation of the chapter provides an intuitive and natural setting in which simple problem situations requiring addition and subtraction of fractional numbers are presented. Following this brief introduction, addition and subtraction are introduced more formally by helping the children see that by selecting fractional number names (in this case, fractions) that have the same denominator they can easily find the sum or difference of the two fractional numbers. A fraction strip which expresses the very essence of this idea is made and used by the children to help them select fractions with the same denominator and to grasp better the fact that they are adding or subtracting fractional numbers which may be expressed by any one of an unlimited number of fractions. These strips are also used in the development of the least common denominator idea. The addition and subtraction algorithms are presented to develop a clear understanding of them and also to develop a certain degree of proficiency in using them. A discussion of mixed numerals and improper fractions is followed by work on the basic principles. This prepares the children for subsequent material involving the addition and subtraction of fractional numbers expressed as mixed numerals.

Continuing the work with mixed numerals, the children are given an opportunity to apply techniques of adding and subtracting fractional numbers to find solutions to word problems. They are also given help in writing mixed numerals for improper fractions.

The final lesson provides the familiar chapter and cumulative reviews to help you evaluate the children's progress and summarize the main ideas of the chapter.

Mathematics

Consider the following definition for the sum of any two fractional numbers:

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two fractional numbers, then

$$\text{(A)}\quad \frac{a}{b}+\frac{c}{d}=\frac{(a\times d)+(b\times c)}{b\times d}.$$

With this definition and the basic principles for fractional numbers, we can deduce the familiar rule,

$$\text{(B)}\quad \frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$$

in this manner,

$$\frac{a}{c}+\frac{b}{c}=\frac{(a\times c)+(b\times c)}{c\times c}$$

$$=\frac{(a+b)\times c}{c\times c}=\frac{a+b}{c}.$$

Conversely, we could have used (B) to deduce (A) as follows.

$$\frac{a}{b}+\frac{c}{d}=\frac{a\times d}{b\times d}+\frac{b\times c}{b\times d}$$

$$=\frac{(a\times d)+(b\times c)}{b\times d}$$

Note that in both instances we use the fact that for any fractional number $\frac{a}{b}$, it is true that if $k\neq 0$, then

$$\frac{a}{b}=\frac{a\times k}{b\times k}.$$

These ideas are presented to children intuitively. For example, it is a simple matter to explain a sum such as

$$\frac{3}{8}+\frac{2}{8}=\frac{5}{8},$$

using the number line or physical objects. Once children understand this general idea, they can find the sum of any two fractional numbers regardless of the fractions used to denote them. When the fractions have different denominators, the task of finding the sum is simply that of finding equivalent fractions for the two numbers that have the same denominator. For example:

$$\begin{array}{ccc}\frac{3}{8} & + & \frac{1}{6}\\ \downarrow & & \downarrow\\ \frac{9}{24} & + & \frac{4}{24}=\frac{13}{24}\end{array}$$

After addition concepts and skills are established, we can apply them to subtraction by using the already familiar inverse relation between addition and subtraction of whole numbers. We point out that the same relation exists between addition and subtraction of fractional numbers as exists between addition and subtraction of whole numbers. After defining this relation, it is a simple matter to justify the following algorithms.

$$\frac{a}{b}-\frac{c}{b}=\frac{a-c}{b}$$

$$\frac{a}{b}-\frac{c}{d}=\frac{(a\times d)-(b\times c)}{b\times d}$$

Teaching the Chapter

Materials

Almanac
Colored strips
Demonstration number line
Felt fractional parts: circular and rectangular regions (*optional*)

Flannelboard (*optional*)
Graph paper (1-cm grid to be cut into strips 1.5-by-15 cm)
Overhead projector (if available) and transparencies
Ruler
Scissors

Vocabulary

improper fraction
least common denominator
mixed numeral
proper fraction

Use of materials to demonstrate addition and subtraction concepts in this chapter should probably be limited to the slower children. Most other children should be ready to do the work of the chapter without physical aids other than the strips in the investigations and a variety of drawings which you may exhibit on the chalkboard. However, if you feel your children do need additional work with materials to help them understand the concepts involved in addition and subtraction of fractional numbers, you should certainly feel free to use such materials. For example, you can cut apart regions and put them back together to illustrate the various fractional number concepts.

Notice the term *mixed numeral* in the vocabulary list. We use mixed numeral in this program in much the same way as traditional programs use the term mixed number. Obviously, it is the notation for the fractional number that is mixed, not the number itself. Since the numeral includes both a symbol for a whole number and a fraction, it is preferable to refer to it as a mixed numeral rather than as a mixed number.

Lesson Schedule

Generally speaking, you should plan to spend about three weeks on this chapter. As usual, adjust your time schedule to the abilities and needs of your children, spending a little more time with manipulative materials for those who need it.

Evaluation of Progress

The chief objectives of the chapter are (1) to help children understand addition and subtraction of fractional numbers, (2) to lead them toward a moderate degree of skill in some of the simple problems provided in the text, and (3) to develop their ability to apply these computational skills to the solution of word problems.

There are three important things to consider in evaluating the children's achievement. (1) It would be relatively easy to give a routine test to find out whether they can add and subtract fractional numbers. (2) In general, it is more difficult to determine whether children understand the concepts involved in the techniques for adding and subtracting fractional numbers. As part of your evaluation, include day-to-day interviews with the children to determine their understanding of the ideas presented in the chapter. (3) You have probably observed that in this chapter we place considerable emphasis on solving word problems. Therefore, a major part of your evaluation should include several word problems similar to those provided in the chapter.

The cumulative reviews on pages 180–181 and 165 should strengthen children's grasp of concepts that were introduced in earlier chapters.

Resources for Active Learning

GENERAL ACTIVITIES

It's a Tangram World, "Fractions," pp. 53–55, Educational Science Consultants
Mathex: Numeration No. 7 Activities 1 and 2, pp. 45–46 (pupil pages 42–46), Encyclopaedia Britannica Publications Ltd.

MANIPULATIVE DEVICES

Cuisenaire Cubes, Squares, and Rods (Cuisenaire Co.)
Fraction Blocks (Creative Publications)
Modulated Cutouts (Encyclopaedia Britannica Publications Ltd.)

COMMERCIAL GAMES

Action Fraction Games (CCM School Materials; Lakeshore; Math Media)
Come Out Even (Holt, Rinehart and Winston)
Fraction Dominoes (Responsive Environments Corp.; Selective Educational Equipment)
Spinner Number Games, Fractions (Heath)

PAGES 156–157

Objective

Given two fractional numbers represented by fractions with common denominators, the child will be able to find the sum or difference of the numbers.

Preparation

If you choose to have a specific preparation for this lesson, you might simply recall with the children the variety of situations in which addition and subtraction may be applied. For example, ask them to give you an instance in which addition with whole numbers might be applicable, such as: Drive 400 km one day and 300 km the next; how many km in all? Similarly, ask the children to suggest situations in which subtraction can be applied. Keep this review brief and use it to recall the basic meaning of addition and subtraction which they will apply to fractional numbers in the investigation.

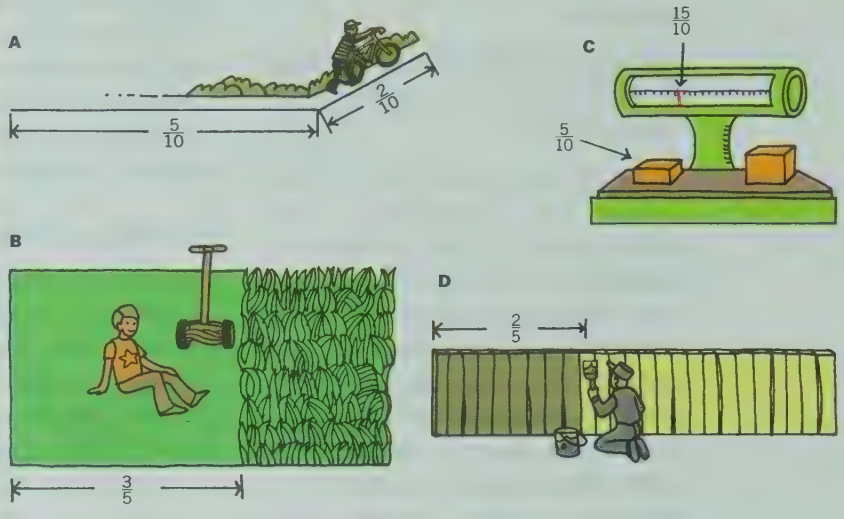
Investigation

Encourage the children to try the investigation independently. Suggest that they write both an addition and a subtraction story problem for a picture if they can. Also, if some finish writing and solving a story problem for one picture, suggest that they try another. Give guidance only when necessary. Note that, in pictures B and D, the child must think of all of the lawn or the whole fence in relation to the fraction indicated on the figure. Observe whether the children who have chosen either of these figures are thinking of the whole in correct terms.

8 Addition and Subtraction of Fractional Numbers

Let's add and subtract fractional numbers.

Investigating the Ideas



? Can you write and solve a story problem for one of these pictures?

See Investigation and Discussion.



Discussing the Ideas

1. A Which pictures suggest addition? A
   B Which pictures suggest subtraction? B, C, D

2. Can you give a simple rule for finding sums and differences such as the ones below? Sample answer: If the denominators are the same, add or subtract numerators.
   A $\frac{4}{5} + \frac{2}{5} = n$ $\frac{6}{5}$
   B $\frac{7}{10} - \frac{3}{10} = m$ $\frac{4}{10}$

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Discussion

Use discussion exercise 1 to have children share the story problems they wrote and solved. Picture A is most clearly seen as addition; picture C, as subtraction. Pictures B and D may be interpreted as either addition or subtraction. Be sure to discuss the possible interpretations for each of these.

B $\frac{5}{5} - \frac{3}{5} = ?$ $\frac{3}{5} + \frac{2}{5} = ?$

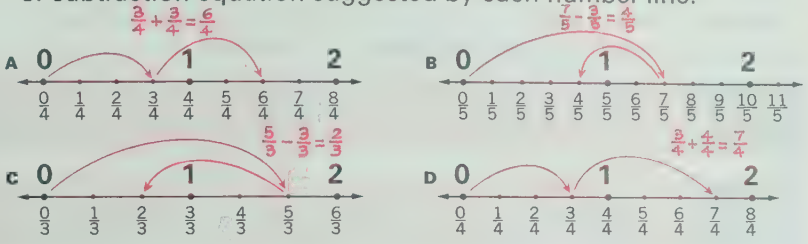
D $\frac{8}{8} - \frac{3}{8} = ?$ $\frac{3}{8} + \frac{5}{8} = ?$

During this discussion, have some children write their equations for each picture on the chalkboard. Use these equations to ask the children if they notice anything about all of the fractions in each equation. Encourage them to observe that all of the fractions in each equation have the same denominator.

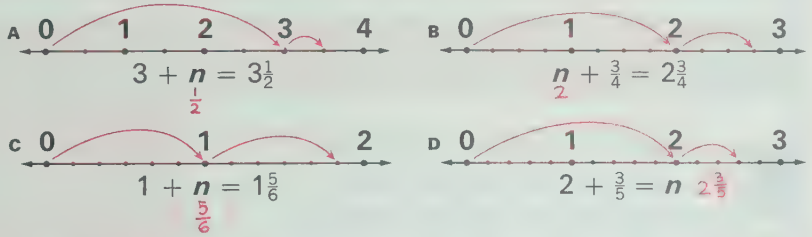
In exercise 2, as children try to find a simple rule, again stress that the fractions in each equation have the same denominators. Be sure children realize that a simple rule, such as add numerators, or subtract numerators, is applicable only to fractions which have common denominators.

Using the Ideas

1. The number line will help you understand addition and subtraction of fractional numbers. Write the addition or subtraction equation suggested by each number line.



2. The number line will help you understand **mixed numerals**, such as $3\frac{1}{2}$, $2\frac{3}{4}$, and $1\frac{5}{6}$. Solve the equations.



3. Find the sums and differences.

A $\frac{2}{8}+\frac{3}{8}$ F $\frac{9}{4}-\frac{2}{4}$ K $\frac{9}{12}-\frac{9}{12}$ M $\frac{5}{50}-\frac{4}{50}$ O $\frac{75}{100}-\frac{38}{100}$

B $\frac{5}{6}-\frac{1}{6}$ G $\frac{6}{10}+\frac{6}{10}$ L $\frac{0}{7}+\frac{5}{7}$ N $\frac{7}{100}+\frac{10}{100}$ P $\frac{62}{200}+\frac{59}{200}$

C $\frac{4}{7}+\frac{2}{7}$ H $\frac{7}{8}+\frac{2}{8}$

D $\frac{6}{10}-\frac{2}{10}$ I $\frac{10}{12}-\frac{3}{12}$

E $\frac{7}{5}+\frac{4}{5}$ J $\frac{3}{9}+\frac{6}{9}$

think

$1 → 1 coin (silver dollar)
$1 → 2 coins (2 half dollars)
$1 → 3 coins (2 quarters, 1 half)
$1 → 4 coins (4 quarters)
$1 → 5 coins (1 half, 1 quarter, 2 dimes, 1 nickel)

Can you continue the pattern up to 20 coins? See Solution: T.E. page 157.

4. Give the correct mixed numeral for each sum.

A $6+\frac{1}{2}$ E $86+\frac{9}{40}$

B $9+\frac{3}{4}$ F $97+\frac{17}{100}$

C $15+\frac{7}{8}$ G $126+\frac{37}{100}$

D $27+\frac{15}{16}$ H $528+\frac{15}{73}$

Using the Exercises

The chief objective of the exercises on page 157 is to strengthen children's understanding of how to add and subtract fractional numbers represented by fractions which have the same denominator. However, when children have finished the assigned exercises, allow sufficient time to discuss the mixed numerals which are presented in exercise 2.

Be sure the children understand that a mixed numeral, such as $3\frac{1}{2}$, stands for the sum $3+\frac{1}{2}$.

Assignments (page 157)

Minimum: 1–4. Average: 1–4.
Maximum: 1–4.

Follow-up

Give more capable children a worksheet using fractional notation to help them review their skill in analyzing problems.

Make up questions based on the given facts. Write one or more equation(s) to show the operations you could use to solve the problem. Do not try to solve your equation if the denominators are different.

1. Ray's house to school: $\frac{1}{2}$ km. School to ball park: $\frac{1}{4}$ km. School to Jack's house: $\frac{3}{8}$ km. Ray's house to Jack's house: $\frac{3}{8}$ km.
2. Fish: $4\frac{3}{4}$ g, $2\frac{1}{2}$ g, $3\frac{1}{8}$ g.
3. Four boys on a cookout. To make 1 cheeseburger: $\frac{1}{2}$ kg ground beef, $\frac{1}{8}$ pkg buns, $\frac{1}{8}$ pkg cheese.

Solutions, Think, page 157

(Abbreviations for coins: H, half dollar; Q, quarter; D, dime; N, nickel; P, penny.)

1 1 Silver Dollar
2 2 H
3 2 Q, 1 H
4 4 Q
5 1 H, 1 Q, 2 D, 1 N
6 1 H, 5 D
7 1 H, 4 D, 2 N
8 1 H, 3 D, 4 N; or 2 Q, 4 D, 2 N
9 1 H, 2 D, 6 N; or 2 Q, 3 D, 4 N
10 10 D; or 1 H, 1 D, 8 N
11 2 Q, 1 D, 8 N; or 9 D, 2 N
12 2 Q, 10 N; or 3 Q, 4 N, 5 P; or 8 D, 4 N
13 7 D, 6 N
14 6 D, 8 N
15 5 D, 10 N
16 10 P, 3 Q, 3 N; or 15 N, 1 Q
17 14 N, 3 D
18 15 P, 1 H, 1 Q, 1 D
19 15 P, 3 Q, 1 D
20 20 N; or 15 P, 3 Q, 2 N

(Other solutions are possible for some numbers.)

PAGES 158–159

Objective

Given addition and subtraction problems involving fractions with unlike denominators, the child will be able to find the sum or difference by choosing fractions with common denominators.

Preparation

Materials

strips of graph paper (1-cm grid) 2-by-12 cm (*Duplicator Masters*, page 69)

To prepare for this lesson, provide children with some review experiences in building sets of equivalent fractions. For example, give them a lowest-terms fraction and ask them to build a set of equivalent fractions.

$\frac{3}{4} \rightarrow \{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \ldots\}$

$\frac{1}{3} \rightarrow \{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \frac{5}{15}, \ldots\}$

Investigation

Give the children as much guidance as necessary as they make the fraction strips. Have them notice that the fractions on the strips in the text have been written with the same spacing so that they can easily be lined up with one another. Although the strips could be made on plain paper, graph paper will facilitate neatness. Encourage the children to make the strips carefully, because they will be required again in the next lesson.

When the children have finished making the strips, be sure they understand how they are to use them. You might want to explain how they are used for the example shown in the text. A child may place the pair of strips he is using flat on his desk and slide them back and forth to find a pair of fractions with the same denominator. You might encourage children to find and record at least 5 or 6 pairs of fractions and their sums or differences. Display how the example in the text should be recorded:

$$\begin{array}{r} \frac{9}{12} \\ +\frac{8}{12} \\ \hline \frac{17}{12} \end{array}$$

How can we add or subtract when the fractions have different denominators?

Investigating the Ideas

Make and cut out equivalent-fraction strips like these. Choose at least one other fractional number and make a strip for it.

One Half	$\frac{1}{2}$	$\frac{2}{4}$	$\frac{3}{6}$	$\frac{4}{8}$	$\frac{5}{10}$	$\frac{6}{12}$	$\frac{7}{14}$	$\frac{8}{16}$	$\frac{9}{18}$	$\frac{10}{20}$
Three Fourths	$\frac{3}{4}$	$\frac{6}{8}$	$\frac{9}{12}$	$\frac{12}{16}$	$\frac{15}{20}$	$\frac{18}{24}$	$\frac{21}{28}$	$\frac{24}{32}$	$\frac{27}{36}$	$\frac{30}{40}$
Four Fifths	$\frac{4}{5}$	$\frac{8}{10}$	$\frac{12}{15}$	$\frac{16}{20}$	$\frac{20}{25}$	$\frac{24}{30}$	$\frac{28}{35}$	$\frac{32}{40}$	$\frac{36}{45}$	$\frac{40}{50}$
Two Thirds	$\frac{2}{3}$	$\frac{4}{6}$	$\frac{6}{9}$	$\frac{8}{12}$	$\frac{10}{15}$	$\frac{12}{18}$	$\frac{14}{21}$	$\frac{16}{24}$	$\frac{18}{27}$	$\frac{20}{30}$

? Can you "line up" pairs of strips so that fractions with the same denominator match and then find the sum? See Investigation.

$\frac{6}{8}$ $\frac{9}{12}$ $\frac{12}{16}$

$\frac{6}{9}$ $\frac{8}{12}$ $\frac{10}{15}$

$\frac{17}{12}$

Discussing the Ideas

1. Use the example above to give this sum: $\frac{3}{4} + \frac{2}{3}$ $\frac{17}{12}$

2. Explain how you could use your equivalent-fraction strips to find $\frac{2}{3} - \frac{1}{2}$. "Line up" $\frac{4}{6}$ and $\frac{3}{6}$. $(\frac{1}{6})$

3. Explain this example. Then write enough fractions to find $\frac{1}{3} + \frac{1}{4}$. $\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}$ $\frac{1}{4}, \frac{2}{8}, \frac{3}{12}$ $n \rightarrow \frac{7}{12}$

$\frac{5}{6} \rightarrow \{\frac{5}{6}, \ldots, \frac{15}{18}, \frac{20}{24}, \ldots\} \rightarrow \frac{10}{12}$

$+\frac{1}{4} \rightarrow \{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \ldots\} \rightarrow \frac{3}{12}$

$n \leftarrow \frac{13}{12}$

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Discussion

Throughout this discussion, you should avoid giving drill or belaboring techniques; also, avoid teaching shortcuts, but if some children remember and prefer them, allow their use. Such children already have a good grasp of the main ideas presented here.

As children discuss exercises 1 and 2, use the built-in characteristics of the strips to stress certain points. For example, remind the children that each strip shows some of the equivalent fractions for one fractional number. Thus, when they line up two of their strips and find fractions with common denominators, the lowest-terms fraction label at the left of the strip should remind them that they are simply finding more convenient names for the two fractional numbers which they are adding or subtracting.

Exercise 3 builds on the ideas developed with the strips. Gradually, the children must learn to make their own lists of equivalent fractions until they can choose, from the lists, two fractions with common denominators. Point out that they need only list as many equivalent fractions for each set as are needed in order to find a pair of fractions with common denominators.

Using the Ideas

1. For each exercise, find the numerators for *a*, *b*, and *c*.

   A To find $\frac{1}{2} + \frac{1}{3}$, we think $\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$.

   $\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \ldots\}$ $\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \ldots\}$

   B To find $\frac{5}{6} - \frac{1}{4}$, we think $\frac{10}{12} - \frac{3}{12} = \frac{7}{12}$.

   $\{\frac{5}{6}, \frac{10}{12}, \frac{15}{18}, \frac{20}{24}, \ldots\}$ $\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \ldots\}$

2. Give the numbers for *a* and *b*. Then give the number for *c*.

   A $\frac{3}{8} = \frac{a}{16}$ (6) $\quad \frac{1}{4} = \frac{b}{16}$ (4) $\rightarrow \frac{3}{8} + \frac{1}{4} = \frac{c}{16}$ (10)

   B $\frac{5}{6} = \frac{a}{12}$ (10) $\quad \frac{1}{4} = \frac{b}{12}$ (3) $\rightarrow \frac{5}{6} - \frac{1}{4} = \frac{c}{12}$ (7)

3. For each exercise, make lists of equivalent fractions to find two fractions with the same denominator. Then find the sum or difference.

   A $\frac{7}{8} + \frac{1}{4}$ (9/8) B $\frac{3}{4} - \frac{1}{6}$ (7/12) C $\frac{1}{10} + \frac{3}{4}$ (17/20) D $\frac{3}{5} - \frac{1}{10}$ (5/10) E $\frac{3}{4} + \frac{1}{10}$ (17/20) F $\frac{1}{5} - \frac{1}{6}$ (1/30)

   G $\frac{1}{2} + \frac{3}{4} = \frac{5}{4}$ H $\frac{1}{4} - \frac{1}{10} = \frac{3}{20}$ I $\frac{1}{3} + \frac{1}{5} = \frac{8}{15}$ J $\frac{1}{5} + \frac{3}{50} = \frac{13}{50}$ K $\frac{3}{4} - \frac{1}{10} = \frac{13}{20}$ L $\frac{1}{2} - \frac{1}{8} = \frac{3}{8}$ M $\frac{1}{6} + \frac{1}{18} = \frac{5}{18}$

   N $\frac{51}{100} + \frac{1}{4} = \frac{76}{100}$ O $\frac{1}{2} - \frac{1}{8} = \frac{3}{8}$ P $\frac{1}{6} + \frac{3}{5} = \frac{23}{30}$ Q $\frac{5}{8} - \frac{1}{2} = \frac{1}{8}$ R $\frac{2}{5} - \frac{1}{4} = \frac{3}{20}$ S $\frac{1}{3} + \frac{1}{6} = \frac{3}{6}$ T $\frac{3}{20} - \frac{1}{10} = \frac{1}{20}$

think

There are 6 more children in room A than there are in room B. 5 children move from room A to room B. Now there are twice as many children in room B as in room A. How many children were in room A before the move? 9





More practice, page A-11, Set 21

Using the Exercises

Assign the exercises on page 159 as independent work. Stress again that fractions with unlike denominators must be replaced by fractions with common denominators before addition or subtraction can be performed. Also, be sure the children realize that they might find the fractions with common denominators in different ways. For example, instead of listing equivalent fractions, some may observe that the denominator of one fraction is a multiple of the denominator of the other, and then find the numerator, which is that factor times the given numerator.

$$\frac{1}{3} \rightarrow \frac{3 \times 1}{3 \times 3} \rightarrow \frac{3}{9}$$

$$\frac{4}{9} \rightarrow \frac{4}{9} \rightarrow \frac{4}{9}$$

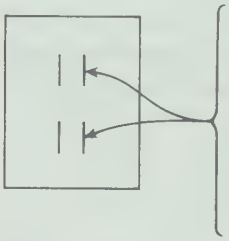
The *Think* problem is sufficiently difficult that you may decide not to have it presented to the entire class. However, many of your children will understand it when the correct answer is carefully explained.

Assignments (page 159)

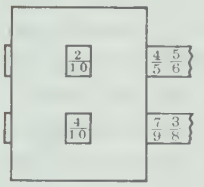
Minimum: 1–3A–L. Average: 1–3. Maximum: 1–3.

Follow-up

Some children might find that using a "strip holder" makes using the strips easier for them. A simple holder may be made by using paper of any size and cutting four slits in it according to the following diagram (The slits can be cut easily if the paper is folded.)



- Cut
- Make slits 1 cm apart.
- Make the length same as width of the strip.



Slide strip through so that only one fraction shows through each window.

To give children an opportunity for further practice, you might prepare a worksheet similar to this:

> Follow the signs to add or subtract. Use your fraction strips to help you.
>
> $\frac{1}{2} + \frac{4}{5}$ $\quad \frac{7}{8} + \frac{3}{4}$ $\quad \frac{9}{10} + \frac{4}{5}$ $\quad \frac{5}{6} + \frac{2}{3}$ $\quad \frac{7}{18} + \frac{2}{3}$
>
> $\frac{13}{15} - \frac{4}{5}$ $\quad \frac{15}{16} - \frac{1}{2}$ $\quad \frac{5}{6} - \frac{2}{3}$ $\quad \frac{3}{4} - \frac{1}{2}$ $\quad \frac{4}{5} - \frac{2}{3}$

Resources for Active Learning

A Cloudburst, Vol. 2, No. 3213, 3223, Midwest Publications.

Franklin Series: *Making and Using Graphs and Nomographs*, "Nomographs: Addition with Fractions," pp. 35–36, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Duplicator Masters, page 28
Workbook, page 49
Skill Masters, page 28

PAGES 160–161

Objective

Given two fractions with unlike denominators, the child will be able to find the least common denominator of the fractions.

Preparation

Materials

fraction strips from previous lesson

To prepare for this lesson, it would be helpful to review *least common multiples*. Ask children to list several multiples of 4; of 5.

{0, 4, 8, 12, 16, 20, 24, 28, 32, . . .}
{0, 5, 10, 15, 20, 25, 30, 35, 40, . . .}

Then ask them to find any common multiples (20). Use examples to help children recall this topic as it was treated in the chapter on number theory, page 134. Remind children that the least common multiple is the least number other than zero which is common to both sets.

If time permits, you might also give children some quick practice in finding missing terms in a pair of fractions. For example, ask them to replace each letter with the correct term:

$\frac{4}{5} = \frac{a}{20}$ $\quad \frac{7}{12} = \frac{d}{24}$

$\frac{2}{3} = \frac{b}{9}$ $\quad \frac{3}{4} = \frac{e}{8}$

$\frac{1}{2} = \frac{c}{10}$ $\quad \frac{5}{6} = \frac{f}{12}$

Investigation

Because the equivalent-fraction strips embody the main idea of finding the least common denominator in order to add or subtract two fractions with unlike denominators, many children will finish this investigation very quickly. Thus, you might want to suggest that they try the same question with these strips and fractions:

$\frac{1}{2}$ and $\frac{3}{4}$ for $\frac{3}{4} - \frac{1}{2}$

$\frac{4}{5}$ and $\frac{2}{3}$ for $\frac{4}{5} + \frac{2}{3}$

$\frac{4}{5}$ and $\frac{1}{2}$ for $\frac{4}{5} - \frac{1}{2}$

Remind the children to carefully record their findings for each pair of fractions.

What is the least common denominator for two fractions?

Investigating the Ideas

Use your equivalent-fraction strips for one half and two thirds in this Investigation.

One Half	$\frac{1}{2}$	$\frac{2}{4}$	$\frac{3}{6}$	$\frac{4}{8}$	$\frac{5}{10}$	$\frac{6}{12}$	$\frac{7}{14}$	$\frac{8}{16}$	$\frac{9}{18}$	$\frac{10}{20}$

Two Thirds	$\frac{2}{3}$	$\frac{4}{6}$	$\frac{6}{9}$	$\frac{8}{12}$	$\frac{10}{15}$	$\frac{12}{18}$	$\frac{14}{21}$	$\frac{16}{24}$	$\frac{18}{27}$	$\frac{20}{30}$

? How many different ways can you "line up" your strips to help you find $\frac{1}{2} + \frac{2}{3}$? | Record your findings.

Answers may vary. See Investigation.

Discussing the Ideas

1. Explain why each sum you found with your strips is correct for $\frac{1}{2} + \frac{2}{3}$. Any fraction from a set of equivalent fractions names the same fractional number.
2. Which sum had the smallest denominator? $\frac{7}{6}$
3. The smallest denominator that is the same for two fractions is called the least common denominator. What is the least common denominator for $\frac{1}{2}$ and $\frac{2}{3}$? 6
4. The least common denominator for two fractions is the least common multiple of the two denominators.
   - A List the multiples (to 54) of 6. 0, 6, 12, 18, 24, 30, 36, 42, 48, 54
   - B List the multiples (to 54) of 9. 0, 9, 18, 27, 36, 45, 54
   - C List the common multiples (to 54) of 6 and 9. 0, 18, 36, 54
   - D What is the least common multiple of 6 and 9? 18
   - E What is the least common denominator of $\frac{1}{6}$ and $\frac{1}{9}$? 18



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Discussion

As you discuss exercise 1, again stress that any name of a fractional number may be used when you wish to add that fractional number to another.

Give children an opportunity to explain their investigation findings, particularly if they worked with fractions other than $\frac{1}{2}$ and $\frac{2}{3}$. Use exercises 2 and 3, and examples from the investigation, to help the children see that, in order to find the sum of two fractional numbers, they can select from the sets of equivalent fractions any pair of fractions having the same denominator.

Exercise 4 identifies the least common denominator of two fractions with the least common multiple of the two unlike denominators. When children realize this, they will understand the convenience of being able to find the least common multiple of two numbers. It would be helpful to work through several examples of pairs of fractions in a similar manner.

$\frac{1}{3}$, $\frac{1}{4}$:
multiples of 3: 3, 6, 9, 12, 15, . . .
multiples of 4: 4, 8, 12, . . .

12 is the least common multiple (LCM), so 12 is the least common denominator (LCD).

Using the Ideas

1. A Is 6 a multiple of 9? No
   B Is 2 × 6 a multiple of 9? No
   C Is 3 × 6 a multiple of 9? Yes
   D What is the least common multiple of 6 and 9? 18
   E What is the least common denominator for $\frac{1}{6}$ and $\frac{1}{9}$? 18

2. A Is 8 a multiple of 6? No
   B Is 2 × 8 a multiple of 6? No
   C Is 3 × 8 a multiple of 6? Yes
   D What is the least common multiple of 6 and 8? 24
   E What is the least common denominator for $\frac{1}{6}$ and $\frac{1}{8}$? 24
   F What is the least common denominator for $\frac{5}{6}$ and $\frac{3}{8}$? 24

3. Find the least common multiple of the two numbers given in red. Then give the least common denominator for the two fractions.

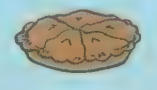
   A $\frac{1}{6}, \frac{7}{9}$ 18,18 B $\frac{7}{8}, \frac{1}{6}$ 24,24 C $\frac{1}{3}, \frac{1}{6}$ 6,6 D $\frac{1}{2}, \frac{5}{8}$ 8,8 E $\frac{3}{4}, \frac{1}{6}$ 12,12 F $\frac{1}{4}, \frac{3}{10}$ 20,20

4. A Give the common multiples (less than 60) of 4, 6, and 8. 0, 24, 48
   B Give the least common multiple of 4, 6, and 8. 24
   C Give the least common denominator for $\frac{1}{4}$, $\frac{5}{6}$, and $\frac{3}{8}$. 24

5. Find the least common denominator for each pair of fractions.

   A $\frac{1}{5}, \frac{1}{10}$ 10
   B $\frac{4}{5}, \frac{7}{10}$ 10
   C $\frac{5}{8}, \frac{1}{6}$ 24
   D $\frac{1}{8}, \frac{5}{6}$ 24
   E $\frac{3}{4}, \frac{1}{3}$ 12
   F $\frac{1}{4}, \frac{2}{3}$ 12
   G $\frac{5}{6}, \frac{7}{10}$ 30
   H $\frac{1}{6}, \frac{9}{10}$ 30

think



What do you know about numerator a if:

1. $\frac{a}{5} = 1$ $a=5$
2. $\frac{a}{5} = 4$ $a=20$
3. $\frac{a}{5} = 2\frac{3}{5}$ $a=13$
4. $\frac{a}{5} = 0$ $a=0$
5. $\frac{a}{5} < 1$ $a<5$
6. $\frac{a}{5} < 2$ $a<10$
7. $\frac{a}{5} > 1$ $a>5$
8. $\frac{a}{5} = a$ $a=0$



Using the Exercises

Assign the exercises on page 161 as independent work. Give guidance to any children who have difficulty. When children finish, allow time for discussion. Point out how the least common multiple of some numbers will be their product, such as 12 for $\frac{1}{3}$ and $\frac{1}{4}$. You might explain that the product of two numbers may always be used as a common denominator but that it rarely will be the least common denominator. Thus, for $\frac{1}{5}$ and $\frac{1}{10}$, 50 could be chosen as a common denominator, but choosing 10 is much more convenient.

Assignments (page 161)
Minimum: 1–5. Average: 1–5.
Maximum: 1–5.

Follow-up

For a game to give children practice in finding least common multiples, cut 80 four-cm squares of tagboard. Write the numerals 1 through 20 on cards to make 4 sets. Shuffle the cards and stack them face down. Each player draws two cards, in turn, and pretends that the two numerals represent denominators for fractions he wishes to add; then he tells the other players the number he would choose as the least common multiple of the two numbers. The others can judge each player's work and help him choose a correct common denominator if he errs. If working with two cards at a time is too easy, have the players choose three cards and find the least common multiple for three denominators.

10	5	8

The least common multiple is 40.

PAGES 162–163

Objective

Given two fractional numbers to add or subtract, the child will be able to find the sum or difference by using fractions with the least common denominator.

Preparation

For this lesson, there are two important skills that should be reviewed. Of first importance, the children should be able to find the least common denominator for two fractions. When you have reviewed this and feel confident that the children understand and can do this, you should review the technique for finding a fraction with a particular denominator that is equivalent to a given fraction in lowest terms. For this review, have the children work problems like these:

$$\frac{1}{3}=\frac{a}{12} \qquad \frac{1}{2}=\frac{c}{10} \qquad \frac{2}{3}=\frac{e}{24}$$

$$\frac{3}{4}=\frac{b}{12} \qquad \frac{1}{5}=\frac{d}{10} \qquad \frac{1}{8}=\frac{f}{24}$$

One of the easiest ways to help children find these numerators is to have them observe what multiple the denominator of the second fraction is of the first. Having found this multiple, the children can multiply it by the numerator of the first fraction to arrive at the numerator for the second fraction.

Investigation

If the children understand the skills reviewed in the preparation, they should not have difficulty in following the chart in the investigation. Although this investigation can be done by the children independently, point out that it is not necessary to recopy each step of the problem. If the example were done in the usual way, it would look like this:

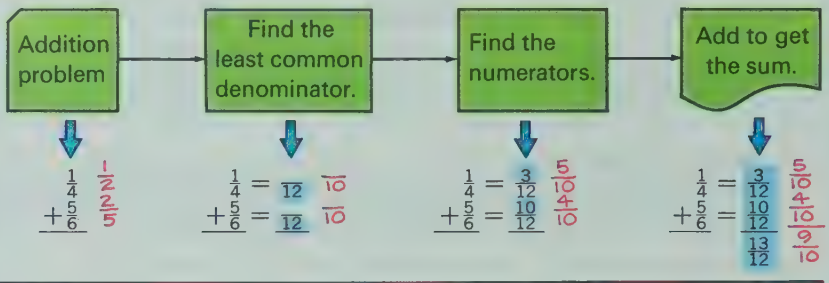
$$\begin{array}{r} \frac{1}{4}=\frac{3}{12} \\ +\frac{5}{6}=\frac{10}{12} \\ \hline \frac{13}{12} \end{array}$$

You might display this example on the chalkboard. If some children finish very quickly, you might suggest that they add $\frac{3}{7}$ and $\frac{2}{5}$, or subtract $\frac{1}{3}$ from $\frac{1}{2}$.

● *Is there a shortcut for adding and subtracting?*

Investigating the Ideas

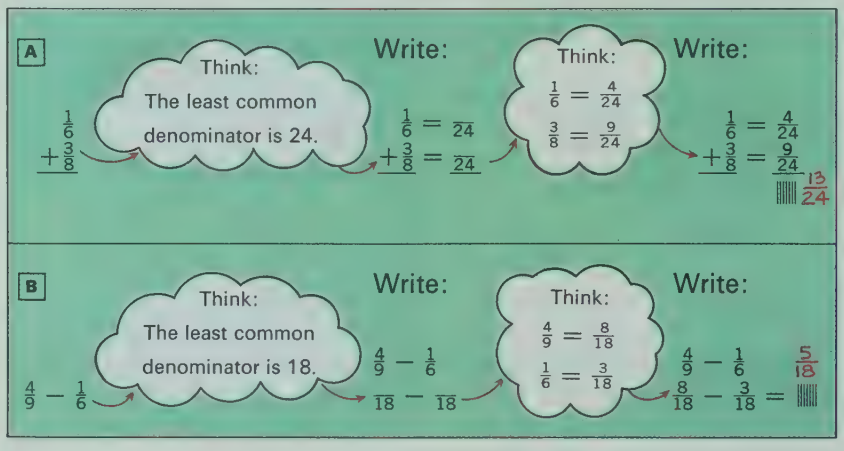
Study the flow chart for finding $\frac{1}{4}+\frac{5}{6}$.



Can you use the flow chart to help you find the sum $\frac{1}{2}+\frac{2}{5}$?
See above and Investigation.

Discussing the Ideas

Explain the steps in each example below. Then find the sum or difference. See Discussion.





Discussion

You might begin the discussion by having children explain each step they followed in the investigation flow chart. Point out that $\frac{13}{12}$ may also be expressed as $1\frac{1}{12}$.

When children respond to the exercise in the discussion section, help them observe that, after they discover that 24 is the least common denominator for $\frac{1}{6}$ and $\frac{3}{8}$, they can write the 24 beside each of the two fractions, as indicated in the example. They then think about the fact that $\frac{1}{6}$ is equal to $\frac{4}{24}$, and $\frac{3}{8}$ is equal to $\frac{9}{24}$, and write these fractions as indicated. Finally, they can find the sum, $\frac{13}{24}$.

Take care that the children do not mistakenly conclude from these two examples that we work all addition problems in vertical form and all subtraction problems in horizontal form. Point out to them that they can use either form for either type of problem. The ideas represented are exactly the same. Also, beginning with this lesson, help the children become accustomed to expressing all their answers as lowest-terms fractions.

Using the Ideas

1. Copy each exercise and give the missing numerators. Then find the sum or difference.

A $\frac{1}{2} = \frac{\square}{6}$ 3
$+\frac{1}{3} = \frac{\square}{6}$ 2
$\frac{5}{6}$

B $\frac{1}{4} = \frac{\square}{8}$ 2
$+\frac{3}{8} = \frac{\square}{8}$ 3
$\frac{5}{8}$

C $\frac{5}{8} = \frac{\square}{8}$ 5
$-\frac{1}{2} = \frac{\square}{8}$ 4
$\frac{1}{8}$

D $\frac{5}{6} = \frac{\square}{18}$ 15
$-\frac{1}{9} = \frac{\square}{18}$ 2
$\frac{13}{18}$

E $\frac{5}{12} = \frac{\square}{12}$ 5
$+\frac{1}{4} = \frac{\square}{12}$ 3
$\frac{8}{12}$

F $\frac{7}{12} + \frac{1}{3}$ $7\frac{\square}{12} + 4\frac{\square}{12}$ $\frac{11}{12}$

G $\frac{5}{6} - \frac{1}{3}$ $5\frac{\square}{6} - 2\frac{\square}{6}$ $\frac{3}{6}$

H $\frac{3}{10} - \frac{1}{4}$ $6\frac{\square}{20} - 5\frac{\square}{20}$ $\frac{1}{20}$

I $\frac{7}{10} + \frac{1}{2}$ $7\frac{\square}{10} + 5\frac{\square}{10}$ $\frac{12}{10}$

J $\frac{1}{8} + \frac{1}{6}$ $3\frac{\square}{24} + 4\frac{\square}{24}$ $\frac{7}{24}$

2. Find the sums and differences. Give your answers in lowest terms.

A $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$

B $\frac{1}{3} - \frac{1}{6} = \frac{1}{6}$

C $\frac{7}{10} - \frac{5}{100} = \frac{13}{20}$

D $\frac{1}{4} + \frac{5}{12} = \frac{2}{3}$

E $\frac{5}{6} - \frac{1}{9} = \frac{13}{18}$

F $\frac{1}{8} + \frac{1}{12} = \frac{5}{24}$

G $\frac{7}{8} - \frac{1}{2} = \frac{3}{8}$

H $\frac{1}{4} + \frac{5}{6} = \frac{13}{12}$

I $\frac{5}{8} - \frac{1}{4} = \frac{3}{8}$

J $\frac{1}{8} + \frac{1}{12} = \frac{5}{24}$

K $\frac{3}{10} - \frac{1}{5} = \frac{1}{10}$

L $\frac{3}{10} - \frac{1}{4} = \frac{1}{20}$

M $\frac{2}{3} + \frac{2}{5} = \frac{16}{15}$

N $\frac{9}{10} - \frac{3}{4} = \frac{3}{20}$

O $\frac{7}{9} + \frac{1}{3} = \frac{10}{9}$

P $\frac{5}{8} - \frac{1}{6} = \frac{11}{24}$

Q $\frac{2}{10} + \frac{7}{100} = \frac{27}{100}$

3. Give the missing numerators in exercises A and B. Then find the sums for exercises C through L.

A $\frac{1}{2} + \frac{1}{3} + \frac{1}{6}$
$\downarrow \quad \downarrow \quad \downarrow$
$\frac{3}{6} + \frac{2}{6} + \frac{1}{6}$

B $\frac{3}{4} + \frac{1}{2} + \frac{1}{6}$
$\downarrow \quad \downarrow \quad \downarrow$
$\frac{9}{12} + \frac{6}{12} + \frac{2}{12}$

C $\frac{1}{2} + \frac{1}{3} + \frac{1}{6}$ $\frac{6}{6}$

D $\frac{3}{4} + \frac{1}{2} + \frac{1}{6}$ $\frac{17}{12}$

E $\frac{2}{3} + \frac{1}{2} + \frac{5}{6}$ $\frac{12}{6}$

F $\frac{1}{4} + \frac{5}{6} + \frac{1}{2}$ $\frac{19}{12}$

G $\frac{1}{4} + \frac{2}{3} + \frac{1}{6}$ $\frac{13}{12}$

H $\frac{3}{8} + \frac{1}{3} + \frac{3}{4}$ $\frac{35}{24}$

I $\frac{1}{4} + \frac{3}{8} + \frac{1}{2}$ $\frac{9}{8}$

J $\frac{1}{4} + \frac{1}{8} + \frac{1}{2}$ $\frac{7}{8}$

K $\frac{1}{2} + \frac{7}{8} + \frac{1}{4}$ $\frac{13}{8}$

L $\frac{1}{2} + \frac{1}{4} + \frac{3}{5}$ $\frac{27}{20}$

think

68 legs on some chickens and horses.
22 animals in all.
How many chickens? 10



More practice, page A-11, Set 22



Using the Exercises

With some groups of children, you might want to work through several of the exercises on page 163 together. Notice any difficulty children have and try to determine whether it results from weakness in a particular skill, such as one reviewed in the preparation, or whether it results from lack of understanding the procedure of putting these skills together. If any child is apparently weak in a particular skill, help him with that before assigning him exercises such as those on page 163.

Although parts of exercise 2 are written in horizontal form, they may be solved vertically if children prefer. Even though exercise 3 deals with three addends, do not try to develop any particular technique. This topic will be treated in a subsequent chapter.

Encourage the children who try the *Think* problem to use a trial-and-error method. They may have to try several combinations of addends of 22 before they discover that 12 and 10 are the correct pair since $12 \times 4 = 48$ and $10 \times 2 = 20$, and $48 + 20 = 68$.

Assignments (page 163)
Minimum: 1, 2. Average: 1–3.
Maximum: 1–3.

Duplicator Masters, page 29
Workbook, page 50
Skill Masters, page 29

PAGES 164–165

Objectives

Given short-story problems which require addition and subtraction of fractional numbers, the child will be able to solve the problems.

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

To prepare for page 164, you might work through a few examples of addition and subtraction with fractions of unlike denominators.

To prepare for page 165, review any of the algorithms which the children find difficult. It would also be helpful to review the basic principles. For example, ask the children to give equations which demonstrate the associative principle for addition, the commutative principle for multiplication, and so on. These basic principles will be studied in a subsequent lesson in relation to fractional numbers, so the time spent on such a review here is appropriate.

Short Stories

1 Studied $\frac{1}{2}$ hour.
Watched TV $\frac{1}{3}$ hour.
How much time in all? $\frac{5}{6}$ hr or 50 min



2 Allan lives $\frac{6}{10}$ km from school.
Cheryl lives $\frac{1}{4}$ km from school.
How much closer does Cheryl live? $\frac{7}{20}$ k

3 Nancy: $\frac{1}{6}$ of a pie.
Bill and Bob: $\frac{3}{4}$ of the pie.
How much of the pie in all? $\frac{11}{12}$

4 Ran $\frac{1}{2}$ km.
Walked $\frac{1}{5}$ km.
Skipped $\frac{1}{10}$ km.
How far in all? $\frac{4}{5}$ k



5 Spent $\frac{1}{2}$ dollar. Spent $\frac{1}{4}$ dollar.
Spent $\frac{1}{10}$ dollar. Spent what part of a dollar in all? $\frac{17}{20}$

6 Screw: 4 centimetres long.
Threaded section: $\frac{3}{4}$ of the length.
How many centimetres without threads? 1 cm



7 Mowed $\frac{2}{3}$ of the lawn.
Then mowed $\frac{1}{4}$ of the lawn.
How much more to mow? $\frac{1}{12}$



8 $\frac{5}{8}$ of class: brown hair.
$\frac{1}{3}$ of class: blonde hair.
Rest of class: red hair.
What part has red hair? $\frac{1}{24}$

9 Walked $\frac{1}{5}$ of a kilometre.
Rode $\frac{3}{4}$ of a kilometre.
How far in all? $\frac{19}{20}$ km

10 Beth: $\frac{7}{10}$ of a dollar.
Larry: $\frac{4}{5}$ of a dollar.
How much more does Larry have? $\frac{1}{10}$

164

Discussion

As you assign page 164, suggest to the children that they write an equation for each problem. Then they can write their work in vertical notation, finding common denominators to use in adding or subtracting. When the children finish, it would be helpful to use several of these problems as a basis for discussion.

Assignments (page 164)

Minimum: 1–5. Average: 1–10
Maximum: 1–10.

Keeping in Touch with

Addition
Subtraction
Multiplication
Division
Measurement
Principles

1. Find the sum, difference, product, and quotient.

A $\begin{array}{r} 27\,643 \\ +54\,896 \\ \hline 82\,539 \end{array}$

B $\begin{array}{r} 57\,384 \\ -32\,641 \\ \hline 24\,743 \end{array}$

C $\begin{array}{r} 64\,237 \\ \times 24 \\ \hline 1\,541\,688 \end{array}$

D $42\overline{)13\,692}$ 326

2. Each of these exercises illustrates one of the basic principles. Give the number of the principle.

A $3278 \times 26 = 26 \times 3278$
Answer: 2

3 B $264 + (278 + 315) = (264 + 278) + 315$

5 C $4 \times (26 + 57) = (4 \times 26) + (4 \times 57)$

6 D $(6428 + 32) + 0 = 6428 + 32$

2 E $276 \times 3426 = 3426 \times 276$

4 F $(76 \times 28) \times 463 = 76 \times (28 \times 463)$

7 G $1 \times (327 + 683) = 327 + 683$

1 H $7 + (326 + 483) = 7 + (483 + 326)$

1 I $(29 + 83) + 65 = 65 + (29 + 83)$

5 J $(3 \times 28) + (3 \times 43) = 3 \times (28 + 43)$

1. Commutative principle (addition)
2. Commutative principle (multiplication)
3. Associative principle (addition)
4. Associative principle (multiplication)
5. Distributive principle
6. Zero principle
7. One principle

3. Solve the equations.

A $648+274=n+648$ 274

B $(23\times65)\times34=23\times(n\times34)$ 65

C $572\times927=927\times n$ 572

D $6\times(254+9)=(6\times n)+54$ 254

E $(82+n)+21=82+(48+21)$ 48

4. Give the measure of each angle.

A $\angle CAB$ 25°

B $\angle DAB$ 40°

C $\angle EAB$ 70°

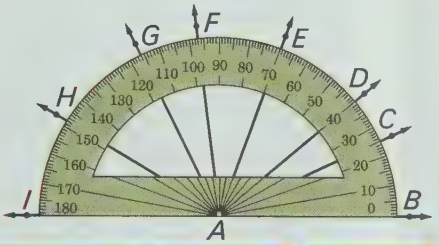
D $\angle CAE$ 45°

E $\angle FAB$ 97°

F $\angle GAB$ 116°

G $\angle HAB$ 148°

H $\angle IAB$ 180°





You are invited to explore

ACTIVITY CARD 8
Page 359

165

Using the Exercises

Assign page 165 as independent work. When the children have finished, help them check their work, carefully discussing any exercise which caused difficulty. As stated above, it would be worthwhile for you to stress a review of the basic principles since they will soon be applied to fractional numbers. Point out how a knowledge of the basic principles will help in finding the solution to the equations in exercise 3.

Workbook, page 51

PAGES 166–167

Objective

Given an improper fraction, the child will be able to write a mixed numeral for it, and given a mixed numeral, the child will be able to express it as an improper fraction.

Preparation

To prepare for this lesson, you might write the following equations on the chalkboard and ask children to supply the missing term.

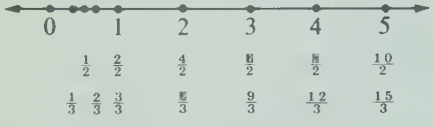
$$5 = \frac{n}{3} \qquad 4 = \frac{n}{7}$$

$$2\tfrac{1}{4} = 2 + ?$$

$$5\tfrac{2}{3} = 5 + ?$$

$$4\tfrac{1}{2} = ? + \tfrac{1}{2}$$

Thus, children should be able to write a whole number as a fraction and should realize that a mixed numeral may be thought of as a sum of a whole number and a fraction. If children have difficulty with the technique of writing a whole number for an improper fraction, you might use a number line to help them recall that whole numbers can be named as fractions.



Then help them recall the technique:

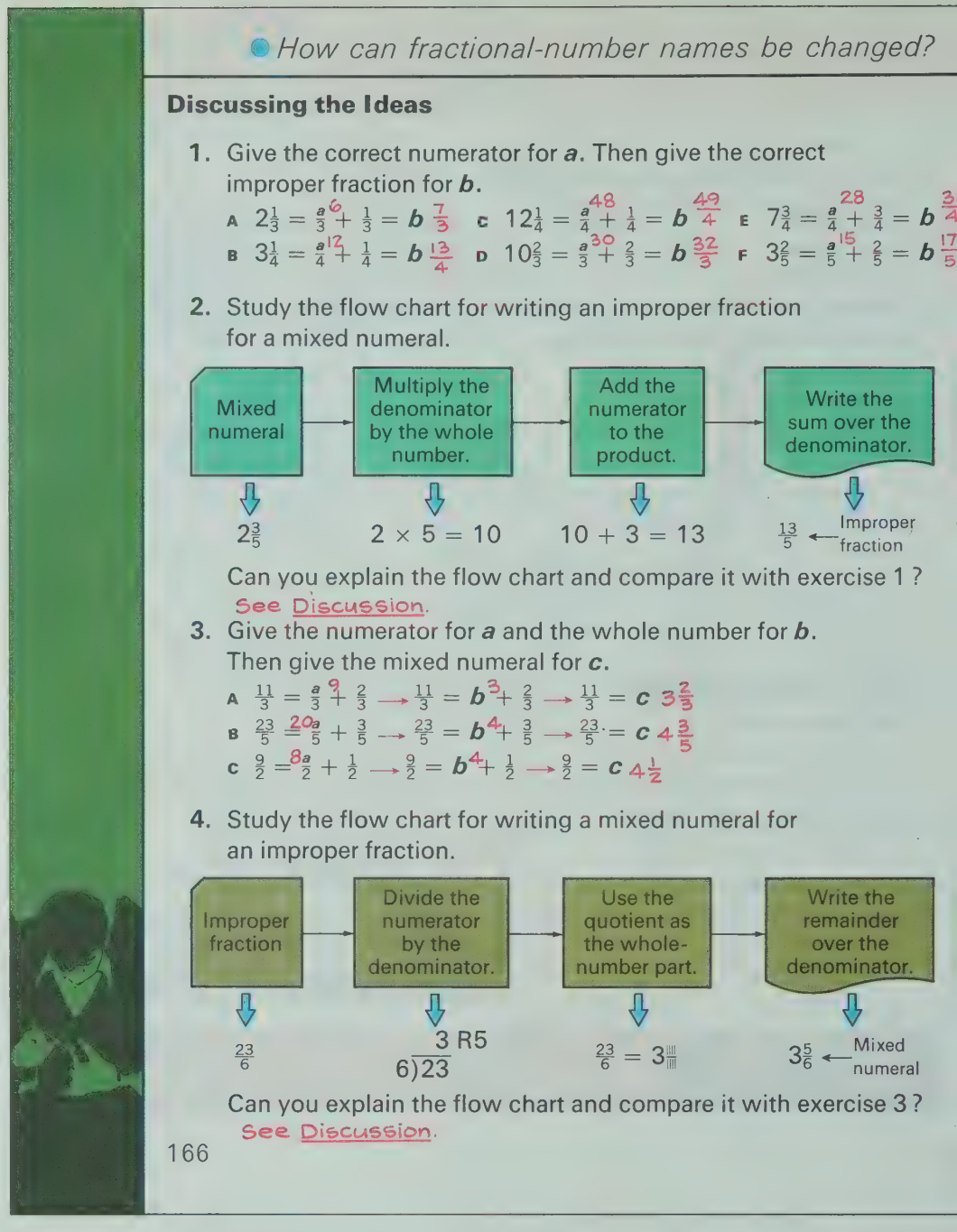
$$5 = \frac{n}{3} \qquad 5 = \frac{5 \times 3}{3} = \frac{15}{3}$$

If necessary, show an intermediate step:

$$5 = 1 + 1 + 1 + 1 + 1$$

$$= \frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{3}{3}$$

$$= \frac{15}{3}$$



How can fractional-number names be changed?

Discussing the Ideas

1. Give the correct numerator for ***a***. Then give the correct improper fraction for ***b***.

   A $2\frac{1}{3} = \frac{a}{3} + \frac{1}{3} = b$ $\frac{7}{3}$ C $12\frac{1}{4} = \frac{a}{4} + \frac{1}{4} = b$ $\frac{49}{4}$ E $7\frac{3}{4} = \frac{a}{4} + \frac{3}{4} = b$ $\frac{31}{4}$

   B $3\frac{1}{4} = \frac{a}{4} + \frac{1}{4} = b$ $\frac{13}{4}$ D $10\frac{2}{3} = \frac{a}{3} + \frac{2}{3} = b$ $\frac{32}{3}$ F $3\frac{2}{5} = \frac{a}{5} + \frac{2}{5} = b$ $\frac{17}{5}$

2. Study the flow chart for writing an improper fraction for a mixed numeral.

   Can you explain the flow chart and compare it with exercise 1? See Discussion.

3. Give the numerator for ***a*** and the whole number for ***b***. Then give the mixed numeral for ***c***.

   A $\frac{11}{3} = \frac{a}{3} + \frac{2}{3} \rightarrow \frac{11}{3} = b + \frac{2}{3} \rightarrow \frac{11}{3} = c$ $3\frac{2}{3}$

   B $\frac{23}{5} = \frac{20}{5} + \frac{3}{5} \rightarrow \frac{23}{5} = b + \frac{3}{5} \rightarrow \frac{23}{5} = c$ $4\frac{3}{5}$

   C $\frac{9}{2} = \frac{8}{2} + \frac{1}{2} \rightarrow \frac{9}{2} = b + \frac{1}{2} \rightarrow \frac{9}{2} = c$ $4\frac{1}{2}$

4. Study the flow chart for writing a mixed numeral for an improper fraction.

   Can you explain the flow chart and compare it with exercise 3? See Discussion.

166

Discussion

Use exercise 1 as a basis for discussion of how a mixed numeral may be thought of in different ways. Stress that a whole number may be named as a fraction with any denominator but, once the denominator is chosen, the numerator is determined. For example, the whole number 2 may be thought of as $\frac{4}{2}$, $\frac{6}{3}$, $\frac{8}{4}$, $\frac{10}{5}$, or $\frac{12}{6}$, and so on. In changing a mixed numeral to an improper fraction, the denominator of the improper fraction is determined by the denominator of the mixed number. Thus, in exercise 1A, $2\frac{1}{3} = \frac{6}{3} + \frac{1}{3} = \frac{7}{3}$. Also, discuss how the number 2 may be thought of as 2×3 thirds; or 2×4 fourths, or 2×9 ninths, because the expressions 3 thirds, 4 fourths, and 9 ninths are all names for one. Thus, in exercise 2, they can express the number of ones ($\frac{3}{3}$, or $\frac{4}{4}$, or $\frac{5}{5}$, and so on) by multiplying the denominator of the given fraction by the whole number:

$$2\frac{3}{5} = \frac{2 \times 5}{5} + \frac{3}{5} = \frac{10}{5} + \frac{3}{5} = \frac{13}{5}$$

Although the primary purpose of this lesson is for children to understand this process, it would be helpful to have them work through several exercises of this kind.

Exercise 3 aims to develop under-

Using the Ideas

1. Write an improper fraction for each mixed numeral.

A $1\frac{1}{2}$ $\frac{3}{2}$ C $1\frac{1}{3}$ $\frac{4}{3}$ E $2\frac{3}{5}$ $\frac{13}{5}$ G $1\frac{3}{4}$ $\frac{7}{4}$

B $4\frac{3}{8}$ $\frac{35}{8}$ D $1\frac{1}{4}$ $\frac{5}{4}$ F $1\frac{2}{3}$ $\frac{5}{3}$ H $6\frac{7}{10}$ $\frac{67}{10}$

2. Write a mixed numeral for each improper fraction.

A $\frac{5}{2}$ $2\frac{1}{2}$ C $\frac{17}{6}$ $2\frac{5}{6}$ E $\frac{8}{3}$ $2\frac{2}{3}$ G $\frac{11}{4}$ $2\frac{3}{4}$

B $\frac{7}{3}$ $2\frac{1}{3}$ D $\frac{9}{4}$ $2\frac{1}{4}$ F $\frac{20}{7}$ $2\frac{6}{7}$ H $\frac{21}{10}$ $2\frac{1}{10}$

3. Give the missing numerators.

A $\frac{8}{5} = 1 + \frac{\square}{5}$ 3

B $\frac{7}{4} = 1 + \frac{\square}{4}$ 3

C $\frac{6}{5} = 1 + \frac{\square}{5}$ 1

D $\frac{5}{3} = 1 + \frac{\square}{3}$ 2

E $\frac{13}{10} = 1 + \frac{\square}{10}$ 3

F $\frac{15}{10} = 1 + \frac{\square}{10}$ 5

G $\frac{9}{5} = 1 + \frac{\square}{5}$ 4

H $\frac{10}{8} = 1 + \frac{\square}{4}$ 1

I $\frac{9}{6} = 1 + \frac{\square}{2}$ 1

J $\frac{8}{6} = 1 + \frac{\square}{3}$ 1

K $\frac{10}{6} = 1 + \frac{\square}{3}$ 2

L $\frac{24}{20} = 1 + \frac{\square}{10}$ 2

M $\frac{32}{20} = 1 + \frac{\square}{5}$ 3

N $\frac{16}{10} = 1 + \frac{\square}{5}$ 3

O $\frac{14}{8} = 1 + \frac{\square}{4}$ 3

P $\frac{60}{50} = 1 + \frac{\square}{25}$ 5

4. Solve the equations. Use lowest-terms fractions for all answers.

A $5\frac{9}{8} = 6 + n$ $\frac{1}{8}$

B $6\frac{15}{10} = 7 + n$ $\frac{1}{2}$

C $3\frac{17}{10} = 4 + n$ $\frac{7}{10}$

D $2\frac{12}{7} = 3 + n$ $\frac{5}{7}$

E $12\frac{6}{5} = 13 + n$ $\frac{1}{5}$

F $15\frac{7}{4} = n + \frac{3}{4}$ 16

G $36\frac{8}{5} = 37 + n$ $\frac{3}{5}$

H $43\frac{9}{7} = 44 + n$ $\frac{2}{7}$

I $95\frac{10}{6} = 96 + n$ $\frac{2}{3}$

J $36\frac{12}{10} = 37 + n$ $\frac{1}{5}$

K $83\frac{9}{6} = n + \frac{1}{2}$ 84

L $46\frac{16}{10} = 47 + n$ $\frac{3}{5}$

M $12\frac{7}{2} = 13 + n$ $\frac{5}{2}$

N $14\frac{12}{9} = n + \frac{1}{3}$ 15

O $13\frac{6}{4} = 14 + n$ $\frac{1}{2}$

5. Copy each exercise. Give the missing numerators.

A $4\frac{1}{2} = 4\frac{a}{4} = 3\frac{b}{4}$ 2 6

B $5\frac{1}{3} = 5\frac{a}{12} = 4\frac{b}{12}$ 4 16

C $6\frac{3}{4} = 6\frac{a}{12} = 5\frac{b}{12}$ 9 21

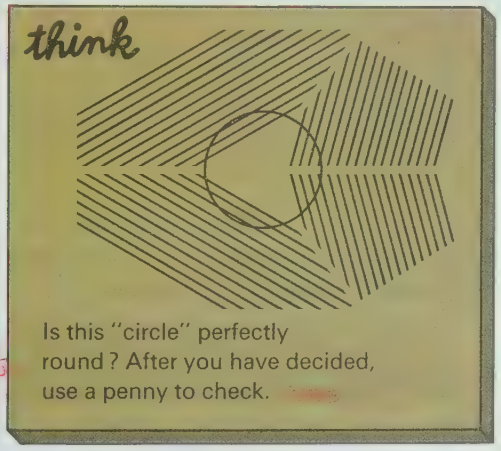
D $7\frac{2}{3} = 7\frac{a}{9} = 6\frac{b}{9}$ 6 15

E $12\frac{2}{5} = 12\frac{a}{10} = 11\frac{b}{10}$ 4 14

F $30\frac{3}{10} = 30\frac{a}{50} = 29\frac{b}{50}$ 15 65

G $1 = 1\frac{a}{3} = \frac{b}{3}$ 0 3

H $9 = 9\frac{a}{4} = 8\frac{b}{4}$ 0 4



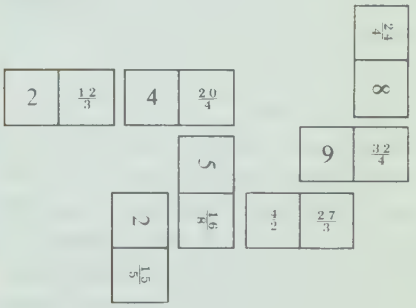


More practice, page A-12, Set 23

Follow-up

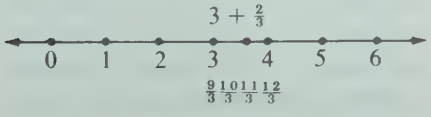
Many children may need considerable experience in converting whole numbers to improper fractions, and vice versa, before tackling mixed numerals.

A small group of these children may enjoy another adaptation of the domino game presented in the follow-up section for pages 136 and 137. Make 40 or more domino cards with at least four equivalent improper fractions for each whole number. To start the game, a card is drawn from the shuffled deck and placed face up on the table. Then the players draw 5 cards apiece from the deck, which is face down. Each child must match a numeral already played with an equivalent from his hand or draw another card from the pack. The first child to play all of his cards correctly wins.



Duplicator Masters, page 30
Workbook, page 52
Skill Masters, page 30

standing of the reverse process, and exercise 4 gives steps to develop the skill. Point out, in exercise 3, that as they examine an improper fraction they should think how many "ones" or "whole numbers" are in this fraction. The use of a number line might help some children understand exercise 3:



Work through several examples to help children discover the pattern:

$\frac{11}{3} = 3\frac{2}{3}$; $\quad 3\overline{)11}$ $\quad 3$ R 2

Using the Exercises

Assign the exercises on page 167 as independent work. However, you might want to work together through exercises 1 and 2 with some groups of children. In parts of exercise 3, help the children see that they can reduce the fractions in the mixed numeral to lowest terms. In exercise 3H, for example,

$\frac{10}{8} = 1 + \frac{2}{8} = 1 + \frac{1}{4}$.

Assignments (page 167)

Minimum: 1, 2. Average: 1–5.
Maximum: 1–5.

PAGES 168–169

Objective

Given fractional numbers in addition problems, the child will demonstrate his understanding of the associative and commutative principles for addition by arranging the addends in any way that is convenient.

Preparation

Briefly review the commutative and associative principles for addition. Emphasize that these two principles, used together, allow the rearrangement of addends in any way that is convenient.

Also, review the regrouping process as shown in exercise 5, page 167. For example, give children practice with these by saying "$6\frac{1}{3}$ may be thought of as 5 and how many thirds?" ($5\frac{4}{3}$) Or, "$2\frac{3}{5}$ may be thought of as "1 and how many fifths?" ($1\frac{8}{5}$). This skill is important to children's success with this lesson.

Investigation

Encourage children to write as many equations as they can. You might explain that the equation $\frac{3}{4} + 7 = 7\frac{3}{4}$ is acceptable even though $7 + \frac{3}{4} = 7\frac{3}{4}$ has already been used. Accept any correct combinations such as $5\frac{1}{2} + 2 + \frac{1}{4} = 7\frac{3}{4}$ or $5\frac{1}{2} + 2\frac{1}{4} = 7\frac{3}{4}$. The variety children record should help to impress upon them the idea that the application of the order and grouping principles is valid with combinations of fractions and whole numbers as well as with whole numbers.

Let's use the basic principles for fractional numbers.

Investigating the Ideas

Check this equation. → $5 + 2 + \frac{1}{2} + \frac{1}{4} = 7\frac{3}{4}$

This equation was formed from the four addends in the first equation. → $7 + \frac{3}{4} = 7\frac{3}{4}$

$5+2+\frac{3}{4}=7\frac{3}{4}$
$7\frac{1}{2}+\frac{1}{4}=7\frac{3}{4}$
$5+2\frac{1}{2}+\frac{1}{4}=7\frac{3}{4}$
$5\frac{1}{2}+2+\frac{1}{4}=7\frac{3}{4}$
$5+2\frac{1}{4}+\frac{1}{2}=7\frac{3}{4}$
$5\frac{1}{4}+2+\frac{1}{2}=7\frac{3}{4}$
$5\frac{1}{2}+2\frac{1}{4}=7\frac{3}{4}$
$5\frac{1}{4}+2\frac{1}{2}=7\frac{3}{4}$
$5\frac{3}{4}+2=7\frac{3}{4}$
$2\frac{3}{4}+5=7\frac{3}{4}$
$7+\frac{1}{2}+\frac{1}{4}=7\frac{3}{4}$
$7\frac{1}{4}+\frac{1}{2}=7\frac{3}{4}$

? How many other equations can you write by combining the addends 5, 2, $\frac{1}{2}$, and $\frac{1}{4}$ in different ways? See Investigation. | Record the equations you find.

Discussing the Ideas

1. What principle is illustrated by each equation?
   - A $\frac{3}{4} + \frac{1}{2} = \frac{1}{2} + \frac{3}{4}$ Commutative
   - B $\frac{5}{6} + (\frac{1}{10} + \frac{3}{5}) = (\frac{5}{6} + \frac{1}{10}) + \frac{3}{5}$ Associative
2. Give numbers for ***a***, ***b***, and ***c***. Explain your answers.
   - A $2\frac{3}{8} + 4\frac{1}{4} = (2 + a) + (4 + \frac{1}{4}) = 6 + \frac{5}{8} = b$ $\frac{3}{8}$, $6\frac{5}{8}$
   - B $5\frac{1}{2} + 2\frac{1}{4} = (5 + 2) + (\frac{1}{2} + a) = 7 + \frac{3}{4} = b$ $\frac{1}{4}$, $7\frac{3}{4}$
   - C $7\frac{1}{4} + 2\frac{1}{6} = (7 + 2) + (\frac{1}{4} + a) = 9 + b = c$ $\frac{1}{6}$, $\frac{5}{12}$, $9\frac{5}{12}$
3. Solve the equations. Explain your answers.
   - A $5\frac{1}{2} + 1\frac{1}{4} = n + \frac{3}{4}$ 6
   - B $2\frac{1}{8} + 3\frac{1}{4} = c + \frac{3}{8}$ 5
   - C $4\frac{1}{8} + 3\frac{1}{2} = 7 + b$ $\frac{5}{8}$
   - D $7\frac{1}{5} + 2\frac{3}{5} = 9 + t$ $\frac{4}{5}$
   - E $8\frac{3}{8} + 1\frac{1}{2} = x + \frac{7}{8}$ 9
   - F $6\frac{2}{7} + 2\frac{4}{7} = 8 + d$ $\frac{6}{7}$
4. Explain how you can use the statement in the box to complete the example and find the sum.

Since you can change the order and the grouping of addends, you can arrange the addends in any way that is convenient.

$2\frac{1}{4} + 3\frac{1}{2} = (2 + \frac{1}{4}) + (3 + \frac{1}{2}) = ?$ $5\frac{3}{4}$

168

Discussion

As children respond to discussion exercise 1, stress that the commutative and associative principles are applicable to fractions. It would be helpful to have the children work through each side of both equations.

$$\frac{3}{4} + \frac{1}{2} \stackrel{?}{=} \frac{1}{2} + \frac{3}{4}$$
$$\frac{3}{4} + \frac{2}{4} \stackrel{?}{=} \frac{2}{4} + \frac{3}{4}$$
$$\frac{5}{4} = \frac{5}{4}$$

and

$$\frac{5}{6} + (\frac{1}{10} + \frac{3}{5}) \stackrel{?}{=} (\frac{5}{6} + \frac{1}{10}) + \frac{3}{5}$$
$$\frac{5}{6} + (\frac{1}{10} + \frac{6}{10}) \stackrel{?}{=} (\frac{25}{30} + \frac{3}{30}) + \frac{3}{5}$$
$$\frac{5}{6} + \frac{7}{10} \stackrel{?}{=} \frac{28}{30} + \frac{3}{5}$$
$$\frac{25}{30} + \frac{21}{30} \stackrel{?}{=} \frac{28}{30} + \frac{18}{30}$$
$$\frac{46}{30} = \frac{46}{30}$$

Have the children work through the parts of exercise 2 with you. Show an example in vertical form. For instance, if you write

$$2\frac{3}{8} = 2\frac{3}{8}$$
$$\underline{4\frac{1}{4} = 4\frac{2}{8}}$$
$$6\frac{5}{8}$$

point out that in the vertical form of addition, we add the fractions first and then the whole numbers, according to the same principles applied in the horizontal example.

Similarly, with exercise 3, you might have children work the problems in vertical form before they try to fill in the missing terms in the equations.

Using the Ideas

Find the sums and differences. Use mixed numerals and lowest-terms fractions for your answers.

1.

Example:

$\frac{5}{6} = \frac{5}{6}$
$+\frac{2}{3} = \frac{4}{6}$
$\frac{9}{6} = 1\frac{1}{2}$

A $\frac{1}{2} + \frac{3}{4} = 1\frac{1}{4}$ B $\frac{5}{8} + \frac{2}{3} = 1\frac{7}{24}$ C $\frac{5}{6} + \frac{1}{2} = 1\frac{1}{3}$ D $\frac{2}{3} + \frac{2}{3} = 1\frac{1}{3}$ E $\frac{7}{10} + \frac{4}{5} = 1\frac{1}{2}$

F $\frac{61}{100} + \frac{7}{10} = 1\frac{31}{100}$ G $\frac{1}{4} + \frac{5}{6} = 1\frac{1}{12}$ H $\frac{11}{20} + \frac{4}{5} = 1\frac{7}{20}$ I $\frac{11}{15} + \frac{7}{10} = 1\frac{13}{30}$ J $\frac{3}{5} + \frac{9}{10} = 1\frac{1}{2}$

2.

Example:

$3\frac{5}{8} = 3\frac{15}{24}$
$+8\frac{2}{3} = 8\frac{16}{24}$
$11\frac{31}{24} = 12\frac{7}{24}$

A $5\frac{1}{2} + 2\frac{3}{4} = 8\frac{1}{4}$ B $6\frac{5}{8} + 7\frac{2}{3} = 14\frac{7}{24}$ C $3\frac{5}{6} + 8\frac{1}{2} = 12\frac{1}{3}$ D $9\frac{2}{3} + 1\frac{2}{3} = 11\frac{1}{3}$ E $6\frac{7}{10} + 7\frac{3}{5} = 14\frac{3}{10}$

F $8\frac{2}{3} + 3\frac{3}{5} = 12\frac{4}{15}$ G $7\frac{1}{4} + 2\frac{5}{6} = 10\frac{1}{12}$

H $61\frac{1}{2} + 91\frac{1}{8} = 152\frac{5}{8}$ I $82\frac{3}{8} + 72\frac{1}{4} = 154\frac{5}{8}$ J $15\frac{7}{8} + 27\frac{5}{6} = 43\frac{17}{24}$ K $83\frac{2}{3} + 36\frac{3}{4} = 120\frac{5}{12}$

think



Fran gave Jan half her cookies and half a cookie. Fran had 7 cookies left. How many did Jan get?

BE CAREFUL ! ! !

3.

Example:

$7\frac{1}{2} = 7\frac{2}{4} = 6\frac{6}{4}$
$-2\frac{3}{4} = 2\frac{3}{4} = 2\frac{3}{4}$
$4\frac{3}{4}$

A $9\frac{1}{2} - 2\frac{3}{4} = 6\frac{3}{4}$ B $8\frac{5}{8} - 1\frac{2}{3} = 6\frac{23}{24}$ C $6\frac{3}{10} - 1\frac{1}{5} = 5\frac{1}{10}$ D $7\frac{1}{4} - 6\frac{5}{6} = \frac{5}{12}$ E $8\frac{7}{10} - 5\frac{4}{5} = 2\frac{9}{10}$ F $4\frac{7}{10} - 2\frac{11}{15} = 1\frac{29}{30}$ G $7\frac{3}{5} - 3\frac{9}{10} = 3\frac{7}{10}$

H $26\frac{1}{2} - 15\frac{1}{4} = 11\frac{1}{4}$ I $47\frac{3}{4} - 39\frac{2}{3} = 8\frac{1}{12}$ J $83\frac{2}{3} - 15\frac{3}{4} = 67\frac{11}{12}$ K $91\frac{5}{6} - 56\frac{1}{8} = 35\frac{17}{24}$ L $65\frac{1}{8} - 38\frac{5}{6} = 26\frac{7}{24}$ M $27\frac{7}{10} - 9\frac{4}{5} = 17\frac{9}{10}$ N $96\frac{5}{6} - 56\frac{9}{10} = 39\frac{14}{15}$

More practice, page A-12, Set 24



As you discuss exercise 4, emphasize how the basic principles allow the rearranging of addends whether they are whole numbers or fractions. Be sure the children understand that when they add, they can add the whole-number parts of the mixed numerals and the fractional parts separately.

Using the Exercises

If you discussed several exercises in vertical notation as suggested in the discussion section, you will probably want to assign the exercises on page 169 as independent work. With some children, it would be best to select only parts of exercises 1 and 2.

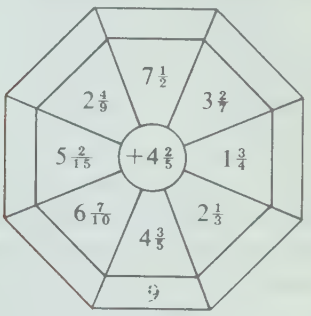
For many children, exercise 3 would provide material for another day's lesson. Be sure to check the children's work carefully and allow ample time for discussion.

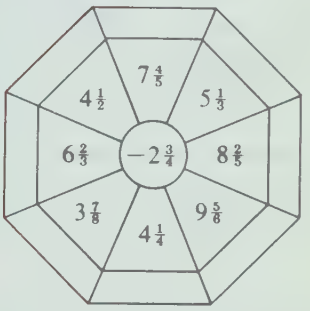
Assignments (page 169)

Minimum: 1A–E, 2A–E, 3A–E.
Average: 1–3. Maximum: 1–3.

Follow-up

Most children would benefit from further practice with addition and subtraction of mixed numerals. For this purpose you might provide some Practagons like the ones below.





Solution, Think, page 169

The clue to solving the *Think* problem lies in observing the fact that, after giving Jan half her cookies and another half cookie, Fran still has 7, a whole number of cookies, left. Therefore, since Fran must have had 15 cookies to begin with $(7 + \frac{1}{2}) \times 2$, Jan got $7\frac{1}{2}$ cookies plus another $\frac{1}{2}$ cookie, or 8 cookies in all.

Duplicator Masters, pages 31, 32
Workbook, page 53
Skill Masters, pages 31, 32

PAGES 170–171

Objective

Given three fractional numbers with different denominators, the child will be able to add them by writing them as fractions with a least common denominator.

Preparation

Materials

colored strips

The best preparation for this lesson would be a review of the techniques for finding the least common multiple of two numbers. For example, write the numbers 5 and 6 on the chalkboard and ask children to find their least common multiple, and to explain how they do it. Some might list the multiples 5, 10, 15, 20, 25, 30, 35, . . . and 6, 12, 18, 24, 30, 36, . . . and then pick out the least common multiple, 30. Others might find factors of 5 and 6 and multiply all factors which appear once:

5, 2, 3

(Since none of the factors of 6 are also factors of 5, the least common multiple is $5 \times 2 \times 3$.)

Work through other pairs of numbers in a similar manner:

6, 8
List multiples:
6, 12, 18, 24, . . .
8, 16, 24, . . .
Or factors of 6: 2, 3
factors of 8: 2, 4
$2 \times 3 \times 4 = 24$

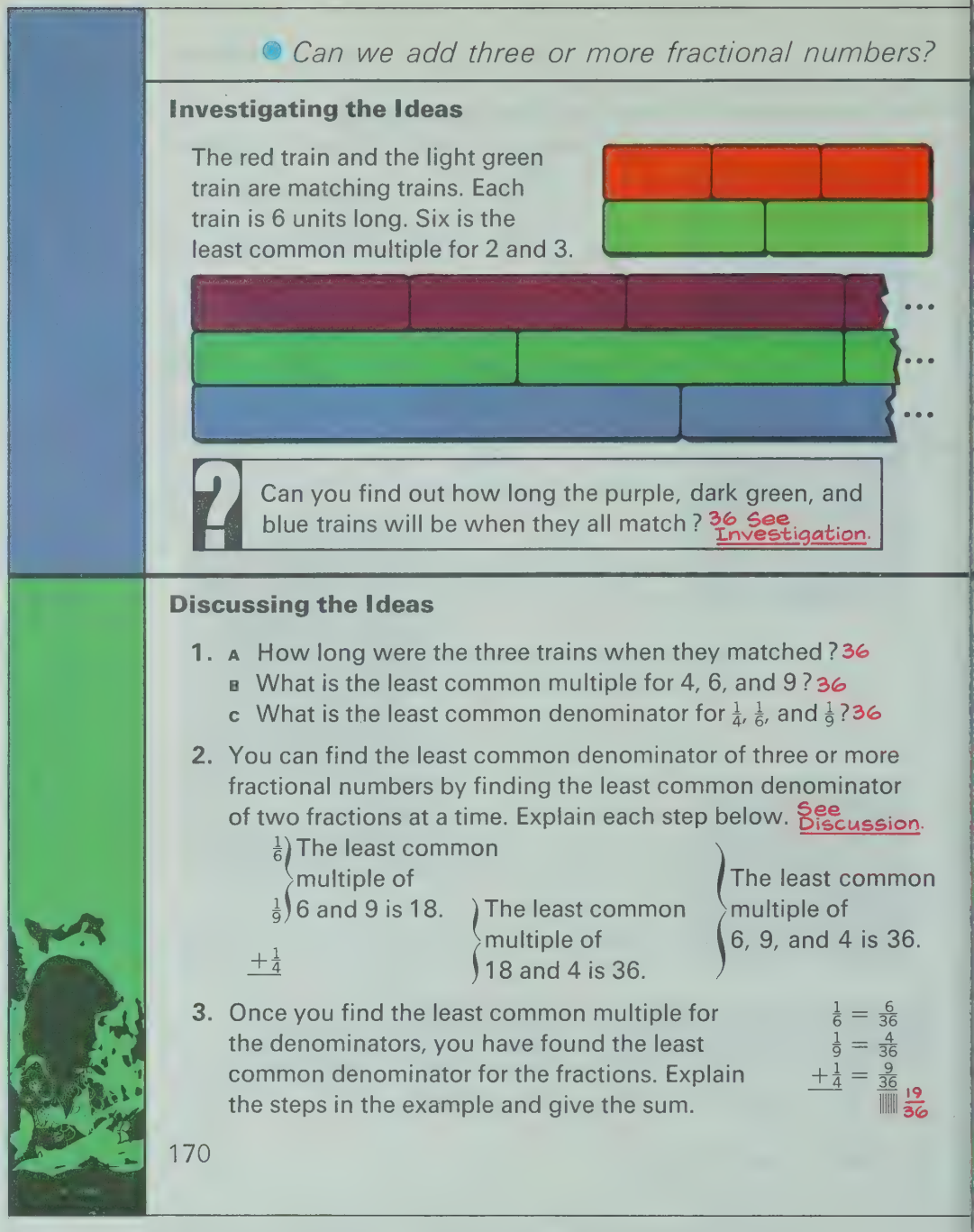
Investigation

Due to the number of strips children will need in this investigation, you might want them to work in pairs, combining their sets of strips. Children might be surprised at the length of the matching trains. Remind them to record the strips they use and the length of the final trains.

For the 4, 6, and 9 strips, the matching train is 36.

You might suggest that those who finish quickly perform a similar investigation with the following set of strips:

3, 4, and 8 (24)
(light green, purple, and brown)



Can we add three or more fractional numbers?

Investigating the Ideas

The red train and the light green train are matching trains. Each train is 6 units long. Six is the least common multiple for 2 and 3.

Can you find out how long the purple, dark green, and blue trains will be when they all match? 36 See Investigation.

Discussing the Ideas

1. A How long were the three trains when they matched? 36
   B What is the least common multiple for 4, 6, and 9? 36
   C What is the least common denominator for $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{9}$? 36

2. You can find the least common denominator of three or more fractional numbers by finding the least common denominator of two fractions at a time. Explain each step below. See Discussion.

   $\frac{1}{6}$, $\frac{1}{9}$: The least common multiple of 6 and 9 is 18.
   $+\frac{1}{4}$: The least common multiple of 18 and 4 is 36.
   The least common multiple of 6, 9, and 4 is 36.

3. Once you find the least common multiple for the denominators, you have found the least common denominator for the fractions. Explain the steps in the example and give the sum.

$\frac{1}{6} = \frac{6}{36}$
$\frac{1}{9} = \frac{4}{36}$
$+\frac{1}{4} = \frac{9}{36}$
$\frac{19}{36}$

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Discussion

Use discussion exercise 1 to encourage children to discuss their findings from the investigation. Be sure they see the relationship of the length of the trains to the least common multiple, and that of the least common multiple to the least common denominator.

As you discuss exercise 2, point out how to find the least common multiple of three numbers. They can find the least common multiple of two of the numbers first, and then find the least common multiple of the third number and the common multiple of the first two numbers.

Next point out that when the least common multiple of the three denominators has been found, this multiple can be used as the least common denominator in order to add the three fractional numbers. Work through several examples with the children, until they feel they are ready to work on their own.

Using the Ideas

1. Find the sums. Give the sums in lowest terms.

A	B	C	D	E	F	G	H
$\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{3}{4}$	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{6}$
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{20}$	$\frac{1}{4}$
$+\frac{1}{8}$ $\frac{7}{8}$	$+\frac{1}{2}$ $1\frac{1}{8}$	$+\frac{1}{2}$ 1	$+\frac{1}{2}$ $1\frac{1}{3}$	$+\frac{5}{8}$ $1\frac{7}{8}$	$+\frac{1}{2}$ $1\frac{7}{12}$	$+\frac{1}{10}$ $\frac{9}{20}$	$+\frac{1}{2}$ $\frac{11}{12}$

I	J	K	L	M	N
$3\frac{1}{2}$	$4\frac{1}{6}$	$19\frac{4}{5}$	$47\frac{5}{6}$	$158\frac{1}{4}$	$785\frac{5}{6}$
$7\frac{1}{4}$	$2\frac{1}{3}$	$26\frac{1}{10}$	$58\frac{1}{8}$	$327\frac{3}{8}$	$654\frac{1}{2}$
$+6\frac{1}{8}$	$+8\frac{2}{3}$	$+35\frac{1}{4}$	$+26\frac{1}{3}$	$+962\frac{1}{2}$	$+327\frac{2}{3}$
$16\frac{7}{8}$	$15\frac{1}{6}$	$81\frac{3}{20}$	$132\frac{7}{24}$	$1448\frac{1}{8}$	1768

Give the missing numbers and function rules.

2. Function Rule: $n + \frac{1}{4} + \frac{1}{2}$

	n	$f(n)$
A	$\frac{1}{6}$	$\frac{11}{12}$
B	$\frac{1}{3}$	$1\frac{1}{12}$
C	$\frac{3}{8}$	$1\frac{1}{8}$
D	$\frac{1}{2}$	$1\frac{1}{4}$
E	$\frac{3}{10}$	$1\frac{1}{20}$

3. Function Rule: $8\frac{7}{8} + n$

	n	$f(n)$
A	$3\frac{1}{4}$	$12\frac{1}{8}$
B	$6\frac{2}{3}$	$15\frac{13}{24}$
C	$5\frac{1}{2}$	$14\frac{3}{8}$
D	$7\frac{3}{10}$	$16\frac{7}{40}$
E	$9\frac{3}{4}$	$18\frac{5}{8}$

4. Function Rule: $n - 4\frac{1}{3}$

	n	$f(n)$
A	$8\frac{5}{6}$	$4\frac{1}{2}$
B	$7\frac{1}{2}$	$3\frac{1}{6}$
C	$6\frac{1}{6}$	$1\frac{5}{6}$
D	$5\frac{1}{4}$	$\frac{11}{12}$
E	$9\frac{3}{5}$	$5\frac{4}{15}$

★ **5.** Function Rule: $(5\frac{2}{3} + n) - 2\frac{2}{3}$

	n	$f(n)$
A	$3\frac{1}{4}$	$6\frac{1}{4}$
B	$4\frac{3}{4}$	$7\frac{3}{4}$
C	$7\frac{1}{6}$	$10\frac{1}{6}$
D	$2\frac{4}{5}$	$5\frac{12}{15}$
E	$1\frac{7}{8}$	$4\frac{7}{8}$

★ **6.** Function Rule: [redacted] $n - \frac{3}{4}$

	n	$f(n)$
A	$4\frac{1}{4}$	$3\frac{1}{2}$
B	$2\frac{1}{8}$	$1\frac{3}{8}$
C	$7\frac{7}{8}$	$7\frac{1}{8}$
D	$6\frac{3}{8}$	$5\frac{5}{8}$
E	$9\frac{3}{4}$	9

★ **7.** Function Rule: [redacted] $1 - n$

	n	$f(n)$
A	$\frac{1}{2}$	$\frac{1}{2}$
B	$\frac{5}{8}$	$\frac{3}{8}$
C	1	0

Using the Exercises

Assign the exercises on page 171 as independent work. Note with the children that in exercise 1 they are to give the sums in lowest terms. You might want to review the use of the function rule and tables before children begin exercise 2. Notice that exercises 5, 6, are 7 are intended to be particularly challenging.

When children finish, check their work with them. Although answers should be mixed numerals, accept correct answers in whatever form they are expressed.

Assignments (page 171)
Minimum: 1A–D, 1I–L.
Average: 1–4. Maximum: 1–7.

Follow-up

To provide variety in working with the concepts presented thus far, present problems in the form of a puzzle such as the following:

1. $\frac{1}{2} + \frac{1}{4}$
2. $\frac{9}{12} + \frac{5}{8}$
3. LCM of 6, 4, and 8
4. LCM of 3, 4, and 6
5. $\frac{6}{15} - \frac{1}{10}$
6. $\frac{1}{2} - \frac{5}{12}$
7. LCM of 4, 5, and 10
8. $5\frac{1}{4} + 7\frac{2}{5}$
9. $7\frac{1}{6} - 6\frac{3}{8}$
10. LCM of 6, 8, and 12

Give the lowest-terms fraction.

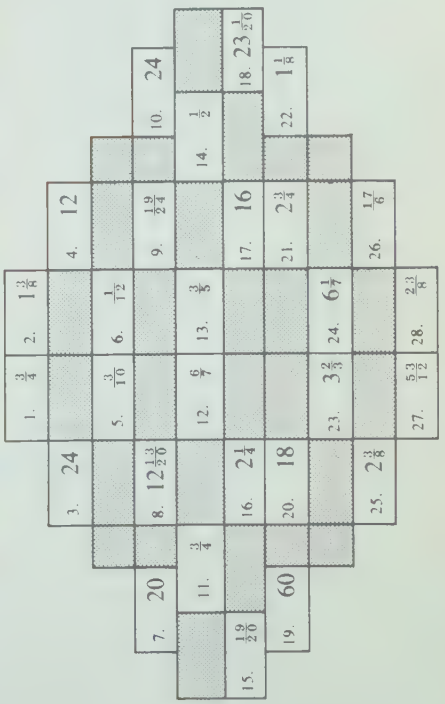
11. $\frac{6}{8}$
12. $\frac{48}{56}$
13. $\frac{27}{45}$
14. $\frac{15}{30}$
15. $\frac{1}{2} + \frac{1}{5} + \frac{1}{4}$
16. $\frac{5}{6} + \frac{2}{3} + \frac{3}{4}$
17. $2\frac{1}{6} + 4\frac{3}{18} + 9\frac{2}{3}$
18. $8\frac{3}{4} + 7\frac{2}{5} + 6\frac{9}{10}$
19. LCM of 3, 4, and 5
20. LCM of 9, 6, and 18
21. $9\frac{7}{12} - 6\frac{5}{6}$
22. $7\frac{3}{8} - 6\frac{1}{4}$

Change to a mixed numeral.

23. $\frac{11}{3}$
24. $\frac{43}{7}$
25. $\frac{19}{8}$

Change to an improper fraction.

26. $2\frac{5}{6}$
27. $4\frac{5}{12}$
28. $2\frac{7}{8}$



(Answers are included for your convenience.)

Workbook, page 54

PAGES 172–173

Objective

Given word problems which require addition and subtraction of fractional numbers, the child will be able to solve the problems by applying the techniques studied in previous lessons.

Preparation

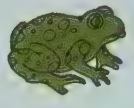
Review any techniques covered thus far in the chapter with which the children had difficulty. You might give children a brief review of regrouping fractional numbers for subtraction since a subtraction problem which requires this technique reviews other related skills as well.

$$3\tfrac{2}{5} = 3\tfrac{12}{30} = 2\tfrac{42}{30}$$
$$-2\tfrac{5}{6} = 2\tfrac{25}{30} = 2\tfrac{25}{30}$$
$$\tfrac{17}{30}$$

Short Stories

1. Fish: First weighed $1\frac{3}{4}$ kg. Second weighed $2\frac{1}{10}$ kg.
   - A How much did they weigh together? $3\frac{17}{20}$ kg
   - B How much more did the second weigh than the first? $\frac{7}{20}$ kg

2. Frog: Jumped $3\frac{7}{10}$ m. Then jumped $2\frac{3}{4}$ m.
   - A How far in all? $6\frac{9}{20}$ m
   - B How much farther was the first jump than the second? $\frac{19}{20}$ m



3. Driving: $57\frac{1}{2}$ km the first hour. $49\frac{7}{10}$ km the second hour. How far in the two hours? $107\frac{1}{5}$ km

4. Stars: Alpha Centauri is $4\frac{1}{3}$ light years* away. Sirius is $8\frac{3}{5}$ light years away. How much farther is Sirius than Alpha Centauri? $4\frac{4}{15}$ lt yr

5. Rectangle: $4\frac{1}{2}$ cm by $6\frac{7}{10}$ cm. Ant crawls from one corner to the opposite corner, moving along the sides of the rectangle. How far? $11\frac{1}{5}$ cm

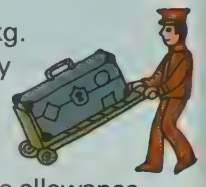
6. Lobster: $3\frac{3}{10}$ kg. Blue crab: $1\frac{1}{4}$ kg. How much more does the lobster weigh? $2\frac{1}{20}$ kg



*A light year is the distance light travels in one year.

7. Grasshopper:
   - A Jumped to $3\frac{5}{8}$. Then jumped to $6\frac{1}{4}$. How far was the second jump? $2\frac{5}{8}$
   - B Jumped to $4\frac{1}{2}$. Then jumped back to $\frac{7}{8}$. How far was the second jump? $3\frac{5}{8}$

8. Steak: T-bone, $\frac{7}{10}$ kg. Rib, $\frac{3}{5}$ kg. Sirloin, $1\frac{1}{4}$ kg. How much do they weigh together? $2\frac{11}{20}$ kg



9. Plane trip: Baggage allowance, 20 kg. Mr. Lo's suitcase weighed $29\frac{6}{10}$ kg. How much overweight was his bag? $9\frac{3}{5}$

10. Tom: $113\frac{1}{2}$ cm tall. Grew $1\frac{1}{4}$ cm. Then grew $2\frac{1}{10}$ cm. How tall now? $116\frac{17}{20}$ cm



11. Reading: $\frac{3}{4}$ hour. Arithmetic: $\frac{2}{3}$ hour. Writing: $\frac{1}{2}$ hour. How long for all three? $1\frac{11}{12}$ hours or 1 hour 55 mins.

12. Triangle's sides: $3\frac{1}{5}$ cm, $5\frac{7}{10}$ cm, $4\frac{3}{4}$ cm. What is the perimeter? $13\frac{13}{20}$ cm

13. Rectangle: $6\frac{7}{10}$ by $2\frac{4}{5}$. What is the perimeter? 19

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Discussion

With most children, assign the problems on page 172 as independent work. When they are finished, check the answers and have them explain to each other how they found their solutions.

Assignments (page 172)

Minimum: 1–6. Average: 1–13.
Maximum: 1–13.



On January 1, 1958, Belgium, France, Italy, Luxembourg, West Germany, and the Netherlands signed a treaty establishing the European Economic Community. This is generally known as the Common Market. The United Kingdom, Denmark, and Ireland joined the Common Market on January 1, 1973, bringing the total membership to nine.

1. What fraction of the total membership
   - A were original members? $\frac{2}{3}$
   - B joined in 1973? $\frac{1}{3}$

2. Denmark, the United Kingdom, the Netherlands, Belgium, and Luxembourg are monarchies (they have a king or queen). What fraction of the Common Market countries are not monarchies? $\frac{4}{9}$

3. Twenty-four European countries are members of the United Nations. All the Common Market countries except West Germany are members of the United Nations. What fraction of the European UN members are not members of the Common Market? $\frac{16}{24}$ or $\frac{2}{3}$

4. The population of Europe is $1\frac{8}{9}$ times as great as the population of the USSR. The population of Europe is $1\frac{5}{6}$ times as great as the population of the Common Market countries. Are there more people in the USSR or in the Common Market countries? By how much? USSR, $\frac{1}{18}$

Follow-up

Giving a data sheet to more capable children may stimulate them to write story problems. Encourage these children to use data from current newspapers and magazines as source material if they prefer.

Direct the children to make up story problems by writing questions for each set of facts, indicating the set for which each question is intended. Ask them to write and solve an equation to answer each question if they can.

Data Sheet

Planes: Flying Time from New York to Paris	
Lindbergh	$33\frac{1}{2}$ hours
Commercial Jet	$5\frac{3}{4}$ hours
B-58 Hustler	$3\frac{1}{3}$ hours

Large Gems: Weight	
Star of Africa Diamond	$530\frac{2}{10}$ carats
Blue Hope Diamond	$44\frac{4}{10}$ carats
Star of India Sapphire	$563\frac{55}{100}$ carats
Devonshire Emerald	1350 carats

Satellites: Orbiting Time	
Explorer I	$114\frac{1}{5}$ minutes
Echo I	$118\frac{3}{10}$ minutes
Tiros	$99\frac{15}{100}$ minutes
Midas II	$94\frac{3}{10}$ minutes
Telstar	158 minutes

Resources for Active Learning

Math Activity Cards, "Recipe," D35, Macmillan.

Using the Exercises

Before assigning page 173, give the children an opportunity to read and discuss the two recipes given at the top of the page. Some may want to use the recipes. They are tested recipes and, with reasonable care, should yield good results.

Following a discussion of the recipes, have the children do the exercises. When they have finished, allow time for discussing the exercises and checking papers. Notice that exercise 5 is primarily for the faster children. They may need to be told that shortening is *not* considered a "dry ingredient."

(Note: The recipe titles are intentional, although some children may think we intended to print "Yellow" Cake and "Cherry" Frosting.)

Assignments (page 173)

Minimum: 1–4, oral. Average: 1–4. Maximum: 1–5.

PAGES 174–175

Objective

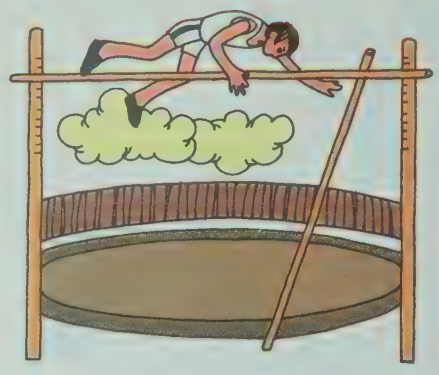
Given tables of data and word problems which require addition and subtraction of fractional numbers, the child will be able to solve the problems.

Preparation

To prepare for these pages of problems, you might exhibit an addition equation on the chalkboard, and ask the children to write a word problem based on this equation. Follow this by having several children read their word problems to the class. Then, present a subtraction equation on the chalkboard and ask the children to write a word problem for this equation. Next, have several other children present their subtraction word problems to the class.

Alternatively, you might review skills used in addition and subtraction exercises, such as changing to lowest terms, or regrouping mixed numerals.

Track Records



Olympic Track Records*	
Running high jump	$224\frac{15}{100}$ cm
Long jump	$8\frac{9}{10}$ m
Pole vault	$5\frac{1}{2}$ m
Javelin	$90\frac{48}{100}$ m
Discus throw	$64\frac{78}{100}$ m
100-metre run	$9\frac{9}{10}$ sec
200-metre run	$19\frac{8}{10}$ sec
800-metre run	1 min $44\frac{3}{10}$ sec
1500-metre run	3 min $34\frac{9}{10}$ sec
5000-metre run	13 min $26\frac{4}{10}$ sec

1. How much higher is the pole-vault record than the high-jump record? 325.85 cm
2. How much farther is the javelin record than the discus record? 25.7 m
3. The Olympic women's record for the long jump is $208\frac{28}{100}$ cm shorter than the men's record. What is this record? 6.8 m
4. The Olympic swimming record for the 100-metre freestyle is $51\frac{2}{10}$ seconds. The same man holds the record for the 200-metre freestyle, 1 minute $52\frac{8}{10}$ seconds. If he could swim the 200 metres at the same speed as the 100, how much time would be cut from the present record? $10\frac{2}{5}$ sec.
5. What is the difference between the time for the 5000-metre run and the time for the 1500-metre run? 9 min $51\frac{1}{2}$ sec.
6. A If the 800-metre runner could run at his record pace for 1500 metres, how fast could he run the 1500 metres? 3 min 23 sec.
   B What is the difference between this time and the record for the 1500-metre run? $11\frac{9}{10}$ sec.

*The records given on this page are Olympic track and field records for men including the 1972 games.

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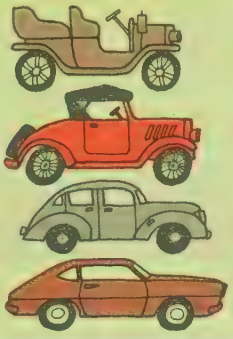
Discussion

Give the children an opportunity to discuss the material at the top of page 174. Notice with them the chart which gives various world track records. You might have some of the children who know of more recent track records give these to the class. Another interesting activity would be to have those who are interested look up the latest world track records and compare them with the records given here.

Assignments (page 174)

Minimum: 1–7, oral. Average: 1–7.
Maximum: 1–7.

Automobiles



Each symbol stands for 1 000 000 cars.

1900 — 4192
1910 — 181,000
1920 — 1 905 560
1930 — 2 787 456
1940 — 3 717 385
1950 — 6 665 863
1960 — 6 674 796
1970 — 6 550 203

In the early 1900's, automobiles were referred to as "horseless carriages." Today the automobile plays a vital role in all our lives.

1. How many more cars were built
   A in 1940 than in 1930? 929 929
   B in 1950 than in 1930? 3 878 407
   C in 1970 than in 1900? 6,546 011
2. There are more than 280 000 000 people in North America. About what is the average number of people per car? about 2
3. In 1925 a new car could be expected to last about $6\frac{1}{2}$ years. Today's cars can be expected to last about $12\frac{1}{4}$ years. How much longer can cars be expected to last today than in 1925? $5\frac{3}{4}$ yr
4. A car travelling 80 km/h travels $16\frac{3}{4}$ metres between the time a driver decides to stop and the time he applies the brakes. It travels another 57 metres after he applies the brakes. What is the total stopping distance from the time the driver decides to stop? $73\frac{3}{4}$
5. In 1911 the winner of the Indianapolis "500" auto race averaged $120\frac{2}{100}$ km/h. In 1972 the winner averaged $262\frac{20}{100}$ km/h. How much faster was the average speed in 1972 than in 1911? $142\frac{9}{50}$ km/h

More practice, page A-13, Set 25

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Using the Exercises

You might want to introduce page 175 before assigning page 174 as independent work. Be sure to emphasize how to interpret a pictograph, such as the one shown here. Point out that each symbol of a small car stands for 1 000 000 cars produced in a certain year. Point out also that in the year 1900 only 4192 cars were built, and therefore, we show only a tiny portion of the car that stands for 1 000 000 cars. Observe that 181 000 cars were built in 1910. This is a little less than $\frac{1}{5}$ of 1 000 000 or of one car symbol, and therefore, we again show only a small portion of the symbol. In 1920, however, nearly 2 000 000 cars were built, so we show two cars.

The children might also enjoy discussing the pictures on the left, which show representative cars for various years.

After introducing these pages, have the children do the exercises. When they finish, allow time for further discussion and checking papers.

Assignments (page 175)

Minimum: 1–5, oral. Average: 1–5. Maximum: 1–5.

Follow-up

Suggest that the children who are interested in racing records use an almanac to find the recent times and speeds for the Indianapolis 500 and the Grand Prix d'Endurance at Le Mans, France. Some children may be interested in the wheeled land-speed records made at the Bonneville Salt Flats in Utah. These records are constantly being broken by jet cars.

Encourage other children to make a new pictograph including current production of automobiles or some other product which is critical in the economy of their region.

PAGES 176–177

Objective

Given an improper fraction, the child will be able to write it as a mixed numeral by dividing the denominator into the numerator and writing the remainder as a fraction.

Preparation

Materials

colored strips *(optional)*

To prepare for this lesson, you might review mixed numerals and improper fractions. For example, write a list of each and have children match equivalent expressions:

$\frac{4}{3}$	$3\frac{2}{3}$
$\frac{7}{5}$	$1\frac{1}{3}$
$\frac{6}{2}$	$2\frac{2}{5}$
$\frac{12}{5}$	$1\frac{2}{5}$
$\frac{11}{3}$	3
$\frac{13}{4}$	$3\frac{1}{4}$

Investigation

If you think the children need the experience of actually using the strips, have them work in groups combining sets of strips in order to make the trains. Many children will not need to use the actual strips; they will be able to picture the situation sufficiently well from the presentation in the text. In either case, suggest that they try to give a mixed numeral for each of the other strips. For instance, you might ask, "Can you give a mixed numeral for the number of purple strips it takes to match the 85 white strips?" Remind them to record their findings carefully. You might suggest a format similar to the following completed table.

White Strips	Matching Strips (unit)	Mixed Numeral
85	red	$42\frac{1}{2}$
85	light green	$28\frac{1}{3}$
85	purple	$21\frac{1}{4}$
85	yellow	17
85	dark green	$14\frac{1}{6}$
85	black	$12\frac{1}{7}$
85	brown	$10\frac{5}{8}$
85	blue	$9\frac{4}{9}$
85	orange	$8\frac{5}{10} = 8\frac{1}{2}$

Let's explore mixed numerals and improper fractions again.

Investigating the Ideas

Suppose you have a train made of 85 white strips.



It takes $42\frac{1}{2}$ red strips to match the white strips.

? Can you give a mixed numeral for the number of light green strips it takes to match the 85 white strips? $28\frac{1}{3}$ See Investigation.

Discussing the Ideas

1. If the red strip is the unit, can you find two ways to express the length of the white train above? $\frac{85}{2}$, $42\frac{1}{2}$

2. A If the light green strip is the unit, what improper fraction gives the length of the white train? $\frac{85}{3}$
   B What mixed numeral gives the length of the train? $28\frac{1}{3}$

3. Think of the purple strip as the unit.
   A Give an improper fraction for the length of the white train. $\frac{85}{4}$
   B Give a mixed numeral for n. $\frac{85}{4} = n$ $21\frac{1}{4}$

4. Solve: $\frac{85}{5} = n$ 17

5. Explain these examples. They show how you can use division to write a mixed numeral for an improper fraction. See Discussion.



A $6\overline{)85}$ 14 R1 $\longrightarrow$ $\frac{85}{6} = \frac{14 \times 6}{6} + \frac{1}{6}$ $\longrightarrow$ $\frac{85}{6} = 14\frac{1}{6}$

B $8\overline{)85}$ 10 R5 $\longrightarrow$ $\frac{85}{8} = \frac{10 \times 8}{8} + \frac{5}{8}$ $\longrightarrow$ $\frac{85}{8} = 10\frac{5}{8}$

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Discussion

Before children respond to discussion exercise 1, you might have them compare their results for the investigation. Then, as you discuss exercise 1, help them see that if the red strip is the unit, each white strip is $\frac{1}{2}$, so 85 white strips is $\frac{85}{2}$. The second way to express this was given in the investigation: $\frac{85}{2} = 42\frac{1}{2}$.

Similarly, if the light green strip is the unit, each white strip is $\frac{1}{3}$, so 85 white strips is $\frac{85}{3}$. From their investigation results, children should supply the equation $\frac{85}{3} = 28\frac{1}{3}$. Work through each of the strips in this manner until you have the following equations on the chalkboard (or overhead projector).

$\frac{85}{2} = 42\frac{1}{2}$ $\quad$ $\frac{85}{7} = 12\frac{1}{7}$

$\frac{85}{3} = 28\frac{1}{3}$ $\quad$ $\frac{85}{8} = 10\frac{5}{8}$

$\frac{85}{4} = 21\frac{1}{4}$ $\quad$ $\frac{85}{9} = 9\frac{4}{9}$

$\frac{85}{5} = 17$ $\quad$ $\frac{85}{10} = 8\frac{5}{10} = 8\frac{1}{2}$

$\frac{85}{6} = 14\frac{1}{6}$

Use exercises 4 and 5 to help children discover the procedure outlined in exercise 5. They should understand that they can simply divide the numerator, 85, by the given denominator, which can then be used to express the remainder of this division as the fraction in the mixed numeral. Use this procedure with other examples as needed.

Using the Ideas

1. Give the numerator for ***a*** and the whole number for ***b***.

A $\frac{35}{2} = \frac{a}{2} + \frac{1}{2} = b + \frac{1}{2}$ (34, 17)

B $\frac{52}{3} = \frac{a}{3} + \frac{1}{3} = b + \frac{1}{3}$ (51, 17)

C $\frac{95}{4} = \frac{a}{4} + \frac{3}{4} = b + \frac{3}{4}$ (92, 23)

D $\frac{83}{6} = \frac{a}{6} + \frac{5}{6} = b + \frac{5}{6}$ (78, 13)

E $\frac{87}{5} = \frac{a}{5} + \frac{2}{5} = b + \frac{2}{5}$ (85, 17)

F $\frac{85}{2} = \frac{a}{2} + \frac{1}{2} = b + \frac{1}{2}$ (84, 42)

G $\frac{191}{8} = \frac{a}{8} + \frac{7}{8} = b + \frac{7}{8}$ (184, 23)

H $\frac{267}{7} = \frac{a}{7} + \frac{1}{7} = b + \frac{1}{7}$ (266, 38)

2. Give the number for ***q*** (quotient) and the number for ***r*** (remainder). Then give a mixed numeral for ***m***.

A $\frac{15}{2} = \frac{q \times 2}{2} + \frac{r}{2} = m$ (q: 7, r: 1; $m = 7\frac{1}{2}$)

B $\frac{25}{4} = \frac{q \times 4}{4} + \frac{r}{4} = m$ (q: 6, r: 1; $m = 6\frac{1}{4}$)

C $\frac{36}{7} = \frac{q \times 7}{7} + \frac{r}{7} = m$ (q: 5, r: 1; $m = 5\frac{1}{7}$)

D $\frac{42}{8} = \frac{q \times 8}{8} + \frac{r}{8} = m$ (q: 5, r: 2; $m = 5\frac{2}{8}$ or $5\frac{1}{4}$)

E $\frac{67}{3} = \frac{q \times 3}{3} + \frac{r}{3} = m$ (q: 22, r: 1; $m = 22\frac{1}{3}$)

F $\frac{84}{5} = \frac{q \times 5}{5} + \frac{r}{5} = m$ (q: 16, r: 4; $m = 16\frac{4}{5}$)

G $\frac{93}{6} = \frac{q \times 6}{6} + \frac{r}{6} = m$ (q: 15, r: 3; $m = 15\frac{3}{6}$ or $15\frac{1}{2}$)

H $\frac{27}{2} = \frac{q \times 2}{2} + \frac{r}{2} = m$ (q: 13, r: 1; $m = 13\frac{1}{2}$)

3. Give the mixed numeral for each fraction.

A $\frac{18}{4}$ ($4\frac{1}{2}$) G $\frac{97}{6}$ ($16\frac{1}{6}$) M $\frac{125}{100}$ ($1\frac{1}{4}$)

B $\frac{42}{5}$ ($8\frac{2}{5}$) H $\frac{53}{4}$ ($13\frac{1}{4}$) N $\frac{164}{8}$ ($20\frac{1}{2}$)

C $\frac{61}{7}$ ($8\frac{5}{7}$) I $\frac{76}{5}$ ($15\frac{1}{5}$) O $\frac{327}{5}$ ($65\frac{2}{5}$)

D $\frac{92}{10}$ ($9\frac{1}{5}$) J $\frac{134}{8}$ ($16\frac{3}{4}$) P $\frac{841}{10}$ ($84\frac{1}{10}$)

E $\frac{81}{50}$ ($1\frac{31}{50}$) K $\frac{113}{5}$ ($22\frac{3}{5}$) Q $\frac{253}{5}$ ($50\frac{3}{5}$)

F $\frac{65}{8}$ ($8\frac{1}{8}$) L $\frac{261}{20}$ ($13\frac{1}{20}$) R $\frac{423}{2}$ ($211\frac{1}{2}$)

think



Suppose that first-class mailing costs 8¢ for each 25 grams, plus 8¢ for any additional weight under 25 grams. Using these rates, give the cost for first-class mailing of items weighing:

1. 126 grams (48¢)
2. $25\frac{1}{10}$ grams (16¢)
3. 87 grams (32¢)
4. 210 grams (72¢)



More practice, page A-13, Set 26

Resources for Active Learning

Discovery, Section II, Units 5/6, 6/5, Encyclopaedia Britannica Educational Corp. [Fraction cross number puzzles].

Duplicator Masters, page 33
Workbook, page 55

Using the Exercises

Before assigning the exercises on page 177, point out that the letter *b* represents the quotient after they divide by the denominator and that the fraction shows the remainder. Similarly, as you assign the exercises, note with the children that in exercise 2, the letters *q*, *r*, and *m* represent quotient, remainder, and mixed numeral, respectively. This notation also illustrates the fact that the fractional part of a mixed numeral should be a proper fraction because the remainder should always be less than the divisor.

The children who complete the page early should enjoy working the *Think* problem.

Assignments (page 177)*

Minimum: 1–3L. Average: 1–3. Maximum: 1–3.

PAGES 178–179

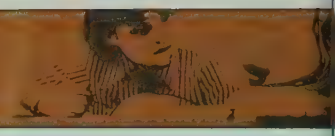
Objective

The child will demonstrate his ability to work with the concepts presented in this chapter.

Preparation

Review any of the material from the chapter that caused particular difficulty. For example, work through a few exercises like exercise 4 on page 178. Point out methods of finding a common denominator, regrouping when necessary, and writing lowest-terms fractions when possible.

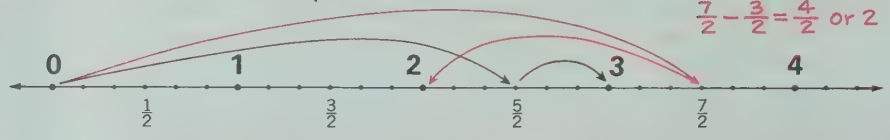
Reviewing the Ideas



1. Write an addition equation for the black arrows. Write a subtraction equation for the red arrows.

$\frac{5}{2}+\frac{1}{2}=\frac{6}{2}$ or 3

$\frac{7}{2}-\frac{3}{2}=\frac{4}{2}$ or 2



2. Find the sums and differences.

A $\frac{1}{4}+\frac{1}{2}$ $\frac{3}{4}$ B $\frac{1}{4}-\frac{1}{6}$ $\frac{1}{12}$ C $\frac{1}{3}-\frac{1}{6}$ $\frac{1}{6}$ D $\frac{7}{10}+\frac{1}{5}$ $\frac{9}{10}$ E $\frac{3}{4}-\frac{3}{8}$ $\frac{3}{8}$ F $\frac{2}{3}+\frac{1}{6}$ $\frac{5}{6}$

G $\frac{1}{8}+\frac{5}{6}$ $\frac{23}{24}$ H $\frac{3}{5}-\frac{1}{10}$ $\frac{1}{2}$ I $\frac{3}{10}+\frac{1}{4}$ $\frac{11}{20}$ J $\frac{9}{10}+\frac{4}{5}$ $1\frac{7}{10}$ K $\frac{1}{4}-\frac{1}{5}$ $\frac{1}{20}$ L $\frac{9}{10}-\frac{3}{4}$ $\frac{3}{20}$ M $\frac{1}{3}+\frac{1}{10}$ $\frac{13}{30}$ N $\frac{5}{6}-\frac{4}{5}$ $\frac{1}{30}$

3. Solve the equations.

A $\frac{1}{4}+n=\frac{3}{4}$ $\frac{2}{4}$ or $\frac{1}{2}$ B $\frac{5}{6}-n=\frac{1}{6}$ $\frac{4}{6}$ or $\frac{2}{3}$ C $n+\frac{1}{2}=\frac{3}{2}$ 1 D $\frac{9}{10}+n=1\frac{1}{10}$

4. Find the sums and differences.

A $17\frac{1}{8}+16\frac{1}{2}$ $33\frac{5}{8}$ B $42\frac{7}{10}+31\frac{1}{2}$ $74\frac{1}{5}$ C $65\frac{3}{4}-23\frac{3}{8}$ $42\frac{3}{8}$ D $84\frac{5}{6}+12\frac{1}{4}$ $97\frac{1}{12}$ E $61\frac{1}{4}-17\frac{1}{5}$ $44\frac{1}{20}$ F $83\frac{3}{4}+98\frac{1}{2}$ $182\frac{1}{4}$

G $56\frac{2}{3}-19\frac{1}{6}$ $37\frac{1}{2}$ H $81\frac{1}{4}-78\frac{1}{2}$ $2\frac{3}{4}$ I $32\frac{9}{10}-19\frac{4}{5}$ $13\frac{1}{10}$ J $64\frac{1}{5}-28\frac{1}{4}$ $35\frac{12}{20}$ K $53\frac{3}{4}+67\frac{9}{10}$ $121\frac{13}{20}$ L $52\frac{1}{5}+47\frac{1}{10}$ $99\frac{3}{10}$

5. Give the missing numbers.

Function Rule $n + 2\frac{1}{2}$	
n	$f(n)$
$3\frac{5}{8}$	A $6\frac{1}{8}$
$6\frac{1}{4}$	B $8\frac{3}{4}$
$9\frac{1}{6}$	C $11\frac{2}{3}$
$5\frac{7}{10}$	D $8\frac{1}{5}$

Function Rule $n - 4\frac{1}{3}$	
n	$f(n)$
$8\frac{1}{2}$	E $4\frac{1}{6}$
$7\frac{5}{6}$	F $3\frac{1}{2}$
G $12\frac{1}{3}$	8
$5\frac{1}{10}$	H $\frac{23}{30}$

think

Copy each equation on your paper and give correct signs (+ or −) for each ●.

1. $(\frac{1}{2}$ ● $\frac{1}{4})$ ● $\frac{1}{8}=\frac{5}{8}$
2. $\frac{1}{2}$ ● $(\frac{1}{2}$ ● $\frac{1}{2})=\frac{1}{2}$
3. $\frac{1}{2}$ ● $(\frac{1}{3}$ ● $\frac{1}{6})=\frac{1}{3}$

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Discussion

Select and assign several of the exercises according to the need and ability of the children. Notice the *Think* problem on page 178. You might suggest to the children that they try it when they have completed the assigned exercises on pages 178 and 179. When they have finished, help them check the answers and discuss any points which need clarification. Have children explain their solutions to the short story word problems, including the steps they took in the algorithm they used.

6. Give a whole number for each improper fraction.

A $\frac{10}{5}$ 2 B $\frac{10}{2}$ 5 C $\frac{12}{3}$ 4 D $\frac{12}{4}$ 3 E $\frac{24}{3}$ 8 F $\frac{36}{3}$ 12 G $\frac{52}{2}$ 26 H $\frac{75}{5}$ 15

7. Give an improper fraction for each number.

A $3\frac{3}{1}$ B $3\frac{1}{2}\frac{7}{2}$ C $3\frac{2}{3}\frac{11}{3}$ D $3\frac{3}{4}\frac{15}{4}$ E $5\frac{1}{2}\frac{11}{2}$ F $6\frac{3}{5}\frac{33}{5}$ G $7\frac{4}{5}\frac{39}{5}$ H $15\frac{3}{10}\frac{153}{10}$

8. Give a mixed numeral for each improper fraction.

A $\frac{15}{2}7\frac{1}{2}$ B $\frac{20}{3}6\frac{2}{3}$ C $\frac{41}{2}20\frac{1}{2}$ D $\frac{83}{10}8\frac{3}{10}$ E $\frac{126}{4}31\frac{1}{2}$ F $\frac{257}{8}32\frac{1}{8}$ G $\frac{356}{5}71\frac{1}{5}$ H $\frac{483}{10}48\frac{3}{10}$

9. Find the sums. Use lowest-terms fractions for all answers.

A $3\frac{1}{2}$, $8\frac{1}{4}$, $+6\frac{1}{2}$ → $18\frac{1}{4}$

B $2\frac{1}{4}$, $9\frac{1}{2}$, $+7\frac{1}{4}$ → 19

C $9\frac{1}{2}$, $6\frac{1}{3}$, $+5\frac{1}{4}$ → $21\frac{1}{12}$

D $14\frac{1}{6}$, $16\frac{1}{2}$, $+18\frac{1}{3}$ → 49

E $32\frac{1}{20}$, $61\frac{1}{10}$, $+15\frac{1}{5}$ → $108\frac{7}{20}$

F $49\frac{1}{2}$, $67\frac{1}{6}$, $+94\frac{1}{4}$ → $210\frac{11}{12}$

G $7\frac{1}{2} + 8\frac{2}{3} + 6\frac{1}{4} + 7\frac{5}{12}$ $29\frac{5}{6}$

H $7\frac{1}{2} + 2\frac{5}{6} + 3\frac{2}{3} + 7\frac{1}{4}$ $21\frac{1}{4}$

Short Stories

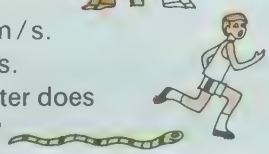


1. First jump: $12\frac{1}{2}$ m.
Second jump: $11\frac{3}{4}$ m.
How much farther was the first jump than the second? $\frac{3}{4}$ m

2. Jim: $161\frac{1}{4}$ cm tall.
Joe: $159\frac{1}{2}$ cm tall.
How much taller is Jim than Joe? $1\frac{3}{4}$ cm



3. Runner: $8\frac{85}{100}$ m/s.
Snake: $\frac{73}{100}$ m/s.
How much faster does the runner go? $8\frac{12}{100}$ mps



4. First suitcase: $16\frac{7}{10}$ kg.
Second suitcase: $25\frac{1}{4}$ kg.
How much do they weigh together? $41\frac{19}{20}$ kg

5. $6\frac{1}{2}$ litres of gasoline in the tank. Put in $8\frac{7}{10}$ litres more.

A How much gasoline is in the tank now? $15\frac{2}{10}$ litres

B If the tank holds $16\frac{1}{4}$ litres, how much more could be put in? $1\frac{1}{20}$ litres

Follow-up

Children might enjoy working a magic square with fractions. The following are suitable. (Spaces with shading should be left blank.)

$\frac{2}{3}$	$\frac{1}{12}$	$\frac{1}{4}$
$\frac{1}{4}$	$\frac{5}{12}$	$\frac{7}{12}$
$\frac{1}{3}$	$\frac{3}{4}$	$\frac{1}{6}$

$\frac{2}{3}$	$\frac{1}{12}$	$\frac{1}{8}$	$\frac{13}{24}$
$\frac{5}{24}$	$\frac{11}{24}$	$\frac{5}{12}$	$\frac{1}{3}$
$\frac{3}{8}$	$\frac{7}{24}$	$\frac{1}{4}$	$\frac{1}{2}$
$\frac{1}{6}$	$\frac{7}{12}$	$\frac{5}{8}$	$\frac{1}{24}$

Workbook, page 56

PAGES 180–181

Objective

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

To prepare for these pages, you might use a "What's My Rule" game or some mental chain games to stress speed in basic facts. Or, you might prefer to use the preparation period to introduce page 181.

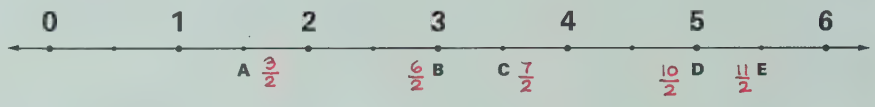
Keeping in Touch with

Computing
Measurement
Inequalities
Fractional numbers

1. Find the sums, differences, products, and quotients.

A $28 + 375 + 89 + 6 + 39$ 537
B $3278 + 9654 + 3612$ 16 544
C $72\,641 - 13\,782$ 58 859
D $52\,006 - 41\,839$ 10 167
E $7 \times 326\,894$ 2 288 258
F 28×5264 147 392
G $387\,522 \div 6$ 64 587
H $2072 \div 74$ 28
I $28\,784 \div 7$ 4112

2. Give improper fractions to name the fractional numbers for the points over letters A through E.



3. Change each fraction to a lowest-terms fraction.

A $\frac{15}{10}$ $\frac{3}{2}$ (or $1\frac{1}{2}$)
B $\frac{75}{100}$ $\frac{3}{4}$
C $\frac{40}{60}$ $\frac{2}{3}$
D $\frac{15}{20}$ $\frac{3}{4}$
E $\frac{18}{30}$ $\frac{3}{5}$
F $\frac{8}{6}$ $\frac{4}{3}$ (or $1\frac{1}{3}$)
G $\frac{125}{100}$ $\frac{5}{4}$ (or $1\frac{1}{4}$)

4. Solve the equations.

A $2 \times n = 16$ 8
B $(2 \times n) + 6 = 16$ 5
C $(2 \times n) - 6 = 16$ 11
D $2 \times n = 12$ 6
E $(2 \times n) + n = 12$ 4
F $(2 \times n) + (2 \times n) = 12$ 3
G $18 = 3 \times n$ 6
H $18 = (3 \times n) + 3$ 5
I $18 = (3 \times n) - 3$ 7
J $24 = n + n + n$ 8
K $24 = (2 \times n) + n$ 8
L $24 = (2 \times n) - 8$ 16

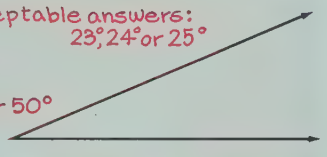
5. Give the correct sign ($<$ or $>$) for each ◍.

A $\frac{1}{2}$ ◍ $\frac{3}{5}$ ($<$)
B $\frac{4}{5}$ ◍ $\frac{7}{8}$ ($<$)
C $\frac{5}{6}$ ◍ $\frac{7}{10}$ ($>$)
D $\frac{7}{12}$ ◍ $\frac{3}{4}$ ($<$)
E $\frac{2}{3}$ ◍ $\frac{8}{10}$ ($<$)
F $\frac{4}{3}$ ◍ $\frac{6}{5}$ ($>$)

6. A Give the degree measure of this angle. Acceptable answers: 23°, 24° or 25°

B Suppose we invent a new unit that is only $\frac{1}{2}$ of a degree. What is the measure of the angle when we use this new unit? 46°, 48°, or 50°

C If we invent and use a unit that is 3 degrees, what is the measure of the angle? About 8°





You are invited to explore

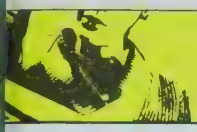
ACTIVITY CARD 9
Page 359

180

Discussion

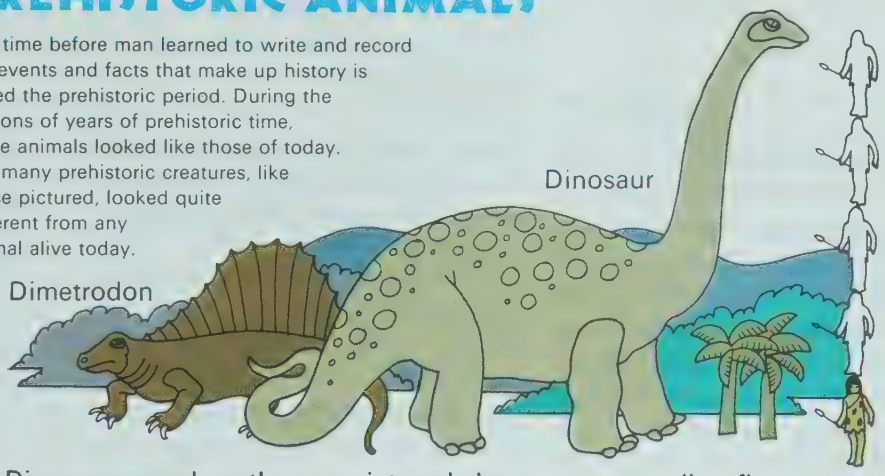
The exercises on page 180 are intended for independent work.

You might point out, before the children begin working the exercises on this page, that they might simply try to solve the equations in exercise 4 by trial and error. By all means, avoid giving the children any rules for solving these equations.



PREHISTORIC ANIMALS

The time before man learned to write and record the events and facts that make up history is called the prehistoric period. During the millions of years of prehistoric time, some animals looked like those of today. But many prehistoric creatures, like those pictured, looked quite different from any animal alive today.



1. Dinosaurs, such as the one pictured above, grew as tall as five 2-metre men, standing as in the picture. How many centimetres tall were these dinosaurs ? 1000 cm

2. Some dinosaurs attained lengths nearly three times the height described above. How many metres long were they ? 30 m

3. The dimetrodon grew to a length of only about 280 centimetres. About how many times shorter than the dinosaur in number two? 11

4. The dinosaur era (called Mesozoic) began about 200 000 000 years ago and lasted about 140 000 000 years. About how long ago did the Mesozoic era end ?
About 60 000 000 years ago

5. Some dinosaurs weighed as much as eighty 570-kilogram cows.
   A How many kilograms is this ? 45 600 kg B How many tonnes ? 45.6

6. Whales, the largest living animals of today, attain weights of as much as 120 tonnes. How many more kilograms is this weight than the weight of the dinosaur ? 74 400

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Using the Exercises

Before assigning page 181, give the children an opportunity to read and discuss the introductory paragraph; then have them do the exercises. When they have finished, allow time for further discussion and checking papers.

CHAPTER 9 | Multiplication and Division of Fractional Numbers

Pages 182–223

General Objectives

To introduce multiplication and division of fractional numbers

To work with strips, regions, and number lines as models for illustrating the product of fractional numbers

To establish the basic principles for multiplication of fractional numbers

To provide a logical development of the usual rule for finding the product of any two fractional numbers

To focus attention on the inverse relation between multiplication and division of fractional numbers

To provide a discovery-type development of the usual rule for division of fractional numbers

To develop skills in multiplying and dividing with fractional numbers

To relate whole-number division to fractional-number concepts

To provide work with functions whose input and output numbers are fractional numbers

To provide appropriate work with estimation and fractional numbers

To provide experiences with word problems requiring all four operations with fractional numbers

The beginning pages of Chapter 9 are designed to give the children experiences that will help them develop an intuitive feeling for finding the product of two fractional numbers denoted by unit fractions ($\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc.) or the product of a whole number and a fractional number expressed as a unit fraction. This objective is accomplished by using strips, regions, and the number line as well as physical examples.

With this background established, the basic principles for multiplication of fractional numbers are then assumed to hold. The basic algorithm for finding the product of any two fractional numbers given in fractional form is developed (1) by using these basic principles and the ability to find the product of two fractional numbers expressed as unit fractions and (2) by using the product of a whole number and a fractional number represented as a unit fraction.

Following this, special attention is devoted to the concept of the reciprocal of a fractional number. Then, material is provided to help the children learn various shortcuts, including what is traditionally known as *cancellation*, though we do not use this term in the text.

The next material is designed to help the children develop skills in finding the product of two fractional numbers. Appropriate sets of word problems are interspersed throughout, not only to assist in the development of skills, but to stimulate interest and maintain problem-solving techniques.

Following a lesson that reviews concepts and skills developed earlier, the work on division of fractional numbers is initiated. Essentially, we approach division of fractional numbers through work with missing factors and reciprocals. Drawing on this background, the children are led to discover that dividing by a fractional number is the same as multiplying by its reciprocal. After the usual division algorithm has been developed, appropriate sets of word problems are provided so that the children will gain experience in actual situations that require the division operation for solutions.

The concluding lessons in the chapter offer a variety of activities, such as working with functions and estimation.

Mathematics

There are many ways to approach the standard definition for the product of any two fractional numbers, such as $\frac{a}{b}$ and $\frac{c}{d}$.

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

The approach shown below parallels the development in the children's text.

(1) Establish the following two facts:

$$\frac{1}{a} \times \frac{1}{b} = \frac{1}{a \times b}$$

and $$a \times \frac{1}{b} = \frac{a}{b}$$

(2) Assume the commutative and associative principles for multiplication of fractional numbers.

(3) Use the principles to arrive at the desired result for the product of any two fractional numbers.

$$\frac{a}{b} \times \frac{c}{d} = \left(a \times \frac{1}{b}\right) \times \left(c \times \frac{1}{d}\right)$$

$$= (a \times c) \times \left(\frac{1}{b} \times \frac{1}{d}\right)$$

$$= (a \times c) \times \left(\frac{1}{b \times d}\right)$$

$$= \frac{a \times c}{b \times d}$$

The two facts in part 1 are presented in the children's text by using physical models (strips, regions, and number lines) to make them plausible. The basic principles for multiplication of fractional numbers are then presented as a parallel to the principles for addition of fractional numbers, and, finally, the steps outlined in part 3 are developed carefully through exercise sets.

The shortcut rule (division algorithm) for division of fractional

numbers can be shown as follows.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a \times d}{b \times c}$$

To develop the familiar division algorithm for fractional numbers, we rely heavily on the inverse relation between multiplication and division. Thus, the children are led to understand that, since $\frac{2}{3} \times \frac{5}{7} = \frac{10}{21}$, then $\frac{10}{21} \div \frac{5}{7} = \frac{2}{3}$. Since the rule for multiplication of fractional numbers is quite simple (multiply numerators and multiply denominators) such examples may lead children to generalize that, to divide fractional numbers, one divides numerator by numerator and denominator by denominator. This works nicely in the example above:

$$\frac{10}{21} \div \frac{5}{7} = \frac{10 \div 5}{21 \div 7} = \frac{2}{3}.$$

However, an example like $\frac{2}{3} \div \frac{4}{5}$ presents difficulties. The numerator 2 is not divisible by 4 and the denominator 3 is not divisible by 5. However, by choosing the appropriate fraction equivalent to $\frac{2}{3}$, the difficulty can be overcome. The necessary fraction is $\frac{2 \times 4 \times 5}{3 \times 4 \times 5}$ because $\frac{2}{3} \div \frac{4}{5} = \frac{2 \times 4 \times 5}{3 \times 4 \times 5} \div \frac{4}{5} = \frac{2 \times 5}{3 \times 4}$ by dividing numerator by numerator and denominator by denominator. Since $\frac{2 \times 5}{3 \times 4} = \frac{2}{3} \times \frac{5}{4}$, this shows that $\frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4}$. This lays the groundwork for the familiar algorithm, that is, the method of multiplying the dividend by the reciprocal of the divisor to find the quotient.

Sufficient practice sets are provided for children to develop understanding and efficiency in dividing with fractional numbers.

Teaching the Chapter

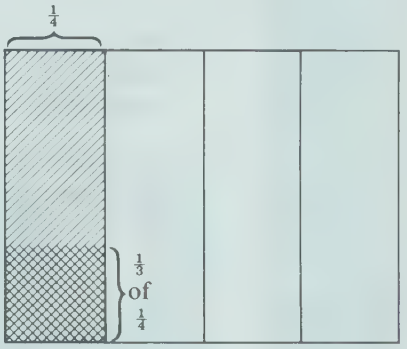
Materials

Colored strips
Crayons or colored pencils
Demonstration number line
Flannelboard
Overhead projector (if available) and transparencies
Ruler, centimetre
Unit regions, felt, to show fractional parts

Vocabulary

reciprocal

Although few materials are listed for this chapter, showing diagrams on the chalkboard or on the overhead projector will be quite helpful in presenting the early lessons involving strips, regions, and number-line illustrations. For example, you can illustrate $\frac{1}{3}$ of $\frac{1}{4}$ for the children by showing a region of which $\frac{1}{4}$ is shaded and then erasing all but $\frac{1}{3}$ of this shading.



Demonstrating number-line ideas on the chalkboard or on a demonstration number line will also be helpful for some children.

Note that *reciprocal* is the only word in the vocabulary list. This simply indicates that most of the background work for the concepts in this chapter has been provided and that new concepts in this chapter are based on those introduced earlier.

Lesson Schedule

Plan to spend four to five weeks on this chapter, adjusting your time schedule according to the needs and abilities of your children. You should be aware that the chapter contains a considerable number of word-problem pages, and you will want to allow enough time for these to give the children an opportunity to discuss the ideas presented.

Evaluation of Progress

As you work with this chapter, note especially the way in which the ideas unfold and culminate in the development of general rules for finding the product and the quotient of two fractional numbers. You should evaluate the children's achievement as you go through this sequential development. Some children will be able to determine quick solutions to routine problems once they find the rules for multiplication and division. However, to evaluate the children's understanding, you will want to take into consideration not only their skill with routine computation and with word problems, but also their daily participation in the activities using concepts developed in this chapter.

Resources for Active Learning

General Activities

It's a Tangram World, "Fractions," pp. 56–61, Educational Science Consultants
Mathex: Numeration No. 7, Activities 3–5, pp. 46–47 (pupil pages 47–51), Encyclopaedia Britannica Publications Ltd.

Manipulative Devices

Cuisenaire Cubes, Squares, and Rods (Cuisenaire Co.)
Fraction Blocks (Creative Publications)
Map measurer and compass (Edmund Scientific; Math Media)
Modulated Cutouts (Encyclopaedia Britannica Publications Ltd.)

Commercial Games

Action Fraction Games (CCM School Materials; Lakeshore; Math Media)
Spinner Number Games, Fractions (Heath)

PAGES 182–183

Objective

Given two fractional numbers expressed as unit fractions, the child will be able to find the product by relating the unit fractions to fractional parts of regions.

Preparation

Materials

crayons or colored pencils; paper (1 sheet per child)

To prepare for this lesson, you might exhibit fractional parts of rectangular and circular regions and review naming these parts with fractions. For example, draw a rectangle on the chalkboard and divide it into halves. Then divide it again, into fourths; then eighths. Each time, have children identify the divided parts.

Since such a review treats concepts with which children should be very familiar, spend only two or three minutes with it.

Investigation

You might want to make sure that the children fold their paper correctly, but then encourage them to do the investigation independently. However, caution them to carefully figure out what region they should color before they start to do any coloring.

If some children finish very quickly and you think they are able, suggest that they also color these regions:

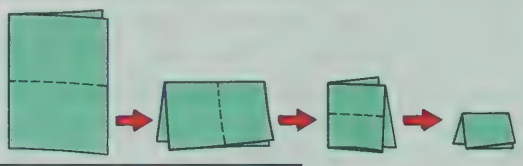
$\frac{1}{2}$ of $\frac{1}{8}$ of the sheet black
$\frac{1}{4}$ of $\frac{1}{2}$ of the sheet orange
$\frac{1}{8}$ of $\frac{1}{4}$ of the sheet purple

9 Multiplication and Division of Fractional Numbers

Let's explore multiplication using unit fractions.

Investigating the Ideas

Fold a piece of notebook paper in half four times.



Can you unfold the paper and color it so that the following statements are true? See Investigation.

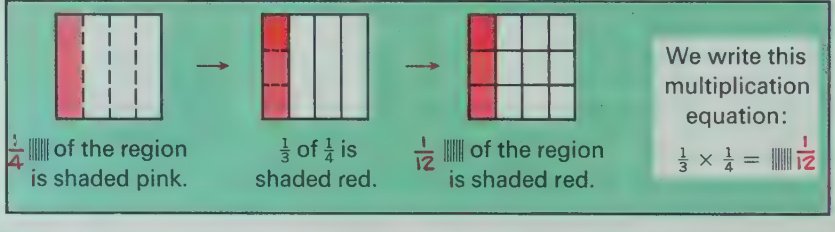
A $\frac{1}{2}$ of $\frac{1}{2}$ of the sheet is red.
B $\frac{1}{4}$ of $\frac{1}{4}$ of the sheet is blue.
C $\frac{1}{2}$ of $\frac{1}{4}$ of the sheet is green.
D $\frac{1}{8}$ of $\frac{1}{2}$ of the sheet is brown.

Discussing the Ideas

1. What fraction of the sheet in the Investigation did you color
   A red? $\frac{1}{4}$ B blue? $\frac{1}{16}$ C green? $\frac{1}{8}$ D brown? $\frac{1}{16}$

2. In the Investigation, you colored $\frac{1}{4}$ of the sheet red.
   We say, "$\frac{1}{2}$ of $\frac{1}{2}$ of the sheet is $\frac{1}{4}$ of the sheet."
   Make statements like this one for parts B, C, and D above.

3. Give the missing fractions.



4. Can you explain how to find these products?
   A $\frac{1}{2} \times \frac{1}{2}$ $\frac{1}{4}$ B $\frac{1}{4} \times \frac{1}{2}$ $\frac{1}{8}$ C $\frac{1}{3} \times \frac{1}{2}$ $\frac{1}{6}$ D $\frac{1}{2} \times \frac{1}{5}$ $\frac{1}{10}$ E $\frac{1}{8} \times \frac{1}{2}$ $\frac{1}{16}$

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Discussion

Use exercises 1 and 2 as a basis for a discussion of the children's investigation results. Write their responses on the chalkboard in the following form:

$\frac{1}{2}$ of $\frac{1}{2}$ is $\frac{1}{4}$. $\frac{1}{4}$ of $\frac{1}{4}$ is $\frac{1}{16}$.
$\frac{1}{2}$ of $\frac{1}{4}$ is $\frac{1}{8}$. $\frac{1}{8}$ of $\frac{1}{2}$ is $\frac{1}{16}$.

As you work through each example, be sure that the children have correctly identified the first fractional part they are dealing with so that they can use it to find that fractional part of it which they colored. Point out that the product represents a fractional part of the whole sheet.

One of the most important points to stress for the children in this lesson is that all of the fractions they are working with are *unit fractions*, that is, their numerator is one. As you work through exercise 3, display the final equation.

$$\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$$

You might then express the four statements from the investigation in equation form.

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$
$\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$ $\frac{1}{8} \times \frac{1}{2} = \frac{1}{16}$

Avoid giving rules for finding such products; many children will discover on their own that they can

Using the Ideas

1. Give the missing numbers. Then copy the equation with the correct product.



$\frac{1}{2} \times \frac{1}{5} = n\ \frac{1}{10}$

$\frac{1}{5}$ of the region is shaded pink. $\frac{1}{2}$ of $\frac{1}{5}$ is shaded red. $\frac{1}{10}$ of the region is shaded red.

2. Study the figures. Then give the product.



A: $\frac{1}{4}$ $\frac{1}{6}$ of $\frac{1}{4}$ $\quad \frac{1}{6} \times \frac{1}{4} = n\ \frac{1}{24}$

B: $\frac{1}{3}$ $\frac{1}{3}$ of $\frac{1}{3}$ $\quad \frac{1}{3} \times \frac{1}{3} = n\ \frac{1}{9}$

C: $\frac{1}{7}$ $\frac{1}{2}$ of $\frac{1}{7}$ $\quad \frac{1}{2} \times \frac{1}{7} = n\ \frac{1}{14}$

3. Give the products.

A $\frac{1}{3} \times \frac{1}{5}\ \frac{1}{15}$ C $\frac{1}{3} \times \frac{1}{3}\ \frac{1}{9}$ E $\frac{1}{6} \times \frac{1}{5}\ \frac{1}{30}$ G $(\frac{1}{2} \times \frac{1}{3}) \times \frac{1}{4}\ \frac{1}{24}$

B $\frac{1}{8} \times \frac{1}{2}\ \frac{1}{16}$ D $\frac{1}{5} \times \frac{1}{4}\ \frac{1}{20}$ F $\frac{1}{10} \times \frac{1}{10}\ \frac{1}{100}$ H $\frac{1}{2} \times (\frac{1}{5} \times \frac{1}{10})\ \frac{1}{100}$

4. Write and solve a multiplication equation for each problem.

A Joan found $\frac{1}{2}$ of a pie in the refrigerator. She ate $\frac{1}{3}$ of it. What fraction of the whole pie did she eat? $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

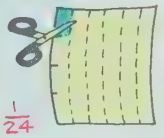


B Gary painted $\frac{1}{4}$ of the side of the garage. When he had repainted $\frac{1}{5}$ of this part, what part of the side of the garage had he painted twice?



$\frac{1}{4} \times \frac{1}{5} = \frac{1}{20}$

C Ann cut a sheet of paper into 6 strips of the same size. Then she colored $\frac{1}{4}$ of one of the strips and cut off the colored piece. What part of the whole sheet was this colored piece? $\frac{1}{4} \times \frac{1}{6} = \frac{1}{24}$



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find the product easily by multiplying the numerators together and the denominators together. Have them discuss this idea as they work through exercise 4.

Using the Exercises

Assign the exercises on page 183 as independent work. With some children, you might want to use physical demonstrations similar to the one described in exercise 4C; or use colored strips of felt on the flannelboard for the same effect.

Assignments (page 183)

Minimum: 1–3. Average: 1–4.
Maximum: 1–4.

Mathematics

A *unit fraction* is any fraction whose numerator is 1. If a is a non-zero whole number, then $\frac{1}{a}$ represents a unit fraction. In this lesson, children study one of the two facts basic to subsequent work in multiplication of fractional numbers, namely, multiplication of unit fractions:

$$\frac{1}{a} \times \frac{1}{b} = \frac{1}{a \times b}$$

In the next lesson, a second basic fact pertaining to unit fractions will be presented.

Follow-up

To give children further practice with multiplication of unit fractions, you might provide exercises like those below.

1. Construct a circle. Mark off 45° angles.
   Color $\frac{1}{2}$ of $\frac{1}{2}$ of the circle blue.
   Color $\frac{1}{4}$ of $\frac{1}{2}$ of the circle red.
   Color $\frac{1}{4}$ of $\frac{1}{4}$ of the circle yellow.
   Color $\frac{1}{2}$ of $\frac{1}{8}$ of the circle green.
2. Draw a rectangle. Divide it into thirds. Then divide the thirds into halves, so that the rectangle is divided into sixths.
   Color $\frac{1}{2}$ of $\frac{1}{3}$ purple.
   Color $\frac{1}{2}$ of $\frac{1}{6}$ green.

You might suggest to more capable or more interested children that they find the area of $\frac{1}{2}$ of $\frac{1}{4}$ of their classroom floor. Or $\frac{1}{4}$ of $\frac{1}{4}$ of the floor.

Resources for Active Learning

Developmental Math Cards, "Pegboard Fractions," J13, Addison-Wesley.

PAGES 184–185

Objective

Given a whole number and a unit fraction, the child will be able to find the product.

Preparation

Materials

colored strips

To prepare for this lesson, you might briefly review the principle studied in the previous lesson:

$$\frac{1}{a} \times \frac{1}{b} = \frac{1}{a \times b}$$

Use various pairs of unit fractions to have children find their products. Keep such a review brief and brisk.

Investigation

Before children begin to work, be sure they understand that the purple strip is being considered as the unit strip. Thus, the red strip is $\frac{1}{2}$ of the unit strip, and the white strip is $\frac{1}{4}$ of the unit strip. Remind the children to record their results.

$5 \times \frac{1}{2} = \frac{5}{2}$ or $2\frac{1}{2}$ $\quad 2 \times \frac{1}{2} = \frac{2}{2}$ or 1
$7 \times \frac{1}{4} = \frac{7}{4}$ or $1\frac{3}{4}$ $\quad 9 \times \frac{1}{4} = \frac{9}{4}$ or $2\frac{1}{4}$
$3 \times \frac{1}{4} = \frac{3}{4}$

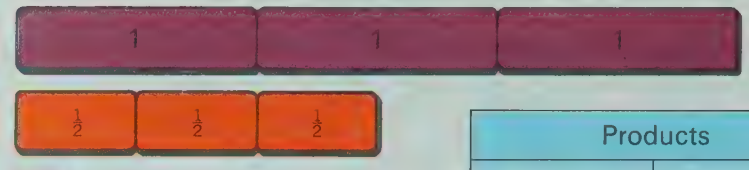
Notice that both the improper fraction and its equivalent mixed numeral are acceptable. One form or the other may be preferred according to the particular emphasis of study or clarity of the answer.

Can you find the product of a whole number and a unit fraction?

Investigating the Ideas

If the purple strip is the unit, this picture shows that:

$3 \times \frac{1}{2} = \frac{3}{2}$

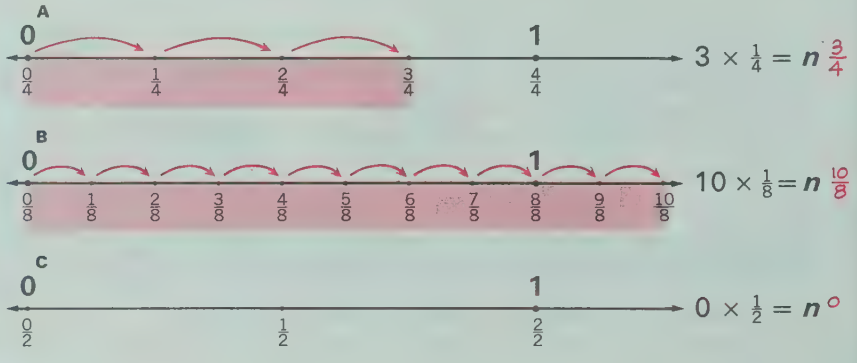


Can you find the products in the table by placing the correct strips along the unit strips? See Investigation.

Products	
$5 \times \frac{1}{2} = \frac{5}{2}$	$2 \times \frac{1}{2} = \frac{2}{2}$
$7 \times \frac{1}{4} = \frac{7}{4}$	$9 \times \frac{1}{4} = \frac{9}{4}$
$3 \times \frac{1}{4} = \frac{3}{4}$	$6 \times \frac{1}{2} = \frac{6}{2}$

Discussing the Ideas

1. How could you use the blue strips and the light green strips to show $4 \times \frac{1}{3}$? See Discussion.
2. Write each product in the Investigation as an improper fraction. See above.
3. Study the number-line picture. Then solve the equation.



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Discussion

Discussion exercise 1 challenges the children to use other strips the way they used the strips called for in the investigation. Be sure they realize that, for exercise 1, the blue strip should be considered to be the unit and the light green strip then represents $\frac{1}{3}$. Suggest other exercises, such as using the brown strips and the red strips to show $3 \times \frac{1}{4}$ (or $5 \times \frac{1}{4}$), or using the orange strips and the yellow strips to show $6 \times \frac{1}{2}$ (or $4 \times \frac{1}{2}$). For each exercise that you give them, have a child record the equation on the chalkboard. Have them relate these exercises and those they did in the investigation to the equation for each. Encourage them to start looking for a pattern which appears in all the equations. If they do find and explain it, use exercise 3 to develop it further. If they do not, continue with exercise 3 and some children will then discover the pattern of multiplying the whole number times the numerator. As you discuss exercise 3, stress the point that multiplication of a whole number by a fraction is like repeated addition; that is, $3 \times \frac{1}{4}$ may be thought of as $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$. Exercise 3C extends the identity property of 0 in multiplication to multiplication of a fractional number by 0.

Using the Ideas

1. Study the examples. Then solve the equations.

$7 \times \frac{1}{4} = \frac{7}{4}$ $6 \times \frac{1}{6} = \frac{6}{6}$ $3 \times \frac{1}{5} = \frac{3}{5}$ $\frac{1}{3} \times \frac{1}{5} = \frac{1}{15}$ $\frac{1}{6} \times \frac{1}{4} = \frac{1}{24}$

A $9 \times \frac{1}{5} = n$ $\frac{9}{5}$

B $4 \times \frac{1}{7} = n$ $\frac{4}{7}$

C $3 \times \frac{1}{3} = n$ $\frac{3}{3}$

D $8 \times \frac{1}{5} = n$ $\frac{8}{5}$

E $7 \times \frac{1}{6} = n$ $\frac{7}{6}$

F $9 \times \frac{1}{9} = n$ $\frac{9}{9}$

G $\frac{1}{2} \times \frac{1}{9} = n$ $\frac{1}{18}$

H $\frac{1}{3} \times \frac{1}{7} = n$ $\frac{1}{21}$

I $\frac{1}{5} \times \frac{1}{5} = n$ $\frac{1}{25}$

J $\frac{1}{4} \times \frac{1}{8} = n$ $\frac{1}{32}$

K $\frac{1}{3} \times \frac{1}{10} = n$ $\frac{1}{30}$

L $\frac{1}{10} \times \frac{1}{10} = n$ $\frac{1}{100}$

M $\frac{1}{10} \times \frac{1}{100} = n$ $\frac{1}{1000}$

N $\frac{1}{10} \times \frac{1}{1000} = n$ $\frac{1}{10\,000}$

O $\frac{1}{100} \times \frac{1}{100} = n$ $\frac{1}{10\,000}$

P $9 \times \frac{1}{10} = n$ $\frac{9}{10}$

Q $13 \times \frac{1}{10} = n$ $\frac{13}{10}$

R $10 \times \frac{1}{10} = n$ $\frac{10}{10}$

S $10 \times \frac{1}{100} = n$ $\frac{10}{100}$

T $(\frac{1}{10})^3 = n$ $\frac{1}{1000}$

Write and solve a multiplication equation for each problem.

2. Andy ran $\frac{1}{2}$ of the way to school. Sue ran $\frac{1}{4}$ as far as Andy. What part of the way to school did Sue run? $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$

3. A bullfrog jumped $\frac{1}{3}$ of the way across a stream. A green frog jumped only $\frac{1}{2}$ as far. What part of the way across the stream did the green frog jump? $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

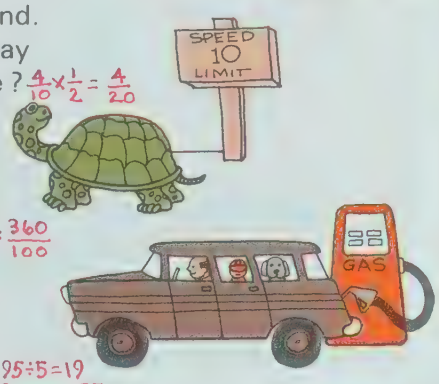
4. Dan's shoes were each $\frac{1}{4}$ metre long. He used a shoe as a segment to measure the width of a room. How wide was the room if Dan counted 20 of his shoe lengths in walking across? $\frac{1}{4} \times 20 = \frac{20}{4}$



5. The school track is $\frac{4}{10}$ km around. Jack and Neil raced $\frac{1}{2}$ of the way around. How far did they race? $\frac{4}{10} \times \frac{1}{2} = \frac{4}{20}$

6. A fast turtle can go $\frac{15}{100}$ km in an hour. At this rate, how far will he go in 24 hours? $\frac{15}{100} \times 24 = \frac{360}{100}$

7. A car needs $\frac{3}{4}$ litre of gasoline to travel 5 km. How many litres will it need to travel 95 kilometres? $95 \div 5 = 19$ $\frac{3}{4} \times 19 = \frac{57}{4}$





More practice, page A-14, Set 27

Using the Exercises

Assign the exercises on page 185 as independent work. You may choose to have the children leave the answers which are improper fractions in that form or you may ask that they change them to mixed numerals. Either form is acceptable.

When you discuss these exercises, bring out the significance of the boldface numerals at the beginning of exercise 1. These problems review the generalization for finding products involving a whole number and a unit fraction and products involving two unit fractions.

Have the children observe that the meaning of the exponent when used with a fraction is the same as for a whole number: the base number (here, $\frac{1}{10}$) is taken as a factor the number of times the exponent indicates (here, 3). You might have them notice the difference in value of 10^3 and $(\frac{1}{10})^3$. For example, they might compare \$1000 to $\frac{1}{1000}$ of a dollar, which is only one tenth of a penny.

Be sure to allow time for a discussion of the word problems as well.

Assignments (page 185)

Minimum: 1. Average: 1–7.
Maximum: 1–7.

Mathematics

The second basic fact for multiplication of fractional numbers is stressed in this lesson. The child discovers, through use of strips and the number line, how the product of a whole number and a fractional number expressed as a unit fraction may be found. The general statement is as follows:

> If a is any whole number, and $\frac{1}{b}$ is a unit fraction, then
>
> $$a \times \frac{1}{b} = \frac{a}{b}.$$

Resources for Active Learning

A Cloudburst, Vol. 2, No. 3313, 3323–4, 3333, Midwest Publications.

Workbook, page 57

PAGES 186–187

Objective

Given fractional-number problems, the child will demonstrate his ability to apply the following basic principles: zero principle for addition, commutative and associative principles, and the 1 principle for multiplication.

Preparation

Review the basic principles for addition of fractional numbers with the children. Be sure to call attention to the fact that using the commutative and the associative principles together allows the rearrangement of addends.

After this review, display several simple mixed-numeral problems that exemplify the basic principles. Examples similar to the following will show the children how the basic principles are applied.

$$\begin{aligned} 2\tfrac{1}{3} + 5\tfrac{1}{4} &= (2 + \tfrac{1}{3}) + (5 + \tfrac{1}{4}) \\ &= (2 + 5) + (\tfrac{1}{3} + \tfrac{1}{4}) \\ &= 7 + \tfrac{7}{12} \\ &= 7\tfrac{7}{12} \end{aligned}$$

● *Let's explore basic principles for fractional numbers.*

Discussing the Ideas

Here are some basic principles that we accept for addition and multiplication of fractional numbers.

Commutative Principles	Associative Principles
You can change the **order** of addends or factors, and the sum or product will be the same.	You can change the **grouping** of the addends or factors, and the sum or product is the same.
Zero Principle	**1 Principle**
When you choose a fractional number and add 0, the sum is the number you have chosen.	When you choose a number and multiply by 1, the product is the number you have chosen.

1. Which principles can help you solve these equations?

   A $\frac{3}{8} \times 1 = n\frac{3}{8}$ 1 principle B $\frac{2}{3} + 0 = n\frac{2}{3}$ Zero principle C $7\frac{1}{3} \times \frac{10}{10} = n7\frac{1}{3}$ 1 principle D $0 + \frac{5}{8} = n\frac{5}{8}$ Zero principle

2. Which principles can help you solve these equations?

   A $4\frac{3}{5} + \frac{2}{3} = \frac{2}{3} + n4\frac{3}{5}$ Comm. principle B $\frac{3}{4} \times \frac{7}{2} = n \times \frac{3}{4}\ \frac{7}{2}$ Comm. principle

3. A If $\frac{1}{2} + (\frac{1}{3} + \frac{1}{4}) = \frac{13}{12}$, what is $(\frac{1}{2} + \frac{1}{3}) + \frac{1}{4}$? $\frac{13}{12}$

   B If $\frac{1}{2} \times (\frac{3}{4} \times \frac{3}{5}) = \frac{9}{40}$, what is $(\frac{1}{2} \times \frac{3}{4}) \times \frac{3}{5}$? $\frac{9}{40}$

   C Which principles did you use in exercises 3A and 3B?
   Assoc. principle

4. Study the statement below. Then solve the equations.

   Since you can change both the **order** and **grouping** of addends or factors, you can arrange the addends or factors in any way that is convenient.

   A If $2 + \frac{4}{5} + 3 + \frac{1}{5} = 6$, then $\frac{4}{5} + \frac{1}{5} + 2 + 3 = n$. 6

   B If $5 \times \frac{1}{4} \times 3 \times \frac{1}{7} = \frac{15}{28}$, then $5 \times 3 \times \frac{1}{4} \times \frac{1}{7} = n$. $\frac{15}{28}$

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Discussion

Note that the introductory material at the top of page 186 speaks of "accepting" basic principles. At this point, the principles for multiplication of fractional numbers are assumptions; we assume that multiplication of fractional numbers is commutative and associative. Exercise careful judgment in deciding how much time to spend discussing this idea.

You will probably find it more advantageous to spend this time discussing each principle and its examples specifically. The zero principle for addition of fractional numbers presents zero as the *identity element*, just as it is for addition of whole numbers. However, the term identity element need not be used with the children. (Note that, even though zero times any fractional number is zero, this fact is not considered a basic principle but, rather, a special property of zero.)

Although the children do not yet have the skills to find the product of any two fractional numbers, their understanding of the basic principles should enable them to find the answers to exercises 2B and 3B. To try to verify these exercises by computation would only confuse the children. This is also true for exercise 4B. One of the most important generalizations children should grasp from this discussion is stated in exercise 4.

Using the Ideas

1. Give the number for ***a***. Then give the number for ***b***.

A $5 \times \frac{1}{6} = a \frac{5}{6} \rightarrow \frac{1}{6} \times 5 = b \frac{5}{6}$

B $9 \times \frac{1}{2} = a \frac{9}{2} \rightarrow \frac{1}{2} \times 9 = b \frac{9}{2}$

C $3 \times \frac{1}{10} = a \frac{3}{10} \rightarrow \frac{1}{10} \times 3 = b \frac{3}{10}$

D $10 \times \frac{1}{10} = a \frac{10}{10} \rightarrow \frac{1}{10} \times 10 = b \frac{10}{10}$

2. Solve the equations.

A $\frac{1}{5} \times 1 = n \frac{1}{5}$ B $1 \times \frac{1}{4} = n \frac{1}{4}$ C $\frac{1}{7} \times \frac{2}{2} = n \frac{1}{7}$ D $0 \times \frac{1}{8} = n$ 0 E $\frac{1}{8} \times \frac{0}{6} = n$ 0

3. Solve the equations.

A $(2 \times \frac{1}{3}) \times \frac{1}{4} = 2 \times (n \times \frac{1}{4})$ $\frac{1}{3}$ C $(9 \times \frac{1}{7}) \times \frac{1}{3} = n \times (\frac{1}{7} \times \frac{1}{3})$ 9

B $\frac{1}{5} \times (\frac{1}{2} \times 7) = (\frac{1}{5} \times n) \times 7$ $\frac{1}{2}$ D $\frac{1}{7} \times (\frac{1}{6} \times 13) = (\frac{1}{7} \times \frac{1}{6}) \times n$ 13

4. Find the products. First find the product of the factors given in red.

A $3 \times \frac{1}{5} \times \frac{1}{2}$ $\frac{3}{10}$ C $\frac{1}{6} \times 6 \times \frac{1}{4}$ $\frac{6}{24}$ E $\frac{1}{6} \times 10 \times \frac{1}{3}$ $\frac{10}{18}$ G $\frac{1}{10} \times \frac{1}{10} \times 100$ $\frac{100}{100}$

B $7 \times \frac{1}{3} \times \frac{1}{4}$ $\frac{7}{12}$ D $\frac{1}{5} \times \frac{1}{6} \times 9$ $\frac{9}{30}$ F $17 \times \frac{1}{10} \times \frac{1}{10}$ $\frac{17}{100}$ H $\frac{1}{5} \times 25 \times \frac{1}{5}$ $\frac{25}{25}$

5. Find the product for each exercise. First find the product of the factors given in heavy black. Then find the product of the factors given in red.

A $2 \times \frac{1}{8} \times 4 \times \frac{1}{2} = p$ $\frac{8}{16}$

B $3 \times \frac{1}{7} \times 5 \times \frac{1}{5} = p$ $\frac{15}{35}$

C $6 \times \frac{1}{6} \times 4 \times \frac{1}{3} = p$ $\frac{24}{18}$

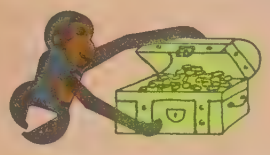
D $2 \times \frac{1}{5} \times 9 \times \frac{1}{8} = p$ $\frac{18}{40}$

E $8 \times \frac{1}{10} \times 7 \times \frac{1}{10} = p$ $\frac{56}{100}$

F $8 \times \frac{1}{10} \times 3 \times \frac{1}{100} = p$ $\frac{24}{1000}$

think

A doubloon is a Spanish coin worth 16 dollars.



A pirate buried $\frac{1}{2}$ of his doubloons and threw $\frac{1}{3}$ of them into the sea. When he counted those remaining, he found 4000 doubloons. How many did he have when he started? How many dollars were they worth?

24000 ; $384 000

Mathematics

In the first two lessons, we provided experiences to help the children find the product of two fractional numbers shown as unit fractions, and the product of a whole number and a fractional number represented by a unit fraction.

The following outline details the three steps needed to make a generalization about the product of any two fractional numbers:

(1) Establish the generalizations concerning products of two unit fractions and products of a whole number and a unit fraction.

(2) Assume that the basic principles hold for fractional numbers.

(3) Use steps 1 and 2 to state a generalization for finding the product of any two fractional numbers.

In other words, by using unit fractions and whole numbers, and the commutative and associative principles, we develop a way to find the product of any two fractional numbers represented by fractions. The third step above will be emphasized in the next lesson.

Using the Exercises

Since exercises 1, 2, and 3 on page 187 involve only unit fractions and whole numbers, they are not a general verification of the principles applied. They are intended only to strengthen children's understanding of these principles. Note that in exercise 4 the children begin by finding the product of two unit fractions, which is a unit fraction also. Then they find the product of this unit fraction and the whole number. This procedure involves work with familiar products. Hence, by using the grouping and order principles, the children can find the type of product given in exercise 4.

In exercise 5, by thinking about rearranging factors, the children can find the product of the whole numbers, the product of the two unit fractions, and finally, the product of the whole-number product and the unit-fraction product. Emphasize that we are able to find such products because we are able to rearrange factors in any way that is convenient.

Assignments (page 187)*

Minimum: 1–3. Average: 1–5.
Maximum: 1–5.

PAGES 188–189

Objective

Given any two fractional numbers, the child will be able to find the product by multiplying numerators together and multiplying denominators together.

Preparation

To prepare for this lesson, you might display equations based on the basic principles and ask children to supply the missing terms.

$$\tfrac{3}{5} \times \tfrac{7}{8} = ? \times \tfrac{3}{5}$$

$$(\tfrac{2}{3} \times \tfrac{1}{4}) \times \tfrac{5}{6} = \tfrac{2}{3} \times (\tfrac{1}{4} \times ?)$$

$$6 \times \tfrac{1}{2} \times 4 \times \tfrac{1}{8} = 6 \times ? \times \tfrac{1}{2} \times ?$$

Observe with the children that in this lesson they will learn how to do the actual multiplication in these equations.

Investigation

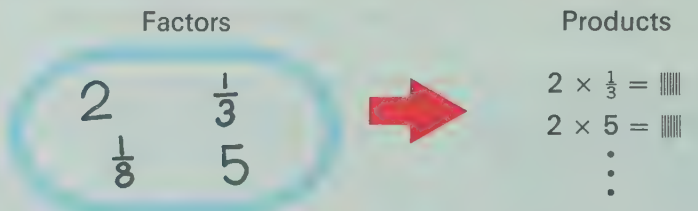
Encourage children to work independently on this investigation. They should try as many combinations as they can find. Some may use 3, or even all 4, factors in one equation. There are eleven different product possibilities.

$$2 \times \tfrac{1}{3} = \tfrac{2}{3}$$
$$2 \times 5 = 10$$
$$2 \times \tfrac{1}{8} = \tfrac{2}{8}$$
$$2 \times \tfrac{1}{3} \times \tfrac{1}{8} = \tfrac{2}{24}$$
$$2 \times \tfrac{1}{3} \times 5 = \tfrac{10}{3}$$
$$2 \times \tfrac{1}{8} \times 5 = \tfrac{10}{8}$$
$$2 \times \tfrac{1}{3} \times \tfrac{1}{8} \times 5 = \tfrac{10}{24}$$
$$\tfrac{1}{8} \times \tfrac{1}{3} = \tfrac{1}{24}$$
$$\tfrac{1}{8} \times 5 = \tfrac{5}{8}$$
$$\tfrac{1}{8} \times \tfrac{1}{3} \times 5 = \tfrac{5}{24}$$
$$\tfrac{1}{3} \times 5 = \tfrac{5}{3}$$

The children may need to be reminded that they should carefully record their products in equation form.

Let's explore multiplication of fractional numbers.

Investigating the Ideas



How many different products can you find by using the factors in the set? See Investigation.

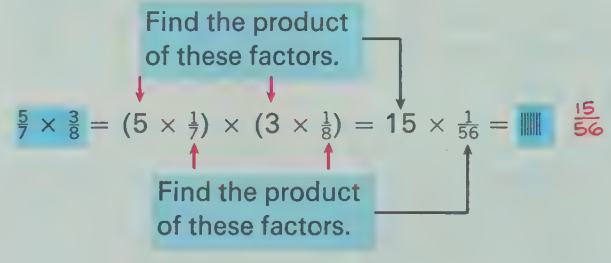
Discussing the Ideas

1. Give the number for *n* in each equation.

   A $2 \times 5 = n$ 10 B $\frac{1}{3} \times \frac{1}{8} = n$ $\frac{1}{24}$ C $(2 \times 5) \times (\frac{1}{3} \times \frac{1}{8}) = n$ $\frac{10}{24}$

   D $2 \times \frac{1}{3} = n$ $\frac{2}{3}$ E $5 \times \frac{1}{8} = n$ $\frac{5}{8}$ F $\frac{2}{3} \times \frac{5}{8} = n$ $\frac{10}{24}$

2. The diagram below will help you review a way to find products such as $\frac{5}{7} \times \frac{3}{8}$. Give the missing product.



3. Can you describe a shortcut for multiplying two fractional numbers? Multiply numerator by numerator, and denominator by denominator.

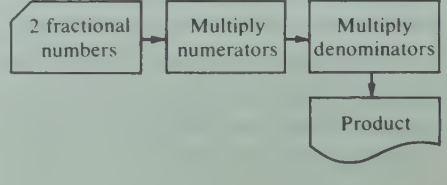
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Discussion

Ask individual children to present their investigation results so that all 11 equations are displayed.

Work through discussion exercise 1 with the children. Note the sequential development of parts A to F. In particular, help children relate part C to part F. Then, in exercise 2, develop the usual procedure for multiplying fractions. Be sure the children understand that the $\frac{5}{7}$ has been broken into $5 \times \frac{1}{7}$, that $\frac{3}{8}$ has been broken into $3 \times \frac{1}{8}$, and that they should think about rearranging factors in order to find the product of 5 and 3 and the product of $\frac{1}{7}$ and $\frac{1}{8}$. The final product is the product of a whole number and a unit fraction.

Lead children to explain in their own words the shortcut rule for multiplying two fractional numbers. Help them to see how the pattern of multiplying numerator times numerator and denominator times denominator is simply a shortcut of the procedure shown in exercise 2. Summarize this discussion by presenting this shortcut flow chart:



Using the Ideas

1. Solve the equations.

A $\frac{3}{4} = n \times \frac{1}{4}$ C $\frac{6}{10} = 6 \times n$ $\frac{1}{10}$ E $\frac{13}{15} = 13 \times n$ $\frac{1}{15}$ G $\frac{9}{10} = 9 \times n$ $\frac{1}{10}$

B $\frac{5}{6} = n \times \frac{1}{6}$ D $\frac{5}{100} = n \times \frac{1}{100}$ F $\frac{0}{7} = n \times \frac{1}{7}$ H $\frac{54}{100} = 54 \times n$ $\frac{1}{100}$

2. Copy the equations. Give the number instead of *n*.

A $\frac{3}{4} \times \frac{1}{3} = n \times \frac{1}{4} \times \frac{1}{3}$ 3 C $\frac{1}{6} \times \frac{3}{8} = \frac{1}{6} \times 3 \times n$ $\frac{1}{8}$ E $\frac{5}{6} \times \frac{1}{5} = n \times \frac{1}{6} \times \frac{1}{5}$ 5

B $\frac{2}{5} \times \frac{1}{4} = n \times \frac{1}{5} \times \frac{1}{4}$ 2 D $9 \times \frac{2}{3} = 9 \times 2 \times n$ $\frac{1}{3}$ F $\frac{1}{2} \times \frac{4}{9} = \frac{1}{2} \times n \times \frac{1}{9}$ 4

3. Solve the equation to find the product of the factors in the red screen.

A $\frac{3}{4} \times \frac{1}{3}$ $3 \times \frac{1}{4} \times \frac{1}{3} = n$ $\frac{3}{12}$

C $\frac{4}{7} \times \frac{1}{5}$ $4 \times \frac{1}{7} \times \frac{1}{5} = n$ $\frac{4}{35}$

E $\frac{1}{6} \times \frac{3}{8}$ $\frac{1}{6} \times 3 \times \frac{1}{8} = n$ $\frac{3}{48}$

G $\frac{1}{3} \times \frac{8}{5}$ $\frac{1}{3} \times 8 \times \frac{1}{5} = n$ $\frac{8}{15}$

B $\frac{2}{5} \times \frac{1}{4}$ $2 \times \frac{1}{5} \times \frac{1}{4} = n$ $\frac{2}{20}$

D $\frac{5}{8} \times \frac{1}{2}$ $5 \times \frac{1}{8} \times \frac{1}{2} = n$ $\frac{5}{16}$

F $9 \times \frac{2}{3}$ $9 \times 2 \times \frac{1}{3} = n$ $\frac{18}{3}$

H $6 \times \frac{9}{10}$ $6 \times 9 \times \frac{1}{10} = n$ $\frac{54}{10}$

4. Give the whole number for *a*.
Give the fraction for *b*. Then give the missing product.

A $\frac{3}{4} \times \frac{2}{3} = (3 \times \frac{1}{4}) \times (2 \times \frac{1}{3}) = a \times b = \square$ a: 6, b: $\frac{1}{12}$, $\square$: $\frac{6}{12}$

B $\frac{2}{5} \times \frac{5}{6} = (2 \times \frac{1}{5}) \times (5 \times \frac{1}{6}) = a \times b = \square$ a: 10, b: $\frac{1}{30}$, $\square$: $\frac{10}{30}$

C $\frac{3}{8} \times \frac{3}{4} = (3 \times \frac{1}{8}) \times (3 \times \frac{1}{4}) = a \times b = \square$ a: 9, b: $\frac{1}{32}$, $\square$: $\frac{9}{32}$

5. Copy each equation. Give the product instead of *n*.

A $\frac{3}{4} \times \frac{2}{3} = n$ $\frac{6}{12}$

B $\frac{2}{5} \times \frac{5}{6} = n$ $\frac{10}{30}$

C $\frac{3}{8} \times \frac{3}{4} = n$ $\frac{9}{32}$

D $\frac{2}{7} \times \frac{7}{2} = n$ $\frac{14}{14}$

E $\frac{5}{8} \times \frac{5}{8} = n$ $\frac{25}{64}$

F $\frac{4}{5} \times \frac{4}{3} = n$ $\frac{16}{15}$

G $\frac{7}{10} \times \frac{5}{2} = n$ $\frac{35}{20}$

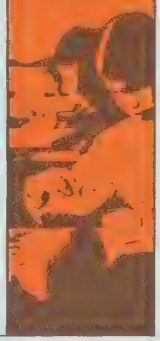
H $\frac{3}{5} \times \frac{5}{7} = n$ $\frac{15}{35}$

think

The weight of an object on the moon is $\frac{1}{6}$ its weight on Earth. Give the total "moon weight" of these objects. (The Earth weights are given.) Total = $7\frac{63}{120}$ kg

Boy: 42 kg 7 kg Hamburger: $\frac{1}{10}$ kg
Cat: 3 kg $\frac{1}{2}$ kg Mouse: $\frac{1}{20}$ kg $\frac{1}{120}$ kg

More practice, page A-14, Set 28



Mathematics

The example below illustrates and explains the various steps used to find the product of two fractional numbers.

(1) $\frac{3}{4} \times \frac{2}{3} = (3 \times \frac{1}{4}) \times (2 \times \frac{1}{3})$

Reason: $\frac{a}{b} = a \times \frac{1}{b}$

(2) $= (3 \times 2) \times (\frac{1}{4} \times \frac{1}{3})$

Reason: Commutative and associative principles

(3) $= 6 \times \frac{1}{12}$

Reason: $\frac{1}{a} \times \frac{1}{b} = \frac{1}{a \times b}$

(4) $= \frac{6}{12}$

Reason: $a \times \frac{1}{b} = \frac{a}{b}$

The type of reason given in step 1 $\left(\frac{a}{b} = a \times \frac{1}{b}\right)$ was established on pages 182 to 185, by using strips and number-line examples. In step 2, the reason given (the commutative and associative principles) is simply that our ability to change the order and the grouping allows us to rearrange factors in any way that is convenient. Obviously, there are several changes of order and grouping that explain the move from step 1 to step 2, which is why we feel it is important, at an early stage, to generalize the rearrangement principle. This principle is the result of both the commutative and the associative principles. In step 3, notice that the reason given $\left(\frac{1}{a} \times \frac{1}{b} = \frac{1}{a \times b}\right)$ is another result derived from lessons on pages 182 to 185, where the children were led (using regions, strips, and the number line) to establish a generalization for finding products involving two fractional numbers expressed as unit fractions. Finally, in step 4, the reason is the same as for step 1. That is, it is based on the generalization for finding the product of a whole number and a fractional number denoted by a unit fraction.

Using the Exercises

Assign the exercises on page 189 as independent work. Notice how they progress developmentally through the steps leading to multiplication of two fractional numbers. For exercises 1 through 4, be sure the children notice how the factors are broken apart, even if they understand the shortcut method of multiplying numerator by numerator and denominator by denominator.

Assignments (page 189)
Minimum: 4–5. Average: 1–5.
Maximum: 1–5.

Duplicator Masters, page 34
Workbook, page 58
Skill Masters, page 34

PAGES 190–191

Objective

Given multiplication of fractional numbers in equations and in short story problems, the child will be able to find the products.

Preparation

A good way to prepare for this lesson is to be sure that the children fully understand that they can rearrange factors in multiplication of fractional numbers. Then, review the generalizations for finding products of two fractional numbers. You might do this simply by asking children to find the product of several equations such as the following:

$\frac{2}{3} \times \frac{5}{6} = ?$

$\frac{3}{4} \times \frac{2}{3} = ?$

$\frac{1}{7} \times \frac{6}{7} = ?$

$\frac{4}{5} \times \frac{5}{6} = ?$

It would also be helpful to point out that some of these answers may be simplified to lowest terms.

Multiplication Practice

1. Find the products.

A $\frac{3}{5} \times \frac{2}{7} = (3 \times 2) \times (\frac{1}{5} \times \frac{1}{7}) = n\ \frac{6}{35}$

B $\frac{3}{4} \times \frac{5}{3} = (3 \times 5) \times (\frac{1}{4} \times \frac{1}{3}) = n\ \frac{15}{12}$

C $\frac{7}{2} \times \frac{2}{5} = (7 \times 2) \times (\frac{1}{2} \times \frac{1}{5}) = n\ \frac{14}{10}$

D $\frac{4}{3} \times \frac{8}{11} = (4 \times 8) \times (\frac{1}{3} \times \frac{1}{11}) = n\ \frac{32}{33}$

E $\frac{3}{5} \times \frac{2}{6} = (3 \times 2) \times (\frac{1}{5} \times \frac{1}{6}) = n\ \frac{6}{30}$

F $\frac{2}{10} \times \frac{9}{10} = (2 \times 9) \times (\frac{1}{10} \times \frac{1}{10}) = n\ \frac{18}{100}$

G $\frac{4}{7} \times \frac{11}{4} = (4 \times 11) \times (\frac{1}{7} \times \frac{1}{4}) = n\ \frac{44}{28}$

H $\frac{5}{9} \times \frac{3}{5} = (5 \times 3) \times (\frac{1}{9} \times \frac{1}{5}) = n\ \frac{15}{45}$

I $\frac{6}{100} \times \frac{4}{10} = (6 \times 4) \times (\frac{1}{100} \times \frac{1}{10}) = n\ \frac{24}{1000}$

J $\frac{0}{7} \times \frac{3}{4} = (0 \times 3) \times (\frac{1}{7} \times \frac{1}{4}) = n\ 0$

2. Find the products.

A $\frac{2}{3} \times \frac{4}{5} \frac{8}{15}$

B $\frac{7}{4} \times \frac{3}{2} \frac{21}{8}$

C $\frac{5}{7} \times \frac{6}{3} \frac{30}{21}$

D $\frac{4}{9} \times \frac{1}{4} \frac{4}{36}$

E $\frac{7}{3} \times \frac{3}{7} \frac{21}{21}$

F $\frac{4}{8} \times \frac{8}{4} \frac{32}{32}$

G $\frac{30}{1} \times \frac{2}{6} \frac{60}{6}$

H $\frac{3}{10} \times \frac{4}{7} \frac{12}{70}$

I $\frac{7}{11} \times \frac{3}{5} \frac{21}{55}$

J $\frac{4}{3} \times \frac{9}{8} \frac{36}{24}$

K $\frac{0}{6} \times \frac{2}{5} 0$

L $\frac{1}{10} \times \frac{1}{10} \frac{1}{100}$

M $\frac{3}{10} \times \frac{8}{100} \frac{24}{1000}$

N $\frac{3}{8} \times \frac{23}{1} \frac{69}{8}$

O $\frac{5}{6} \times \frac{5}{6} \frac{25}{36}$

P $\frac{6}{7} \times \frac{7}{6} \frac{42}{42}$

Q $\frac{4}{10} \times \frac{5}{10} \frac{20}{100}$

R $\frac{7}{20} \times \frac{8}{30} \frac{56}{600}$

S $\frac{5}{5} \times \frac{11}{30} \frac{55}{150}$

T $\frac{25}{100} \times \frac{4}{10} \frac{100}{1000}$

★ **3.** Give the symbol (<, >, =) for each ◯.

A $\frac{4}{5} \times \frac{2}{3}$ ◯ $\frac{4}{5} + \frac{2}{3}$ <

B $\frac{1}{3} \times \frac{1}{3}$ ◯ $\frac{1}{3}$ <

C $\frac{3}{4} \times 4$ ◯ $\frac{3}{7} \times 7$ =

D $\frac{7}{5} \times \frac{4}{3}$ ◯ $\frac{5}{7} \times \frac{3}{4}$ >

E $3 - \frac{3}{4}$ ◯ $3 \times \frac{3}{4}$ =

F $\frac{3}{2} \times 2$ ◯ $\frac{3}{2} + 2$ <

G $\frac{4}{3} \times 4$ ◯ $\frac{4}{3} + 4$ =

H $\frac{5}{4} \times 5$ ◯ $\frac{5}{4} + 5$ =

think

Give the pair of fractional numbers for *a* and *b* in each exercise.

Products

Sums

1.	1	$a\frac{1}{2}$	$b\frac{1}{2}$	$\frac{1}{4}$
2.	$\frac{2}{3}$	$a\frac{1}{3}$	$b\frac{1}{3}$	$\frac{1}{9}$
3.	$\frac{1}{2}$	$a\frac{1}{4}$	$b\frac{1}{4}$	$\frac{1}{16}$
4.	1	$a\frac{1}{4}$	$b\frac{3}{4}$	$\frac{3}{16}$
5.	$\frac{5}{6}$	$a\frac{1}{3}$	$b\frac{1}{2}$	$\frac{1}{6}$

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Discussion

Have the children do the exercises on page 190. In this lesson, you should begin to ask the children to give answers in lowest-terms fractions or in mixed numerals. Only a minimum amount of attention has been given to this in prior lessons, since the chief stress was on understanding the processes and on steps involved in arriving at the product of two fractional numbers. Now that these skills have been developed and understood, begin to encourage the children to change their answers to lowest-terms fractions and mixed numerals.

Notice that exercise 3 is particularly challenging. Also, there is a stimulating *Think* problem for the children who are interested in trying to solve it.

Assignments (page 190)

Minimum: 1–2. Average: 1–2. Maximum: 1–3.

Short Stories

1 Bowling ball: weighs 6 kg. Baseball: weighs $\frac{1}{40}$ as much.
What does the baseball weigh ? $\frac{6}{40}$ or $\frac{3}{20}$ kg



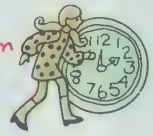
2 $\frac{3}{4}$ metre of ribbon. Cut off $\frac{1}{3}$ of it.
What part of a metre was cut off ?
$\frac{3}{12}$ or $\frac{1}{4}$ m

3 $\frac{3}{4}$ kg of butter. Used $\frac{2}{3}$ of it.
Used what part of a kilogram ?
$\frac{6}{12}$ or $\frac{1}{2}$ kg

4 $\frac{1}{5}$ of the girls are blondes.
20 girls. How many blondes ? 4



5 1 m: $\frac{1}{1000}$ of a kilometre. $\frac{500}{1000}$ or $\frac{1}{2}$ Km
500 m: what part of a kilometre ?



6 Long kangaroo jump: 12 m.
Long frog jump: $\frac{1}{3}$ as far.
How far ? 4m

7 Walk $\frac{5}{2}$ kilometres per hour.
Walk for $\frac{3}{4}$ hour. How far ?
$\frac{15}{8}$ or $1\frac{7}{8}$ Km

8 10 km. Walked $\frac{2}{3}$ of the way.
Walked how far ? $\frac{20}{3}$ or $6\frac{2}{3}$ Km

9 An apron contains $\frac{7}{10}$ of a square metre of cloth. 40 aprons.
How many square metres ? 28 m^2

10 Running track: $\frac{4}{10}$ km. 16 laps: how many kilometres ?
$\frac{64}{10}$ or $6\frac{2}{5}$ km

11 Drive 45 minutes.
$\frac{2}{3}$ km each minute.
How many km ?
$\frac{90}{3}$ or 30 km

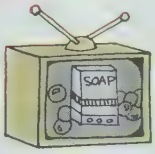


12 1 day: $\frac{1}{7}$ of a week.
$\frac{3}{4}$ of a day: what part of a week ? $\frac{3}{28}$

13 Iceberg: $\frac{1}{8}$ of it shows above the surface. If the iceberg is 210 cubic metres, how many cubic metres show on the surface ?
$\frac{210}{8}$ or $26\frac{1}{4}$ m^3



14 Television program: $\frac{1}{2}$ hour.
Commercials: $\frac{1}{10}$ of the time. How many minutes of commercials ?
$\frac{30}{10}$ or 3 min



★ 15 $24. Left $\frac{1}{3}$ of it at home. Spent $\frac{3}{4}$ of the rest. How much was spent ? $12

Using the Exercises

It would be helpful to discuss some of the short story problems on page 191. For instance, you might work through problems 1 and 5, showing equations for each:

1. $\frac{1}{40} \times 6 = ?$
5. $500 \times \frac{1}{1000} = ?$

Encourage children to do the remaining problems by writing and solving equations for each. Give directions such as this with discretion, however, because it is not necessary that all the children write an equation for every word problem they work.

Notice that exercise 15 is intended as a challenge primarily for the faster children.

When the class has finished these exercises, allow time for further discussion and checking papers.

Assignments (page 191)
Minimum: 1–10. Average: 1–14.
Maximum: 1–15.

Follow-up

Converting quantities of ingredients from a recipe for a family to one for a large group provides a number of uses for multiplication.

Resources for Active Learning

A Cloudburst, Vol. 2, No. 3343, Midwest Publications.

PAGES 192–193

Objective

Given two fractional numbers expressed as mixed numerals, the child will be able to find their product.

Preparation

It would be helpful to review the distributive principle for whole numbers with the children. You might do this by illustrating the use of the distributive principle in finding the product 4×23. For such an example, exhibit the following equation on the chalkboard or on the overhead projector.

$4 \times (20 + 3) = (4 \times 20) + (4 \times 3)$

This should prepare the children for the steps shown on page 192, where the product is found by using mixed-numeral notation.

Let's use the distributive principle to multiply fractional numbers.

Discussing the Ideas

The examples that follow show how the **distributive principle for fractional numbers** can help you find products.

1. Explain each step of this example. See Discussion.

$3 \times 4\frac{1}{5} = 3 \times (4 + \frac{1}{5}) = (3 \times 4) + (3 \times \frac{1}{5})$

1	2	3
$4\frac{1}{5} \times 3$	$4\frac{1}{5} \times 3$	$4\frac{1}{5} \times 3$
12	$12\frac{3}{5}$	$12\frac{3}{5}$
$3 \times 4 = 12$	$3 \times \frac{1}{5} = \frac{3}{5}$	$12 + \frac{3}{5} = 12\frac{3}{5}$

Multiply whole number by whole number. Multiply fraction by whole number. Add partial products.

2. Explain each step of this example.

$\frac{1}{2} \times 4\frac{1}{5} = \frac{1}{2} \times (4 + \frac{1}{5}) = (\frac{1}{2} \times 4) + (\frac{1}{2} \times \frac{1}{5})$

1	2	3
$4\frac{1}{5} \times \frac{1}{2}$	$4\frac{1}{5} \times \frac{1}{2}$	$4\frac{1}{5} \times \frac{1}{2}$
2	$2\frac{1}{10}$	$2\frac{1}{10}$
$\frac{1}{2} \times 4 = 2$	$\frac{1}{2} \times \frac{1}{5} = \frac{1}{10}$	$2 + \frac{1}{10} = 2\frac{1}{10}$

Multiply whole number by fraction. Multiply fraction by fraction. Add partial products.

3. The products found in exercises 1 and 2 will help you find the product below. Explain each step of this example.

$3\frac{1}{2} \times 4\frac{1}{5} = (3 + \frac{1}{2}) \times 4\frac{1}{5} = (3 \times 4\frac{1}{5}) + (\frac{1}{2} \times 4\frac{1}{5})$

1	2	3
$4\frac{1}{5} \times 3\frac{1}{2}$	$4\frac{1}{5} \times 3\frac{1}{2}$	$4\frac{1}{5} \times 3\frac{1}{2}$
$12\frac{3}{5}$	$12\frac{3}{5}$	$12\frac{3}{5}$
	$2\frac{1}{10}$	$2\frac{1}{10}$
		$14\frac{7}{10}$
$3 \times 4\frac{1}{5} = 12\frac{3}{5}$	$\frac{1}{2} \times 4\frac{1}{5} = 2\frac{1}{10}$	$12\frac{3}{5} + 2\frac{1}{10} = 14\frac{7}{10}$

Multiply whole number and fraction by whole number. Multiply whole number and fraction by fraction. Add partial products.



Discussion

Remind the children that the distributive principle relates multiplication to addition. As you work through the exercises and children explain them, call attention to the equation at the top of each box. Also, use the color coding in each problem to help you point out each step. Be sure the children see the relationship of exercises 1 and 2 to exercise 3.

Using the Ideas

1. Solve the equations.

A $5 \times 3\frac{1}{4} = (5 \times 3) + (5 \times n)$ $\frac{1}{4}$

B $6 \times 2\frac{1}{5} = (6 \times n) + (6 \times \frac{1}{5})$ 2

C $8 \times 5\frac{2}{3} = (n \times 5) + (8 \times \frac{2}{3})$ 8

D $4\frac{1}{6} \times 3 = (4 \times n) + (\frac{1}{6} \times 3)$ 3

E $\frac{1}{2} \times 3\frac{1}{7} = (\frac{1}{2} \times n) + (\frac{1}{2} \times \frac{1}{7})$ 3

F $\frac{3}{5} \times 5\frac{2}{3} = (\frac{3}{5} \times 5) + (\frac{3}{5} \times n)$ $\frac{2}{3}$

2. Use the distributive principle to find the products.

A $5 \times 3\frac{1}{4}$ $16\frac{1}{4}$ B $6 \times 2\frac{1}{5}$ $13\frac{1}{5}$ C $8 \times 5\frac{2}{3}$ $45\frac{1}{3}$ D $\frac{1}{2} \times 3\frac{1}{7}$ $1\frac{4}{7}$ E $\frac{3}{5} \times 5\frac{2}{3}$ $3\frac{2}{5}$

F $3\frac{1}{8} \times 5 = 15\frac{5}{8}$ G $9\frac{1}{4} \times \frac{1}{3} = 3\frac{1}{12}$ H $53\frac{1}{2} \times 9 = 481\frac{1}{2}$ I $3\frac{1}{4} \times 2\frac{1}{3} = 7\frac{7}{12}$ J $5\frac{2}{3} \times 3\frac{1}{4} = 18\frac{5}{12}$

3. Find the products below by first replacing each mixed numeral with an improper fraction as in the example.

$1\frac{2}{3} \times 2\frac{1}{2} = 4\frac{1}{6}$

$\frac{5}{3} \times \frac{5}{2} = \frac{25}{6}$

A $1\frac{1}{2} \times 1\frac{1}{3}$ $\frac{12}{6}$

B $2\frac{1}{2} \times 1\frac{1}{4}$ $\frac{25}{8}$

C $3\frac{1}{2} \times 1\frac{3}{4}$ $\frac{49}{8}$

D $3\frac{1}{5} \times \frac{2}{1}$ $\frac{32}{5}$

E $\frac{3}{4} \times 2\frac{1}{2}$ $\frac{15}{8}$

F $1\frac{1}{8} \times 2\frac{1}{3}$ $\frac{63}{24}$

G $\frac{3}{4} \times 3\frac{2}{3}$ $\frac{33}{12}$

H $\frac{1}{6} \times 1\frac{2}{5}$ $\frac{7}{30}$

I $3\frac{1}{2} \times 2\frac{1}{7}$ $\frac{105}{14}$

J $4\frac{1}{3} \times 5\frac{1}{2}$ $\frac{143}{6}$

K $8 \times 4\frac{1}{4}$ $\frac{136}{4}$

L $1\frac{1}{6} \times 3\frac{2}{3}$ $\frac{77}{18}$

M $2\frac{1}{3} \times 5\frac{1}{3}$ $\frac{112}{9}$

N $1\frac{1}{10} \times 2\frac{1}{5}$ $\frac{121}{50}$

O $\frac{5}{2} \times 3\frac{3}{4}$ $\frac{75}{8}$

P $6\frac{1}{2} \times 1\frac{1}{7}$ $\frac{104}{14}$

(Mixed numeral equivalents are also acceptable.)

think

First odd number
$1 = 1$

First 2 odd numbers
$1 + 3 = 4$

First 3 odd numbers
$1 + 3 + 5 = 9$

First 4 odd numbers
$1 + 3 + 5 + 7 = 16$

What is the sum of the first 100 odd numbers? 10,000

★ **4.** Ned drew this shortcut for finding products like those in exercise 3. Explain Ned's shortcut.

$4 \quad 1\frac{1}{5}$

$\times 3 \quad \frac{1}{2}$

See Using the Exercises.

More practice, page A-15, Set 29



Using the Exercises

Encourage children to do most of the exercises on page 193 independently. Although exercise 4 is starred, it would be helpful to have someone show the shortcut on the chalkboard.

$4\frac{1}{5}$

$\times 3\frac{1}{2}$

$12\frac{3}{5} = (3 \times 4) + (3 \times \frac{1}{5})$

$2\frac{1}{10} = (\frac{1}{2} \times \frac{1}{5}) + (\frac{1}{2} \times 4)$

$14\frac{7}{10}$

Point out that Ned was able to do most of the multiplying mentally. You might present the same example in this manner:

$3\frac{1}{2} \times 4\frac{1}{5} = n$

$\frac{7}{2} \times \frac{21}{5} = \frac{147}{10} = 14\frac{7}{10}$

Explain to the children that in using this method they simply change each mixed numeral to an improper fraction and then follow the procedure for multiplying fractional numbers. Work through other examples according to the need of the children.

The solution to the *Think* problem depends upon the children's recognizing the pattern: the sum of the first **n** odd integers is $\mathbf{n}^2$.

Assignments (page 193)

Minimum: 1–3L. Average: 1–3. Maximum: 1–4.

Mathematics

The distributive principle for multiplication over addition for fractional numbers holds that:

> For all fractional numbers a, b, and c,
> $a \times (b + c) = (a \times b) + (a \times c)$.

The following example shows how the distributive principle may be used to find the product of two fractional numbers.

$$3 \times 4\frac{1}{5} = 3 \times (4 + \frac{1}{5})$$
$$= (3 \times 4) + (3 \times \frac{1}{5})$$

Follow-up

Some children might be interested in improving their skills in multiplying whole numbers and mixed numerals. The distributive principle can be very helpful in finding certain special products. For example:

$$8 \times 3\frac{1}{2} = (8 \times 3) + (8 \times \frac{1}{2})$$
$$= 24 + 4 = 28$$
$$6 \times 5\frac{1}{3} = (6 \times 5) + (6 \times \frac{1}{3})$$
$$= 30 + 2 = 32$$

Obviously, success in working such problems mentally depends upon the child's ability to find the products $8 \times \frac{1}{2}$ and $6 \times \frac{1}{3}$ without pencil and paper. Here are some other sample problems the children could use for practice.

1. $10 \times 4\frac{1}{5}$
2. $9 \times 2\frac{1}{3}$
3. $8 \times 7\frac{1}{4}$
4. $12 \times 1\frac{1}{6}$
5. $100 \times 2\frac{1}{10}$
6. $15 \times 10\frac{1}{5}$
7. $6 \times 4\frac{1}{2}$
8. $4 \times 7\frac{1}{2}$
9. $14 \times 2\frac{1}{7}$
10. $20 \times 6\frac{1}{5}$
11. $16 \times 2\frac{1}{8}$
12. $9 \times 5\frac{1}{9}$
13. $25 \times 4\frac{1}{5}$
14. $6 \times 1\frac{2}{3}$

Answers:

1. 42
2. 21
3. 58
4. 14
5. 210
6. 153
7. 27
8. 30
9. 30
10. 124
11. 34
12. 46
13. 105
14. 10

Duplicator Masters, pages 35, 36
Workbook, page 59
Skill Masters, pages 35, 36

PAGES 194–195

Objective

Given a fractional number other than zero, the child will be able to find its reciprocal.

Preparation

Review any of the skills children have had difficulty developing. You might want to have them practice changing fractions to lowest terms, or working some multiplication equations. If you feel such a review is not necessary, begin immediately with the investigation.

Investigation

Have the children observe that the function machine rule is not given and that they are to figure out what to do by studying the input-output cards printed by the machine. If they have trouble getting started, you might suggest several input numbers for which the children could try to give the appropriate outputs, such as:

$\frac{1}{5}$	
$\frac{5}{6}$	
$\frac{2}{7}$	
9	
6	
$\frac{7}{9}$	

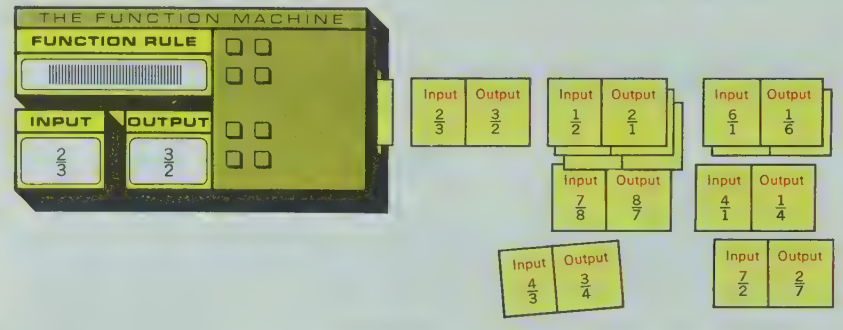
$\frac{4}{5}$	
$\frac{2}{9}$	
$\frac{1}{10}$	
10	
8	
$\frac{1}{3}$	

The children should have no difficulty with the input numbers that are expressed as simple fractions. If any have difficulty when the input is a whole number, encourage them to study the input-output cards again until they can find a model to follow. Note that they are not asked in the investigation to state the rule, but only to follow a pattern and make other input-output cards.

● *Let's explore reciprocals.*

Investigating the Ideas

For each input number the function machine finds the output number and records the input and output on cards like these.



? Can you give some more input-output numbers that this function machine might produce? See Investigation.

Discussing the Ideas

1. Can you describe the rule the function machine is using? See Discussion.
2. What is the product of each of the input-output pairs on the cards? 1 See Discussion.
3. When the product of two numbers is 1, we say that each of the numbers is a **reciprocal** of the other. Does the number **0** have a reciprocal? Explain. No. See Discussion.

F F P
$\frac{5}{1} \times \frac{1}{5} = \frac{5}{5} = 1$

F F P
$\frac{3}{4} \times \frac{4}{3} = \frac{12}{12} = 1$

4. What is the reciprocal of each of these numbers?
   A $\frac{4}{5}$ $\frac{5}{4}$ B $\frac{1}{2}$ $\frac{2}{1}$ C $\frac{3}{13}$ $\frac{13}{3}$ D $7\ \frac{1}{7}$ E $1\ 1$ F $\frac{11}{5}$ $\frac{5}{11}$ G $6\frac{1}{3}$ $\frac{3}{19}$

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Discussion

Ask several children to give an example of one of the input-output cards which they made. When you ask them to respond to discussion exercise 1, accept any answer which yields the correct result. For instance, if some children respond that the rule is to turn the input number "upside down" accept this even though the word choice is awkward. Actually, the rule is "give the reciprocal of the input number," but for now you might speak of exchanging the positions of the numerator and denominator.

Have children multiply each pair of numbers shown on the cards. They should discover that all have the same product, 1. Stress this as the characteristic of these pairs, emphasizing the statement and examples given in exercise 3. Also emphasize that a *reciprocal* expresses a relationship between two factors: each factor is said to be a reciprocal of the other. We say that "$\frac{3}{4}$ *is the reciprocal of* $\frac{4}{3}$," and "$\frac{4}{3}$ *is the reciprocal of* $\frac{3}{4}$." Zero has no reciprocal because there is no number by which zero can be multiplied to give the product 1.

For exercise 4D, be sure children understand that since 7 may be thought of as $\frac{7}{1}$, the reciprocal of 7 is $\frac{1}{7}$.

Using the Ideas

1. Give the reciprocal of each number.

A $\frac{3}{5}$ $\frac{5}{3}$ B $\frac{7}{4}$ $\frac{4}{7}$ C $6\frac{1}{6}$ D $\frac{1}{6}$ 6 E 1 1 F $2\frac{1}{4}$ $\frac{4}{9}$ G $\frac{9}{10}$ $\frac{10}{9}$

2. Find the products.

A $\frac{5}{8} \times \frac{8}{5}$ 1 B $\frac{6}{1} \times \frac{1}{6}$ 1 C $\frac{9}{2} \times \frac{2}{9}$ 1 D $4 \times \frac{1}{4}$ 1 E $\frac{1}{8} \times 8$ 1

3. The missing factor is the reciprocal of the factor given. Find the missing factor.

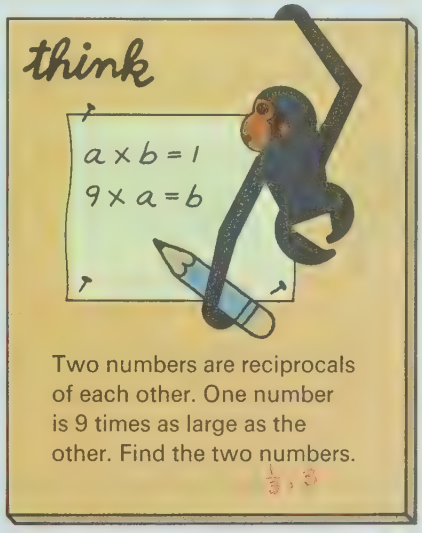
A $\frac{n}{\frac{3}{2}} \times \frac{2}{3} = 1$ B $\frac{n}{3} \times \frac{1}{3} = 1$ C $\frac{5}{\frac{6}{5}} \times n = 1$ D $\frac{1}{7} \times \frac{n}{7} = 1$ E $2\frac{1}{2} \times \frac{n}{\frac{5}{2}} = 1$

4. Give the number for the ▥. Then give the product *n*.

A $12 \times \frac{1}{4} = ▥^{3} \times (4 \times \frac{1}{4}) = n^{3}$
B $15 \times \frac{1}{3} = 5 \times (▥^{10} \times \frac{1}{3}) = n^{4}$
C $40 \times \frac{1}{10} = 4 \times (▥^{4} \times \frac{1}{10}) = n^{4}$
D $20 \times \frac{1}{5} = ▥^{4} \times (5 \times \frac{1}{5}) = n^{4}$
E $18 \times \frac{1}{6} = 3 \times (▥^{6} \times \frac{1}{6}) = n^{3}$
F $27 \times \frac{1}{3} = ▥^{9} \times (3 \times \frac{1}{3}) = n^{9}$
G $56 \times \frac{1}{7} = 8 \times (▥^{7} \times \frac{1}{7}) = n^{8}$
H $72 \times \frac{1}{9} = ▥^{8} \times (9 \times \frac{1}{9}) = n^{8}$

5. Give the number for *a*. Then give the number for *b*.

A $5 \times \frac{1}{5} = a^{1} \rightarrow 10 \times \frac{1}{5} = b^{2}$
B $7 \times \frac{1}{7} = a^{1} \rightarrow 14 \times \frac{1}{7} = b^{2}$
C $3 \times \frac{1}{3} = a^{1} \rightarrow 9 \times \frac{1}{3} = b^{3}$
D $6 \times \frac{1}{6} = a^{1} \rightarrow 24 \times \frac{1}{6} = b^{4}$
E $9 \times \frac{1}{9} = a^{1} \rightarrow 9 \times \frac{2}{9} = b^{2}$
F $10 \times \frac{1}{10} = a^{1} \rightarrow 10 \times \frac{2}{10} = b^{2}$
G $4 \times \frac{1}{4} = a^{1} \rightarrow 4 \times \frac{3}{4} = b^{3}$
H $5 \times \frac{3}{5} = a^{3} \rightarrow 10 \times \frac{3}{5} = b^{6}$
I $7 \times \frac{4}{7} = a^{4} \rightarrow 14 \times \frac{4}{7} = b^{8}$
J $5 \times \frac{3}{5} = a^{3} \rightarrow 20 \times \frac{3}{5} = b^{12}$
K $10 \times \frac{7}{10} = a^{7} \rightarrow 30 \times \frac{7}{10} = b^{21}$



Two numbers are reciprocals of each other. One number is 9 times as large as the other. Find the two numbers.

$\frac{1}{3}$, 3



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Mathematics

Two fractional numbers $\frac{a}{b}$ and $\frac{c}{d}$ are reciprocals of one another, if, and only if,

$$\frac{a}{b} \times \frac{c}{d} = 1.$$

Another name that is sometimes used instead of reciprocal is *multiplicative inverse*.

The fact that each non-zero fractional number has a unique multiplicative inverse, or reciprocal, is an important feature which distinguishes the set of whole numbers from the set of fractional numbers. The quotient of two whole numbers is not necessarily a whole number. For example, $2 \div 3$ is not a whole number. But, applying the known fact that every fractional number other than zero has a reciprocal, we can show that the quotient of two fractional numbers is also a fractional number. In other words, the set of fractional numbers is *closed* with respect to division, while the set of whole numbers is not.

Workbook, page 60

Using the Exercises

Exercises 1, 2, and 3 on page 195 emphasize the basic meaning of the reciprocal relationship. In exercise 4, children must recognize that the whole number has been broken apart into two factors, one of which is the reciprocal of the original fractional factor: $12 \times \frac{1}{4} = (3 \times 4) \times \frac{1}{4} = 3$. Some children may discover that the final product is the same as the missing factor of the whole number.

In exercise 5, note that the children must do some reasoning involving products of reciprocals. In exercise 5A, the children first find that $5 \times \frac{1}{5}$ is 1, and should reason from this that $10 \times \frac{1}{5}$ is 2 because 10 is 2×5. This type of thinking is necessary throughout the exercise set. Encourage the children to discuss their reasoning in finding product *b* in each case.

The *Think* problem will probably be solved by trial and error even by more capable children, but most of the children will understand an explanation of the solution.

Assignments (page 195)
Minimum: 1–3. Average: 1–5.
Maximum: 1–5.

PAGES 196–197

Objective

Given fractional numbers to multiply, the child will first divide out any factors common to the numerators and denominators and then find the product.

Preparation

To prepare for this lesson, you might simply review multiplication with proper fractions. For example, write the following equations on the chalkboard and ask children to find the products.

$\frac{3}{4} \times \frac{1}{5} = ?$ $\quad \frac{4}{7} \times \frac{1}{5} = ?$

$\frac{7}{8} \times \frac{1}{3} = ?$ $\quad \frac{6}{7} \times \frac{3}{5} = ?$

$\frac{5}{7} \times \frac{2}{9} = ?$ $\quad \frac{1}{2} \times \frac{2}{3} = ?$

$\frac{1}{3} \times \frac{8}{11} = ?$ $\quad \frac{4}{5} \times \frac{15}{16} = ?$

● *Let's explore a shortcut for multiplying.*

Discussing the Ideas

1. Can you explain why the products in both examples A and B are the same? What are the products?
   $\frac{20}{120}$ or $\frac{1}{6}$

   A $\frac{5}{15} \times \frac{4}{8}$
   B $\frac{5}{8} \times \frac{4}{15}$

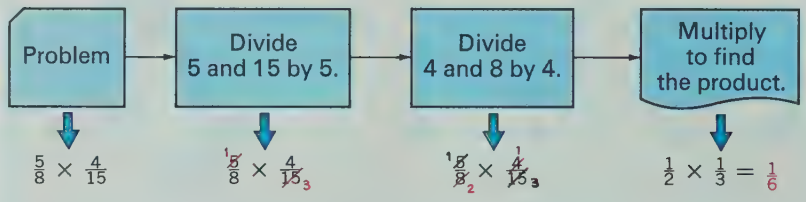
2. Jody said that example A above would be easier if the fractions were simplified before multiplying. Give the lowest-terms fractions for ***a*** and ***b*** and the product ***c***.

   A $\frac{5}{15} \times \frac{4}{8} = \overset{\frac{1}{3}}{a} \times \overset{\frac{1}{2}}{b} = c\ \frac{1}{6}$

3. Brian said, "I can do example B by simplifying before I multiply." Can you explain Brian's method?
   See exercise 4.

   B $\frac{\cancel{5}^{1}}{\cancel{8}_{2}} \times \frac{\cancel{4}^{1}}{\cancel{15}_{3}} = \frac{1}{6}$

4. Study the flow chart to understand Brian's shortcut.



Try the shortcut for these problems.

A $\frac{2}{3} \times \frac{3}{4}$ $\frac{1}{2}$ $\quad$ B $12 \times \frac{2}{3}$ 8 $\quad$ C $\frac{8}{27} \times \frac{9}{20}$ $\frac{2}{15}$

5. Can the shortcut be used to find the product $\frac{2}{3} \times \frac{5}{7}$? No
   What is the product? $\frac{10}{21}$

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Discussion

Allow children an opportunity to figure out examples A and B in discussion exercise 1. During their discussion, you might write the products in the following manner.

$$\text{A} \quad \frac{5}{15} \times \frac{4}{8} = \frac{5 \times 4}{8 \times 15}$$

$$\text{B} \quad \frac{5}{8} \times \frac{4}{15} = \frac{5 \times 4}{8 \times 15}$$

Ask children to point out the difference between the denominators and ask if they can explain why 8×15 is the same as 15×8 (commutative principle). As you work through exercises 2 and 3, stress that in exercise 2, we can think of $\frac{4}{8}$ and $\frac{5}{15}$ and simplify the fractions accordingly. Be sure children see that the problem then becomes $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.

As you ask children to explain each step of the flow chart in exercise 4, be sure they realize that both numerator and denominator were divided by 5 because 5 is a common factor of 5 and 15, and by 4 because 4 is a common factor of 4 and 8. Present other examples and have children work through several of them as needed.

During the development of this lesson, we urge you to avoid using the traditional term "cancellation." Instead, say, "*divide out* common factors."

Using the Ideas

1. Find the products. You can use the shortcut discussed on the previous page to make your work easier.

A	$\frac{4}{9} \times \frac{3}{8}$ $\frac{1}{6}$	E	$\frac{5}{6} \times \frac{12}{5}$ 2	I	$\frac{4}{15} \times \frac{10}{28}$ $\frac{2}{21}$	M	$\frac{27}{32} \times \frac{4}{9}$ $\frac{3}{8}$
B	$\frac{5}{12} \times \frac{3}{10}$ $\frac{1}{8}$	F	$\frac{3}{4} \times \frac{20}{27}$ $\frac{5}{9}$	J	$\frac{7}{12} \times \frac{8}{21}$ $\frac{2}{9}$	N	$\frac{4}{5} \times \frac{15}{16}$ $\frac{3}{4}$
C	$\frac{6}{5} \times \frac{15}{18}$ 1	G	$\frac{6}{9} \times \frac{18}{36}$ $\frac{1}{3}$	K	$\frac{9}{10} \times \frac{25}{21}$ $\frac{15}{14}$	O	$\frac{18}{25} \times \frac{15}{81}$ $\frac{2}{15}$
D	$\frac{3}{8} \times \frac{8}{9}$ $\frac{1}{3}$	H	$\frac{5}{8} \times \frac{6}{15}$ $\frac{1}{4}$	L	$\frac{4}{5} \times \frac{15}{8}$ $\frac{3}{2}$	P	$\frac{30}{33} \times \frac{11}{20}$ $\frac{1}{2}$

2. Find the products. Use the easiest possible method.

A	$24 \times \frac{5}{24}$ 5	C	$100 \times \frac{3}{100}$ 3	E	$10 \times \frac{7}{100}$ $\frac{7}{10}$	G	$24 \times \frac{5}{6}$ 20	I	$\frac{7}{8} \times \frac{12}{14}$ $\frac{3}{4}$
B	$10 \times \frac{3}{10}$ 3	D	$100 \times \frac{3}{10}$ 30	F	$\frac{5}{10} \times \frac{20}{100}$ $\frac{1}{10}$	H	$\frac{9}{15} \times \frac{7}{9}$ $\frac{7}{15}$	J	$\frac{7}{10} \times \frac{100}{21}$ $\frac{10}{3}$

Complete the following function tables.

3.

Function Rule: $\frac{3}{4} \times n$

	n	$f(n)$
A	$\frac{4}{27}$	$\frac{1}{9}$
B	$\frac{20}{21}$	$\frac{5}{7}$
C	24	18
D	$\frac{4}{3}$	1
E	$2\frac{2}{3}$	2

4.

Function Rule: $n \times \frac{2}{3}$

	n	$f(n)$
A	$\frac{6}{5}$	$\frac{4}{5}$
B	0	0
C	9	6
D	$6\frac{3}{4}$	$4\frac{1}{2}$
E	$\frac{3}{2}$	1

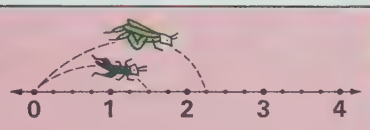
5.

Function Rule: $3\frac{1}{4} \times n$

	n	$f(n)$
A	8	26
B	$\frac{1}{3}$	$1\frac{1}{12}$
C	$5\frac{5}{8}$	$18\frac{9}{32}$
D	$8\frac{4}{5}$	$28\frac{3}{5}$
E	3	$9\frac{3}{4}$

think

The picture shows the sizes of the jumps as a grasshopper and a cricket jump along a number line.



Grasshopper: 7 jumps.
Cricket: 11 jumps.
How far apart are they?

Can you find 3 different points on which both insects will land?

See Solution, T.E. page 197.



More practice, page A-15, Set 30

Using the Exercises

Encourage the children to use the shortcut method as they work the exercises on page 197. Point out that, if they do not divide out a factor before they multiply, they will have to do so in expressing the product in lowest terms. The shortcut enables them to simplify before they multiply.

Although exercises 3, 4, and 5 are presented as function tables, they provide further opportunity to use the same shortcut in multiplying.

Solution, Think, page 197

Grasshopper jumps 7: $7 \times 2\frac{1}{4} = 15\frac{3}{4}$
Cricket jumps 11: $11 \times 1\frac{1}{2} = 16\frac{1}{2}$
Difference: $16\frac{1}{2} - 15\frac{3}{4} = \frac{3}{4}$
Points each will land on:
Grasshopper: $2\frac{1}{4}$, $4\frac{1}{2}$, $6\frac{3}{4}$, 9, $11\frac{1}{4}$, $13\frac{1}{2}$
Cricket: $1\frac{1}{2}$, 3, $4\frac{1}{2}$, 6, $7\frac{1}{2}$, 9, $10\frac{1}{2}$, 12, $13\frac{1}{2}$
Points both will land on: $4\frac{1}{2}$, 9, $13\frac{1}{2}$

Assignments (page 197)*
Minimum: 1. Average: 1–5.
Maximum: 1–5.

Mathematics

In this lesson, a topic traditionally known in elementary arithmetic as "cancellation" is developed through use of an interesting theorem, stated as follows:

$$\frac{a}{b} \times \frac{c}{d} = \frac{a}{d} \times \frac{c}{b}$$

If, in working with products of fractional numbers, we note that the denominators are interchangeable, we can develop the idea that cancellation means simply that if the numerator of one fraction and the denominator of the other fraction have a common factor, we can divide out this common factor by simply thinking of the denominators as being reversed. The examples on page 196 illustrate this idea. For instance, in considering the fractions $\frac{5}{8}$ and $\frac{4}{15}$, note that we have a factor of 5 in the numerator of the first fraction and in the denominator of the second fraction. So, by using the theorem just stated, we are able to think of this product as $\frac{5}{15} \times \frac{4}{8}$ rather than as $\frac{5}{8} \times \frac{4}{15}$. When we think about the product in this new way, we can simplify the fractions $\frac{5}{15}$ and $\frac{4}{8}$ so that they are in lowest terms. To shorten this involved process, we can use the steps in exercise 4 to "divide out" the common factor, 5, from the numerator of the first fraction and the denominator of the second fraction. In the same way, the common factor 4 can be divided out.

Duplicator Masters, page 37
Workbook, page 61
Skill Masters, page 37

PAGES 198–199

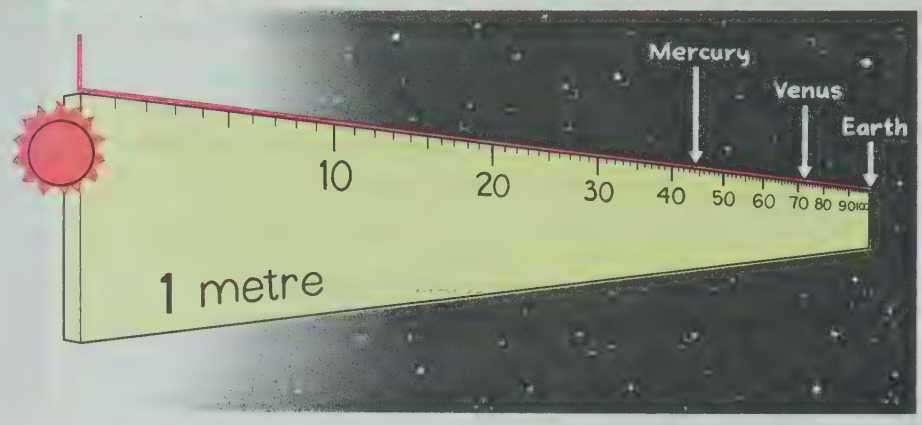
Objective

Given a scale, related information with fractional numbers, and word problems, the child will be able to solve the word problems by applying his skills in working with fractions and his understanding of scale measurements.

Preparation

To prepare for this lesson, you might conduct a short practice session to review multiplication with mixed numerals. You might also give children some oral problems such as, "If Tim has 6 dollars and Sadie has $2\frac{1}{2}$ times as much, how much money does Sadie have?" ($2\frac{1}{2} \times 6 = \$15$) Or, "If Ted hiked 4 km and Jim hiked $1\frac{1}{3}$ times as far, how far did Jim hike?" ($4 \times 1\frac{1}{3} = 5\frac{1}{3}$). A few problems of this kind should help the children interpret the chart on page 198.

OUR SOLAR SYSTEM



To give an idea of the size of our solar system, we will let one metre (100 cm) represent the distance from Earth to the sun.

Scale: 1 metre = 150 million kilometres
or 1 cm = 1.5 million kilometres

On this scale the sun would be about the size shown above. Mercury, the closest planet, would be a speck about 44 centimetres away from the sun. The table below gives the distances of the planets from the sun.

Planet	Average distance (kilometres) from the sun
Mercury	66 000 000
Venus	108 000 000
Earth	150 000 000
Mars	228 000 000
Jupiter	778 000 000
Saturn	1 427 000 000
Uranus	2 870 000 000
Neptune	4 500 000 000
Pluto	5 909 000 000

198

Discussion

This lesson might be handled in a variety of ways. Since it is a problem set with much material for discussion, you might have children work in groups of three, four, or five. Suggest that they carefully study the material on page 198, and then share their ideas on how to solve the problems on page 199. Finally, they might work the problems independently, but check with each other to see if they are proceeding correctly. When they have worked for a considerable amount of time, check answers and discuss any points with which children had difficulty.

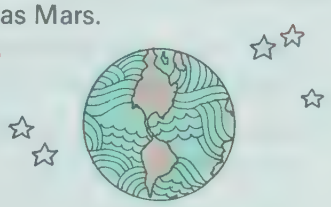
If you prefer, use page 198 as a basis for class discussion. The children should already have some ideas about scale drawings and should be able to understand the given scale. Point out that Earth is 150 000 000 km from the sun and that this is indicated by showing, in perspective, a metrestick between the sun and Earth.

1. Use the scale and the table on page 198 for these exercises.

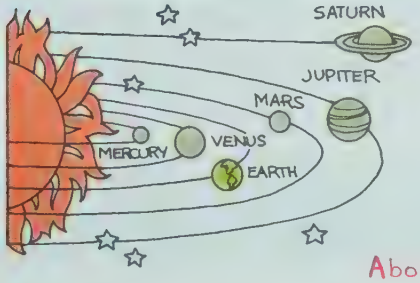


- A Is Earth closer to Venus or to Mars ? How much closer ? Venus; 36 000 000
- B On a scale where 1 centimetre = 1 500 000 km, about how many centimetres should Venus be shown from the sun ? about 72 cm
- C Which planet would be nearly 10 metres from the sun ? Saturn
- D How many times as far from the sun is the planet Pluto than Earth ? About 40 times as far
- E Using the scale on page 198, how many kilometres would Pluto be from the sun ? .04 or $\frac{1}{25}$
- F How much farther is Uranus from Neptune than from Saturn ? 187 000 000

2. Venus is about the same size as Earth. Mars is $\frac{1}{7}$ as large (in volume) as Earth. Mercury is $\frac{7}{16}$ as large as Mars. What fraction of Earth's size is Mercury ? $\frac{1}{16}$



3. Saturn is 730 times as large as Earth. Neptune is about $\frac{1}{12}$ as large as Saturn. About how many times as large as Earth is Neptune ? About 61 times as large



4. Some astronomers believe there is a tenth planet, Planet X, as yet undiscovered. In the scale drawing on page 198, this planet would be about 60 metres from the sun. About how many kilometres is Planet X from the sun ? About 9 000 000 000 km

Using the Exercises

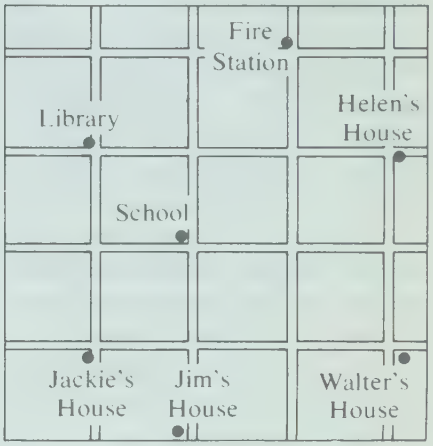
As suggested in the discussion section, encourage children to work these problems together in groups. Some of these questions simply require straightforward use of the basic operations; others require understanding of scales. For example, questions such as B, C, and E of exercise 1 might require further discussion of the scale shown on page 198.

Assignments (page 199)
Minimum: 1–4, oral. Average: 1–4. Maximum: 1–4.

Follow-up

Let the class work on a map like that below to provide drill on addition, subtraction, and multiplication of fractional numbers as well as review the use of scale in drawing.

More capable children may wish to draw this map to a larger scale or make one of their own neighborhood and create original problems about it.



Scale: 1 cm = 1 km

1. Helen asked Jackie over to listen to records after school. How far will Jackie have to walk home from Helen's house?
2. Walter rode his bike to the library and back. How far did he ride?
3. If Walter stopped off to see Jackie's brother on the way home from the library, would he ride farther than if he came home past the school?
4. Jim's father is a volunteer fireman. How far does he have to go to reach the fire station?

Resources for Active Learning

Applied Mathematics Cards, Group 2/25; Group 3/6, 24–25, Schofield and Sims. (Available from Mafex Associates)

Developmental Math Cards, $J^{1}8$, $K^{3}7$, $K^{2}17$, Addison-Wesley.

Math Activity Cards, D10–11; E9, Macmillan.

Mathex: Graphing and Probability No. 6, p. 7 (pupil pages 15–22), Encyclopaedia Britannica Publications Ltd.

Measure and Find Out, Book 3, "Some Earth Science," pp. 5–36, Scott Foresman. (Available from Gage Educational Publishing)

PAGES 200–201

Objectives

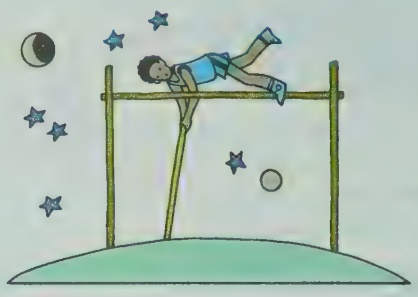
Given word problems concerning a chart of information, the child will be able to solve the problems by applying his skills and understanding of fractional numbers.

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

You may find it helpful to provide the children with a short practice session on the various algorithms for fractional-number computation covered so far. Follow this by reviewing any of the topics in the text that the children have found particularly difficult. You might relate this review work to the exercises on page 201. If you feel that such a practice session is unnecessary, have children begin immediately with the text material.

TRACK RECORDS IN SPACE???



Each planet in our solar system pulls objects toward it with a different amount of force. In most cases, the smaller the planet, the smaller this force and the easier it would be to jump away from the surface of the planet.

Near-Record Earth Jumps	
High jump	$2\frac{1}{4}$ m
Running broad jump	$8\frac{4}{5}$ m
Pole vault	$5\frac{1}{2}$ m

Planet Jumps Compared with Earth Jumps*

A person's jump on Earth would be about:

6 times as high on the **moon**
$2\frac{3}{5}$ times as high on **Mercury**
$2\frac{1}{2}$ times as high on **Mars**
$1\frac{1}{7}$ times as high on **Venus**
$\frac{5}{6}$ times as high on **Neptune**
$\frac{5}{12}$ times as high on **Jupiter**

1. If the holder of the pole-vault record were on the moon, about how high could he vault? 33m
2. The largest planet is Jupiter. What would be a near-record pole vault on Jupiter? $2\frac{7}{24}$
3. If the pole-vault record holder were on each of the other planets listed in the table, about how high could he vault? Mercury $14\frac{3}{10}$; Mars $13\frac{3}{4}$; Venus $6\frac{2}{7}$; Neptune $4\frac{7}{12}$
4. About how far could the long-jump record holder jump on each of the planets listed in the table? Moon $52\frac{4}{5}$; Mercury $22\frac{22}{25}$; Mars 22; Venus $10\frac{2}{3}$; Neptune $7\frac{1}{3}$; Jupiter $3\frac{3}{8}$
5. About how high could the high-jump record holder jump on each of the planets listed in the table? Moon $13\frac{1}{2}$; Mercury $5\frac{17}{20}$; Mars $5\frac{5}{8}$; Venus $2\frac{4}{7}$; Neptune $1\frac{7}{8}$; Jupiter $\frac{15}{16}$
- ★ 6. If you could pole vault 8 metres on Mars, how high could you pole vault on Earth? $3\frac{1}{5}$

*For the purpose of these exercises, we assume that the multiples given in the table can be applied to the given Earth jumps.

200

Discussion

Page 200 may be treated in a variety of ways: by small groups working and discussing together; as independent work; or through class discussion. In any case, you might want to emphasize some significant points. For example, be sure children interpret the chart correctly. It gives information on other planets compared to Earth. Thus, a person would weigh only $\frac{1}{6}$ as much on the moon as he does on Earth; hence, we are assuming that he could jump six times as high on the moon as he can jump on Earth. Of course, you should point out during your discussion of these ideas that the figures used are conjectural, and it is highly improbable that the multiple is exactly six.

When the children have finished, allow time for further explanation of specific concepts or skills.

Assignments (page 200)

Minimum: 1–5, oral. Average: 1–5. Maximum: 1–6.

Keeping in Touch with Computing
Fractional numbers

1. Find the sums and differences.

A $\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$ B $\frac{5}{8} + \frac{3}{8} = 1$ C $1 - \frac{7}{100} = \frac{93}{100}$ D $1 - \frac{3}{10} = \frac{7}{10}$ E $\frac{3}{4} + 0 = \frac{3}{4}$ F $1 + \frac{9}{10} = 1\frac{9}{10}$ G $5 - \frac{1}{5} = 4\frac{4}{5}$

2. Find the sums and differences. Give your answers in lowest terms.

A $\frac{1}{5} + \frac{3}{4} = \frac{19}{20}$ B $\frac{5}{8} - \frac{1}{4} = \frac{3}{8}$ C $2\frac{1}{3} + 5\frac{1}{4} = 7\frac{7}{12}$ D $4\frac{3}{10} + 8\frac{4}{5} = 13\frac{1}{10}$ E $\frac{5}{6} - \frac{1}{7} = \frac{29}{42}$ F $9\frac{1}{5} - 4\frac{4}{10} = 4\frac{4}{5}$ G $18\frac{1}{3} + 23\frac{4}{6} = 42$ H $4\frac{1}{2} + 19\frac{3}{8} = 23\frac{7}{8}$

I $\frac{1}{5} + \frac{3}{15} = \frac{2}{5}$ J $4\frac{2}{5} + 5\frac{3}{6} = 10\frac{1}{15}$ K $5\frac{31}{50} - 4\frac{1}{100} = \frac{61}{100}$ L $6\frac{1}{10} - 4\frac{2}{5} = \frac{7}{10}$ M $32\frac{1}{4} - 8\frac{3}{4} = 23\frac{1}{2}$

3. Find the sums.

A $4\frac{1}{8} + 3\frac{1}{2} + 6\frac{1}{4} = 13\frac{7}{8}$ B $5\frac{1}{10} + 8\frac{4}{5} + 9\frac{1}{4} = 23\frac{3}{20}$ C $89\frac{3}{5} + 95\frac{7}{15} + 67\frac{2}{3} = 252\frac{11}{15}$

D $5\frac{1}{10} + 8\frac{21}{100} = 13\frac{31}{100}$ E $138\frac{7}{10} + 57\frac{4}{5} = 196\frac{1}{2}$ F $5\frac{3}{4} + 9\frac{1}{5} + 7\frac{3}{20} = 22\frac{1}{10}$ G $7\frac{1}{3} + 8\frac{1}{2} + 3\frac{5}{6} = 19\frac{2}{3}$

4. Write an improper fraction in place of each mixed numeral.

A $2\frac{1}{2} = \frac{5}{2}$ B $3\frac{1}{3} = \frac{10}{3}$ C $4\frac{1}{4} = \frac{17}{4}$ D $5\frac{2}{5} = \frac{27}{5}$ E $9\frac{1}{6} = \frac{55}{6}$ F $12\frac{3}{7} = \frac{87}{7}$ G $8\frac{5}{9} = \frac{77}{9}$ H $15\frac{3}{10} = \frac{153}{10}$

5. Write a mixed numeral in place of each improper fraction.

A $\frac{9}{2} = 4\frac{1}{2}$ B $\frac{13}{3} = 4\frac{1}{3}$ C $\frac{19}{4} = 4\frac{3}{4}$ D $\frac{39}{4} = 9\frac{3}{4}$ E $\frac{78}{5} = 15\frac{3}{5}$ F $\frac{139}{7} = 19\frac{6}{7}$ G $\frac{242}{10} = 24\frac{1}{5}$ H $\frac{167}{100} = 1\frac{67}{100}$

6. Write and solve an equation for each problem.

A Jack: $87\frac{1}{2}$ kilograms.
Jim: $96\frac{3}{4}$ kilograms.
On the scales together.
How many kilograms?
$87\frac{1}{2} + 96\frac{3}{4} = 184\frac{1}{4}$

B Gasoline tank holds $17\frac{1}{2}$ litres.
Takes $8\frac{7}{10}$ litres to fill it.
How many litres were already in the tank? $17\frac{1}{2} - 8\frac{7}{10} = 8\frac{4}{5}$



You are invited to explore

ACTIVITY CARD 10
Page 360

201

Using the Exercises

Page 201 reviews fractional number concepts and skills introduced in previous chapters. You will probably want to have the children work through the exercises independently, but reserve ample time for questions and discussions after the children have completed their work.

Follow-up

Encourage interested children to do research on the recent records for the high jump, the running broad jump, and the pole vault. Some may make assignment cards using the new records and the information given in the chart comparing planet jumps with Earth jumps.

You might spark the interest of some children by relating the story of Astronaut Alan B. Shepard, who attempted to see how far he could hit a golf ball on the surface of the moon. Shepard was one of the astronauts of the Apollo 14 mission (January 31–February 9, 1971).

Duplicator Masters, page 38
Workbook, page 62

PAGES 202–203

Objective

Given division equations and related multiplication equations with fractional numbers, the child will be able to find the missing factor by applying his understanding of the inverse relation between multiplication and division.

Preparation

To prepare for this lesson, it would be helpful to give children practice with missing factors in equations with whole numbers. For example, ask them to explain the relationship between the following equations:

$$42 \div 7 = n$$
$$n \times 7 = 42$$

Be sure it is clear to the children that both problems may be thought of as finding a missing factor.

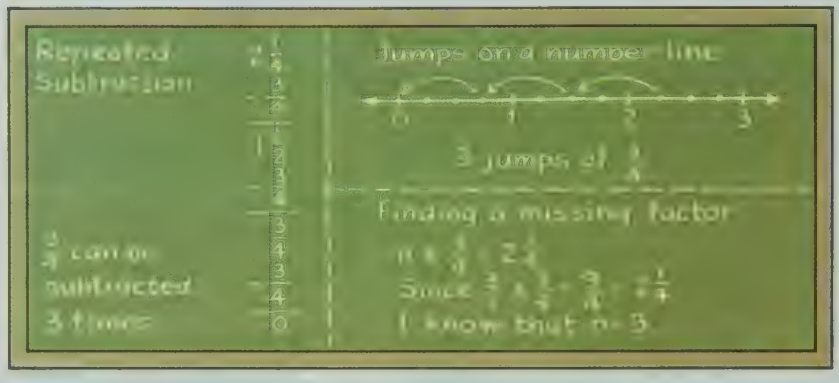
Investigation

You might want children to work in small groups for this investigation, although some might benefit more from independent work. With children who have had difficulty grasping the fractional-number skills, you might read through the three examples together, asking them to describe each one. Note that in the missing-factor method, the first choice of a missing factor might not be the correct one. If they use this method, they may have to make several attempts before the missing factor is found. Suggest that those who finish quickly try one of the other methods to see if they can get the same answer.

● *Do you understand division of fractional numbers?*

Investigating the Ideas

Here are three ways that some students used to find how many times $\frac{3}{4}$ is contained in $2\frac{1}{4}$.



? Can you use one of these methods to find how many times $\frac{5}{8}$ is contained in $2\frac{1}{2}$? 4

See Investigation.

Discussing the Ideas

1. $\frac{3}{4}$ is contained 3 times in $2\frac{1}{4}$. You can write this fact as the division equation $2\frac{1}{4} \div \frac{3}{4} = 3$. What division equation can you write to answer the Investigation question? $2\frac{1}{2} \div \frac{5}{8} = 4$

2. You can think of division as finding a missing factor. Explain how you can find the number for n in the example.

You find this quotient when you find this factor

$\frac{8}{15} \div \frac{4}{5} = n$; $n \times \frac{4}{5} = \frac{8}{15}$

Think: What number times 4 gives 8? What number times 5 gives 15?

3. Jane showed how she solved this division problem.

$\frac{8}{15} \div \frac{4}{3} = \frac{8 \div 4}{15 \div 3} = \frac{2}{5}$

A Do you think Jane's method is correct? How can you check?

Yes

B Would this method work for $\frac{3}{8} \div \frac{2}{5}$? Explain.

No, because 2 and 5 are not factors of 3 and 8. See Discussion.

Discussion

As you discuss exercise 1, it would be helpful to have children show their methods on the chalkboard. Be sure children relate their methods to the division equation $2\frac{1}{4} \div \frac{3}{4}$. Although it is important for children to see the validity of each method suggested in the investigation, the main emphasis of this lesson is on the inverse relation between multiplication and division, that is, on the missing factor method. This should be emphasized in discussing exercise 2.

Exercise 3 is designed to foster discussion concerning a particular type of division problem in which the quotient can be found by "dividing numerator by numerator and dividing denominator by denominator." Although part B shows that this method does not appear feasible in all cases, the following lesson will show a way to adapt the method so that it will apply to any division problem.

Using the Ideas

1. For each multiplication equation, write two division equations.
Example: $\frac{2}{3} \times \frac{1}{5} = \frac{2}{15} \rightarrow \frac{2}{15} \div \frac{1}{5} = \frac{2}{3}$; $\frac{2}{15} \div \frac{2}{3} = \frac{1}{5}$

A $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$ $\frac{3}{8} \div \frac{1}{2} = \frac{3}{4}$; $\frac{3}{8} \div \frac{3}{4} = \frac{1}{2}$ B $\frac{1}{4} \times \frac{3}{4} = \frac{3}{16}$ $\frac{3}{16} \div \frac{1}{4} = \frac{3}{4}$; $\frac{3}{16} \div \frac{3}{4} = \frac{1}{4}$ C $3 \times \frac{2}{5} = \frac{6}{5}$
$\frac{6}{5} \div 3 = \frac{2}{5}$; $\frac{6}{5} \div \frac{2}{5} = 3$

D $\frac{2}{3} \times \frac{3}{2} = 1$ $1 \div \frac{3}{2} = \frac{2}{3}$; $1 \div \frac{2}{3} = \frac{3}{2}$ E $\frac{1}{3} \times 5 = \frac{5}{3}$ $\frac{5}{3} \div \frac{1}{3} = 5$; $\frac{5}{3} \div 5 = \frac{1}{3}$ F $\frac{1}{2} \times \frac{4}{5} = \frac{4}{10}$
$\frac{4}{10} \div \frac{4}{5} = \frac{1}{2}$; $\frac{4}{10} \div \frac{1}{2} = \frac{4}{5}$

2. In each exercise, when you find the **missing factor** in the first equation, you will have found the **quotient** in the second equation. Write the second equation with the correct quotient.

A $n \times \frac{1}{3} = \frac{1}{6}$ B $n \times \frac{1}{4} = \frac{1}{12}$ C $n \times \frac{3}{4} = \frac{9}{4}$ D $n \times \frac{2}{5} = \frac{14}{5}$

$\frac{1}{6} \div \frac{1}{3} = n$ $\frac{1}{2}$ $\frac{1}{12} \div \frac{1}{4} = n$ $\frac{1}{3}$ $\frac{9}{4} \div \frac{3}{4} = n$ 3 $\frac{14}{5} \div \frac{2}{5} = n$ 7

E $n \times \frac{3}{8} = 1$ F $n \times \frac{5}{3} = \frac{10}{27}$ G $n \times \frac{7}{5} = 1$ H $n \times 1 = \frac{4}{13}$

$1 \div \frac{3}{8} = n$ $\frac{8}{3}$ $\frac{10}{27} \div \frac{5}{3} = n$ $\frac{2}{9}$ $1 \div \frac{7}{5} = n$ $\frac{5}{7}$ $\frac{4}{13} \div 1 = n$ $\frac{4}{13}$

I $n \times \frac{1}{9} = \frac{5}{18}$ J $n \times \frac{6}{1} = \frac{12}{13}$ K $n \times \frac{1}{9} = 1$ L $n \times \frac{4}{7} = \frac{12}{14}$

$\frac{5}{18} \div \frac{1}{9} = n$ $\frac{5}{2}$ $\frac{12}{13} \div \frac{6}{1} = n$ $\frac{2}{13}$ $1 \div \frac{1}{9} = n$ 9 $\frac{12}{14} \div \frac{4}{7} = n$ $\frac{3}{2}$

3. Find each quotient. Check your work by multiplying.

A $\frac{10}{3} \div \frac{2}{3}$ 5
B $\frac{9}{16} \div \frac{3}{4}$ $\frac{3}{4}$
C $\frac{3}{8} \div \frac{1}{4}$ $\frac{3}{2}$
D $\frac{20}{9} \div \frac{4}{3}$ $\frac{5}{3}$
E $\frac{8}{21} \div \frac{2}{3}$ $\frac{4}{7}$
F $\frac{3 \times 5}{4 \times 7} \div \frac{5}{7}$ $\frac{3}{4}$
G $\frac{6 \times 2}{5 \times 11} \div \frac{6}{11}$ $\frac{2}{5}$
H $\frac{2 \times 3 \times 5}{2 \times 5 \times 8} \div \frac{2 \times 5}{3 \times 8}$ $\frac{9}{10}$

think



Using each of this year's digits once, parentheses, and the signs +, −, ×, and ÷, find as many more whole numbers in the list as you can.

$(7 + 3) - (9 + 1) = 0$
$1 \times (7 + 3) - 9 = 1$
$(3 + 1) - (9 - 7) = 2$
$(7 - 1) - (9 \div 3) = 3$
⋮

See Solutions, T.E. page 203.



Using the Exercises

With most groups of children, you will want to assign the exercises on page 203 as independent work. Call attention to the fact that several of the exercises involve work with reciprocals. However, point out that the method discussed in exercise 3 on page 202 may be applied to the problems in exercise 3 on page 203.

When children finish and you help them check their work, continue to emphasize that they can find quotients by thinking about missing factors.

Solutions, Think, page 203

Some of the possible solutions are:

$(9 + 7) \div (3 + 1) = 4$
$(9 - 7) + (3 \times 1) = 5$
$(9 - 7) \times (3 \times 1) = 6$
$3 \times (9 - 7) + 1 = 7$
$(9 - 7) \times (3 + 1) = 8$
$19 - (7 + 3) = 9$
$(91 \div 7) - 3 = 10$

Assignments (page 203)

Minimum: 1, 2. Average: 1–3.
Maximum: 1–3.

Follow-up

To give children more practice in recognizing and finding reciprocals, give them worksheets similar to the ones below.

Find the missing fractional numbers.	
Number	Reciprocal
2	
$\frac{1}{2}$	
$\frac{8}{3}$	
5	
$\frac{3}{8}$	
	$\frac{7}{5}$
3	
	12

Find the missing factors or products	
$7 \times \square = 1$	$\square \times \frac{7}{8} = 1$
$\frac{3}{4} \times \frac{4}{3} = \square$	$3 \times \square = 1$
$4 \times \square = 1$	$4 \times \frac{2}{4} = \square$
$\square \times 6 = 1$	$10 \times \frac{3}{10} = \square$

Resources for Active Learning

Franklin Series: *Making and Using Graphs and Nomographs*, "Nomographs: Multiplication and Division with Fractions," pp. 87-88, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Workbook, page 63

PAGES 204–205

Objective

Given division problems with fractional numbers, the child will be able to find the correct quotient by using the basic division algorithm for fractional numbers.

Preparation

To prepare for this lesson, again review the reciprocal relationship of appropriate pairs of numbers. Also, use examples in which this relationship is expressed in division equations with 1:

$1 \div \frac{7}{8} = ?$
$1 \div \frac{1}{2} = ?$
$1 \div 7 = ?$
$1 \div \frac{5}{6} = ?$
$1 \div \frac{1}{3} = ?$
$1 \div \frac{1}{4} = ?$
$1 \div \frac{3}{4} = ?$

Is there a shortcut for dividing fractional numbers?

Discussing the Ideas

1. Answer the questions about the division problem $\frac{2}{3} \div \frac{5}{7}$.

   A Can 2 be divided exactly by 5? No $\frac{2}{3} \div \frac{5}{7}$

   B Can 2×5 be divided by 5? Yes $\frac{2}{3} = \frac{2 \times 5}{3 \times 5}$ $\frac{2 \times 5}{3 \times 5} \div \frac{5}{7}$
   Is 3×5 divisible by 7? NO

   C Is $2 \times 5 \times 7$ divisible by 5? Yes $\frac{2}{3} = \frac{2 \times 5}{3 \times 5} = \frac{2 \times 5 \times 7}{3 \times 5 \times 7}$
   Is $3 \times 5 \times 7$ divisible by 7? Yes $\frac{2 \times 5 \times 7}{3 \times 5 \times 7} \div \frac{5}{7}$

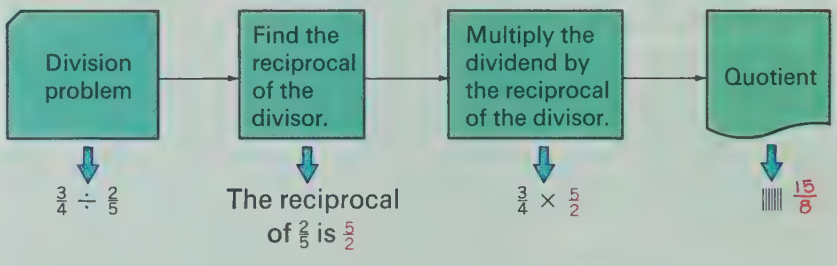
   D Explain how you can use the inverse relation of division and multiplication to check this division problem. $\frac{2 \times 5 \times 7}{3 \times 5 \times 7} \div \frac{5}{7} = \frac{2 \times 7}{3 \times 5}$

   E Explain these equations. $\frac{2}{3} \div \frac{5}{7} = \frac{2 \times 7}{3 \times 5} = \frac{2}{3} \times \frac{7}{5}$

2. Patty said that she saw a shortcut for dividing. Can you use Patty's method to find the quotient and then check your work?

   $\frac{1}{3} \div \frac{4}{5} = \frac{1}{3} \times \frac{5}{4} = \frac{5}{12}$
   Reciprocals

3. Follow the steps in the flow chart. Then give the quotient.



4. Explain how you can use the flow chart in exercise 3 to find these quotients.

   A $\frac{1}{2} \div \frac{2}{3}$ $\frac{3}{4}$ B $\frac{7}{8} \div \frac{2}{5}$ $\frac{35}{16}$ C $\frac{1}{2} \div \frac{1}{10}$ 5 D $\frac{3}{4} \div \frac{5}{8}$ $\frac{24}{20}$ or $\frac{6}{5}$

204

Discussion

Work carefully through each section of discussion exercise 1. In parts B and C, be sure children realize that the expressions $\frac{2 \times 5}{3 \times 5}$ and $\frac{2 \times 5 \times 7}{3 \times 5 \times 7}$ are simply fractional equivalents for $\frac{2}{3}$. Also, point out that 5 and 7 were chosen to be used as factors of the numerator and denominator because the divisor of the problem is $\frac{5}{7}$. Some children will benefit more than others from this presentation of the logical development which leads to the shortcut shown in discussion exercise 3. Do not overemphasize the steps in exercise 1 with those children who have difficulty grasping this development. These children might be helped by the explanation given in "Using the Exercises."

Using the Ideas

1. Solve the equations.

A $6 \div \frac{5}{9} = n \times \frac{9}{5}$ 6

B $12 \div \frac{3}{4} = n \times \frac{4}{3}$ 12

C $5 \div \frac{3}{2} = n \times \frac{2}{3}$ 5

D $\frac{3}{5} \div \frac{1}{2} = n \times 2\frac{3}{5}$

E $\frac{1}{3} \div \frac{1}{4} = n \times 4\frac{1}{3}$

F $\frac{2}{5} \div \frac{3}{4} = \frac{2}{5} \times n\frac{4}{3}$

G $\frac{4}{5} \div 5 = \frac{4}{5} \times n\frac{1}{5}$

H $\frac{7}{4} \div \frac{4}{5} = \frac{7}{4} \times n\frac{5}{4}$

I $\frac{5}{8} \div \frac{3}{2} = \frac{5}{8} \times n\frac{2}{3}$

2. Complete this sentence: To divide a first number by a second number, multiply the first number by the __?__ of the second number.

reciprocal

3. Write each division problem as a multiplication problem. Then find the number for *n*.

Example: $\frac{4}{5} \div \frac{3}{2} = n$ Solution: $n = \frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$

A $\frac{1}{2} \div \frac{3}{4} = n\ \frac{1}{2} \times \frac{4}{3} = \frac{2}{3}$

B $\frac{2}{3} \div \frac{3}{5} = n\ \frac{2}{3} \times \frac{5}{3} = \frac{10}{9}$

C $\frac{5}{6} \div \frac{3}{4} = n\ \frac{5}{6} \times \frac{4}{3} = \frac{20}{18}$

D $\frac{7}{8} \div \frac{1}{2} = n\ \frac{7}{8} \times \frac{2}{1} = \frac{7}{4}$

E $\frac{4}{5} \div \frac{1}{5} = n\ \frac{4}{5} \times \frac{5}{1} = 4$

F $\frac{1}{3} \div \frac{5}{9} = n\ \frac{1}{3} \times \frac{9}{5} = \frac{3}{5}$

G $\frac{5}{12} \div \frac{1}{3} = n\ \frac{5}{12} \times \frac{3}{1} = \frac{5}{4}$

H $\frac{7}{10} \div \frac{2}{5} = n\ \frac{7}{10} \times \frac{5}{2} = \frac{7}{4}$

I $\frac{5}{8} \div \frac{5}{8} = n\ \frac{5}{8} \times \frac{8}{5} = 1$

4. Find the quotients. Answers may be given in higher-terms equivalents.

A $\frac{3}{8} \div \frac{1}{2}$ $\frac{3}{4}$

B $\frac{1}{4} \div \frac{2}{3}$ $\frac{3}{8}$

C $\frac{5}{3} \div \frac{5}{4}$ $\frac{4}{3}$

D $\frac{4}{7} \div 8$ $\frac{1}{14}$

E $6 \div \frac{2}{3}$ 9

F $\frac{3}{10} \div \frac{7}{10}$ $\frac{3}{7}$

G $7 \div \frac{1}{3}$ 21

H $7 \div 3\frac{7}{3}$

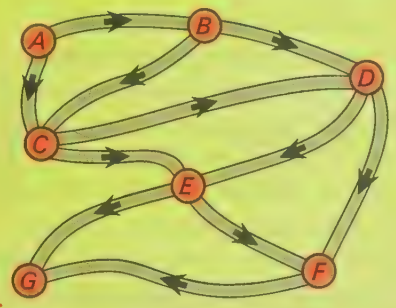
I $\frac{5}{8} \div 10$ $\frac{1}{16}$

J $5 \div 10$ $\frac{1}{2}$

think

If you travel along each "one-way" street only in the direction shown by the arrow, how many different routes from *A* to *G* can you find? 13

See Solution, T.E. page 205.



More practice, page A-16, Set 31

Using the Exercises

If children have understood the development on page 204, encourage them simply to apply the shortcut rule of discussion exercise 3. However, some might benefit from the following kind of explanation for the first exercise on page 205.

We know, from our study of reciprocals, that $1 \div \frac{5}{9} = 1 \times \frac{9}{5}$ since $\frac{5}{9} \times \frac{9}{5} = 1$. Since we know that $1 \div \frac{5}{9} = 1 \times \frac{9}{5}$, it is easy to see that $6 \div \frac{5}{9} = 6 \times \frac{9}{5}$.

Similarly, in part B, since we know that $1 \div \frac{3}{4} = 1 \times \frac{4}{3}$, then we know that $12 \div \frac{3}{4} = 12 \times \frac{4}{3}$.

Exercise 2 summarizes the rule for division with fractional numbers. Exercises 3 and 4 assume that children have developed such an understanding of the rule that they can apply it without more specific directions.

Assignments (page 205)

Minimum: 1–4. Average: 1–4. Maximum: 1–4.

Mathematics

The method of dividing fractional numbers illustrated in discussion exercise 1 was presented in the mathematics section at the beginning of Chapter 9. Although the method is quite general, it is not intended that the children master the technique and use it extensively. Instead, it is meant to be a power-skill method for finding quotients which will help the children understand the usual shortcut algorithm given in the flow chart of discussion exercise 3.

The quotient $\frac{a}{b} \div \frac{c}{d}$ is easy to find if c is a factor of a **and** d is a factor of b, for then the quotient is simply $\frac{a \div c}{b \div d}$. But, in general we are not likely to be so fortunate in the matter of factors. However, we can always choose a fraction equivalent to $\frac{a}{b}$ so that the division process will work nicely. The choice for $\frac{a}{b}$ is $\frac{a \times d \times c}{b \times d \times c}$. Then

$$\frac{a}{b} \div \frac{c}{d} = \frac{a \times d \times c}{b \times d \times c} \div \frac{c}{d}$$

$$= \frac{a \times d \times c \div c}{b \times d \times c \div d} = \frac{a \times d}{b \times c}$$

$$= \frac{a}{b} \times \frac{d}{c}.$$

The final result shows the familiar rule for dividing fractional numbers: To divide $\frac{a}{b}$ by $\frac{c}{d}$, multiply $\frac{a}{b}$ by the reciprocal of $\frac{c}{d}$.

Solution, Think, page 205

The *Think* problem may develop into an interesting extension problem. There are 13 different paths from *A* to *G*. To discover this, children might count the number of points: to *B*, 1; to *C*, 2; to *D*, 3; to *E*, 5; to *F*, 8; to *G*, 13. The number of paths forms the beginning of the pattern known as the Fibonacci sequence.

Duplicator Masters, page 39
Workbook, page 64
Skill Masters, page 39

PAGES 206–207

Objective

Given division exercises which involve mixed numerals and whole numbers, the child will be able to find the quotients.

Preparation

It would be helpful to review the method for expressing mixed numerals as improper fractions. For example, write the following mixed numerals on the chalkboard and ask children to give the improper fraction for each and explain their method.

$3\frac{2}{5}$	$(\frac{17}{5})$	$7\frac{3}{4}$	$(\frac{31}{4})$
$6\frac{1}{4}$	$(\frac{25}{4})$	$9\frac{1}{7}$	$(\frac{64}{7})$
$2\frac{1}{8}$	$(\frac{17}{8})$	$4\frac{2}{3}$	$(\frac{14}{3})$

How are other numerals used to divide fractional numbers?

Discussing the Ideas

1. Explain the steps used to find the quotient.
See Discussion.

Problem: $2\frac{1}{2} \div 1\frac{3}{4}$

Solution: $\frac{5}{2} \div \frac{7}{4} = \frac{5}{2_1} \times \frac{\overset{2}{4}}{7} = \frac{10}{7}$, or $1\frac{3}{7}$.

2. The fractional number $\frac{3}{10}$ is the quotient for $3 \div 10$. Since it will be helpful in later work with fractional numbers, we shall define expressions such as

$$\frac{\frac{4}{5}}{\frac{7}{10}} \text{ to mean } \frac{4}{5} \div \frac{7}{10}.$$

You can think of $\frac{4}{5}$ as the numerator and $\frac{7}{10}$ as the denominator of this "fraction." These more complicated "fractions" have the same properties as the fractions with whole-number numerators and denominators.

Explain how you can use division to "simplify" this fraction: $\frac{\frac{2}{3}}{\frac{5}{6}}$
See Discussion.

3. For any division problem you choose (other than division by 0), there is always a fractional-number quotient. The example below shows that the quotient for $29 \div 3$ is the fractional number $\frac{29}{3}$. $29 \div 3 = 29 \times \frac{1}{3} = \frac{29}{3}$

Quotients like $\frac{29}{3}$ are usually represented by a mixed numeral. You can use the division process shown in example A to represent $\frac{29}{3}$ as the mixed numeral $9\frac{2}{3}$. You can complete all this work and find the fractional-number quotient more easily by arranging your work as in example B. Explain how to find fractional-number quotients for each of the following.

A
$$3\overline{)29}\quad 9$$
$$27$$
$$2$$

B
$$3\overline{)29}\quad 9\frac{2}{3}$$
$$27$$

A $12 \div 5$ $2\frac{2}{5}$ B $38 \div 7$ $5\frac{3}{7}$ C $53 \div 8$ $6\frac{5}{8}$ D $77 \div 9$ $8\frac{5}{9}$ E $237 \div 10$ $23\frac{7}{10}$

206

Discussion

Each of the three discussion exercises treats an important, distinct but related concept. In exercise 1, the method of division with mixed numerals is studied. If children recall the procedure for changing from mixed numerals to improper fractions, they should not have much difficulty with this exercise.

You might stress to the children the importance of changing the mixed numeral before thinking about solving the division equation. Then they can simply apply the rule that they learned in the previous lesson.

As you work through discussion exercise 2 with the children, point out that the expression

$$\frac{\frac{4}{5}}{\frac{7}{10}}$$

is read "four fifths divided by seven tenths." Help children realize that the division bar may be read "divided by" no matter what type of numerals are in the numerator and denominator. Thus, when fractional numbers are expressed as one of these complex fractions, they can easily be changed to the more familiar form:

$$\frac{\frac{2}{3}}{\frac{5}{6}} = \frac{2}{3} \div \frac{5}{6}$$

Using the Ideas

1. Find the quotients.

   A $1\frac{1}{2} \div \frac{3}{4}$ 2 C $3\frac{1}{4} \div \frac{5}{6}$ $3\frac{9}{10}$ E $\frac{4}{5} \div 2\frac{2}{3}$ $\frac{3}{10}$ G $3\frac{1}{2} \div 2\frac{1}{4}$ $1\frac{5}{9}$ I $5\frac{2}{3} \div 6\frac{3}{4}$ $\frac{68}{81}$

   B $2\frac{1}{3} \div 1\frac{1}{5}$ $1\frac{2}{3}$ D $\frac{3}{8} \div 1\frac{1}{3}$ $\frac{9}{32}$ F $\frac{1}{6} \div 3\frac{1}{2}$ $\frac{1}{21}$ H $1\frac{3}{8} \div 4\frac{1}{3}$ $\frac{33}{104}$ J $4\frac{2}{3} \div 2\frac{1}{7}$ $2\frac{8}{15}$

2. Simplify each expression. Example: $\frac{\frac{4}{5}}{\frac{7}{10}} = \frac{4}{5} \div \frac{7}{10} = \frac{40}{35} = 1\frac{1}{7}$

   A $\frac{\frac{1}{3}}{\frac{1}{2}}$ $\frac{2}{3}$ B $\frac{\frac{2}{3}}{\frac{3}{4}}$ $\frac{8}{9}$ C $\frac{\frac{3}{10}}{\frac{1}{10}}$ 3 D $\frac{\frac{3}{5}}{1}$ $\frac{3}{5}$ E $\frac{\frac{6}{5}}{\frac{6}{5}}$ 1 F $\frac{1}{\frac{3}{10}}$ $3\frac{1}{3}$ G $\frac{\frac{1}{2}}{100}$ $\frac{1}{200}$ H $\frac{8\frac{1}{3}}{100}$ $\frac{1}{12}$

3. Solve the equations. (* Any nonzero whole number is also correct.)

   A $\frac{\frac{1}{2} \times \frac{1}{3}}{\frac{2}{5} \times \frac{1}{3}} = n$ $\frac{5}{4}$ B $\frac{\frac{2}{3} \times n}{\frac{1}{4} \times n} = \frac{\frac{8}{3}}{14^*}$ C $\frac{8\frac{1}{3} \times n}{100 \times n} = \frac{25}{3003^*}$ D $\frac{\frac{2}{3}}{\frac{4}{5}} \times \frac{\frac{3}{2}}{\frac{5}{6}} = n$ $\frac{1}{2}$

4. Find the quotients. Check your work.

   A $54 \div 7$ $7\frac{5}{7}$ C $168 \div 9$ $18\frac{2}{3}$ E $495 \div 8$ $61\frac{7}{8}$ G $569 \div 11$ $51\frac{8}{11}$ I $467 \div 23$ $20\frac{7}{23}$

   B $72 \div 5$ $14\frac{2}{5}$ D $3576 \div 5$ $715\frac{1}{5}$ F $347 \div 3$ $115\frac{2}{3}$ H $2372 \div 25$ $94\frac{22}{25}$ J $896 \div 13$ $68\frac{12}{13}$

think

Why is a 25-cent piece called a quarter? The answer may help you find an easy way to solve the two problems below.

1. A restaurant ordered the supplies shown on the right. What was the total bill ?
2. If hamburgers cost 75¢, how many could they buy for $3000 ?

BILL

2564	hot dogs	IIII
720	steaks	IIII
182 kg	potatoes	IIII
54	bottles ketchup	IIII
	Total	IIII

Hot dogs 25¢ Potatoes 15¢ kg
Steaks $1.60 Ketchup $1.50



More practice, page A-16, Set 32

Similarly, when whole numbers are expressed as a quotient, they may be changed into the form of a fractional-number quotient:

$$29 \div 3 = \frac{29}{3}$$

But since an improper fraction is usually expressed as a mixed numeral, we write

$$\frac{29}{3} = 9\frac{2}{3}.$$

Emphasize that, to find $29 \div 3$, the children can use the dividing process and the remainder, arriving immediately at the correct mixed-numeral quotient. Work through as many examples as necessary to help children clarify these ideas.

Using the Exercises

Assign the exercises on page 207 as independent work. However, continue to help any children who have difficulty with these expressions. According to the children's need, have various examples presented at the chalkboard.

Assignments (page 207)
Minimum: 1–2. Average: 1–4.
Maximum: 1–4.

Duplicator Masters, page 40
Workbook, page 65
Skill Masters, page 40

PAGES 208–209

Objective

Given word problems requiring division with fractional numbers, the child will show his understanding of division by solving the problems.

Preparation

Since the children have just completed a rather intensive study of finding the quotient of two fractional numbers, it might help, as preparation for this lesson, to review these skills and techniques briefly.

You might use several of these types of problems:

$5 \div \frac{3}{4} = ?$ $(\frac{20}{3}$ or $6\frac{2}{3})$
$\frac{5}{9} \div 3 = ?$ $(\frac{5}{27})$
$29 \div 6 = ?$ $(4\frac{5}{6})$
$\frac{1}{3} \div \frac{2}{5} = ?$ $(\frac{5}{6})$

Using division to solve problems

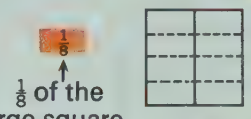
Each problem below can be solved by division of fractional numbers. For each problem, first use any method you wish to find the solution. Then choose the division equation that you can use to solve the problem. The solution to the division equation should agree with your solution to the problem.



1. How many $\frac{1}{4}$-hour periods are there in 8 hours? 32

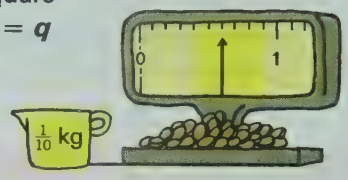
A $\frac{1}{4} \div 8 = q$ (B) $8 \div \frac{1}{4} = q$ C $8 \div 4 = q$

2. How many pieces of red paper $\frac{1}{8}$ as large as the square does it take to cover $\frac{1}{4}$ of the square? 2



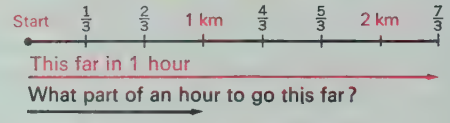
(A) $\frac{1}{4} \div \frac{1}{8} = q$ B $\frac{1}{8} \div \frac{1}{4} = q$ C $4 \div 8 = q$

3. How many cups of peanuts, each weighing $\frac{1}{10}$ kg, are in a pile of peanuts weighing $\frac{3}{5}$ kg? 6



A $\frac{3}{5} \div 10 = q$ B $\frac{1}{10} \div \frac{3}{5} = q$ (C) $\frac{3}{5} \div \frac{1}{10} = q$

4. Joe walks at a rate of $\frac{7}{3}$ km/h. How long does it take him to walk 1 km? $\frac{3}{7}$ hr



A $1 \div \frac{3}{7} = q$ B $\frac{7}{3} \div 1 = q$ (C) $1 \div \frac{7}{3} = q$

5. 32 candy bars. Same amount to each of 5 children. How many candy bars does each child get? $6\frac{2}{5}$



A $5 \div 32 = q$ (B) $32 \div 5 = q$ C $32 \div \frac{1}{5} = q$

6. Jan walks $\frac{3}{4}$ km each hour. How many hours will it take her to go $\frac{5}{2}$ km? $3\frac{1}{3}$



(A) $\frac{5}{2} \div \frac{3}{4} = q$ B $\frac{5}{2} \div \frac{4}{3} = q$ C $\frac{3}{4} \div \frac{5}{2} = q$

208

Discussion

It would be helpful to have children discuss the problems on page 208, either with the entire class or in small groups. Keep in mind, as you work through these exercises with the children, that the most important purpose of this page is to help the children see a variety of situations which require division of fractional numbers.

The answers from your class might be similar to those shown in the right-hand column on T.E. page 209.

Assignments (page 208)

Minimum: 1–4, oral. Average: 1–6. Maximum: 1–6.

Solving Short Stories

Write and solve a division equation to answer each exercise.

1. 5 hours. How many:



- A $\frac{1}{2}$-hour periods
- B $\frac{1}{3}$-hour periods
- C $\frac{1}{5}$-hour periods

See Answers at right.

2. $\frac{3}{4}$ hour. How many:

- A $\frac{1}{4}$-hour periods
- B $\frac{1}{2}$-hour periods

See Answers at right.

3. Square piece of paper. Red piece is $\frac{1}{8}$ as large as the square. How many red pieces are needed to cover:



- A $\frac{1}{2}$ of the square
- B $\frac{3}{4}$ of the square
- C 2 squares
- D $\frac{1}{16}$ of the square

See Answers at right.

4. Nuts. A cup weighs $\frac{4}{10}$ kg. How many cups of nuts are in a pile weighing:

A 1 kg B 2 kg C 10 kg
D 100 kg E $\frac{4}{5}$ kg F $\frac{1}{5}$ kg

See Answers at right.

5. $\frac{2}{5}$ kg of nuts in each bag. How many bags of nuts from:

A 1 kg B 30 kg C $1\frac{1}{5}$ kg

$1 \div \frac{2}{5} = 2\frac{1}{2}$ $30 \div \frac{2}{5} = 75$ $1\frac{1}{5} \div \frac{2}{5} = 3$

6. Walk 12 kilometres. How long does it take going:

A $\frac{3}{2}$ km/h B $\frac{7}{3}$ km/h C 5 km/h

$12 \div \frac{3}{2} = 8$ $12 \div \frac{7}{3} = 5\frac{1}{7}$ $12 \div 5 = 2\frac{2}{5}$

7. A strange coincidence. Six hens, each the same weight. Total weight, 35 kilograms. $35 \div 6 = 5\frac{5}{6}$ What does each hen weigh?



8. $\frac{4}{5}$ km to school and back. $15 \div \frac{4}{5} = 18\frac{3}{4}$ How many trips to go: A 15 km

B 24 km $24 \div \frac{4}{5} = 30$ C 1 km $1 \div \frac{4}{5} = 1\frac{1}{4}$ D $\frac{5}{4}$ km $\frac{5}{4} \div \frac{4}{5} = 1\frac{9}{16}$



9. Rope: $5\frac{1}{4}$ metres long. How many $\frac{3}{4}$-metre pieces can be cut from it?

$\frac{21}{4} \div \frac{3}{4} = 7$

10. Running track: $\frac{4}{10}$ km. How many laps in:

A 1 km $1 \div \frac{4}{10} = 2\frac{1}{2}$ B 10 km $10 \div \frac{4}{10} = 25$

11. Area of rectangle: $\frac{7}{10}$ square centimetre. Width: $\frac{3}{5}$ cm. What is the length?

$\frac{7}{10} \div \frac{3}{5} = \frac{7}{6}$ or $1\frac{1}{6}$

12. Takes $1\frac{3}{4}$ minutes to make a gadget. How many gadgets can be made in 14 minutes? $14 \div 1\frac{3}{4} = 8$

13. Total weight of 9 children: 329 kg. What is the average weight of the children?

$329 \div 9 = 36\frac{5}{9}$

Using the Exercises

The problems on page 209 are intended to be worked independently following the discussion of problems on page 208. Since it is the first real opportunity the children will have had to apply their experiences with the dividing process to word problems on an independent basis, you will want to be sure to allow plenty of time for discussion, including explanations by the children of how they were able to arrive at solutions.

Assignments (page 209)

Minimum: 1–6. Average: 1–13.
Maximum: 1–13.

Answers, exercises 1–4, page 209

1. A $5 \div \frac{1}{2} = 10$
   B $5 \div \frac{1}{3} = 15$
   C $5 \div \frac{1}{5} = 25$
2. A $\frac{3}{4} \div \frac{1}{4} = 3$
   B $\frac{3}{4} \div \frac{1}{2} = 1\frac{1}{2}$
3. A $\frac{1}{2} \div \frac{1}{8} = 4$
   B $\frac{3}{4} \div \frac{1}{8} = 6$
   C $2 \div \frac{1}{8} = 16$
   D $\frac{1}{16} \div \frac{1}{8} = \frac{1}{2}$
4. A $1 \div \frac{4}{10} = 2\frac{1}{2}$
   B $2 \div \frac{4}{10} = 5$
   C $10 \div \frac{4}{10} = 25$
   D $100 \div \frac{4}{10} = 250$
   E $\frac{4}{5} \div \frac{4}{10} = 2$
   F $1\frac{1}{2} \div \frac{4}{10} = 3\frac{3}{4}$

Resources for Active Learning

A Cloudburst, Vol. 2, No. 3353–83, Midwest Publications.

PAGES 210–211

Objectives

Given word problems involving division with fractional numbers, the child will solve the problems by applying his skill with operations involving fractions.

Given a line graph, the child will be able to solve related problems by interpreting the data given by the line graph.

Preparation

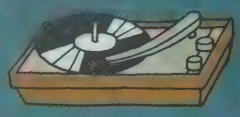
To prepare for this lesson, you might review skills and techniques in dividing with fractions, giving children practice with a variety of division situations. You might prefer to begin immediately with the introductory material at the top of page 210.

DIAMONDS

Uses of diamonds



Jewelry



Phonograph needles

Cutting and drilling



Weighing diamonds

One cubic centimetre of water weighs 1 gram. Diamonds are weighed by using a unit called a **carat.**



A diamond weighing **1 carat** weighs $\frac{1}{5}$ **gram.**

1. A diamond this size  weighs about 4 carats. What part of a gram is this? $\frac{4}{5}$

2. The largest blue diamond in the world is the Hope diamond, which weighs $44\frac{2}{5}$ carats. Another famous diamond was the Timken stone, which weighed $28\frac{3}{4}$ carats. How much more than the Timken stone does the Hope diamond weigh? $15\frac{13}{20}$ carats

3. The Cullinan crystal, the world's largest, weighed about 585 grams. How many carats did it weigh? 2925 carats $605\frac{1}{3}$

4. The world's largest cut diamond, the Star of South Africa, was cut from the Cullinan crystal. The Star weighed about 530 carats. Did the Star of South Africa weigh closer to $\frac{1}{10}$kg, $\frac{2}{10}$kg, or $\frac{3}{10}$kg? $\frac{1}{10}$ kg

5. Diamonds, which are pure carbon, are the hardest natural substance. If measured for hardness, a fingernail might get a rating of $2\frac{1}{3}$; a copper coin, $2\frac{3}{4}$; a window glass, $5\frac{1}{2}$; and a diamond, 10. Since $5\frac{1}{2} \div 2\frac{3}{4} = 2$, we could say that glass is twice as hard as a copper coin. Diamonds are how many times as hard as
   A your fingernail? $4\frac{2}{7}$ B a copper coin? $3\frac{7}{11}$ C glass? $1\frac{9}{11}$

Discussion

You might treat these pages in a variety of ways. For example, you might have children work on one of them in small groups and then give explanations to the class. Or you might use parts of them as a basis for a teacher-led discussion and assign remaining parts as independent work.

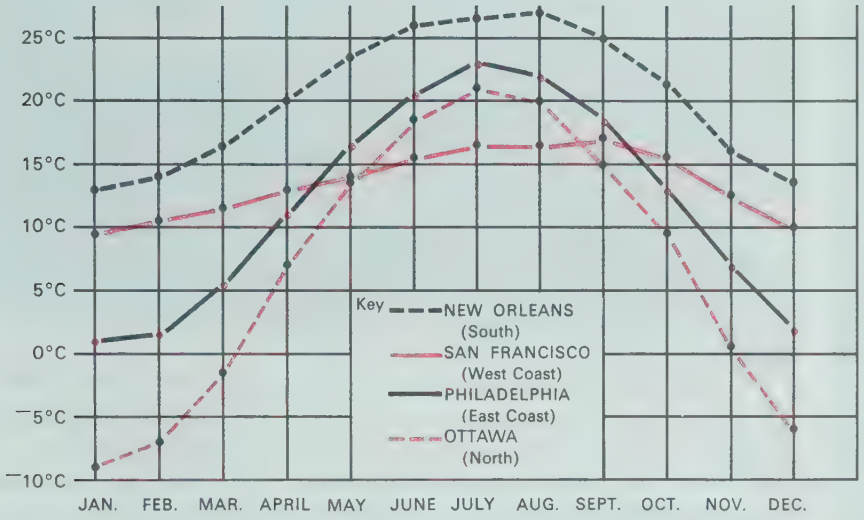
For page 210, be sure the children understand the equivalents between gram and carat. Stress with them that a carat is a unit of weight equivalent to one fifth of a gram. Also, help them see that a weight of one gram is the weight of one cubic centimetre of water. Help them think through any problems which cause difficulty.

Assignments (page 210)

Minimum: 1–5, oral. Average: 1–5. Maximum: 1–5.

AVERAGE TEMPERATURES

Monthly normal temperatures (degrees Celsius)



1. Give the normal July temperature for each city. N.O. $26\frac{1}{2}°$; S.F. $16\frac{1}{2}°$; Phil. 23°; Ottawa 21°
2. How much higher is the July temperature in New Orleans than in:
   A Ottawa $5\frac{1}{2}°$ B San Francisco 10°
3. Give the average summer (June, July, August) temperature for:
   A San Francisco $16\frac{1}{6}°$ B Philadelphia $21\frac{5}{6}°$ C Ottawa $19\frac{5}{6}°$ D New Orleans $26\frac{1}{2}°$
4. How much greater is the average summer temperature in Ottawa than San Francisco? $3\frac{4}{6}°$
5. Give the average winter (December, January, February) temperature for each of the four cities. N.O. $13\frac{1}{2}°$; S.F. 10°; Phil. $1\frac{1}{2}°$; Ottawa $-7\frac{1}{3}°$
6. How much greater is the average winter temperature in San Francisco than in Ottawa? $17\frac{1}{3}°$
7. Give the average yearly temperature for San Francisco. $13\frac{1}{2}°$

More practice, page A-17, Set 33

Using the Exercises

With many groups of children, you will want to study the line graph on page 211 as a class activity. Choose one of the graphs and ask children to make several readings from it. For example, you might choose the one that represents Philadelphia and have the children make various readings from this line graph for specific months. You might also have them find the average temperature of Philadelphia for 3 or 4 given months.

When children have finished these exercises, allow time to check their answers and discuss those that caused difficulty.

Assignments (page 211)

Minimum: 1–3. Average: 1–7. Maximum: 1–7.

Follow-up

As an extension of problem 5 on page 210, you might gather sample materials and have children test the hardness of them. If a first object can make a scratch mark on a second, then the first object is harder than the second. The fingernail, penny, and a piece of glass (not sharp-edged) can be used to measure the hardness of various objects, particularly other coins, metals, and rocks. Children might categorize and chart their findings. (Numerical values can be assigned to items according to a scale of hardness, such as: fingernail – 1; penny – 3; glass – 5.) The more capable children might then make up problems concerning the relative hardness of the items.

The use of line graphs also provides many possibilities for activity. For example, children might devise line graphs for the temperature, rainfall, or hours of sunshine for a given week. Other graphs could be made from data on population, exports, imports, industrial or food production, space probes, educational costs, growth of your local community, sports records, or similar topics of current interest to the children.

Resources for Active Learning

Measure and Find Out, Book 2, "The High for Today," Activity 3/3, Scott Foresman. [See "Resources for Active Learning," on page 99 for other temperature activities.] (Available from Gage Educational Publishing Ltd.)

Developmental Math Cards, K²10, Addison-Wesley. [Using line graphs]

Duplicator Masters, page 41
Workbook, page 66

PAGES 212–213

Objectives

Given a drawing of an animal and the scale of the drawing, the child will be able to find the actual length of the animal.

Preparation

Materials

colored strips

If you want to give the children a brief practice on fractional-number division skills, you might review equations with a missing factor.

$$7 \div ? = \tfrac{2}{3}$$
$$\tfrac{2}{5} \times ? = \tfrac{1}{2}$$
$$\tfrac{4}{7} \div ? = 6$$

Review with the children the idea that these equations are the same as these:

$$7 \div \tfrac{2}{3} = n$$
$$\tfrac{2}{5} \div \tfrac{1}{2} = n$$
$$\tfrac{4}{7} \div n = 6$$

They know how to solve the second group by multiplying the dividend by the reciprocal of the divisor.

If you prefer, you might replace such a review with an introductory discussion regarding some of the animals pictured on these pages.

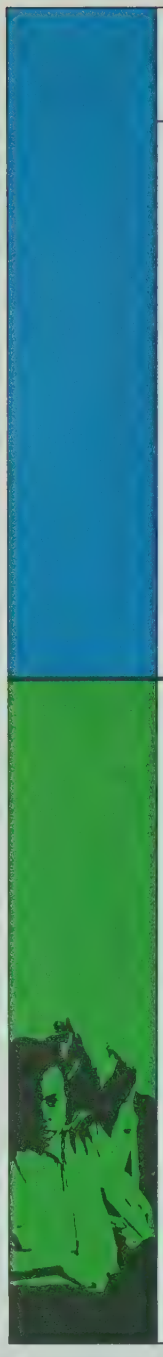
Investigation

Most children would benefit from doing this investigation in small groups. The main activity involved is lining up the strips called for in the text; this gives concrete reinforcement to the idea of scale. Remind children to carefully record the results. Help those who have difficulty by discussing the example of the armadillo. If one light green strip is $\frac{1}{16}$ of the actual length, then 16 light green strips will represent the actual length. If we recall that a light green strip is 3 centimetres, then we can figure that 16 light green strips is 3 times 16, or 48 centimetres. This can be expressed as

$$\tfrac{1}{16} \times \text{actual length} = 3.$$

Thus:

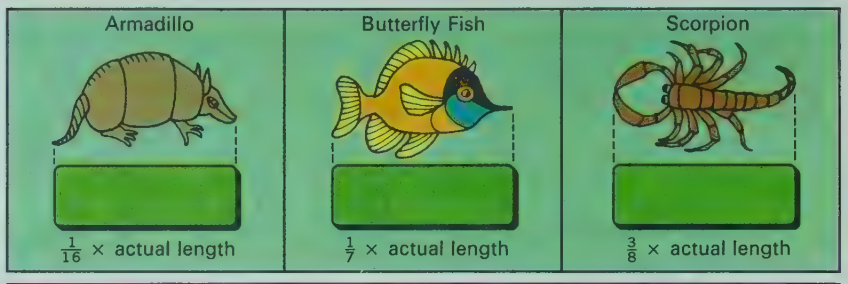
$$3 \div \tfrac{1}{16} = \text{actual length}$$



● *Can you find actual lengths from scale drawings?*

Investigating the Ideas

The picture of each creature has been reduced until it is just as long as the light green strip.



?

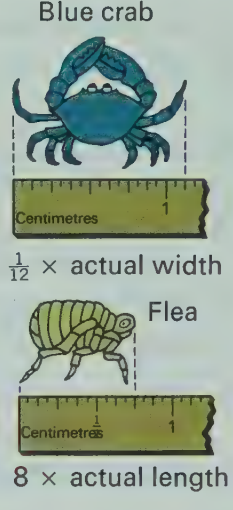
Can you lay down strips to show the actual length of each creature? 48 cm (16 strips) 21 cm (7 strips) See above and Investigation.

Record the number of strips you used for each part. 8 cm (2⅔ strips)

Discussing the Ideas

1. When objects are drawn to scale, you can find their actual lengths by division. For example, the picture of the blue crab is $\frac{1}{12}$ the width of a real crab. You can write the multiplication equation
$\frac{1}{12} \times w = 3$.
   - A Explain how to find the missing factor. See Discussion.
   - B What is the actual width of a blue crab? 13½ cm

2. A Is a flea larger or smaller than the picture? smaller
   B Explain how to find the actual length of a flea. $\frac{3}{32}$ cm



212

Discussion

Ask children to explain their findings from the investigation. Relate the discussion to equations for each problem and stress the relationships between the different ways of expressing the same problem. Use exercise 1 as a basis for discussion. In particular, show that

$$\tfrac{1}{12} \times w = 1\tfrac{1}{8}$$

is related to

$$1\tfrac{1}{8} \div \tfrac{1}{12} = w$$

and

$$1\tfrac{1}{8} \times 12 = w.$$

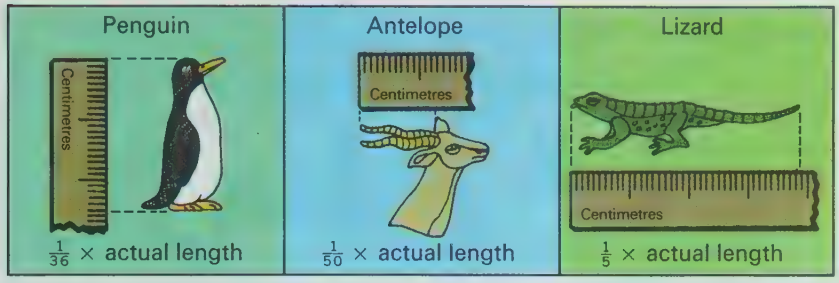
Note that in exercise 2, since the scale drawing of the flea is larger than the actual length of the flea, the actual length may be found by dividing by 8. Again show the relationship of the equations:

$$8 \times \text{actual length} = \tfrac{3}{4}$$
$$\tfrac{3}{4} \div 8 = \text{actual length}$$
$$\tfrac{3}{4} \times \tfrac{1}{8} = \text{actual length}$$

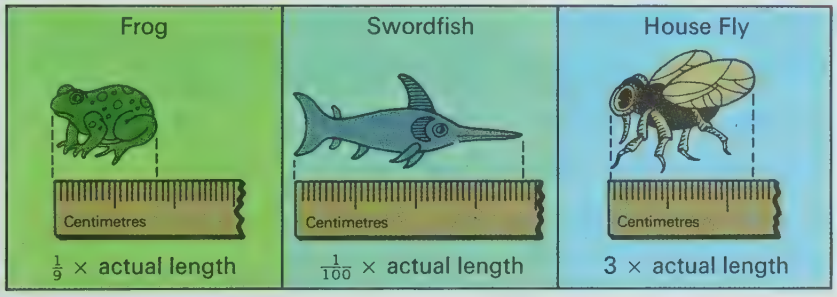
Throughout this discussion, stress that the indicated multiples or fractions refer to lengths or widths of the animals and not to volume or overall size.

ANIMAL LENGTHS

Using the Ideas



1. How tall is a penguin
   A in centimetres? 90 cm B in metres? 0.9 m
2. Find the length of an antelope's horns
   A in centimetres. 60 cm B in metres. 0.6 m
3. One of the longest elephant tusks is about $4\frac{3}{5}$ times as long as the antelope's horn. How long is the tusk? 276 cm
4. Find the length of a lizard. 19 cm



5. How long is a frog's body? $15\frac{3}{10}$ cm
6. How long is the swordfish? 380 cm
7. How long is the house fly? $\frac{4}{3}$ cm
8. If a frog can jump $17\frac{1}{2}$ times as far as the length of his body, how many metres can he jump? $2\frac{73}{100}$ m

★ 9. The lizard's tail is $\frac{3}{4}$ the length of his body. How long is the lizard's tail? $14\frac{1}{4}$ cm

213



Follow-up

To extend the work on these pages, you might suggest that the children examine dictionaries or encyclopaedias for information concerning the sizes of other animals that interest them. Dictionaries sometimes show small pictures of animals and give the relationship of the picture to the actual size of the animal. Scientific books and drawings may also be helpful in this respect. Encourage them to report their findings to the class. Some might make a chart showing small drawings of the animals and recording the scale used and the actual size of each.

Using the Exercises

Encourage the children to write equations for each of the exercises on page 213. Some may choose the division expression and others may interpret the scale drawings in terms of multiplication. When children have finished, allow time to check their solutions and ask children to explain how they found their answers. Again, show the relationships between the appropriate multiplication and division equations.

Assignments (page 213)

Minimum: 1–4. Average: 1–8.
Maximum: 1–9.

PAGES 214–215

Objective

Given products and quotients involving fractional numbers, the child will compare them to 1 and indicate the relationships by using the inequality and equality symbols.

Preparation

According to the need of the children, review some of the skills used in the multiplication and division of fractional numbers. In particular, you might review the use of the terms *product* and *quotient*. In a pair of related equations, the product and the number being divided are the same.

$$\overset{\text{P}}{\frac{1}{3} \times \frac{4}{5} = \frac{4}{15}}$$

$$\overset{\text{P}}{\frac{4}{15} \div \frac{4}{5} = \frac{1}{3}}$$

In some equations, as with equations involving missing factors, the number being divided may be referred to as the product.

Investigation

If some children conclude that in order to answer the investigation question they must find the product and quotient of every pair in the set, point out that they will have a great many to find. Encourage them to think first about the two numbers they choose to try for each question, and to figure what the two numbers are for each question before multiplying. For example, since multiplication is repeated addition, the largest product will clearly be the product of the two largest numbers. Suggest that they record not just the two numbers they use for each question, but that they describe these two numbers as "the two largest numbers," or "the two smallest," or "the smallest and the largest," and so on.

A $8 \times 10 = 80$ (two largest numbers)

B $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$ (two smallest numbers)

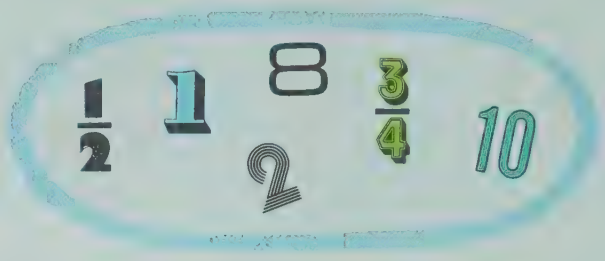
C $10 \div \frac{1}{2} = 20$ (largest number divided by the smallest)

D $\frac{1}{2} \div 10 = \frac{1}{20}$ (smallest number divided by the largest)

Let's compare products and 1.

Investigating the Ideas

Use only the numbers in this set.



? Can you find the two different numbers in the set that give See Investigation.

A the largest product?
B the smallest product?
C the largest quotient?
D the smallest quotient?

Discussing the Ideas

1. Explain how you found the largest product.
   Multiply the two largest numbers.
2. How can you find the smallest product?
   Multiply the two smallest numbers.
3. Which quotient is larger, $2 \div \frac{1}{2}$ or $2 \div \frac{1}{4}$?
4. Which phrase (**greater than, less than,** or **equal to**) would you choose to complete each sentence?
   - A The product of two fractional numbers (each less than 1) is __? __ either of the factors. less than
   - B The product of two fractional numbers (each greater than 1) is __? __ either of the factors. greater than
   - C When dividing by a fractional number less than 1, the missing factor is __? __ the product. greater than
   - D When dividing by a fractional number greater than 1, the missing factor is __? __ the product. less than

214



Discussion

For discussion exercises 1 and 2, have children explain their answers to the questions in the investigation. As they respond to exercise 3, help them recall that division may be thought of as a subtractive process; it would take more steps to get to 0 by subtracting $\frac{1}{4}$ from 2 than by subtracting $\frac{1}{2}$ from 2.

One of the main points in this lesson is to help children compare the products of whole numbers with the products involving fractional numbers. For example, children should realize that the product of two whole numbers, each greater than 1, is greater than either factor. However, as stated in exercise 4A, the product of two fractional numbers, each less than 1, is less than either factor. Stress this and the statements in parts B, C, and D. If further explanation is needed, provide additional examples to illustrate the ideas underlying each statement.

Using the Ideas

These exercises will help you compare fractional-number products and factors.

1. Give the symbol (<, =, >) for each ◯.

A $8 \times 8 \bigcirc 8$ >	H $\frac{1}{2} \times 8 \bigcirc \frac{1}{2}$ >	O $5 \times 1000 \bigcirc 5$ >
B $8 \times 4 \bigcirc 8$ >	I $\frac{1}{2} \times 4 \bigcirc \frac{1}{2}$ >	P $5 \times 100 \bigcirc 5$ >
C $8 \times 2 \bigcirc 8$ >	J $\frac{1}{2} \times 2 \bigcirc \frac{1}{2}$ >	Q $5 \times 10 \bigcirc 5$ >
D $8 \times 1 \bigcirc 8$ =	K $\frac{1}{2} \times 1 \bigcirc \frac{1}{2}$ =	R $5 \times 1 \bigcirc 5$ =
E $8 \times \frac{1}{2} \bigcirc 8$ <	L $\frac{1}{2} \times \frac{1}{2} \bigcirc \frac{1}{2}$ <	S $5 \times \frac{1}{10} \bigcirc 5$ <
F $8 \times \frac{1}{4} \bigcirc 8$ <	M $\frac{1}{2} \times \frac{1}{4} \bigcirc \frac{1}{2}$ <	T $5 \times \frac{1}{100} \bigcirc 5$ <
G $8 \times \frac{1}{8} \bigcirc 8$ <	N $\frac{1}{2} \times \frac{1}{8} \bigcirc \frac{1}{2}$ <	U $5 \times \frac{1}{1000} \bigcirc 5$ <

2. Give the symbol (<, >, =) for each ◯.

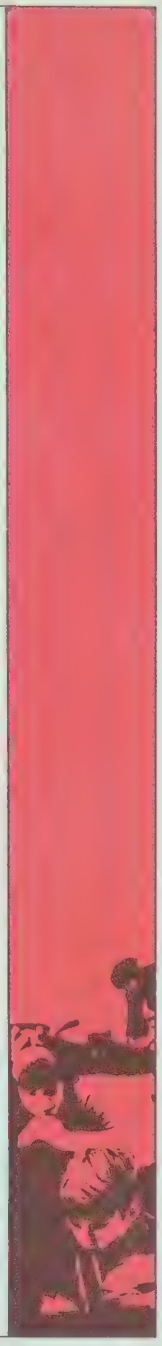
A $24 \div 4 \bigcirc 24$ <	H $6 \div 12 \bigcirc 6$ <	O $1 \div 1000 \bigcirc 1$ <
B $24 \div 3 \bigcirc 24$ <	I $6 \div 6 \bigcirc 6$ <	P $1 \div 100 \bigcirc 1$ <
C $24 \div 2 \bigcirc 24$ <	J $6 \div 3 \bigcirc 6$ <	Q $1 \div 10 \bigcirc 1$ <
D $24 \div 1 \bigcirc 24$ =	K $6 \div 1 \bigcirc 6$ =	R $1 \div 1 \bigcirc 1$ =
E $24 \div \frac{1}{2} \bigcirc 24$ >	L $6 \div \frac{1}{3} \bigcirc 6$ >	S $1 \div \frac{1}{10} \bigcirc 1$ >
F $24 \div \frac{1}{3} \bigcirc 24$ >	M $6 \div \frac{1}{6} \bigcirc 6$ >	T $1 \div \frac{1}{100} \bigcirc 1$ >
G $24 \div \frac{1}{4} \bigcirc 24$ >	N $6 \div \frac{1}{12} \bigcirc 6$ >	U $1 \div \frac{1}{1000} \bigcirc 1$ >

3. Give the symbol (<, >, =) for each ◯.
Do as little computing as possible.

A $128 \div 4 \bigcirc 128 \div 2$ <	D $\frac{5}{6} \times \frac{3}{8} \bigcirc \frac{5}{6}$ <	G $24 \div \frac{5}{4} \bigcirc 24$ <
B $128 \div 2 \bigcirc 128 \div 1$ <	E $\frac{5}{6} \times \frac{3}{8} \bigcirc \frac{3}{8}$ <	H $\frac{1}{2} \div \frac{1}{2} \bigcirc \frac{4}{7} \div \frac{4}{7}$ =
C $128 \div 1 \bigcirc 128 \div \frac{1}{2}$ <	F $1 \div \frac{1}{3} \bigcirc 1$ >	I $\frac{2}{3} \times \frac{5}{7} \bigcirc \frac{5}{7} \times \frac{2}{3}$ =

4. A If, in the equation $a \times b = 1$, the number for a is less than 1, what do you know about the number for b? It is greater than 1. It is a's reciprocal.

B Can the numbers for both a and b be less than 1? No



Using the Exercises

Before children begin the exercises on page 215, suggest that they look for a pattern in each column of exercises 1 and 2. Be sure they realize that these exercises, as implied in the directions for exercise 3, are not intended simply as practice in computation. By studying the patterns in exercises 1 and 2 and reasoning through exercises 3 and 4, children should have a better understanding of multiplication and division involving fractional numbers. For example, children often mistake $24 \div \frac{1}{2}$ for the multiplication problem $24 \times \frac{1}{2}$; this exercise set is designed to help children gain a feeling for the fact that $24 \div \frac{1}{2}$ is greater than 24.

Assignments (page 215)*
Minimum: 1–2. Average: 1–4.
Maximum: 1–4.

PAGES 216–217

Objective

Given a fractional number, the child will be able to give the whole number nearest that fractional number.

Preparation

To prepare for this lesson, you might provide a review of the function rule. For example, exhibit the following tables and ask children to see whether they can figure out your rule and supply the missing numbers.

?	
$\frac{1}{2}$	1
$2\frac{1}{2}$	3
5	$5\frac{1}{2}$
$4\frac{1}{2}$	?
$\frac{1}{4}$	?

(add $\frac{1}{2}$)

?	
2	$1\frac{3}{4}$
$\frac{3}{4}$	$\frac{1}{2}$
$\frac{1}{2}$	?
$3\frac{1}{4}$	?
5	?

(subtract $\frac{1}{4}$)

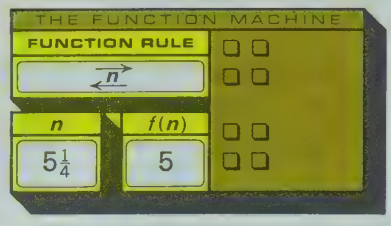
Investigation

For this investigation, many children would benefit from working in groups and sharing their ideas about the rule they are to find. You might want to read the introductory material with all the children before they form groups. Observe with the children that, previously, a function rule told them what to do with a given number, n. That is, if they had a function rule $n + 5$, or $f(n) = n + 5$, for a given number n, they would add five to the number in order to find $f(n)$. Explain to the children that, now, they must find out what this double-arrow function does to the number n. Then, encourage them to study the tables and see if they can figure out Tim's rule. Since the rule is basically a simple one (though difficult to discover, perhaps), children should be able to make several function tables once they have discovered that the rule is to give the whole number that is nearest to the fractional-number input. Do not tell them the rule, but if necessary, show other sample tables.

Let's explore a special function.

Investigating the Ideas

Tim thought of a special function. Since he could not use addition, subtraction, multiplication, or division to describe the function rule, Tim used the symbol $\overrightarrow{\underleftarrow{n}}$. Study the tables below to understand Tim's function.



A

Function Rule $\overrightarrow{\underleftarrow{n}}$

n	$f(n)$
5	5
$5\frac{1}{8}$	5
$5\frac{3}{8}$	5
$5\frac{5}{8}$	6
$5\frac{7}{8}$	6

B

Function Rule $\overrightarrow{\underleftarrow{n}}$

n	$f(n)$
$8\frac{1}{4}$	8
$8\frac{1}{2}$	9
$8\frac{3}{4}$	9
9	9
$9\frac{1}{4}$	9

C

Function Rule $\overrightarrow{\underleftarrow{n}}$

n	$f(n)$
$\frac{8}{3}$	3
$\frac{7}{2}$	4
$\frac{1}{4}$	0
$\frac{17}{2}$	9
$23\frac{5}{6}$	24

? Can you discover Tim's function rule and use the rule to make a table of your own?

See Investigation and Discussion.

Discussing the Ideas

1. Describe Tim's rule in your own words. See Discussion.
2. Explain how to find the missing numbers.
   A $\overrightarrow{\underleftarrow{3}} =$ ▮ 3 B $\overrightarrow{\underleftarrow{3\frac{1}{4}}} =$ ▮ 3 C $\overrightarrow{\underleftarrow{3\frac{1}{2}}} =$ ▮ 4 D $\overrightarrow{\underleftarrow{3\frac{3}{4}}} =$ ▮ 4 E $\overrightarrow{\underleftarrow{4\frac{2}{5}}} =$ ▮ 4
3. Explain how to find the sums. See Discussion.
   A $\overrightarrow{\underleftarrow{1\frac{1}{3} + 2\frac{1}{3}}} =$ ▮ 4 B $\overrightarrow{\underleftarrow{2\frac{3}{4} + 3\frac{1}{2}}} =$ ▮ 7
4. Can you explain why the two sums below are not the same?
   A $\overrightarrow{\underleftarrow{5\frac{3}{8}}} + \overrightarrow{\underleftarrow{4\frac{3}{8}}} = 9$ B $\overrightarrow{\underleftarrow{5\frac{3}{8} + 4\frac{3}{8}}} = 10$ See Discussion.

216

Discussion

Encourage children to show examples from the function tables they made and to explain what they think Tim's rule is. The function rule rounds each input number to the nearest whole number. Be sure children understand the agreement that, if the input number is halfway between two whole numbers, then we round *up* to the next larger whole number. Thus, an input of $2\frac{1}{2}$ results in an output of 3. Note the differences between exercises 3A and 3B and between 4A and 4B. When the line of the symbol extends unbroken over and under both addends, the rounding rule is to be applied after the sum is found. When the symbol appears over and under each addend separately, the rule is to be applied to each addend separately before they are added. Thus:

$$\overrightarrow{\underleftarrow{5\frac{3}{8}}} + \overrightarrow{\underleftarrow{4\frac{3}{8}}} = 5 + 4 = 9$$

$$\overrightarrow{\underleftarrow{5\frac{3}{8} + 4\frac{3}{8}}} = \overrightarrow{\underleftarrow{9\frac{6}{8}}} = 10$$

Provide other examples of this kind, as needed, to help children discover and understand this difference.

Using the Ideas

Complete the function tables in exercises 1 through 3.

1.

Function Rule $\overrightarrow{n}$

	n	$f(n)$
A	$\frac{7}{8}$	1
B	$1\frac{2}{3}$	2
C	$\frac{3}{2}$	2
D	$\frac{8}{2}$	4
E	$\frac{22}{7}$	3

2.

Function Rule $\overrightarrow{n}$

	n	$f(n)$
A	$\frac{1562}{1561}$	1
B	$\frac{1562}{1500}$	1
C	$\frac{287}{288}$	1
D	$\frac{49}{10}$	5
E	$\frac{65}{7}$	9

3.

Function Rule $\overrightarrow{n}$

	n	$f(n)$
A	$7\frac{3}{8}$	7
B	$13\frac{7}{16}$	13
C	$1\frac{8}{5}$	3
D	$5\frac{33}{65}$	6
E	$\frac{66}{12}$	6

4. Give the missing numbers.

A $\overrightarrow{\frac{9}{2}} =$ 5

B $\overrightarrow{\frac{7}{8}} =$ 1

C $\overrightarrow{\frac{73}{10}} =$ 7

D $\overrightarrow{\frac{324}{10}} =$ 32

E $\overrightarrow{\frac{546}{100}} =$ 5

F $\overrightarrow{\frac{494}{10}} =$ 49

G $\overrightarrow{\frac{25}{2}} =$ 13

H $\overrightarrow{18\frac{3}{4}} =$ 19

I $\overrightarrow{\frac{5500}{1000}} =$ 6

J $\overrightarrow{5\frac{13}{27}} =$ 5

K $\overrightarrow{9\frac{23}{45}} =$ 10

L $\overrightarrow{\frac{6852}{100}} =$ 69

5. Give the missing numbers. Do as little computing as possible.

A $\overrightarrow{2\frac{3}{4}} + \overrightarrow{3\frac{1}{2}} =$ 7

B $\overrightarrow{2\frac{3}{4} + 3\frac{1}{2}} =$ 6

C $\overrightarrow{7\frac{5}{11}} + \overrightarrow{6\frac{5}{8}} =$ 14

D $\overrightarrow{7\frac{5}{10} + 6\frac{5}{8}} =$ 14

E $\overrightarrow{68\frac{1}{3}} + \overrightarrow{99\frac{1}{16}} =$ 167

F $\overrightarrow{58\frac{1}{2} + 69\frac{8}{17}} =$ 128

G $\overrightarrow{9\frac{1}{2}} \times \overrightarrow{6\frac{3}{8}} =$ 60

H $\overrightarrow{12\frac{5}{6}} \times \overrightarrow{12\frac{1}{3}} =$ 156

I $\overrightarrow{35\frac{6}{13}} \times \overrightarrow{7\frac{7}{13}} =$ 280

J $\overrightarrow{4\frac{3}{4} \times 3} =$ 14

K $\overrightarrow{9 \times 7\frac{3}{8}} =$ 66

L $\overrightarrow{32\frac{1}{8}} - \overrightarrow{19\frac{3}{4}} =$ 12

M $\overrightarrow{58\frac{3}{4} - 8\frac{1}{2}} =$ 50

N $\overrightarrow{24} \div \overrightarrow{\frac{7}{8}} =$ 24

O $\overrightarrow{35\frac{1}{4}} \div \overrightarrow{7\frac{3}{8}} =$ 5



Using the Exercises

Assign the exercises on page 217. When the children have finished, allow time for checking papers and discussion. It will help to have several parts of each table presented and explained to the class. For example, ask the children to explain how they got the number 5 for exercise 2D or the number 9 for exercise 2E.

For an exercise such as 2E, the children should observe that $\frac{65}{7}$ is $9\frac{2}{7}$, and $\frac{2}{7}$ is less than $\frac{1}{2}$; hence, 9 is the output number for $\frac{65}{7}$.

Note that in exercise 5, children must again distinguish between applying the function rule before or after performing the operation. The arrows over the combined expression mean *compute first* and then find the *nearest whole number;* the individual arrows mean *find the nearest whole number* first and then *compute*. Ideally, exercises 4 and 5 should be done by mental calculation only. Encourage as many of your children as you think can do so to complete these exercises without using pencil and paper except to write their answers.

Assignments (page 217)*

Minimum: 1–4. Average: 1–5.
Maximum: 1–5.

PAGES 218–219

Objective

Given short story problems which involve fractional numbers, the child will be able to estimate answers to the problems.

Preparation

To prepare for this lesson, it would be helpful to show products and sums of fractions and ask children to tell whether they are equal to, greater than, or less than 1. For example:

$\frac{2}{3} \times \frac{1}{3}$ (< 1)	$\frac{1}{2} + 1\frac{1}{2}$ (> 1)
$\frac{2}{3} + \frac{1}{3}$ $(= 1)$	$\frac{1}{2} \times 1\frac{1}{2}$ (< 1)
$\frac{2}{3} \div \frac{1}{3}$ (> 1)	$\frac{1}{2} \div 1\frac{1}{2}$ (< 1)

You might also review estimation with whole numbers and with mixed numbers. For example, point out how the rule "find the nearest whole number" would help children make the estimates below.

Problem	Estimate
$5\frac{1}{3} \times 6\frac{2}{3}$	35
$9 \times 7\frac{3}{8}$	63
$53\frac{3}{4} - 7\frac{1}{4}$	47
$4\frac{3}{4} \div 3$	$\frac{5}{3}$

Short Stories ESTIMATION

For each exercise the answer given may or may not be correct. You are to: A Choose whole numbers nearest to the numbers in the problem and **estimate** the answer. B Then tell whether or not the given answer is correct. Use your estimate to determine this if possible. Exercise 1 is completed correctly.



1 $3\frac{5}{8}$¢ for each gram. 10 grams. Costs how much ? ($42\frac{1}{8}$¢)
A Estimate: 10×4, or 40¢ B The answer is not correct.

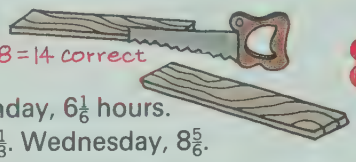
2 Packet costs 36¢. Contains $5\frac{9}{10}$ grams. How much per gram ? ($4\frac{3}{4}$¢)
$36 \div 6 = 6$ not correct



3 Drove 165 km. $3\frac{3}{4}$ hours. Speed ? (38 km/h)
$165 \div 4 = 41$ not correct

4 Man weighs $93\frac{1}{5}$ kg. Another weighs $69\frac{3}{4}$ kg. What is the difference of their weights ? (13 kg)
$93 - 70 = 23$ not correct

5 Cut a board. One piece, $5\frac{4}{5}$ m long. Other piece, $8\frac{1}{4}$ m long. How long before cut ? ($14\frac{1}{20}$ m)
$6 + 8 = 14$ correct

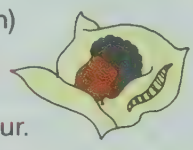


6 Ribbon, $\frac{9}{10}$ metre long. Cut into 5 equal pieces. How long is each piece ? ($\frac{1}{4}$ m)
$1 \div 5 = \frac{1}{5}$ not correct

7 Sleep: Monday, $6\frac{1}{6}$ hours. Tuesday, $7\frac{1}{3}$. Wednesday, $8\frac{5}{6}$. Thursday, $10\frac{2}{3}$. Friday, $8\frac{1}{2}$. Saturday, $9\frac{2}{3}$. Sunday, $7\frac{5}{6}$. What is the average number of hours slept each night ? ($6\frac{3}{7}$ h)
$6+7+9+11+9+10+8 = 60$
$60 \div 7 = 8\frac{4}{7}$ not correct

8 Length of field, $39\frac{2}{5}$ metres. Width of field, 75 metres. What is the area of the field ? (2955 square metres)
40×75 3000 correct

9 $\frac{9}{20}$ km to school. To school and back for 30 days. How far ? ($31\frac{7}{20}$ km)
$1 \times 30 = 30$ not correct



10 Walk $\frac{9}{4}$ km in an hour. Walked $20\frac{1}{4}$ km. How many hours ? (11 h)
$20 \div 2 = 10$ not correct

★**11** Satellite: Around Earth in $1\frac{2}{5}$ hours. How many trips in one week ? (165)
$168 \div 1 = 168$ not correct

218

Discussion

Pages 218 and 219 may be treated in a variety of ways. You might use page 218 as a group activity to review estimation and then assign page 219. Alternatively, page 219 might be used for group work or a discussion, after children have worked the problems on page 218 independently.

Note in exercise 2, page 219, that the difference in the two given temperatures is 15°. The children will probably see that, since $1\frac{1}{2}$ divisions on the gauge represents 15°, 1 division must represent 10°. Then they can reason that the higher temperature must be 85°.

When you discuss these problems, keep in mind that one of the important features of an exercise set on estimation is the opportunity it gives the children to put into words how they thought about rounding in order to arrive at their estimates.

Assignments (page 218)

Minimum: 1–6. Average: 1–10. Maximum: 1–11.

ESTIMATION FOR FUN

1. The gasoline gauge on a car looked like this when there were 2 litres of gasoline in the tank. Estimate the number of litres of gasoline in the tank for each gauge-reading below.



A 

B 

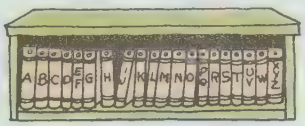
C 

D 

2. When the temperature gauge on the car looked like this, the temperature was 60°C. When the gauge looked like this, the temperature was 75°C. What was the temperature when the gauge looked like this? 85°



3. The thickest book in a set of encyclopedias is $4\frac{3}{4}$ cm. The thinnest book is $1\frac{3}{4}$ cm. Estimate the length of shelf needed to hold the complete set of 20 books. 65 cm

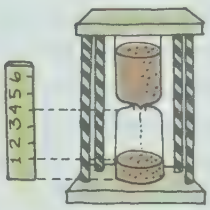


4. The small can holds $19\frac{3}{4}$ grams. Estimate the number of grams the large can holds. 60g



5. A If it has taken $1\frac{3}{4}$ hours for the sand shown to go into the bottom part of the time glass, about how much longer will it take for all the sand to go down? 5¼ hr.

   B Suppose it takes only 1 hour for all the sand to go down. Estimate the number of minutes that have passed when the sand has reached the point midway between 2 and 3 on the ruler. 37½ min



219

Follow-up

To follow up these pages, urge children to make up some estimation problems on their own. They might estimate the number of children in the class, the number of classrooms in the entire school, and then the total number of children in the school. Or, you might have them count the number of cars that pass in front of the school in a five-minute period, and then estimate the number that pass in an hour or during the entire school day.

A variety of estimation problems like these should give added stimulus to the lesson.

Duplicator Masters, page 42
Workbook, page 67

Assignments (page 219)

Minimum: 1–5, oral. Average: 1–5.
Maximum: 1–5.

PAGES 220–221

Objective

The child will demonstrate his ability to work with the concepts presented in this chapter.

Preparation

Review any of the material from the chapter with which the children have had particular difficulty. Since much of the chapter is devoted to developing skills in multiplication and division of fractional numbers and to applications of those skills you will want to devote most of your time to reviewing these areas.

Reviewing the Ideas

1. Find the products.

A $9 \times \frac{1}{5}$ $1\frac{4}{5}$ C $\frac{1}{2} \times \frac{1}{5}$ $\frac{1}{10}$ E $\frac{3}{4} \times \frac{4}{5}$ $\frac{3}{5}$ G $4 \times 3\frac{3}{4}$ 15 I $5\frac{2}{3} \times 3\frac{1}{5}$ $18\frac{2}{15}$

B $\frac{1}{10} \times 4\frac{2}{5}$ D $\frac{1}{3} \times \frac{1}{8}$ $\frac{1}{24}$ F $\frac{2}{3} \times \frac{1}{6}$ $\frac{1}{9}$ H $5 \times 6\frac{2}{3}$ $33\frac{1}{3}$ J $7\frac{1}{6} \times 8\frac{1}{4}$ $59\frac{1}{8}$

2. Find the quotients.

A $1 \div \frac{5}{6}$ $1\frac{1}{5}$ C $5 \div \frac{3}{4}$ $6\frac{2}{3}$ E $6 \div 7$ $\frac{6}{7}$ G $\frac{8}{5} \div \frac{2}{5}$ 4 I $\frac{3}{5} \div 4$ $\frac{3}{20}$

B $1 \div 3$ $\frac{1}{3}$ D $8 \div 3$ $2\frac{2}{3}$ F $\frac{1}{9} \div \frac{1}{3}$ $\frac{1}{3}$ H $\frac{3}{10} \div \frac{4}{5}$ $\frac{3}{8}$ J $3\frac{3}{4} \div 1\frac{2}{3}$ $2\frac{1}{4}$

3. Match each quotient with a product that is equal to it.

A $\frac{5}{6} \div \frac{5}{6}$ 2 1 $\frac{5}{6} \times \frac{5}{6}$

B $\frac{6}{5} \div \frac{5}{6}$ 4 2 $\frac{5}{6} \times \frac{6}{5}$

C $\frac{5}{6} \div \frac{6}{5}$ 1 3 $\frac{6}{5} \times \frac{5}{6}$

D $\frac{6}{5} \div \frac{6}{5}$ 3 4 $\frac{6}{5} \times \frac{6}{5}$

4. Simplify each expression.

A $\frac{\frac{1}{2}}{\frac{2}{3}}$ $\frac{3}{4}$ B $\frac{\frac{3}{5}}{\frac{1}{10}}$ 6 C $\frac{\frac{3}{10}}{\frac{1}{10}}$ 3 D $\frac{12\frac{1}{2}}{100}$ $\frac{1}{8}$

E $\frac{\frac{1}{4}}{\frac{1}{5}}$ $1\frac{1}{4}$ F $\frac{\frac{3}{2}}{\frac{2}{3}}$ $2\frac{1}{4}$ G $\frac{\frac{3}{4}}{\frac{3}{4}}$ 1 H $\frac{\frac{1}{3}}{100}$ $\frac{1}{300}$

5. Find the quotients. Write them as mixed numerals.

A $49 \div 7$ 7 B $85 \div 3$ $28\frac{1}{3}$ C $104 \div 9$ $11\frac{5}{9}$ D $276 \div 5$ $55\frac{1}{5}$ E $5963 \div 24$ $248\frac{11}{24}$

6. Give the symbol ($<$, $>$, $=$) for each ◍.

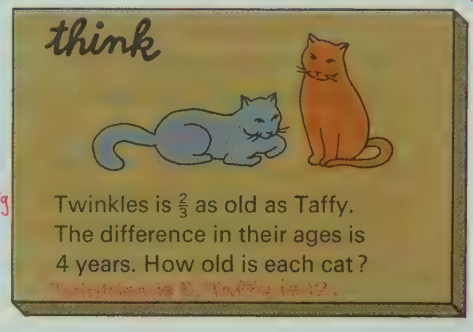
A $5 \div 2$ ◍ $5 \div \frac{1}{2}$ $<$ B $8 \div \frac{1}{4}$ ◍ $8 \div \frac{1}{2}$ $>$ C $\frac{3}{4} \div \frac{2}{3}$ ◍ $\frac{2}{3} \div \frac{3}{4}$ $>$

7. Suppose a can contains $67\frac{1}{2}$ grams of nuts.

A How many grams of nuts are in 5 such cans? $337\frac{1}{2}$ g

B How many grams in $\frac{1}{2}$ can? $33\frac{3}{4}$ g

C How many $6\frac{3}{4}$-gram paper cups can you fill from 1 can of nuts? 10



Discussion

You might want to work through several exercises on these pages with the children as a review, or you might assign the pages as independent work and use the results to evaluate their understanding of the concepts treated.

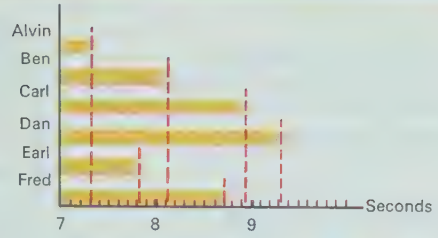
In either case, remind the children to examine each answer and ask themselves if it seems reasonable. Here, the estimating skills they have just studied will prove very helpful. Work through problems in any area which children found difficult. Some children will need two days if you want them to do all the exercises and problems. Encourage children to share explanations for their solutions.

A tabular form is a convenient way to record the children's guesses for the *Think* problem. Such a table may help clarify the explanations for those who could not solve the problem. For example:

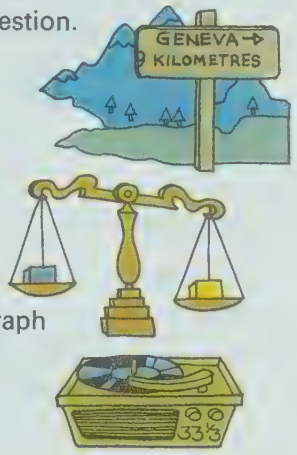
If – Twinkles	Then – Taffy
$\frac{1}{3}$ of 2 or $1\frac{1}{3}$	3
$\frac{2}{3}$ of 6 or 4	6
$\frac{2}{3}$ of 9 or 6	9
$\frac{2}{3}$ of 12 or 8	12

Since $12 - 8 = 4$, Twinkles is 8 and Taffy is 12.

8. The graph gives the time it took each boy to run the 50-metre dash. Give the average (arithmetic mean) of these times. $8\frac{7}{20}$



9. Write and solve an equation to answer each question.



A An average walker can cover 1 kilometre in about $\frac{1}{6}$ hours. How long would it take him to reach Geneva ? 1½ hours

B A cubic centimetre of gold weighs $2\frac{2}{5}$ as much as a cubic centimetre of iron, which weighs about $4\frac{6}{10}$ g. What is the 11½ g weight of a cubic centimetre of gold ?

C A $33\frac{1}{3}$-rpm (revolutions per minute) phonograph record turns around $33\frac{1}{3}$ times each minute. If it takes 21 minutes to play the record, how many times does it turn around ? 700

D Each $\frac{3}{10}$ on a map represents 1 km on land. The distance between two cities on the map is $2\frac{4}{10}$ cm. How many kilometres apart are these cities ? 8

E On a scale drawing, a room is 9 cm long. If $\frac{2}{5}$ cm represents 1 metre, how long is the room ? 22½ m

F A diamond about this size  weighs $\frac{1}{3}$ carat. An object that weighs 2500 carats weighs 500 g. How many diamonds of the size shown does it take to make 1 kilogram ? 15000

10. Estimate the number of $4\frac{7}{10}$-metre pieces of rope you can cut from a piece of rope $89\frac{3}{10}$ metres long. 19

Workbook, page 68

PAGES 222–223

Objective

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

Since a large portion of this lesson is devoted to units of measure, it should be helpful to review these by presenting exercises similar to exercises 4 and 5 on the chalkboard. You may need to list some of the equivalents on the chalkboard, or have the children look them up in their Tables of Measures at the back of the book.

Keeping in Touch with

Computing
Measurement
Fractional numbers

1. Solve the equations.

A $9 \times n = 56$ $6\frac{2}{9}$ B $27 + n = 34$ 7 C $48 \div n = 6$ 8 D $98 - 29 = n$ 69

2. Solve the equations.

A $(9 \times 6) + n = 60$ 6 C $(4 \times 8) + t = 36$ 4 E $(3 \times n) + 5 = 29$ 8

B $(8 \times 7) + s = 59$ 3 D $(9 \times 7) + r = 70$ 7 F $(5 \times p) + 7 = 42$ 7

3. Find the sums, differences, products, and quotients.

A 9672 + 4865 = 14,537

B 7004 − 879 = 6125

C 598 × 63 = 37,674

D 529 × 304 = 160,816

E 9)6347 = 705 R2

F 58)429 = 7 R23

think

Suppose you snap your fingers 2 minutes from now; wait 4 minutes, do it again; wait twice as long (8 minutes), do it again. If you continue this, will you snap your fingers about 2000, 200, or 20 times during the next year?

20

4. Find the totals. Change each answer so that you have the greatest number of the larger unit.

A 6 h 54 min + 8 h 10 min = 15 hr 4 min

B 6 min 30 sec + 9 min 40 sec = 16 min 10 sec

C 8 yr 7 mo + 23 yr 9 mo = 32 yr 4 mo

D 12 wk 2 days + 9 wk 6 days = 22 wks 1 day

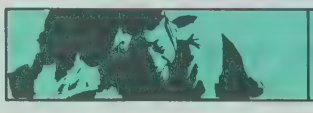
5. Find the differences.

A 15 wk 4 days − 6 wk 5 days = 8 wk 6 days

B 5 h 1 min − 3 h 2 min = 1 hr 59 min

C 24 min 17 sec − 15 min 35 sec = 8 min 42 sec

D 39 yr 7 mo − 19 yr 10 mo = 19 yr 9 mo



You are invited to explore

ACTIVITY CARD 11
Page 360

222

Discussion

If children need quite a bit of review for exercises 4 and 5 on page 222, you might plan to have children spend two math periods on these pages. When they have finished with the exercises on page 222, have them explain their solutions to those exercises which caused the most difficulty.

On page 223, use the illustrations of devices for measuring time as a basis for discussion. Explain to the children that the device illustrated beneath exercise 13 is a sun dial.

You will probably find that all children will enjoy participating in the activity suggested by the *Think* problem on page 222, even though some may not be able to do all the arithmetic needed to determine which answer is correct.



Short Stories

TIME



1 2044 seconds. How many minutes? $34\frac{1}{15}$

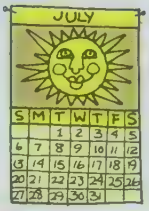
2 $\frac{1}{2}$ hour. How many seconds? 1800

3 9 hours. How many minutes? 540

4 $5\frac{1}{4}$ days. How many hours? 126

5 1 week. How many minutes? 10 080

6 3312 minutes. How many days? $2\frac{3}{10}$



7 11 years old. How old
A in days? 4015
B in hours? 96 360
C in minutes? 5 781 600
D in seconds? 346 896 000

8 324 months. How many years? 27

9 Semimonthly means twice a month. Meet a friend semimonthly for 12 years. How many meetings? 288

10 100 years: 1 century. How many full centuries have passed since the year 1 A.D.? 19

11 10 years: 1 decade. How many decades of the 20th century have passed? 7

12 Bimonthly means once very 2 months. Play golf bimonthly for 12 years. How many games? 72

13 How long is twice as long as: 3 weeks, 4 days, 8 hours, 35 minutes, 40 seconds? Use the smallest number of each unit.



7 wk, 1 day, 17 hr, 11 min, 20 sec

14 1000 years: 1 millenium. Write a mixed numeral that describes the milleniums that have passed since the year 1 A.D. $1\frac{9**}{1000}$ (*digits depend on current year)



15 1 millenium, 9 centuries, 7 decades, 6 years, and 8 months. How many years? $1976\frac{8}{12}$

16 The Great Sphinx in Egypt was built in 2900 B.C. How many years ago? 2900 + 19** (*digits depend on current year)

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Follow-up

If some children found page 223 difficult, you might have them do research on various time measuring devices, or on how clocks are made. If possible, show the film "About Time," produced by Bell System Science Series. (Contact your local phone company to find out about your nearest Telephone Film Library.)

CHAPTER 10 | Decimals

Pages 224–257

General Objectives

To introduce decimal notation
To provide additional experience with fractional numbers
To extend concepts of place value to include tenths, hundredths, thousandths, and so on
To introduce addition and subtraction of fractional numbers using decimal notation
To introduce multiplication and division of fractional numbers using decimal notation
To relate notation for money to decimal notation
To provide a variety of word problems using decimal notation
To introduce scientific notation
To relate concepts of decimals, fractional numbers, and the long-division process
To introduce repeating decimals
To provide estimation experiences using decimals

The opening pages of this chapter explain what decimal notation means and how numerals are written and read in this notation. Following this, the use of decimals to label points on the number line is described, and inequalities using decimal notation are introduced.

Then, addition and subtraction using decimal notation are presented, and a variety of word problems follow. Before the children are introduced to multiplication and division of decimals, they round numbers represented by decimals and approximate products in decimal notation to strengthen their understanding of decimals. The children are then introduced to multiplication of decimals, including a variety of multiplication activities, which leads to the shortcut for placing decimal points. After this, the process of dividing, using decimal notation, is introduced and developed. More word problems follow.

Toward the end of the chapter, the children learn how to use the dividing process to arrive at a decimal quotient. This kind of work also establishes the relationship between fractional numbers represented as fractions and fractional numbers represented as decimals. Further, it helps the children understand how to use the dividing process to find the decimal representation of a fraction. Next, repeating decimals are introduced, and finally, the children are exposed to several applications of decimals, including scientific notation.

Mathematics

The following equation illustrates a specific example of the general definition of decimal notation.

$$3.672 = 3 + \tfrac{6}{10} + \tfrac{7}{100} + \tfrac{2}{1000}$$

In order to read 3.672 as *three and six hundred seventy-two thousandths*, we must show that, given the basic definition of 3.672 above,

$$3 + \tfrac{6}{10} + \tfrac{7}{100} + \tfrac{2}{1000} = 3\tfrac{672}{1000}.$$

The proof is simple; nevertheless, it is an important step and should not be omitted:

$$\begin{aligned} 3 + \tfrac{6}{10} + \tfrac{7}{100} + \tfrac{2}{1000} &= \\ &= 3 + \tfrac{600}{1000} + \tfrac{70}{1000} + \tfrac{2}{1000} \\ &= 3\tfrac{672}{1000} \end{aligned}$$

The mathematics section for this chapter may seem oversimplified for the amount of work provided. However, decimal notation for fractional numbers constitutes the only new material. The mathematical concepts associated with fractional numbers and the various operations are the same.

Some fractional numbers cannot be represented by fractions having denominators of 10, 100, 1000, and so on. These fractional numbers can, however, be expressed as repeating decimals. Some of the most obvious examples are the decimal representations of $\frac{1}{3}$ (0.333. . .), $\frac{2}{3}$(0.666. . .), $\frac{1}{6}$(0.1666. . .), and so on. Admittedly, we could also consider a decimal such as 0.5000 . . . as a repeating decimal because it repeats zeros, but it is usually thought of as a terminating decimal and is simply written as 0.5.

We emphasize again that the mathematics for this chapter is the same as that for previous chapters on fractional numbers; only the notation is different.

Teaching the Chapter

Materials

Colored strips
Container for water and oil, such as a glass beaker or a glass graduate
Graph paper, 1-cm grid or larger
Metre stick
Metric weights, assorted, gram to kilogram
Oil, vegetable or light motor oil (about 3-5 cu cm)
Overhead projector (if available) and transparencies
Pails for water, 3 litre and 8 litre
Ruler, centimetre
Scales, balance type (if available)
Scissors
Slips of paper, 2-cm square
Two cubes of different density (metal or wood, and styrofoam)
Water

Vocabulary

decimal	millionth
decimal point	repeating decimal
gram	scientific notation
hundredth	tenth
hundred thousandth	ten thousandth
metre	thousandth

Although the children have been introduced to words such as tenth, hundredth, and thousandth prior to this chapter, these words are listed as new vocabulary because of the context in which they are used. In

this chapter, these words refer to notation using decimals. Point out that the difference between tenths written as a fraction and tenths written as a decimal is simply a matter of notation. Explain to the children that tenths are not really new to them, because they have worked with fractions including tenths for a long time. This chapter simply shows a new way to write fractions for tenths, hundredths, thousandths, and so on—as decimals.

Lesson Schedule

Plan to spend about three and a half to four weeks on this chapter, adjusting your schedule to the special needs and abilities of your children and to the amount of time left in the school year. All the material from Chapter 10 through the end of the book is covered thoroughly again in the seventh-, eighth-, and ninth-grade programs. Therefore, you may wish to be selective in your use of the remaining topics in the text, depending on how much time remains in the school year.

Evaluation of Progress

In this chapter, evaluation presents a special problem. One of the primary objectives is to help the children understand decimal notation; therefore, your evaluation should center on the degree to which they have attained such an understanding. However, it is not easy to determine whether children grasp decimal concepts, because much of the work with decimals involves computing that is based on the techniques learned for whole numbers. For example, in decimal multiplication, the children eventually are led to the algorithm for multiplying with decimals: they simply multiply as if they were using whole numbers and then determine the location of the decimal point for the final product. Naturally, a great deal of work is provided to show why the decimal point is located in a certain place. Therefore, you will need to evaluate the children's daily participation and understanding, as well as their ability to work with the final algorithm.

In studying this material, one goal should be to convince children that they are working with exactly the same fractional numbers that they have worked with all along and that the difference is only in the way they represent these numbers on their papers. Once you have convinced them of this, it should be a simple matter to point out the convenience of working with decimals. This convenience is especially obvious in working with addition and subtraction, where the decimal points are lined up to assure computing with common-denominator fractions. In multiplication and division, the convenience of working with decimals also becomes apparent, for calculating with fractional numbers is no more difficult than calculating with whole numbers.

The ideas that form the nucleus of this chapter are reviewed on pages 254 and 255. A cumulative review is supplied on pages 256 and 257.

Resources for Active Learning

GENERAL ACTIVITIES

Mathex: Numeration No. 7, "Decimals," pp. 47–48 (pupil pages 52–60), Encyclopaedia Britannica Publications Ltd.

Nuffield Project: *Computation and Structure* 4, ". . . Place Value," pp. 1–12, Wiley

MANIPULATIVE DEVICES

Centimetre Decimal Set (Math Media)

Decimal Abacus Board (Selective Educational Equipment)

Metre Stick (Geyer Instructional Aids)

Metric Measure Set (Gamco)

Metric System Materials (Metric Association)

PAGES 224–225

Objectives

Given fractions with denominators which are hundreds or thousands, the child will be able to write decimals for the fractions.

Given a decimal, the child will be able to give the name of each place of the decimal.

Preparation

Materials

graph paper, 1-cm grid or larger (*Duplicator Masters*, page 62); scissors

In order for children to benefit from this investigation, it is important that they understand that units are arbitrary and may be chosen for a particular need or convenience. For example, you might have them recall how different strips were used as the unit strip when they studied fractional numbers. Then show them samples of the unit used in this lesson. Since children will be cutting out and using the small squares of graph paper for the hundredths, you might duplicate grids that are at least 1 square cm. The children will also be working with 10-by-10 squares (unit) and 10-by-1 strips (tenths).

Investigation

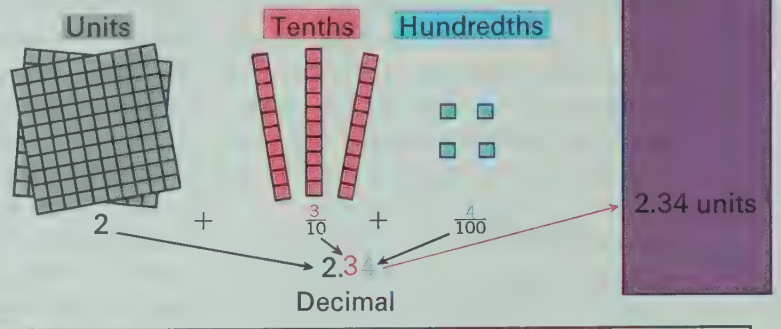
Once children understand how the units, tenths, and hundredths are to be represented with the graph paper, encourage them to work on the investigation independently or with a classmate. It is intended that the children first cut out the graph paper and then use various pieces to make an interesting region. Direct them to trace around the pieces they have chosen in order to show the region. They should then try to figure out how to represent its area as a decimal. As the children work, move around the room and ask occasional questions to direct their attention to the relationships between units, tenths, and hundredths, such as, "Which is greater, 4 hundredths or 4 tenths?" They should be able to see that 4 1-by-1 squares (hundredths) are clearly fewer than 4 strips (tenths).

10 Decimals

● *Let's explore decimal names for fractional numbers.*

Investigating the Ideas

The pieces below will just cover this region.
We can show the region's area as a decimal.

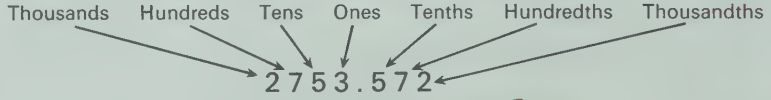


Can you cut from graph paper some pieces like those above, put them together to form an interesting region, and give the region's area as a decimal? See Investigation.

Discussing the Ideas

1. The decimal 2.34 is another way of writing $2 + \frac{3}{10} + \frac{4}{100}$.
   What decimal would you write for $3 + \frac{5}{10} + \frac{7}{100}$? 3.57

2. Study the diagram. Give the missing fractions.
   For the sum $2753 + \frac{5}{10} + \frac{7}{100} + \frac{2}{1000}$, we write:



   A The 5 in the tenths' place means __? __. $\frac{5}{10}$
   B The 7 in the hundredths' place means __? __. $\frac{7}{100}$
   C The 2 in the thousandths' place means __? __. $\frac{2}{1000}$



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Discussion

Before beginning the discussion exercises in the text, it would be helpful to have several children show their regions; tell how many units, tenths, and hundredths they used; and write the decimal for their region's area.

During the discussion, there are two important points to stress and explain. Be sure children realize that a decimal is simply another way of writing a fractional number. Explain that whenever a fractional number can be written as a fraction with 10, 100, 1000, and so on, as the denominator, it is often convenient to use a decimal to name this number. Provide examples of fractional numbers represented first by fractions or mixed numerals and then by decimals.

$\frac{6}{10} = 0.6$ $\quad 24\frac{5}{10} = 24.5$
$\frac{9}{100} = 0.09$ $\quad 36\frac{7}{100} = 36.07$
$\frac{7}{1000} = 0.007$ $\quad 56\frac{2}{1000} = 56.002$

Also, as you work through exercise 2, emphasize the place value of each digit. Again, provide other examples. Note that the example in the book shows how the ones' place may be thought of as the central digit and that the digits to its right (and to the right of the decimal point) follow a pattern similar to the digits on the left. A chart such

Using the Ideas

1. Copy the sentence. Give the missing word and number.

A 537.29: The 3 in the __?__ place means ||||. tens, 30

B 537.29: The 2 in the __?__ place means ||||. tenths, $\frac{2}{10}$

C 537.29: The 5 in the __?__ place means ||||. hundreds, 500

D 537.29: The 9 in the __?__ place means ||||. hundredths, $\frac{9}{100}$

E 8462.157: The 8 in the __?__ place means ||||. thousands, 8000

F 8462.157: The 7 in the __?__ place means ||||. thousandths, $\frac{7}{1000}$

G 8462.157: The 5 in the __?__ place means ||||. hundredths, $\frac{5}{100}$

2. Write each fractional number as in the example. See Answers, T.E. page 225.

$54.21 = 54 + \frac{2}{10} + \frac{1}{100}$

A 356.3	D 89.05	G 836.4	J 0.999	M 5.490
B 49.86	E 3.008	H 83.64	K 8.364	N 8.603
C 2.475	F 50.7	I 5.607	L 0.836	O 29.07

3. Write the correct decimal for each sum.

A $9 + \frac{4}{10}$ 9.4

B $9 + \frac{4}{10} + \frac{3}{100}$ 9.43

C $9 + \frac{4}{10} + \frac{3}{100} + \frac{7}{1000}$ 9.437

D $50 + \frac{7}{10}$ 50.7

E $50 + \frac{7}{10} + \frac{2}{100}$ 50.72

F $5 + \frac{6}{10}$ 5.6

G $5 + \frac{0}{10} + \frac{6}{100}$ 5.06

H $5 + \frac{0}{10} + \frac{0}{100} + \frac{6}{1000}$ 5.006

I $5 + \frac{3}{100}$ 5.03

J $5 + \frac{7}{1000}$ 5.007

K $8 + \frac{3}{10} + \frac{9}{1000}$ 8.309

L $17 + \frac{4}{100} + \frac{6}{1000}$ 17.046

M $80 + \frac{8}{10}$ 80.8

N $600 + \frac{6}{100}$ 600.06

O $9000 + \frac{9}{1000}$ 9000.009

★**4.** Use what you know about place value to answer questions about the decimal 5 496 000.007281
Give the number of:

A thousands 6 B thousandths 7

C ten thousands 9

D ten thousandths 2

E hundred thousands 4

F hundred thousandths 8

G millions 5 H millionths 1

think

Give the missing numbers so this will be a magic square.

$\frac{8}{12}$	$\frac{9}{12}$	$\frac{4}{12}$
$\frac{3}{12}$	$\frac{7}{12}$	$\frac{11}{12}$
$\frac{10}{12}$	$\frac{5}{12}$	$\frac{6}{12}$



as the following might help children with this:

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
$\frac{1000}{1}$	$\frac{100}{1}$	$\frac{10}{1}$	$\frac{1}{1}$	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
1000	100	10	1	.1	.01	.001

Using the Exercises

You may need to provide a special discussion of exercise 3, parts I through O, before having the children attempt the exercises. A thorough discussion of the children's work is important as preparation for the next lesson, so allow time for checking answers.

Children who finish quickly may wish to try to solve the *Think* problem. Some of them may need to be reminded that the magic sum holds for each column, row, and diagonal.

Assignments (page 225)

Minimum: 1–3. Average: 1–3.
Maximum: 1–4.

Follow-up

To provide extra practice, you might prepare a worksheet like this.

Write the decimal for each of the following.

1. 87 and 59 hundredths ____
2. 3 and 958 thousandths ____
3. 601 and 7 hundredths ____
4. 4 and 37 thousandths ____
5. 23 and 211 thousandths ____
6. 376 and 9 tenths ____

Write each decimal as a sum of fractions.

1. 0.34 = _____
2. 0.701 = _____
3. 2.16 = _____
4. 57.98 = _____
5. 201.102 = _____
6. 39.39 = _____

Answers, exercise 2, page 225

2.A $356.3 = 356 + \frac{3}{10}$

B $49.86 = 49 + \frac{8}{10} + \frac{6}{100}$

C $2.475 = 2 + \frac{4}{10} + \frac{7}{100} + \frac{5}{1000}$

D $89.05 = 89 + \frac{5}{100}$

E $3.008 = 3 + \frac{8}{1000}$

F $50.7 = 50 + \frac{7}{10}$

G $836.4 = 836 + \frac{4}{10}$

H $83.64 = 83 + \frac{6}{10} + \frac{4}{100}$

I $5.607 = 5 + \frac{6}{10} + \frac{7}{1000}$

J $0.999 = \frac{9}{10} + \frac{9}{100} + \frac{9}{1000}$

K $8.364 = 8 + \frac{3}{10} + \frac{6}{100} + \frac{4}{1000}$

L $0.836 = \frac{8}{10} + \frac{3}{100} + \frac{6}{1000}$

M $5.490 = 5 + \frac{4}{10} + \frac{9}{100}$

N $8.603 = 8 + \frac{6}{10} + \frac{3}{1000}$

O $29.07 = 29 + \frac{7}{100}$

Resources for Active Learning

Experiences in Mathematics Ideas, Volume 1, "Physical Models for Decimals," pp. 168–197, NCTM.

Workbook, page 69

PAGES 226–227

Objectives

Given a decimal using tenths, hundredths, or thousandths, the child will be able to read it.

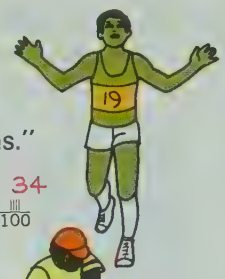
Given a fraction with a denominator of 10, 100, or 1000, the child will be able to write it as a decimal.

Preparation

To prepare for this lesson, you might give children practice in identifying the place value of digits in a decimal. For example, present several decimals on the chalkboard or overhead projector and ask children to give the place value of specific digits you name.

How are decimals read?

Discussing the Ideas

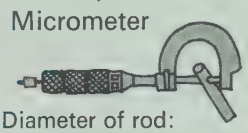


1. Jack ran a kilometre in 2.54 minutes.
   $2.54 = 2 + \frac{5}{10} + \frac{4}{100} = 2 + \frac{50}{100} + \frac{4}{100} = 2\frac{54}{100}$
   For 2.54 we read "two and fifty-four hundredths."
   - A Give the missing numerators.
     $8.34 = 8 + \frac{3}{10} + \frac{4}{100} = 8 + \frac{30}{100} + \frac{4}{100} = 8 + \frac{34}{100}$
   - B How would you read the decimal in part A?



2. Tony's batting average is 0.355
   $0.355 = \frac{3}{10} + \frac{5}{100} + \frac{5}{1000}$
   $= \frac{300}{1000} + \frac{50}{1000} + \frac{5}{1000} = \frac{355}{1000}$
   For 0.355 we read "three hundred fifty-five thousandths."
   - A Give the missing numerators.
     $0.317 = \frac{3}{10} + \frac{1}{100} + \frac{7}{1000} = \frac{300}{1000} + \frac{10}{1000} + \frac{7}{1000} = \frac{317}{1000}$
   - B How would you read the decimal in part A?

3. The examples show some uses of decimals. Read each decimal.
   - A Micrometer



Diameter of rod: 1.241 cm

"One and two hundred forty-one thousandths"

   - B Odometer



Distance: 34.5 kilometres

"Thirty-four and five tenths"

   - C Barometer



Air pressure: 76.13 cm

"Seventy-six and thirteen hundredths"

   - D Racing car



Speedway record: 257.735 km/h

"Two hundred fifty seven and seven hundred thirty five thousandths"

4. Read these decimals.

A	27.8	E	6.38	I	4.07	M	125.7	Q	7.08	U	0.007
B	8.76	F	19.56	J	0.36	N	32.09	R	0.023	V	0.654
C	5.123	G	35.872	K	0.743	O	4.008	S	0.08	W	0.054
D	4.364	H	2.654	L	96.04	P	16.017	T	3.64	X	0.09

Follow patterns of exercise 3.

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Discussion

The main purpose of this discussion section is to enable children to read decimals. Exercises 1 and 2 help to show why we read decimals as we do. Some children may find this difficult, so do not seek mastery or spend too much time on this explanation. As you discuss exercise 1, point out that $\frac{54}{100}$ is the sum of $\frac{5}{10}$ and $\frac{4}{100}$ and that $\frac{5}{10}$ had to be changed to $\frac{50}{100}$ in order to add it to $\frac{4}{100}$. Similarly, in exercise 2, $\frac{3}{10}$ and $\frac{5}{100}$ were changed to $\frac{300}{1000}$ and $\frac{50}{1000}$ before finding the sum $\frac{355}{1000}$. Children who understand these exercises will have a clearer concept of why decimals such as 6.369 can be read as "six and three hundred sixty-nine thousandths."

Use exercises 3 and 4 to help all children learn to read decimals, especially those who found exercises 1 and 2 difficult. Provide other examples as needed.

Using the Ideas

1. Copy each exercise and give the missing numerator or denominator.

A $5.7 = 5\frac{7}{10}$ B $6.54 = 6\frac{54}{100}$ C $8.37 = 8\frac{37}{100}$ D $6.50 = 6\frac{50}{100}$

E $9.06 = 9\frac{6}{100}$ F $30.43 = 30\frac{43}{100}$ G $8.276 = 8\frac{276}{1000}$ H $3.495 = 3\frac{495}{1000}$

I $0.407 = \frac{407}{1000}$ J $0.560 = \frac{560}{1000}$ K $0.560 = \frac{56}{100}$ L $0.078 = \frac{78}{1000}$

M $5.060 = 5\frac{60}{1000}$ N $0.024 = \frac{24}{1000}$ O $3.004 = 3\frac{4}{1000}$ P $14.07 = 14\frac{7}{100}$

2. Find the correct decimal.

A $\frac{13}{100} =$ 0.13 B $\frac{63}{100} =$ 0.63 C $\frac{572}{1000} =$ 0.572

D $\frac{17}{1000} =$ 0.017 E $6\frac{25}{100} =$ 6.25 F $8\frac{56}{100} =$ 8.56

G $1\frac{5}{10} =$ 1.5 H $\frac{15}{10} =$ 1.5 I $\frac{125}{100} =$ 1.25

3. Copy and complete each exercise.

A $\frac{1}{2} = \frac{5}{10} =$ 0.5 B $\frac{3}{25} = \frac{12}{100} =$ 0.12 C $\frac{8}{20} = \frac{40}{100} =$ 0.40

D $\frac{1}{8} = \frac{125}{1000} =$ 0.125 E $\frac{7}{8} = \frac{875}{1000} =$ 0.875 F $\frac{4}{5} = \frac{8}{10} =$ 0.8

G $\frac{17}{50} = \frac{34}{100} =$ 0.34 H $\frac{1}{4} = \frac{25}{100} =$ 0.25 I $\frac{5}{8} = \frac{625}{1000} =$ 0.625

J $\frac{3}{4} = \frac{75}{100} =$ 0.75

4. Give the correct number for n.

A n thousandths = 1 hundredth: $n = 10$

B n hundredths = 1 tenth: $n = 10$

C n tenths = 1: $n = 10$

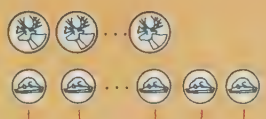
5. Write a decimal for each number.

A $8\frac{7}{10}$ 8.7 B $53\frac{27}{100}$ 53.27 C $\frac{75}{100}$ 0.75 D $54\frac{357}{1000}$ 54.357

E $76\frac{7}{100}$ 76.07 F $873\frac{9}{1000}$ 873.009 G $49\frac{76}{1000}$ 49.076

H $\frac{7}{20}$ 0.35 I $\frac{9}{50}$ 0.18 J $\frac{6}{25}$ 0.24 K $\frac{3}{5}$ 0.6 L $\frac{17}{25}$ 0.68

think



30 + 30 ... 30 + 5 + 5 = 280

In a collection of quarters and nickels, there are 2 more nickels than quarters. Use the diagram to tell how many of each coin if the collection is worth $2.80.

9 quarters 11 nickels



Using the Exercises

Before assigning the exercises on page 227, you might want to work parts of exercise 3 with the children. Point out how each of the fractions can be written first as an equivalent fraction with a denominator of 10, 100, or 1000, and then as a decimal. It is hoped that after children work exercise 3, they will be able to apply their understanding to exercises 5H through 5L. You might want to use these as discussion exercises for some children.

When children have finished, allow time for checking their work, encouraging them to give explanations for some parts of each exercise.

Assignments (page 227)

Minimum: 1–3. Average: 1–5. Maximum: 1–5.

Follow-up

A worthwhile follow-up activity for a lesson such as this is simply to provide more practice in reading decimals.

Exhibit a variety of decimals on the chalkboard or overhead projector, and give the children an opportunity to read them. As they say these decimals aloud, review with them why they can read a decimal such as 5.249 as *five and two hundred forty-nine thousandths*. Emphasize, especially, why $\frac{200}{1000} = \frac{20}{100} = \frac{2}{10} = 0.2$; why $\frac{360}{1000} = \frac{36}{100} = 0.36$; and so on.

Workbook, page 70

PAGES 228–229

Objective

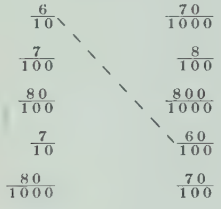
Given two fractional numbers in decimal notation, the child will be able to compare them and indicate the greater of the two by writing an inequality.

Preparation

Materials

slips of paper, about 2 cm square, (4 per child)

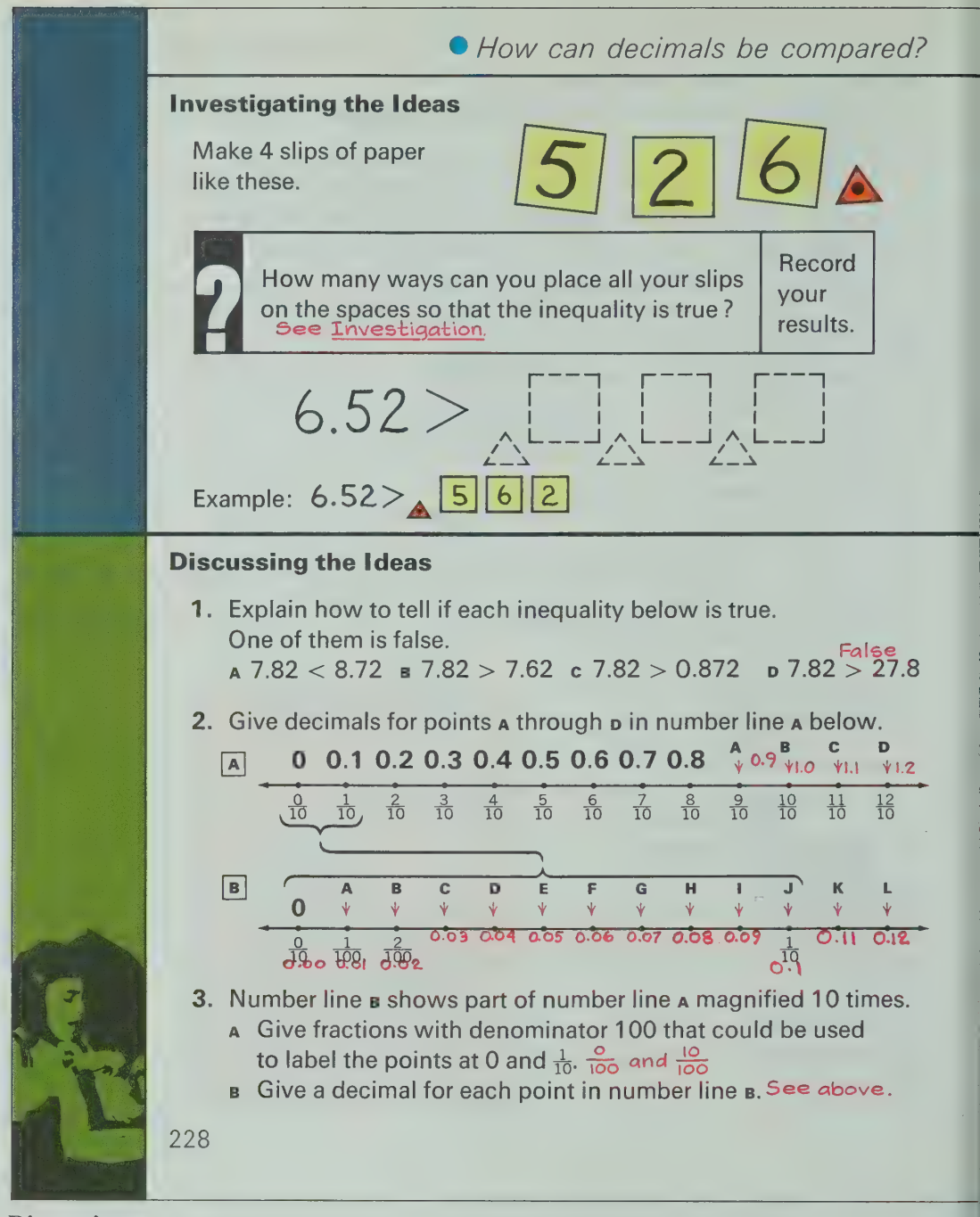
According to the need of the children, list several decimals for a review of reading decimals. You might also review equivalences of fractions with denominators of 10, 100, and 1000. For example, have children match equivalent fractions you have written in two lists.



Investigation

Stress with the children how important it is for them to record their investigation results. Encourage them to use the space provided in the text and place their pieces of paper where they would put the digits. The possible true inequalities are as follows:

$6.52 > 0.256$	$6.52 > 2.56$
> 0.265	> 2.65
> 0.526	> 5.26
> 0.562	> 5.62
> 0.625	> 6.25
> 0.652	



Discussion

Ask several children to give one of the inequalities they found in the investigation and explain why each is true. In particular, stress that $6.52 > 6.25$ because $\frac{52}{100} > \frac{25}{100}$; $6.52 > 0.652$ because 0.652 is less than 1.

As you work through discussion exercise 1, it would be helpful to have some children write inequalities with mixed numerals as well:

$7.82 < 8.72$	$7.82 > 7.62$
$7\frac{82}{100} < 8\frac{72}{100}$	$7\frac{82}{100} > 7\frac{62}{100}$

Continue to stress place-value concepts and the relationship of decimals to fractional numerals.

For exercises 2 and 3, it is essential that children realize what part of a number line they are studying. For example, be sure they realize that point B in number line 2A is the whole number 1, labelled as $\frac{10}{10}$.

For number line B, focus attention primarily on the labels between $\frac{0}{10}$ and $\frac{1}{10}$. Help the children see that they can think of this as a magnified portion of the part of number line A between 0 and $\frac{1}{10}$. It would also be helpful to relate the hundredths on number line B to those on number line A by writing inequalities. For example, ask children to compare point E with $\frac{1}{10}$ ($\frac{5}{100} < \frac{1}{10}$).

Using the Ideas

1. Give the correct sign (<, =, or >) for each ◍.

A	0.5 ◍ 0.3 >	F	28.9 ◍ 29.1 <	K	0.004 ◍ 0.04 <
B	18.7 ◍ 18.9 <	G	36.08 ◍ 36.80 <	L	0.680 ◍ 0.68 =
C	47.6 ◍ 46.6 >	H	0.3 ◍ 0.30 =	M	0.080 ◍ 0.08 =
D	0.28 ◍ 0.38 <	I	0.30 ◍ 0.300 =	N	1.010 ◍ 0.999 >
E	12.54 ◍ 12.45 >	J	0.4 ◍ 0.04 >	O	4.009 ◍ 4.090 <

2. Give the number that is **one tenth** more than:
A 0.3 0.4 B 0.9 1.0 C 1.5 1.6 D 0.38 0.48 E 0.004 0.104 F 5.95 6.05

3. Give the number that is **one hundredth** more than:
A 0.05 0.06 B 0.09 0.10 C 0.29 0.30 D 0.037 0.047 E 0.6 0.61 F 0.99 1.00 G 99.99 100.00

4. Give the number that is **one thousandth** more than:
A 0.003 0.004 B 0.009 0.010 C 0.569 0.570 D 0.6 0.601 E 0.38 0.381 F 0.099 0.100 G 0.999 1.000

5. Give the correct sign (<, =, >) for each ◍.

<	A	$54.237 \text{ ◍ } 54+\frac{3}{10}+\frac{3}{100}+\frac{7}{1000}$
<	B	$3.058 \text{ ◍ } 3+\frac{5}{10}+\frac{8}{1000}$
=	C	$9.7042 \text{ ◍ } 9+\frac{7}{10}+\frac{4}{1000}+\frac{2}{10.000}$
=	D	$61.359 \text{ ◍ } 60+\frac{13}{10}+\frac{5}{100}+\frac{9}{1000}$
<	E	$5.070 \text{ ◍ } 50+\frac{70}{1000}$
<	F	$7.008 \text{ ◍ } 7+\frac{8}{100}$
=	G	$5.280 \text{ ◍ } 5+\frac{2}{10}+\frac{8}{100}$
>	H	$8.064 \text{ ◍ } 8+\frac{6}{100}$
>	I	$4.000 \text{ ◍ } 3+\frac{9}{10}+\frac{9}{100}+\frac{9}{1000}$
<	J	7.999 ◍ 8
>	K	$6.204 \text{ ◍ } 6+\frac{200}{1000}$
=	L	$5.832 \text{ ◍ } 5+\frac{800}{1000}+\frac{32}{1000}$

think

■■■■.■■■

Each digit in this decimal is covered by a red screen. Give the decimal if

A it names the largest number possible when no two digits are alike.

B it names the smallest number possible when no digits are alike.

C it names a number that is closer to 10 000 than any other such decimal.

See below.

A 9876.543 B 1023.456 C 999.999

229



Follow-up/Hi-Lo

To provide practice in reading and comparing decimals, suggest to the children that they play a game like the following.

Direct all players to print the numerals 0 through 9 on appropriate playing cards. They should also draw some picture on one unnumbered card; this will be the "joker." In this game of "Hi-Lo," the joker will represent a decimal point. Each player keeps one joker throughout the game.

Four players should combine their numeral cards to make a 40-card deck. The dealer shuffles and deals four cards face down to each player, and calls "Hi" or "Lo." Each player places his joker among these and rearranges his five cards as he wishes while they are still face down. Then the dealer asks the players to show their cards, reminding them whether he called "Hi" or "Lo." Each player turns over his cards as they are sitting before him and reads off his number. The player with the highest (or lowest) number gets a point. The player with the most points after a predetermined number of rounds wins.

Duplicator Masters, page 43
Workbook, page 71
Skill Masters, page 43

Using the Exercises

On page 229, suggest to the children that if they are in doubt about any part of exercises 1 through 4, they should write the decimal as a fraction and then make the comparison. You might want to use several of these as discussion material with some children.

You can use the *Think* problem at the bottom of the page for those who complete their work quickly. In this problem, it may help to observe that zero cannot be the digit in either the thousands' place or the thousandths' place. The children should also notice that part C does not limit them to unlike digits.

Assignments (page 229)
Minimum: 1. Average: 1–4.
Maximum: 1–5.

PAGES 230–231

Objective

Given decimals in addition and subtraction exercises which require regrouping, the child will be able to find the sums and differences.

Preparation

According to the need of the children and your method of class organization, provide the children with a chalkboard practice session on adding and subtracting 3- and 4-digit numbers. You might give several children an opportunity to explain the algorithm to the class. Be sure that proper emphasis is placed on the regrouping required in each exercise. Probably the most important objective of such practice is to ensure that the children are quite skillful at this whole-number algorithm before they begin their work on decimals, so that full attention can be focussed on the decimal concepts.

Investigation

Encourage children to work independently on this investigation. Remind them to express the sums and differences of the first set as fractions or mixed numerals. Although it would be better if they did not change fractions to lowest terms, it would be better if they changed improper fractions to mixed numerals. Guide them accordingly, as the need arises. Note that some of the examples treat regrouping skills. If you notice which children seem to understand these examples involving regrouping, you might ask them to give explanations of their procedures.



● *Let's add and subtract using decimals.*

Investigating the Ideas

Find these sums and differences.

$\begin{array}{r} \frac{8}{10} \\ +\frac{7}{10} \\ \hline 1\frac{5}{10} \end{array}$ $\begin{array}{r} \frac{7}{10} \\ -\frac{5}{10} \\ \hline \frac{2}{10} \end{array}$ $\begin{array}{r} \frac{18}{100} \\ +\frac{27}{100} \\ \hline \frac{45}{100} \end{array}$ $\begin{array}{r} 4\frac{6}{10} \\ -1\frac{7}{10} \\ \hline 2\frac{9}{10} \end{array}$ $\begin{array}{r} 2\frac{9}{10} \\ +5\frac{17}{100} \\ \hline 8\frac{7}{100} \end{array}$

? Can you use the problems above to help you find the sums and differences below?

$\begin{array}{r} 0.8 \\ +0.7 \\ \hline 1.5 \end{array}$ $\begin{array}{r} 0.7 \\ -0.5 \\ \hline 0.2 \end{array}$ $\begin{array}{r} 0.18 \\ +0.27 \\ \hline 0.45 \end{array}$ $\begin{array}{r} 4.6 \\ -1.7 \\ \hline 2.9 \end{array}$ $\begin{array}{r} 2.9 \\ +5.17 \\ \hline 8.07 \end{array}$

Discussing the Ideas

Explain the steps in the two examples. See Discussion.

A

Adding thousandths	Adding hundredths	Adding tenths
$\begin{array}{r} {\scriptstyle 1\ \ \ } \\ 0.563 \\ +0.749 \\ \hline 2 \end{array}$	$\begin{array}{r} {\scriptstyle 1\ 1\ } \\ 0.563 \\ +0.749 \\ \hline 12 \end{array}$	$\begin{array}{r} {\scriptstyle 1\ 1\ } \\ 0.563 \\ +0.749 \\ \hline 1.312 \end{array}$
$\frac{3}{1000} + \frac{9}{1000} = \frac{12}{1000}$	$\frac{1}{100} + \frac{6}{100} + \frac{4}{100} = \frac{11}{100}$	$\frac{1}{10} + \frac{5}{10} + \frac{7}{10} = 1\frac{3}{10}$

B

Subtracting thousandths	Subtracting hundredths	Subtracting tenths
$\begin{array}{r} 0.7{\scriptstyle 2}3{\scriptstyle 14}4 \\ -0.578 \\ \hline 6 \end{array}$	$\begin{array}{r} 0.{\scriptstyle 6}7{\scriptstyle 12}3{\scriptstyle 14}4 \\ -0.578 \\ \hline 56 \end{array}$	$\begin{array}{r} 0.{\scriptstyle 6}7{\scriptstyle 12}3{\scriptstyle 14}4 \\ -0.578 \\ \hline 0.156 \end{array}$
$\frac{14}{1000} - \frac{8}{1000} = \frac{6}{1000}$	$\frac{12}{100} - \frac{7}{100} = \frac{5}{100}$	$\frac{6}{10} - \frac{5}{10} = \frac{1}{10}$

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Discussion

Use the exercises in the investigation side by side to show and explain the procedures involved. If possible, have children give explanations for each. Be sure children realize in the last example that 2.9 is the same as 2.90 so 2.9 + 5.17 may be thought of as 2.90 + 5.17, though in addition it is not necessary to write the zero.

Work through example A, calling attention to the equation as well as the algorithm for each.

For the first step in the addition algorithm shown here, you might observe with the children that, when they add $\frac{3}{1000}$ and $\frac{9}{1000}$ and get $\frac{12}{1000}$, they may think of this as $\frac{1}{100}$ and $\frac{2}{1000}$, since $\frac{12}{1000} = \frac{1}{100} + \frac{2}{1000}$. This idea explains why we are able to regroup the "1" to the hundredths' column and then add this along with $\frac{6}{100}$ and $\frac{4}{100}$. Work through other examples with the children relating the procedures to addition and subtraction using fractions. For example:

A $\frac{5}{10} + \frac{7}{10} = n$
$0.5 + 0.7 = n$

B $\frac{8}{10} - \frac{7}{10} = n$
$0.8 - 0.7 = n$

C $\frac{35}{100} - \frac{17}{100} = n$
$0.35 - 0.17 = n$

D $\frac{18}{100} + \frac{27}{100} = n$
$0.18 + 0.27 = n$

E $\begin{array}{r} \frac{2}{10} \\ +\frac{3}{10} \\ \hline \end{array}$ $\begin{array}{r} 0.2 \\ 0.3 \\ \hline \end{array}$

F $\begin{array}{r} \frac{8}{10} \\ -\frac{5}{10} \\ \hline \end{array}$ $\begin{array}{r} 0.8 \\ -0.5 \\ \hline \end{array}$

Using the Ideas

1. Find the sums and differences.

A $0.65 + 0.86 = 1.51$	B $5.67 + 8.4 = 14.07$	C $0.68 - 0.23 = 0.45$	D $29.6 + 8.93 = 38.53$	E $29.37 - 5.62 = 23.75$
F $5.37 - 1.28 = 4.09$	G $34.96 + 59.45 = 94.41$	H $8.763 - 0.045 = 8.718$	I $17.078 + 6.974 = 24.052$	J $206.4 - 53.32 = 153.08$
K $0.067 + 0.836 = 0.903$	L $57.38 + 94.073 = 151.453$	M $102.35 - 41.17 = 61.18$	N $0.876 - 0.199 = 0.677$	O $700.3 - 267.5 = 432.8$
P $5.004 - 1.333 = 3.671$	Q $3.142 + 7.128 = 10.270$	R $80.637 + 109.85 = 190.487$	S $0.054 - 0.026 = 0.028$	T $55.62 - 7.99 = 47.63$

U $0.3 + 0.5 + 0.9 + 0.6 + 0.8 = 3.1$

V $8.6 + 7.4 + 9.3 + 6.9 = 32.2$

W $0.34 + 0.63 + 0.79 + 0.64 = 2.40$

X $0.521 + 0.467 + 0.816 = 1.804$

Y $4.37 + 8.56 + 7.38 = 20.31$

Z $5.6 + 3.48 + 29.6 + 0.387 = 39.067$

2. Find the missing amounts.

A Since $9.67 + 5.38 = 15.05$, we know that \$9.67 and \$5.38 is ▮.

B Since $20.00 - 7.35 = 12.65$, we know that \$20.00 less \$7.35 is ▮. \$15.05, \$12.65

3. Solve each money problem.

A Find the total cost of the items shown in this budget book. \$40.21

DATE	DESCRIPTION	AMOUNT	
1/17	Gas	16	13
1/17	Electricity	9	63
1/17	Water	4	17
1/20	Telephone	7	68
1/21	Other	2	60

B Tom bought a pair of gym shoes for \$6.98. How much change did he get back from a \$20 bill? \$13.02

C Jan received a check for \$10.55. She spent \$4.98 for a record and \$2.75 for a book. How much did she have left? \$2.82

D Mr. Smith bought two adult tickets and one child's ticket for a play. He paid \$3.25. Mr. Jones bought two adult tickets and two children's tickets. He paid \$4.00. What is the price of an adult ticket? \$1.25

More practice, page A-17, Set 34



Using the Exercises

When you assign the exercises on page 231, be aware that parts such as 1J may require special attention. Sometimes you will want to point out such a problem after children have tried it on their own. With children who have little self-confidence, you might discuss it before they begin; such children need as many experiences of success as possible. Exercises 1U–1Z should be treated similarly; when the addends are changed to column form, all digits representing the same place value should be in the same column.

Assignments (page 231)

Minimum: 1A–T. Average: 1–3. Maximum: 1–3.

Follow-up

Worksheets similar to those below might be of help to some children.

Study the examples. Change each fractional sum to a single decimal.

$\frac{3}{10} + \frac{3}{100} + \frac{6}{1000} + = 0.3 + 0.03 + 0.006 = 0.336$

$\frac{7}{10} + \frac{8}{100} + \frac{9}{1000}$ = ____ + ____ + ____ = ____

$\frac{9}{10} + \frac{5}{100} + \frac{8}{1000}$ = ____ + ____ + ____ = ____

Study the examples. Try to change the other common fractions to decimals.

$\frac{2}{5} = \frac{4}{10} = 0.4$ $\frac{3}{4} = \frac{75}{100} = 0.75$

$\frac{1}{10} =$ ____ = ____ $\frac{1}{2} =$ ____ = ____

$\frac{1}{5} =$ ____ = ____ $\frac{7}{20} =$ ____ = ____

$\frac{n}{25} =$ ____ = ____ $\frac{3}{10} =$ ____ = ____

Resources for Active Learning

Franklin Series: *Making and Using Graphs and Nomographs*, "Nomographs: Addition with Decimals," pp. 37–38, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Duplicator Masters, page 44
Workbook, page 72
Skill Masters, page 44

PAGES 232–233

Objective

Given word problems requiring the use of decimals, the child will be able to solve the problem by adding or subtracting decimals.

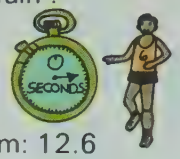
Preparation

To stimulate interest in pages 232 and 233, use demonstration materials to give examples involving the concept of density. For example, pour some oil into a graduated cylinder that has been partially filled with water. Encourage the children to discuss the fact that the oil rises above the water. Use this demonstration to introduce the text pages.

Short Stories

1 April: 4.7 cm of rain. May: 3.9 cm of rain. How much rain? 8.6

2 Travel. Monday: 238.4 kilometres
Tuesday: 467.5 kilometres. Wednesday: 537.8 kilometres. How far? 1243.7



3 100-metre dash. Tom: 12.6 seconds. Joe: 10.9 seconds.
How much longer did it take Tom? 1.7 sec

4 18.7 km per litre.
2 litres. How many km? 37.4



5 Average depth of river: 15.9 m.
Average depth of lake: 24.3 m.
How much deeper is the lake? 8.4 m

6 Pour in 25.35 cubic centimetres. Then pour in 37.68 cubic centimetres more. How much liquid? 63.03 cu cm

7 Expenses. Monday: $3.95. Tuesday: $6.23. Wednesday: $5.67. Thursday: $3.94. Friday: $4.58. Total? $24.37

8 Micrometer reading: 1.53 centimetres. Polishing removes 0.67 centimetres. What does the micrometer read now? 0.86 cm

9 Old speedway record: 256.867 km/h.
New record: 257.735 km/h.
How much greater is the new record? 0.868 km/h

10 Normal body temperature: 37°C.
High fever: 2.7C° higher.
How many degrees? 39.7°

11 Magic square. Row, column, and diagonal sums the same. Give the missing numbers. 1.4 1.5 1.3

A	0.9	1.0
0.7	1.1	B
1.2	C	0.8

12 Highest recorded temperature: 58.2°C. Lowest recorded temperature: 118.3C° **below zero.** Find the change in degrees from one temperature to the other. 176.5°

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Discussion

Since an important purpose of word-problem sets is to stimulate the children's imagination and to create additional interest in arithmetic, encourage discussion of topics suggested by problems on both pages according to the interest of the children. For example, the suggested preparation introducing density should give rise to discussion. You might include in the discussion of density the old question of which weighs more, a kilogram of feathers or a kilogram of gold. Of course, they weigh the same, but ask children to imagine the volume of gold that is needed compared to the volume of feathers needed to make a kilogram. Relate this idea to the fact that the density of the feathers is less than the density of the gold. Let each child examine the container of water and oil you used in the demonstration. Point out that the density of oil is less than the density of water, and relate this to their weights given in the table on page 233.

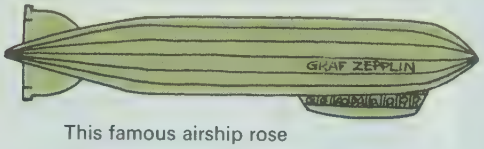
As children work on these problems, try to have available two blocks of equal volume and obviously different weights, such as metal and styrofoam, or wood and styrofoam, for children to examine.

When the children have finished

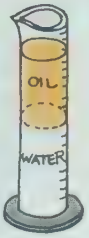
DENSITY



A cubic metre of balsa wood weighs much less than a cubic metre of copper. We say that the density of copper is greater than the density of balsa wood.



This famous airship rose above the ground because it contained hydrogen gas. We say the density of hydrogen gas is less than the density of air.



Since oil rises to the top when mixed with water, it must be lighter. We say the density of oil is less than the density of water.

1. How much more does a litre of sea water weigh than a litre of pure water ? 25.6
2. Add to find the weight of 5 litres of gasoline. 3408.5
3. Which weighs more, 2 millilitres of acid or 3 millilitres of oil ? How much more ? 0.395 g (Acid)
4. Find the difference in the weights of a litre of mercury and a litre of kerosene. 12 725.5
5. Which is heavier, a cubic metre of cork or a cubic metre of glass ? How much heavier ? Glass 2317 kg
6. How much more or less does a cubic centimetre of gold weigh than a cubic centimetre of iron and a cubic centimetre of lead ? 0.162
7. ★ Estimate:
   - A The number of litres of water in a cubic metre of water. 1000
   - B The number of litres of water needed to weigh about the same as a cubic decimetre of gold. 19.3

Liquid	Grams per litre
Water	1000.0
Gasoline	681.7
Kerosene	820.7
Mercury	13 546.2
Oil	1082.41
Sulfuric acid	1821.13
Sea water	1025.6

Material	Grams per cubic centimetre
Water	1.0
Balsa wood	0.112
Copper	8.92
Cork	0.225
Glass	2.542
Gold	19.31
Iron	7.86
Lead	11.288

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the problems on both pages, allow time for further discussion and checking papers. Discuss in particular any problems which seemed to cause difficulty. Notice that exercise 7 on page 233 is intended for more capable children. If they think of the whole-number values given in both charts, they should be able to relate the two sets of information.

Assignments (page 232)
Minimum: 1–6. Average: 1–12.
Maximum: 1–12.

Assignments (page 233)
Minimum: 1–6, oral. Average: 1–6.
Maximum: 1–7.

Resources for Active Learning

Developmental Math Cards, "Weight and Water," "L-13, Addison-Wesley.

Freedom to Learn "Weight and Density," pp. 143–147, Addison-Wesley.

Measure and Find Out, Book 2, "How dense are you?" Activity 2/8, Scott Foresman. (Available from Gage Education Publishing Ltd.)

Duplicator Masters, page 45

PAGES 234–235

Objective

Given a decimal, the child will be able to round it to a specified decimal place.

Preparation

Materials

centimetre rulers

To prepare for this lesson, it would be helpful to review rounding of whole numbers. For example, list several numerals on the chalkboard and ask children to round them to the nearest ten, the nearest hundred, or the nearest thousand.

243 128 054	785 612 987
496 701 985	648 793 299
820 511 538	312 495 564
522 346 109	598 300 726

Remind children of the agreement that 5 is rounded to the greater ten, 50 is rounded to the greater hundred, and so on.

Investigation

Since centimetres are not divided into tenths, hundredths, and thousandths, this investigation requires that children figure out the value of each decimal as related to the ordinary marks on a centimetre ruler. Due to the values of the decimals that are given, children must estimate or round these decimals to draw the correct line segments. Do not give any direction regarding how to round the decimals. If children have difficulty, encourage them to think of the ruler as a number line or actually draw a number line in order to place each decimal correctly.

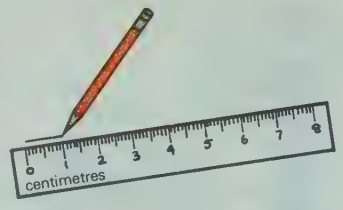


● *How are decimals rounded?*

Investigating the Ideas

Can you draw line segments that are very close to
- 4.52 centimetres long ?
- 6.017 centimetres long ?
- 5.249 centimetres long ?

See Investigation.

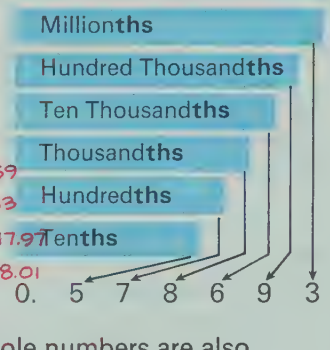


Discussing the Ideas

1. Explain how you thought about each decimal before you drew your segment. See Discussion.

2. The chart shows place-value names through millionths. Give each of these decimals rounded to the nearest hundredth.

Answer: A 0.5372 → 0.54; B 0.6211 → 0.62; C 0.7849 → 0.78; D 0.6954 → 0.70; E 0.2886 → 0.29; F 0.2549 → 0.25; G 0.2550 → 0.26; H 0.6247 → 0.62; I 0.62476 → 0.62; J 3.5914 → 3.59; K 7.8265 → 7.83; L 17.96501 → 17.97; M 38.0095 → 38.01; N 99.9999 → 100.00



3. The rules you learned for rounding whole numbers are also used for rounding decimals. Give each decimal above rounded to the nearest: See Answers, T.E. page 235.
   - A tenth
   - B thousandth

4. Study each example. Then read the decimals in exercise 2.
   - A For 0.5786, we read "five thousand seven hundred eighty-six ten thousandths."
   - B For 0.57869, we read "fifty-seven thousand eight hundred sixty-nine hundred thousandths."
   - C For 0.578693, we read "five hundred seventy-eight thousand, six hundred ninety-three millionths."

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Discussion

As children discuss how they thought about each decimal in the investigation, help them see that 4.52 is close to 4.50 or 4.5, so the first line segment should be $4\frac{1}{2}$ cm. 6.017 is close to 6.02. which is closer to 6.0 cm than to $6\frac{1}{4}$ or $6\frac{1}{8}$ cm; and 5.249 is close to 5.25 which some may realize is halfway between $5\frac{1}{2}$ (5.50) and 5 (5.0), so this line segment should be about $5\frac{1}{4}$ cm. Stress the usefulness of being able to round decimals. Use exercise 2 to stress that the rule for rounding whole numbers also apply to decimals. Explain that when 4.52 is rounded, we write 4.5 and say "4.52 rounded to the nearest tenth is 4.5."

You might use several examples from exercise 2 to explain how decimals are rounded to a nearest place just as whole numbers are.

Have children carefully study the chart in exercise 2. You might then have children do exercises 2 and 3 as a class activity, writing their answers on the chalkboard. However, if you prefer to treat exercises 2 and 3 as oral exercises, it would be better to do exercise 4 first. Then, when children give their rounded numbers orally, they will have had some practice in reading these decimals of 4, 5, and 6 places.

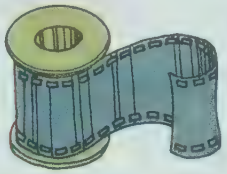
Using the Ideas

1. Give the place value of each red digit.

A 3.57
Answer: $\frac{7}{100}$
B 8.7642 4 (1000)
C 35876 5 (10)
D 8.76953 3 (100000)
E 0.00058 5 (10000)
F 3.7045 0 (100)
G 0.00617 6 (1000)
H 4.82378 8 (100000)
I 2.700 (2000)
J 3.07610 1 (10000)
K 8.00026 6 (100000)
L 3.479 7 (100)
M 2.3604 4 (10000)
N 0.0083 8 (1000)

2. Give the missing word for each of these interesting decimals.

A Thin metallic film: 0.00002 cm
We read: two ___ ? ___ cm hundred thousandths
B Human hair: about 0.005 cm
We read: five ___ ? ___ cm Thousandths



3. Give the missing numbers.

A 0.28 rounded to the nearest tenth is ▒. 0.3
B 0.745 rounded to the nearest hundredth is ▒. 0.75
C 0.57362 rounded to the nearest hundredth is ▒. 0.57
D 39.654 rounded to the nearest whole number is ▒. 40

4. Give each decimal:

2.540005
30.48006
91.44018
0.3937

A rounded to the nearest hundredth
2.54, 30.48, 91.44, 0.39
B rounded to the nearest tenth
2.5, 30.5, 91.4, 0.4
C rounded to the nearest whole number
3, 30, 91, 0

5. Balloons that rise into the air contain a gas that is lighter than air. Hydrogen, the lightest gas, weighs 0.089882 grams per litre.
Give the weight of hydrogen gas:



A rounded to the nearest hundred thousandth 0.089882
B rounded to the nearest ten thousandth 0.0899
C rounded to the nearest thousandth 0.090
D rounded to the nearest hundredth 0.09

More practice, page A-18, Set 35

Follow-up

To provide more practice in rounding decimals, give the children a worksheet like the one shown below; or suggest that they use information from their science, social studies, or current events classes.

Round each group of decimals as directed.
1. Round to the nearest tenth. A 0.87 0.23 0.64 B 0.37 0.18 0.06 C 0.91 0.44 0.59
2. Round to the nearest hundredth. A 23.147 9.482 16.597 B 16.997 75.335 80.171 C 57.481 43.086 90.999
3. Round to the nearest thousandth. A 3.1416 B 72.9763 C 2.2222

Workbook, page 73

Answers, discussion exercise 3, page 234

A A 0.5 B 0.6 C 0.8 D 0.7 E 0.3 F 0.3 G 0.3 H 0.6 I 0.6 J 3.6 K 7.8 L 18.0 M 38.0 N 100.0

B A 0.537 B 0.621 C 0.785 D 0.695 E 0.289 F 0.255 G 0.255 H 0.625 I 0.625 J 3.591 K 7.827 L 17.965 M 38.010 N 100.000

Using the Exercises

For exercise 1 on page 235, encourage children to use the chart in exercise 2, page 234, if they need to. Both exercises 1 and 2 provide readiness for rounding the decimals in exercises 3, 4, and 5. With some groups of children, you might want to work through several of these exercises together.

Assignments (page 235)*

Minimum: 1. Average: 1–4.
Maximum: 1–5.

PAGES 236–237

Objective

Given a decimal, the child will be able to multiply it by 10 or 100 mentally.

Preparation

Materials

colored strips

To prepare for this lesson, you might briefly review how the value of the strips can vary according to the unit chosen. You might have the children use their strips to answer questions such as these:

"If the red strip is the unit, what is the purple strip?" (2) ". . . the brown strip?" (4)

"If the light green strip is the unit, what is the white strip?" ($\frac{1}{3}$)

"If the yellow strip is the unit, what is the red strip?" ($\frac{2}{5}$) ". . . the yellow strip?" (1)

Finally, introduce the investigation by posing the question, "If the orange strip is the unit, what is the white strip?" ($\frac{1}{10}$ or 0.1)

Investigation

Point out to the children that for the entire investigation they should think of the orange strip as the unit strip. If children understand that the white strip then becomes 0.1 and the red strip 0.2, they should not have difficulty in giving the decimals for the other two strips. However, the main question of the investigation may require a little more thinking for some. You will probably want the children to work in groups of at least 3, since they will not have, in one set of strips, 10 of each kind. By combining 3 sets of strips, they should have at least 10 of each kind. As they begin to work, you might move around the room and ask questions like these: "Will all the trains be the same length?" "How much longer will the red train be than the white?"

If the children have difficulty with the basic problem of finding the length of each train, ask them to find out how many 10-strips they can match with each train they have made. Several children should discover the pattern and realize they are multiplying by 10.



Is there an easy rule for multiplying decimals by 10 and 100?

Investigating the Ideas

Suppose the orange strip is the unit.



Can you find the length of a train of 10 strips for each of the four colors? See above and Investigation.

Discussing the Ideas

1. You can find these products by thinking about fractions. Give your answer by using a whole number or the simplest possible decimal.

A	B	C	D
$10 \times 0.1 = a$ 1	$100 \times 0.01 = a$ 1	$10 \times 0.01 = a$ 0.1	$100 \times 0.1 = a$ 10
$10 \times 0.2 = b$ 2	$100 \times 0.02 = b$ 2	$10 \times 0.02 = b$ 0.2	$100 \times 0.2 = b$ 20
$10 \times 0.3 = c$ 3	$100 \times 0.03 = c$ 3	$10 \times 0.04 = c$ 0.4	$100 \times 0.6 = c$ 60
$10 \times 0.8 = d$ 8	$100 \times 0.07 = d$ 7	$10 \times 0.09 = d$ 0.9	$100 \times 0.9 = d$ 90

2. The distributive principle is useful in finding the product of 10 or 100 times a fractional number named by a decimal. Explain the examples and give the products. See Discussion.

A
$10 \times 4.6 = (10 \times 4) + (10 \times 0.6)$
Since $10 \times 4 = 40$, $10 \times 0.6 = 6$, we know that $10 \times 4.6 = n$.

B
$100 \times 3.07 = (100 \times 3) + (100 \times 0.07)$
Since $100 \times 3 = 300$, $100 \times 0.07 = 7$, we know that $100 \times 3.07 = n$

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Discussion

Encourage children to share their investigation results. Relate each train to a multiplication equation:

10 white strips: $10 \times 0.1 = 1$
10 red strips: $10 \times 0.2 = 2$
10 light green strips: $10 \times 0.3 = 3$
10 purple strips: $10 \times 0.4 = 4$
10 orange strips: $10 \times 1.0 = 10$

Relate this discussion to exercise 1 and to multiplication with fractions. Show how the children might think: since $10 \times \frac{1}{10} = 1$, $10 \times \frac{2}{10} = 2$, which is the same as $10 \times 0.2 = 2$. Develop the other parts of exercise 1 in relation to multiplication with fractions. For example, help children realize that 100×0.03 is the same as $100 \times \frac{3}{100}$, or 3.

Use exercise 2 to help the children see how they can use this principle to add the two products to get the final product. To multiply 10×4.6, for example, they can multiply 10×4, and 10×0.6, and add the two products to get the final answer. Both exercises 1 and 2 are intended to develop the generalizations summarized in exercise 4 of page 237. However, allow children to come to this realization in their own time. If they have not discovered it by the time they reach exercise 4 on page 237, use that exercise to help them.

Using the Ideas

1. Find the products.

A	10×0.4 4	E	100×0.055	I	0.2×10 2	M	10×0.06 0.6
B	10×0.7 7	F	100×0.088	J	0.07×100 7	N	100×0.09 9
C	10×0.04 0.4	G	100×0.330	K	0.05×10	O	10×0.5 5
D	10×0.08 0.8	H	100×0.660	L	0.5×100 50	P	100×0.033

2. Copy each exercise and give the number for each letter.

A $10 \times 5 = a$ 50 → $10 \times 0.3 = b$ 3 → $10 \times 5.3 = c$ 53

B $10 \times 0.7 = a$ 7 → $10 \times 0.04 = b$ 0.4 → $10 \times 0.74 = c$ 7.4

C $100 \times 6 = a$ 600 → $100 \times 0.2 = b$ 20 → $100 \times 6.2 = c$ 620

D $100 \times 0.5 = a$ 50 → $100 \times 0.04 = b$ 4 → $100 \times 0.54 = c$ 54

3. Find the products.

A	10×6.4 64	C	100×9.3 930	E	10×8.56 85.6	G	10×51.3 513
B	10×0.87 8.7	D	100×0.67 67	F	100×9.73 973	H	100×26.4 2640

4. Complete each sentence correctly.
   - A To write the product of 10 times a number named by a decimal, write the decimal point __ ? __ place farther to the right. one
   - B To write the product of 100 times a number named by a decimal, write the decimal point __ ? __ places to the right. two

5. Find the products.

A	10×8.43 84.3	K	100×0.088
B	10×29.3 293	L	10×97.2 972
C	100×1.97 197	M	100×9.72 972
D	100×5.8 580	N	100×97.2 9720
E	100×53.7 5370	O	100×0.56 56
F	10×0.36 3.6	P	100×0.056 5.6
G	100×0.862 86.2	Q	10×0.056 0.56
H	100×34.23 3423 ★	R	1000×0.001 1
I	10×27.64 276.4 ★	S	1000×0.01 10
J	100×54.8 5480 ★	T	1000×0.1 100

think

Suppose you have these pails.



There are no markings on either pail. How can you use these pails to get 4 litres of water in the largest pail?

See Solution, T.E. page 237.



More practice, page A-18, Set 36

Using the Exercises

Assign the exercises on page 237 as independent work. The exercises follow a developmental sequence. Exercise 1 is intended as preparation for exercise 2, which develops the power skill for multiplying decimals by 10 and 100. Exercise 3 develops readiness for the generalizations given in exercise 4. You might want to discuss exercise 4 with all the children, but be sure to do so with those who have not quite understood the pattern.

The *Think* problem might be challenging even for the more capable children. If possible, encourage children to use actual containers of these sizes to help them find the solution. (See *Solution* at the right.)

Assignments (page 237)

Minimum: 1, 5A–Q. Average: 1–4, 5A–Q. Maximum: 1–5.

Follow-up

For more practice with the decimal system, let the children explore the metric system of measurement, which is based on multiples of ten.

Solution, Think, page 237

Fill 8-litre container; from it, fill 3-litre container; empty it. Fill 3-litre container again; empty it. Pour remaining 2 litres into 3-litre container. Fill 8-litre container. From it, fill the remaining empty section of the 3-litre container; empty it. Since there are now 7 litres left in the 8-litre container, fill the 3-litre container once more and you will have 4 litres remaining in the larger pail.

Duplicator Masters, page 46
Workbook, page 74
Skill Masters, page 46

PAGES 238–239

Objective

Given two decimals with no more than 3 places to the right of the decimal, the child will be able to find their product.

Preparation

Review with the children a variety of products such as: $\frac{1}{10} \times \frac{1}{10}$, $\frac{3}{10} \times \frac{4}{10}$, $\frac{4}{10} \times \frac{6}{100}$, $\frac{5}{10} \times \frac{3}{10}$, $\frac{4}{10} \times \frac{8}{100}$, $\frac{7}{100} \times \frac{3}{10}$, $\frac{1}{10} \times \frac{1}{100}$, $\frac{1}{100} \times \frac{1}{100}$, $\frac{7}{100} \times \frac{2}{100}$, $\frac{4}{10} \times \frac{3}{100}$.

You need not have the children use lowest-terms fractions in such exercises, since the lesson concerns the placement of the decimal point in products of fractional numbers using decimal notation.



● *Let's multiply decimals.*

Discussing the Ideas

1. Study each example. Then give the product ***P***.

A $\frac{3}{10}$ $\frac{4}{10}$ $0.3 \times 0.4 = P$ 1.2

To find this product, think:
1 $3 \times 4 = 12$
2 tenths × tenths = hundredths

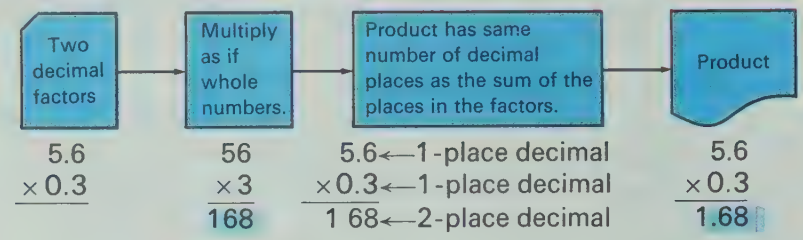
B $\frac{4}{10}$ $\frac{6}{100}$ $0.4 \times 0.06 = P$ 0.024

To find this product, think:
1 $4 \times 6 = 24$
2 tenths × hundredths = thousandths

C $\frac{5}{100}$ $\frac{13}{100}$ $0.05 \times 0.13 = P$ 0.0065

To find this product, think:
1 $5 \times 13 = 65$
2 hundredths × hundredths = ten thousandths

2. Study the flow chart below.



5.6	56	5.6 ← 1-place decimal	5.6
× 0.3	× 3	× 0.3 ← 1-place decimal	× 0.3
	168	1 68 ← 2-place decimal	1.68

A Use the flow chart to help you find the product 7.2×0.6 4.32

B Explain how you would place the decimal in the product for 0.72×0.6. Count number of decimal places in the factors; product has same number of places

C How many decimal places are there in the product 0.72×0.06? 4

3. How would you complete each sentence?

A When each factor is named by a 1-place decimal, the product is named by a __?__ -place decimal. 2

B When each factor is named by a 2-place decimal, the product is named by a __?__ -place decimal. 4

C To find the number of decimal places in the product, we __?__ the number of decimal places in each factor. add

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Discussion

Discuss with the children how they can think about fractions to find the product of two fractional numbers named by decimals. Point out that their multiplying work with decimals will be easier if they can quickly find each of these products: $\frac{1}{10} \times \frac{1}{10}$, $\frac{1}{10} \times \frac{1}{100}$, $\frac{1}{100} \times \frac{1}{100}$. Develop parts A, B, and C of exercise 1 and emphasize that tenths times tenths is hundredths, $\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$; tenths times hundredths is thousandths, $\frac{1}{10} \times \frac{1}{100} = \frac{1}{1000}$; and hundredths times hundredths is ten thousandths, $\frac{1}{100} \times \frac{1}{100} = \frac{1}{10\,000}$. Provide examples for the children to use this method.

Next, carefully work through the flow chart in exercise 2. Be sure children know what the terms 1-place decimal, 2-place decimal, and so on, mean. You might want to show and discuss the following examples.

tenths		hundredths	
0.3	1-place	0.54	2-place
8.6	decimals	0.08	decimals
34.5		5.99	

thousandths	
0.124	3-place
0.050	decimals
8.007	

Relate the pattern developed here to the "tenths times tenths" approach used in exercise 1. As you work through other examples using the shortcut method presented in exercise 2, include one like this:

$$0.05 \times 0.06 = 0.0030$$

Explain that the zeros in the factors must be considered, so here there are four places. Then the zero which is in the last place should also be included in determining place value, 0.0030; and the zeros must be used in the tenths' and hundredths' places to make the required 4-place decimal. In problems such as this, encourage chil-

Using the Ideas

1. Do not compute the product. Simply tell whether the product will be written as tenths, hundredths, thousandths, or ten thousandths.

A 0.2×0.4 Answer: hundredths

B 0.5×0.3 h

C 4×0.08

D 0.23×0.9 th.

E 0.12×0.25 ten th.

F 4×0.2 tenths

G 0.07×0.38 ten th.

H 0.49×0.87 ten th.

I 0.26×0.3 th.

J 0.4×0.04 th.

K 0.7×0.39 th.

2. Find the products.

A 0.5×0.7 0.35

B 0.9×0.6 0.54

C 0.8×0.5 0.40

D 0.3×0.52 0.156

E 0.2×3 0.6

F 0.49×0.2 0.098

G 0.12×0.3 0.036

H 0.5×0.24 0.120

I 0.68×0.7 0.476

J 0.9×0.8 0.72

K 0.59×0.67 0.3953

L 0.41×0.09 0.0369

M 0.03×0.12 0.0036

N 0.09×0.07 0.0063

O 0.8×0.06 0.048

P 0.57×0.40 0.228

3. Find the products.

A 3.7×0.06 0.222	C 24.3×9 218.7	E 6.32×0.8 5.056	G 2.64×0.057 0.15048	I 6.308×9 56.772
B 5.28×0.31 1.6368	D 61.5×4.3 264.45	F 70.2×0.08 5.616	H 93.04×0.033 3.07032	J 6.275×2.6 16.3150

4. Find the products.

A 83.6×0.5 41.80

B 97.1×3.14 304.894

C 72.1×0.004 0.2884

D 0.007×5.98 0.04186

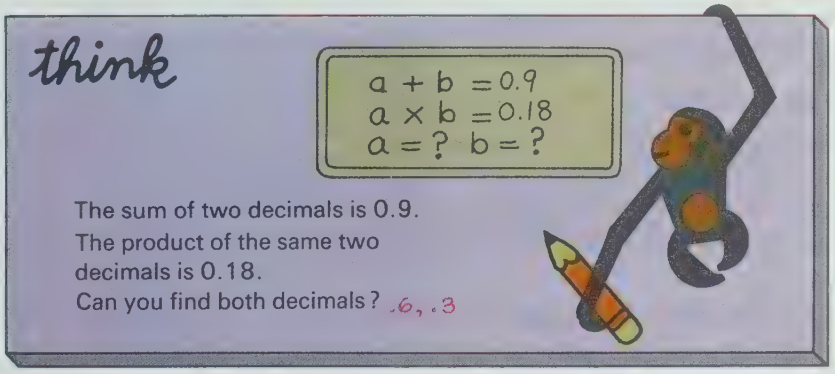
E 0.023×0.24 0.00552

F 53.4×2.6 138.84

G 6.38×24.5 156.310

H 9.76×8.06 78.6656

I 2.03×4.04 8.2012



More practice, page A-19, Set 37

dren to think through the method shown in exercise 1 as well as the shortcut method.

Summarize the discussion by asking children to complete the statements in exercise 3.

Using the Exercises

Assign the exercises on page 239 as independent work, but help those children who still have difficulty. When they have finished, you may want to allow time for placing several exercises on the chalkboard and having various children explain them to the class. Be sure that they explain the placement of the decimal in the answer in terms of the decimals that appear in the factors.

Assignments (page 239)

Minimum: 1, 2A–H, 3A–H, 4A–D.
Average: 1–4. Maximum: 1–4.

Resources for Active Learning

Developmental Math Cards, "Multiplying with the Abacus," J-13, Addison-Wesley.

Franklin Series: *From Fingers to Computers*, "Multiplying Fractional Numbers" (using a slide rule), pp. 57–58, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Duplicator Masters, page 47
Workbook, page 75
Skill Masters, page 47

PAGES 240–241

Objective

Given word problems and charts containing data expressed in decimal notation, the child will be able to solve the problems by applying his understanding of addition, subtraction, and multiplication with decimals.

Preparation

Since much of the work in these word-problem sets is devoted to multiplication with decimal notation, the children should review the algorithm for multiplying with decimals that was presented in the last lesson. You might select several examples similar to those in exercise 1 on page 239 and have some children write them on the chalkboard and explain them to the class. Emphasize that the children can always think of decimal multiplication simply as whole-number multiplication and ignore the placement of the decimal point until they have found the product.

Short Stories SPACE

1. Earth travels around the sun at 29.77 km/sec. Mars travels around the sun at 0.81 times Earth's speed. How fast? 24.11 km/sec

2. Planets. The closer to the sun, the faster they travel around it. Pluto's orbital speed: 4.8 km/sec. Venus: 7.25 times as fast. How fast? 34.8 km/sec

3. Earth's diameter: 12 755 km. Diameter of the moon: 0.27 times that of Earth. What is the diameter of the moon? 3443.85 km

4.

Planet	Diameter
Earth	12 755 kilometres.
Mercury	0.38 × Earth's diameter
Venus	0.91 × Earth's diameter
Mars	0.52 × Earth's diameter
Jupiter	10.97 × Earth's diameter
Saturn	9.03 × Earth's diameter
Uranus	3.73 × Earth's diameter
Neptune	3.38 × Earth's diameter
Pluto	0.45 × Earth's diameter

   A Find the diameter of each planet.

   B Round each diameter to the nearest whole number.

See Answers, T.E. page 240.

5. Day on Mars: 24.6 Earth hours. 10 days on Mars. How many Earth hours? 246

6. Year on Mars: 1.9 Earth years. 8.3 years on Mars. How many Earth years? 15.77

7. Satellite. Takes 95.57 minutes to orbit Earth.
   A How many minutes for 9.5 orbits? 907.915 min
   B How many minutes for 100 orbits? 9557 min
   C About how many hours for 100 orbits? About 160 hr
   D About how many days for 100 orbits? About 7 days

8. Rocket. To leave Mercury, must go 4.2 km/sec. To leave Earth, must go 1.28 km/sec. less than 3 times that for Mercury. How fast? 11.32 km/sec

think

The weight of an object on Mars is $\frac{2}{5}$ of its weight on Earth. Give the total Earth weight of these objects. (The Mars weights are given.)
Girl: 13.6 kg 34 kg Poodle: 4.6 kg 11.5 kg
Purse: 0.28 kg 0.7 kg Package: 3.75 kg 9.4 kg

Discussion

Use one of the pages as basis for an introductory discussion. For example, if many children have a special interest in science and space facts, use three or four problems as discussion material.

Children may need help in pronouncing some of the names for the monetary units. You might also want children to locate each of the listed countries on a world map.

When children finish the problems allow time for discussion. Note that the starred problems on page 241 are intended as a special challenge. Encourage those who try them to compare answers and discuss solutions. In all of these problems, the children need only multiply to change from the foreign monetary unit to dollars. They should not be expected to change dollars into foreign units here.

Answers, exercise 4, page 240

4.	A	B
Earth	12755	12755
Mercury	4846.90	4847
Venus	11 607.05	11 607
Mars	6632.60	6633
Jupiter	139 922.35	139 922
Saturn	115 177.65	115 118
Uranus	47 576.15	47 576
Neptune	43 111.90	43 112
Pluto	5739.75	5740

MONEY AROUND THE WORLD

Country	Monetary unit	Value in Canadian dollars
U.S.A.	Dollar	1.003
Great Britain	Pound	2.50
India	Rupee	0.13
Japan	Yen	0.0038
Mexico	Peso	0.08
Switzerland	Franc	0.31



1. One Mexican peso is worth 0.08 of a dollar or $0.08.
Two pesos are worth 2 × 0.08, or $0.16. Find the value in dollars of:
A 3 pesos $0.24 B 4 pesos $0.32 C 5 pesos $0.40 D 7 pesos $0.56 E 10 pesos $0.80

2. One Swiss franc is worth 0.31 of a dollar.
Find the value in dollars of:
A 2 francs $0.62 B 3 francs $0.93 C 5 francs $1.55 D 8 francs $2.48
E 10 francs $3.10 F 100 francs $31.00 G 1000 francs $310.00

3. Suppose you bought these articles in Switzerland:
watch, 128 francs; music box, 48 francs; Swiss chocolate,
16 francs. What would be the total cost in dollars ? $59.52

4. In Canadian dollars give the value of:
A 2 U.S. dollars $2.006 B 3 U.S. dollars $3.009
C 10 U.S. dollars $10.03 D 100 U.S. dollars $100.30

5. How many Canadian dollars can you get for 378 Mexican pesos ? $30.24

6. Suppose you had 5000 yen. How many Canadian dollars would they be worth ? $19.00

★ 7. If you had 12 Mexican pesos, about how many Swiss francs would they be worth ? About 4

★ 8. Which will buy most, 1000 yen, 1 pound, or 10 rupees ? 1000 yen

*These money values are subject to change because of political and economic factors.

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Follow-up

More capable children may be interested in extending the metric system to weight, since this system is used for most scientific work. In the metric system, the unit of weight is the gram (g). The prefixes used for lengths in the follow-up section on page 237 are also used for weights.

Resources for Active Learning

Math Activity Cards, "Large and Small," E26, Macmillan.

Measure and Find Out, Book 2, "Measuring Length," Activity 1/1, Scott Foresman. (Available from Gage Educational Publishing Ltd.)

Nuffield Project: *Computation and Structure* 4, pp. 57–58, Wiley. [Numbers in space]

Assignments (page 240)
Minimum: 1–6. Average: 1–8.
Maximum: 1–8.

Assignments (page 241)
Minimum: 1–6, oral. Average: 1–6.
Maximum: 1–8.

PAGES 242–243

Objective

Given a decimal, the child will be able to divide it by a whole number.

Preparation

Depending on the need of the children, review the long division algorithm or the rules for multiplication with decimals. Both skills are needed for this lesson.

Investigation

The statement introducing this investigation should immediately remind children of the relationship between multiplication and division. It is intended that this reminder suggest to the children that by multiplying the divisor times the quotient in each problem they can find the correct quotients. If some children finish quickly, challenge them to try the following problems and check their answers by multiplication.

$4\overline{)12.64}$ $\quad$ $4\overline{)1.264}$

$7\overline{)318.5}$ $\quad$ $7\overline{)31.85}$

Can decimals be divided by whole numbers?

Investigating the Ideas

Use multiplication to check this quotient.

$$6\overline{)1458}\quad 243$$

$6\overline{)145.8}$ 24.3 Correct $\quad$ $6\overline{)145.8}$ 2.43 Incorrect

$6\overline{)14.58}$ 2.43 Correct $\quad$ $6\overline{)14.58}$ 24.3 Incorrect

$6\overline{)14.58}$ 0.243 Incorrect $\quad$ $6\overline{)1.458}$ 0.243 Correct

Can you use what you know about multiplying decimals to find which quotients on the paper are correct? See Investigation.

Discussing the Ideas

1. How could you use estimation to help you decide which quotients above are correct? See Discussion.
2. Can you give a shortcut for placing the decimal point in the quotient? See Discussion.
3. Explain each example. See Discussion.

A

Dividing **ones**	Dividing **tenths**	Dividing **hundredths**
$8\overline{)58.^{2}48}$ 7	$8\overline{)58.^{2}48}$ 7.3	$8\overline{)58.^{2}48}$ 7.31

B

Dividing **hundredths**	Dividing **thousandths**	Dividing **ten thousandths**
$34\overline{)0.9214}$ 0.02 68 24	$34\overline{)0.9214}$ 0.027 68 241 238 3	$34\overline{)0.9214}$ 0.0271 68 241 238 34 34



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Discussion

Allow children an opportunity to explain their answers for the investigation. As you discuss exercise 1, children should see again how rounding decimals proves to be useful. For example, for the problem

$$6\overline{)14.58}\quad 24.3$$

the child might reason that 6×20 is 120, so 24.3 cannot be correct because 14.58 is not close to 120. However, since 6×2 is 12, the answer in this quotient is reasonable:

$$6\overline{)14.58}\quad 2.43$$

If necessary, provide other examples of estimation, similar to those in exercise 1 on page 243. Since this lesson is crucial to the work that will be done in division with decimals, special emphasis should be given to exercise 2. Help the children discover the general rule for placing the decimal point: Put the decimal point directly above where it is placed in the dividend, and keep the quotient figures lined up according to the place-value scheme for division of whole numbers. Stress that this method obvi-

Using the Ideas

1. Estimating products will help you check your quotients when dividing decimals. Give the missing numbers.

To estimate this product: →	Round to the nearest whole number. →	Find or estimate the whole-number product.
3.2 × 9	3 × 9	27
4.76 × 58	5 × 58	5 × 60, or 300
9.8 × 37	10 A × 37	B 370
28.6 × 8	29 C × 8	D 232
3.954 × 62	4 E × 62	F 248
53.51 × 4	54 G × 4	H 216

2. Estimate each product. Then find the product and compare it with your estimate.

A 35.6×8 284.8 C 0.72×47 33.84 E 0.059×8 .472 G 0.0628×34 2.1352

B 7.85×62 486.7 D 0.835×6 5.01 F 0.043×67 2.881 H 0.742×5 3.71

3. The correct digits are given for the quotient.

A Estimate the products 82×6, 8.2×6, and 0.82×6.

B Give the correct quotient for $49.2 \div 6$.

$$6\overline{)49.2}\quad \text{quotient: } 8\,2$$

A 480, 48.0, 4.80
B 8.2

4. The correct digits, other than zero, are given for each quotient. Find the correct quotient.

A $8\overline{)34.4}$ quotient: 4.3 B $28\overline{)9.52}$ quotient: .34 C $6\overline{)2.622}$ quotient: .437 D $76\overline{)2.128}$ quotient: .028 E $9\overline{)47.07}$ quotient: 5.23

5. Find the quotients. Check your work.

A $4\overline{)34.0}$ 8.5 B $43\overline{)30.1}$ .7 C $5\overline{)1.285}$ .257 D $67\overline{)361.8}$ 5.4

E $8\overline{)198.08}$ 24.76 F $20\overline{)174.0}$ 8.7 G $50\overline{)3.350}$ .067 H $79\overline{)2.686}$ .034

More practice, page A-19, Set 38



ously holds only for whole-number divisors.

As you discuss exercise 3, it would be helpful to present each example on the chalkboard and work through it step by step with the children. Observe that there are only whole-number divisors in division problems having decimals. They will change any exercise that does not have a whole-number divisor to an equivalent problem that does have a whole-number divisor.

Using the Exercises

The exercises on page 243 give the child an opportunity to work developmentally from estimation to actual long division with decimals as the dividends. With some children, you will want to use many of these exercises as a basis for discussion. Exercises 3 and 4 are appropriate for such use. Remind the children that even after they learn the rule of how to place the decimal point, they should try to at least mentally check their answer.

Assignments (page 243)*
Minimum: 1, 2. Average: 1–4.
Maximum: 1–5.

Duplicator Masters, page 48
Workbook, page 76
Skill Masters, page 48

PAGES 244–245

Objective

Given two fractional numbers expressed as decimals, the child will be able to find their quotient.

Preparation

To prepare for this lesson, review the rules for multiplying decimals by 10 and 100. For example, list several decimals on the chalkboard and ask children to write or give the product of each multiplied by 10, then by 100, without computing. You might also briefly review how multiplication can be used to check quotients, as shown in the previous lesson.

● *Let's explore dividing decimals again.*

Discussing the Ideas

See Discussion.

1. Explain how the diagram shows that the quotient for $0.4\overline{)3.2}$ is the same as the quotient for $4\overline{)32}$. Find this quotient.

$$0.4\overline{)3.2} \rightarrow \boxed{\frac{3.2}{0.4} = \frac{3.2 \times 10}{0.4 \times 10} = \frac{32}{4}} \rightarrow 4\overline{)32}$$

2. Explain how the diagram shows that you can find the quotient for $0.21\overline{)6.741}$ by finding the quotient for $21\overline{)674.1}$. Find the quotient.

$$0.21\overline{)6.741} \rightarrow \boxed{\frac{6.741}{0.21} = \frac{6.741 \times 100}{0.21 \times 100} = \frac{674.1}{21}} \rightarrow 21\overline{)674.1}$$

3. Explain how to find the missing number so that the two problems each have the same quotient.

A $0.4\overline{)2.4}$ $4\overline{)\blacksquare\blacksquare}$ 6 C $0.06\overline{)3.42}$ $6\overline{)\blacksquare\blacksquare\blacksquare}$ 57 E $1.1\overline{)693}$ $11\overline{)\blacksquare\blacksquare\blacksquare}$ 630

B $0.5\overline{)0.35}$ $5\overline{)\blacksquare\blacksquare}$ 0.7, 24, 3.5 D $0.19\overline{)0.874}$ $19\overline{)\blacksquare\blacksquare\blacksquare}$ 46, 342, 8.74 F $0.08\overline{)58.4}$ $8\overline{)\blacksquare\blacksquare\blacksquare\blacksquare}$ 730, 6930, 5840

4. Explain steps 1 and 2. Then copy step 3 and complete the dividing.

A

1	2	3
Multiply the divisor by 100. Multiply the dividend by 100. $0.21_\wedge\overline{)6.74_\wedge1}$	Think about the problem: $21\overline{)674.1}$ $0.21_\wedge\overline{)6.74_\wedge1}$	Complete the dividing. $0.21_\wedge\overline{)6.74_\wedge1}$ 32.1

B

1	2	3
10 × divisor 10 × dividend $8.3_\wedge\overline{)0.5\,81}$	Think about the problem: $83\overline{)5.81}$ $8.3_\wedge\overline{)0.5_\wedge81}$	Complete the dividing. $8.3_\wedge\overline{)0.5_\wedge81}$ 0.07, $5\,81$

244

Discussion

The main purpose of this lesson is to help children discover and understand that they move the decimal point in the dividend to the right the same number of places as they move the decimal in the divisor to the right, in order to make the divisor represent a whole number.

As you work through discussion exercise 1, note that children should think first of the division in fractional form; then by multiplying the numerator and denominator by 10, they will arrive at a whole-number problem. The important thing for the children to realize is that the answer is the same for $32 \div 4$ as it is for $3.2 \div 0.4$. They could multiply the numerator and denominator by 100 or by 1000 as long as they multiply both the numerator and the denominator by the same number.

Although the children have been introduced to equivalent fractions previously, the point to stress is that they can rewrite a problem that does not contain a whole-number divisor as a problem in which the divisor is a whole number.

You might show examples such as the following and have children check by multiplication that the quotients are the same.

$$\begin{array}{cccc} 3 & 3 & 3 & 3 \\ 0.06\overline{)0.18} & 0.6\overline{)1.8} & 6\overline{)18} & 60\overline{)180} \end{array}$$

Work through exercise 3 as indicated in exercises 1 and 2.

Use exercise 4 to help children discover a shortcut rule. Help them see that they can think of moving the decimal point the same number of places to the right in both the divisor and the dividend. Point out that the caret (∧) is simply a device for helping them remember where to place the decimal point. Capable children should understand the concepts involved in this method, while slower children will be able to follow the process once they are told how to go about it.

Using the Ideas

1. Copy the second part of each exercise. Give the missing number so that both division problems have the same quotient.

A $0.5\overline{)15.5}$ $5\overline{)\blacksquare\blacksquare\blacksquare}$ 155

B $0.8\overline{)3.28}$ $8\overline{)\blacksquare\blacksquare\blacksquare}$ 32.8

C $6.3\overline{)504}$ $63\overline{)\blacksquare\blacksquare\blacksquare}$ 5040

D $0.23\overline{)92.23}$ $23\overline{)\blacksquare\blacksquare\blacksquare}$ 9223

E $0.72\overline{)3.6}$ $72\overline{)\blacksquare\blacksquare\blacksquare}$ 360

F $0.07\overline{)0.224}$ $7\overline{)\blacksquare\blacksquare\blacksquare}$ 22.4

2. Find the quotients.

A $0.03\overline{)0.12}$ B $0.3\overline{)1.2}$ C $3\overline{)12}$ D $30\overline{)120}$

E $6\overline{)30}$ F $0.6\overline{)3}$ G $60\overline{)300}$ H $0.06\overline{)0.3}$

3. Each quotient is given without a decimal point. Copy the problem and put in the decimal point correctly.

A $0.7\overline{)245}$ 350 B $0.04\overline{)0.1248}$ 3.12 C $1.2\overline{)4.32}$ 3.6 D $0.62\overline{)1.4322}$ 2.31

4. Find the quotients. Check your work.

A $0.4\overline{)29.6}$ 74 B $0.08\overline{)30.56}$ 382

C $0.7\overline{)58.10}$ 83.0 D $4.1\overline{)36.9}$ 9

E $0.03\overline{)8.562}$ 285.4 F $5.8\overline{)0.406}$ 0.07

G $0.09\overline{)47.7}$ 530 H $0.67\overline{)5.36}$ 8

I $8.4\overline{)16.80}$ 2.0 J $0.25\overline{)0.625}$ 2.5

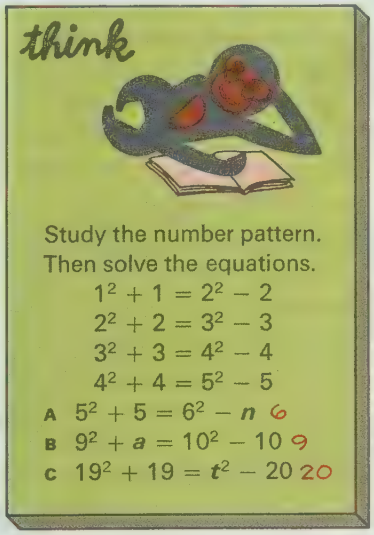
K $0.7\overline{)308.0}$ 440 L $0.11\overline{)0.638}$ 5.8

M $9.6\overline{)1824}$ 190 N $0.71\overline{)326.6}$ 460

O $6.5\overline{)351.0}$ 54 P $0.008\overline{)3.568}$ 446

Q $0.029\overline{)0.493}$ 17 R $0.84\overline{)3.5868}$ 4.27

S $3.45\overline{)24.15}$ 7 T $61.8\overline{)5.562}$ 0.09





More practice, page A-20, Set 39

Resources for Active Learning

Franklin Series: *Making and Using Graphs and Nomographs*, pp. 89–92, Lyons and Carnahan. [Multiplication and division with decimals] (Available from McGraw-Hill Ryerson)

Duplicator Masters, page 49
Workbook, page 77
Skill Masters, page 49

Using the Exercises

With many children, you will want to provide further discussion material. Both exercises 1 and 2 on page 245 are excellent material for this. You might have some children work them on the chalkboard. Continue to stress that they are to multiply divisor and dividend by the same multiple of ten and that this may be shown by the use of the caret. If necessary, use the method in exercises 1 and 2 for many examples to help children think of equivalent fractions with denominators of 10, 100, or 1000.

Assignments (page 245)

Minimum: 1–3, 4A–H.
Average: 1–4. Maximum: 1–4.

PAGES 246–247

Objective

Given word problems involving decimals and multiplication and division, the child will be able to solve the problems by using the division algorithm for decimals.

Preparation

Since much of the material in this lesson involves working with decimal notation and multiplication and division, one of the best preparations is to review the decimal algorithm presented in the last lesson. Many of the children may need extra help in understanding this algorithm. Stress that moving the decimal point in the divisor and the dividend necessitates multiplying both the divisor and the dividend by the same multiple of ten.

Short Stories



1 Large wire: 0.58-cm diameter. Cable: 3.48-cm diameter. How many times as thick? 6

2 Drove 16.1 km. Used 0.9 litres of gas. How many km/litre? 17⅞ Km/Litre

3 Drove 193.8 km in 3.4 hours. How many km/h? 57

4 World's smallest motor: weighs about 0.000001 kg. About how many needed to weigh 1 kg? 1000000

5 1 shoelace: 39.37 cm. 100 shoelaces. How many centimetres? 3937

6 Divide $58.86 equally among 4 people. How much for each person? $14.64

7 Gasoline record: Mon., 17.3 litres; Tues., 18.7 litres; Wed., 16.4 litres; Thurs., 19.5 litres; Fri., 17.6 litres.
- A Give the average number of litres per day. 17.9
- B Find the total cost at 22.9 cents a litre. $20.50
- C Travelled 12.6 km/litre. How many kilometres in all? 1082.3 Km

8 Litre of gasoline: about 660 g. 43.56 kg of gasoline. How many litres? 66 Litres

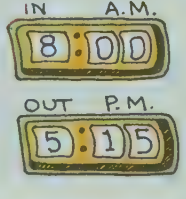
9 Litre of milk: about 1.56 times as heavy as a litre of gasoline. How heavy? 1029.6 g

10 Box of cereal: contains 315 grams. Costs 39¢. How many cents per gram? 0.12¢

11 Total cost: $38.34.
- A Bought 9 items, each costing the same amount. How much? $4.26
- B How much change received from a 50-dollar bill? $11.66

12 Square: each side 4.69 cm.
- A What is the area? 21.9961
- B What is the perimeter? 18.76

★13 Salary: $2.32 per hour. Time clock: $19.14



Deduct 1 hour for lunch and then find the exact pay for the day.

246

Discussion

Some of the problems on page 246 might be challenging for many children. In accord with the problem-solving approach recommended throughout this series, encourage the children to confront each problem individually, think it through, try to solve it, and then check to see if their answer is reasonable. Since most of the problems require either multiplication or division, children will need sufficient time to think through each problem; do not expect them to finish all problems in a single lesson. Stress, rather, that they understand how to solve each problem they try and use the algorithms correctly, even though they may not complete them all.

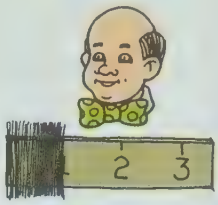
You might assign all the odd-numbered problems to some children and the even-numbered to others and have children explain their problems and solutions to each other.

The problems on page 247 will also challenge many children. Again, stress that for many it is more important to work carefully through 2 or 3 problems rather than try to finish all 6.

Assignments (page 246)

Minimum: 1–6. Average: 1–12. Maximum: 1–13.

HOW SMALL IS SMALL



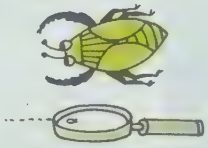
1. It is estimated that it would take about 200 "average" human hairs laid side by side to cover 1 cm. Assuming that this is true, what part of a centimetre is the width of a human hair ? Give this as a decimal. 0.005



2. The diameter of the wire used to make coat hangers is about 0.036 cm.
How many times the diameter of a human hair is this ? $7\frac{1}{2}$



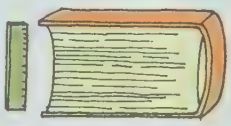
3. One of the smallest watches is about 1 cm long and 0.5 cm wide and weighs 7.08 g.
   - A How many times as wide is this watch as the thickness of a human hair ? 100
   - B How many of these watches would it take to weigh 1 kilogram ? 141.24
   - C One of the largest clocks in the world has a width of 18.28 m. How many times the width of the small watch is this ? 3656



4. One of the smallest insects is the minute beetle, which is 0.02 cm long. One of the largest insects is the goliath beetle, which is 14.86 cm long. How many minute beetles laid end to end would be the same length as the goliath beetle ? 743



5. The human eye can see a bright light coming from an opening as small as 0.00036 cm in diameter. The diameter of this opening is about 6 times the diameter of the fine platinum wire used in a telescope.
   - A What is the diameter of the platinum wire ? 0.00006
   - B How many times as thick as this wire is a human hair ? 83.3



6. One of the largest books is 30.6 cm thick (not including the cover). If each page is 0.006 cm thick, how many pages does the book have ? 5100

247

Assignments (page 247)
Minimum: 1–4, oral. Average: 1–6.
Maximum: 1–6.

Duplicator Masters, page 50
Workbook, page 78

PAGES 248–249

Objective

Given a fraction, the child will be able to express it as a repeating decimal.

Preparation

To prepare for this lesson, you might give the children an opportunity to practice using the division algorithm. Or, review any concept which the children have found particularly troublesome.

Investigation

In this investigation, children discover that some fractions cannot be changed to a fraction which has some multiple of ten as its denominator. Encourage them to try different ways of finding a whole number of hundredths that is "very close" to $\frac{1}{3}$. Many might think of finding the missing factor for the equation $3 \times n = 100$. This, of course, can be solved by dividing 100 by 3. However, since no multiple of 3 is even a power of 10, there is no whole-number factor. Since few investigation questions have a negative response, as does this one, you might hint that the question reads, "*Can* you find . . ."; it does not simply say, "*Find* a whole number. . . ."

Is there a decimal for every fraction?

Investigating the Ideas

Find the whole number for **n**. → $\frac{1}{4} = \frac{n}{100}$ 25

There is **no** whole number for **m**. → $\frac{1}{3} = \frac{m}{100}$

? Can you find a number of hundredths that is **very close** to $\frac{1}{3}$?

Answers may vary. See Investigation.

Discussing the Ideas

1. The fact that $\frac{3}{8} = 3 \div 8$ suggests that you use the dividing process to find a decimal for $\frac{3}{8}$.

1	2	3	4
8) 3	8) 3.0	8) 3.0⁶0	8) 3.0⁶0⁴0
	0.3	0.3 7	0.3 7 5

Now divide to find the decimal for $\frac{5}{8}$. .625 See Discussion.

2. Study the example for finding a decimal for $\frac{2}{3}$. See Discussion.

1 Dividing tenths	2 Dividing hundredths	3 Dividing thousandths
3) 2.0	3) 2.0²0	3) 2.0²0²0
0.6 R2	0.6 6 R2	0.6 6 6 R2

Explain these sentences about each step in the example above.

1 $\frac{2}{3}$ is **approximately** 0.7 (to the nearest tenth).
2 $\frac{2}{3}$ is approximately 0.67 (to the nearest hundredth).
3 $\frac{2}{3}$ is approximately 0.667 (to the nearest thousandth).

3. The decimal for $\frac{2}{3}$ is called a repeating decimal. We write $\frac{2}{3} = 0.66\ldots$ Explain how 0.66 . . . is different from 0.66.

$0.66 = \frac{66}{100}$ while $0.66\ldots = \frac{66}{100} + \frac{6}{1000} + \frac{6}{10\,000} + \frac{6}{100\,000} \cdots$



Discussion

Ask the children to explain how they found the number **n** in the investigation, stressing the relation $4 \times 25 = 100$. Then work through the fractions:

$$\frac{1}{3} = \frac{a}{10} \qquad \frac{1}{3} = \frac{c}{1000}$$
$$\frac{1}{3} = \frac{b}{100} \qquad \frac{1}{3} = \frac{d}{10\,000}$$

Help the children to realize that no multiple of 3 is ever a power of 10. The powers of 10 have only 2 and 5 as factors, not 3. Explain to the children that fractions such as $\frac{1}{3}$ can be written as repeating decimals, discussed in later exercises.

For exercise 1, remind the children that the division line in a fraction may be read "divided by." It would be helpful to relate the conversion of $\frac{3}{8}$ to a fraction with a denominator which is a power of 10.

$$\frac{3}{8} = \frac{3 \times 125}{8 \times 125} = \frac{375}{1000} = 0.375$$

Work through both processes also for the fraction $\frac{5}{8}$.

$$\frac{5}{8} = \frac{5 \times 125}{8 \times 125} = \frac{625}{1000} = 0.625$$

or

$$\frac{5}{8} = 8\overline{)5.000}\ .625$$

Point out that this division has no

Using the Ideas

1. Fractions which have denominators that are factors of 10 or 100 are easily written as decimals. Study the example. Then find decimals for the fractions given.

Examples: $\frac{4}{5} = \frac{4\times2}{5\times2} = \frac{8}{10} = 0.8$

$\frac{11}{25} = \frac{11\times4}{25\times4} = \frac{44}{100} = 0.44$

A $\frac{1}{5}$ 0.2 B $\frac{3}{20}$ 0.15 C $\frac{3}{4}$ 0.75 D $\frac{3}{5}$ 0.60 E $\frac{19}{25}$ 0.76 F $\frac{43}{50}$ 0.86

2. Use division to find decimals for these fractions.

A $\frac{1}{8}$ 0.125 B $\frac{7}{8}$ 0.875 C $\frac{3}{16}$ 0.1875 D $\frac{7}{16}$ 0.4375 E $\frac{15}{32}$ 0.46875

3. The repeating decimal for $\frac{2}{3}$ repeats one digit at a time.

$\frac{2}{3} = 0.66\ldots$

Other repeating decimals may repeat blocks of two or more digits. Study the example for $\frac{13}{33}$. Then find the repeating decimal for $\frac{3}{11}$. 0.2727...

```
   0.3939 ...
33)13.0000 ...
   99
   310
   297
   130
    99
   310
   297
    13
```

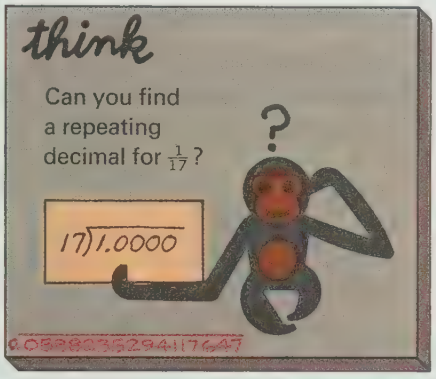
4. Find the repeating decimal for each of these.

0.33... A $\frac{1}{3}$ 0.1666... B $\frac{1}{6}$ 0.833... C $\frac{5}{6}$

0.1111... D $\frac{1}{9}$ 0.222... E $\frac{2}{9}$ 0.555... F $\frac{5}{9}$

0.4545... G $\frac{5}{11}$ 0.8181... H $\frac{9}{11}$ 0.9090... I $\frac{10}{11}$

5. Some decimals repeat in "large" blocks. Find the repeating decimal for $\frac{1}{7}$.





More practice, page A-20, Set 40

remainder, so there is a whole number factor for $8 \times n = 1000$.

Repeating decimals are introduced in exercise 2. Help the children realize that, since no multiple of 3 is ever a power of 10, $\frac{2}{3}$ cannot be changed to a fraction with a denominator that is a power of 10. Thus, the process that is shown in exercise 2 is used. Be sure they realize that no matter how long they continue to divide 2 by 3, they will keep getting sixes. In other words, they are now encountering a decimal which "will never come out even." Point out that the three dots following the decimal indicate a repeating decimal.

Using the Exercises

The exercises on page 249 develop both methods of changing a fraction to a decimal. As in exercise 1, if a fraction has a denominator that is a power of 10, the equivalent-fraction method may be used; if the denominator is not a power of 10, the dividing process can be used.

Children should be intrigued by the decimals which repeat in large blocks, such as those whose denominator is 7.

Assignments (page 249)*

Minimum: 1, 2. Average: 1–4. Maximum: 1–5.

Mathematics

The fact that every fractional number has a repeating decimal representation and, conversely, every repeating decimal represents a fractional number is sometimes given as a kind of definition of fractional numbers. Numbers which have decimal representations which are not repeating decimals are called *irrational* numbers. Some examples of non-repeating decimals are the decimals for the square root of 2 ($\sqrt{2} = 1.4142\ldots$) or the decimal for π, which is 3.14159265358979. . . .

Fractions whose denominators contain only powers of 2 or 5 (such as $\frac{1}{2}$, $\frac{1}{10}$, $\frac{3}{4}$, $\frac{7}{50}$, etc.) obviously have decimals which repeat, after some place, in zeros. To see that other fractions will also have repeating decimals, consider the fraction $\frac{3}{7}$ and the usual division process used to find the decimal for the fraction.

In the division process we see that, if we get the same remainder twice, then clearly we will get a repeating decimal. In the case of the divisor 7, we cannot have more than 6 different remainders before getting one of the remainders twice; hence, $\frac{3}{7}$ has a repeating decimal representation. Since the same kind of argument could be made for any fraction, every fractional number has a repeating decimal representation.

```
   .428571...
7)3.000000
  28
  (20)
  14
  (60)
  56
  (40)
  35
  (50)
  49
  (10)
   7
   (3)
```

Follow-up

To provide practice in comparison of fractional and decimal notation, give children problems like these:

1. Find the missing numbers.

Fraction	$\frac{4}{5}$		$\frac{1}{8}$	$\frac{2}{25}$	
Decimal		0.75			0.85

2. Change the fraction to a decimal; then round it to the nearest tenth.

$\frac{3}{8}$ = ____ = ____

$\frac{99}{100}$ = ____ = ____

PAGES 250–251

Objective

Given an appropriate fraction, the child will be able to express it as a mixed decimal numeral.

Preparation

To prepare for this lesson, you might review how to change a fraction to a decimal. For example, list several fractions and ask children to find the decimal equivalent. For such a review, list the more commonly used fractions, such as $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{2}{3}$, $\frac{1}{2}$. Stress that for some fractions either method may be used.

$$\frac{3}{4} = 3 \div 4 = 4\overline{)3.00}\ .75$$

or $$\frac{3}{4} = \frac{3 \times 25}{4 \times 25} = \frac{75}{100}$$

● *Let's explore mixed-decimal numerals.*

Discussing the Ideas

1. Study the examples below to see how you can write **mixed-decimal numerals** for $\frac{1}{3}$. Give the missing names in the chart.

Dividing process	$3\overline{)1.0}$ 0.3, 9, 1	$3\overline{)1.00}$ 0.33, 9, 10, 9, 1	$3\overline{)1.000}$ 0.333, 9, 10, 9, 10, 9, 1
Mixed-decimal numeral	$.3\frac{1}{3}$	$.33\frac{1}{3}$	$0.333\frac{1}{3}$

2. We read $.3\frac{1}{3}$ as "3 and $\frac{1}{3}$ tenths." How would you read the other mixed-decimal numerals in exercise 1? See Discussion.

3. A When we use A, what is the mixed numeral in **tenths** for $\frac{5}{6}$? $0.8\frac{1}{3}$
   B When we use B, what is the mixed numeral in **hundredths** for $\frac{5}{6}$? $0.83\frac{1}{3}$

A	B
0.8, $6\overline{)5.0}$, 48, 2	0.83, $6\overline{)5.00}$, 48, 20, 18, 2

4. The examples show two ways to express the quotient for problems such as $0.3\overline{)0.136}$. Explain the examples.



A

0.453, $0.3\overline{)0.1\wedge360}$, 12, 16, 15, 10, 9, 1

We say, "The quotient, rounded to the nearest hundredth, is 0.45."

B

0.45, $0.3\overline{)0.1\wedge36}$, 12, 16, 15, 1

The quotient contains 45 hundredths. The remainder is 1 hundredth. We say, "The quotient is $45\frac{1}{3}$ hundredths ($0.45\frac{1}{3}$)."

250

Discussion

In the previous lesson, children learned how to indicate by three dots a repeating decimal or to round the decimal to a particular place. In this lesson, they learn how to express the remainder as a fraction in a mixed decimal numeral. Work through the sections of the chart. In discussing exercise 2, be sure children know how to read the expressions properly:

$0.3\frac{1}{3}$ – "three and one-third tenths"

$0.33\frac{1}{3}$ – "thirty-three and one-third hundredths"

$0.333\frac{1}{3}$ – "three hundred thirty-three and one-third thousandths"

Note in exercise 3 that the fraction formed by the remainder and the divisor, $\frac{2}{6}$, should be expressed in lowest terms, $\frac{1}{3}$. It may be helpful to select several other exercises and present them on the chalkboard for children to explain. Each time, emphasize for the children how the remainder is used to arrive at a fractional number with a fraction in the quotient, that is, a mixed decimal numeral.

Using the Ideas

1. Use division to find a mixed-decimal numeral in **tenths** for each fraction.

   A $\frac{1}{6}$ 0.1$\frac{2}{3}$ B $\frac{2}{3}$ 0.6$\frac{2}{3}$ C $\frac{5}{9}$ 0.5$\frac{5}{9}$ D $\frac{3}{8}$ 0.3$\frac{3}{4}$ E $\frac{4}{7}$ 0.5$\frac{5}{7}$

2. Find a mixed-decimal numeral in **hundredths** for each fraction.

   A $\frac{5}{8}$ 0.62$\frac{1}{2}$ B $\frac{7}{12}$ 0.58$\frac{1}{3}$ C $\frac{9}{16}$ 0.56$\frac{1}{4}$ D $\frac{2}{7}$ 0.28$\frac{4}{7}$ E $\frac{4}{15}$ 0.26$\frac{2}{3}$

3. Find a mixed-decimal numeral in **thousandths** for each fraction.

   A $\frac{5}{6}$ 0.833$\frac{1}{3}$ B $\frac{5}{11}$ 0.454$\frac{6}{11}$ C $\frac{3}{16}$ 0.187$\frac{1}{2}$ D $\frac{2}{3}$ 0.666$\frac{2}{3}$ E $\frac{1}{99}$ 0.010$\frac{10}{99}$

4. Give each quotient rounded to the nearest hundredth.

   A 0.44 9)4 B 1.33 0.4)0.53 C 0.22 6)1.33 D 0.92 9)8.243

   E 0.74 1.3)0.967 F 0.16 3.5)0.56342 G 55.38 0.07)3.8764 H 10.29 0.63)6.4798

5. Give each quotient expressed as a mixed-decimal numeral in hundredths.

   A 0.66$\frac{2}{3}$ 3)2 B 13.56$\frac{1}{2}$ 0.4)5.426 C 1.45$\frac{2}{3}$ 0.6)0.874 D 0.73 0.9)0.657

   E 1.54$\frac{1}{3}$ 3)4.63 F 14.09$\frac{1}{2}$ 0.04)0.5638 G 0.45$\frac{1}{8}$ 8)3.61 H 92.68 0.05)4.634

6. Solve the problems.

   A A 5-centimetre rod is cut into 11 pieces of the same size. How long is each piece (to the nearest hundredth)? 0.45 cm

   B Give the cost of 1 gram of this cereal (to the nearest tenth of a cent). 1.2¢

   C A car is driven 96.4 km and uses 8.1 litres of gasoline. Give the average number of km for 1 litre of gasoline. 11.9 km





More practice, page A-21, Set 41

Mathematics

At this stage, a problem occurs with the dual use of the word *quotient*. Note in the examples that the word quotient can refer to two different numbers in the same problem.

4 quotient	$4\frac{2}{3}$ quotient
3)14	3)14
12	12
2 remainder	2

Therefore, when you talk about *the quotient* for a given division exercise, be sure to make clear which type is being discussed. Usually, this will be apparent from the context. It is important to recognize the ambiguity and make certain that this does not confuse the children.

Duplicator Masters, page 51
Workbook, page 79
Skill Masters, page 51

Using the Exercises

Assign the exercises on page 251 as independent work, unless you consider it necessary to work them together with the children. With some groups, you might want to have the children present several exercises on the chalkboard. When you help them check their papers, allow time for discussion of exercise 3. Be sure they understand that 5 may be divided by 11, and that the answer will simply be a fractional number expressed as a decimal. And in exercise 6B, be sure they see that "to the nearest tenth" means to the nearest tenth of a cent; this answer may be expressed in various ways: \$0.32 ÷ 6 = \$0.05$\frac{1}{3}$ or 5.3¢.

Assignments (page 251)*
Minimum: 1–3. Average: 1–5.
Maximum: 1–6.

PAGES 252–253

Objective

Given a decimal numeral for a number greater than 1, the child will be able to express it in scientific notation.

Preparation

Due to the nature of the investigation in this lesson, you might omit a preparation and begin immediately with the investigation.

Investigation

Although this investigation is brief, it displays the kind of reasoning children will be required to use in developing an understanding of scientific notation. Encourage them to use multiplication to check their answers. You might use this investigation as an oral introduction to the discussion section. The answers may be expressed in equation form, but this will be treated in the discussion exercises.

$1000 \times 9\text{ kg} = 9000\text{ kg}$
$10\ 000 \times 1.2\text{ m} = 12\ 000\text{ m}$
$1000 \times \$1.25 = \$1250$

How are decimals used in scientific notation?

Investigating the Ideas

Can you answer these questions?
See Investigation.

How many times as heavy?



Truck: 9000 kg
1000 times
Toy truck: 9 kg

How many times as high?



Jet: 12 000 m
10 000 times
Bee: 1.2 m

How many times as much?



Diamond: $1250
1000 times
Glass: $1.25

Discussing the Ideas

1. Solve the equations.
   A $9000 = 9 \times n$ 1000 B $35\ 000 = 3.5 \times n$ 10 000 C $1250 = 1.25 \times a$ 1000
2. Give the exponent.
   A $9000 = 9 \times 10^{n}$ 3 B $35\ 000 = 3.5 \times 10^{n}$ 4 C $1250 = 1.25 \times 10^{n}$ 3
3. To name a number in scientific notation, we write it as the product of a number between 1 and 10 and a power of ten. Study the table and give the missing numbers.

Number		Number between 1 and 10		Power of ten
593	=	5.93	×	10^2
64	=	6.4	×	10
4627	=	4.627	×	10^3
895	=	8.95	×	10^2
1634.7	=	1.6347	×	10^3



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Discussion

Help children relate the equations in discussion exercise 1 to those in exercise 2, which use exponents. As you work through the chart in exercise 3, emphasize the fact that in scientific notation, we write a number as the product of a number between one and ten and a power of ten. Note that expressions such as 9×1000 or $1.2 \times 10\ 000$ are not written in scientific notation. You might list expressions like the following and ask children to determine which are written in scientific notation.

42.3×100 (no, $42.3 > 10$)
56×10^2 (no, $56 > 10$)
7.3×10^3 (yes)
8.102×10^5 (yes)
81.02×10^5 (no, $81 > 10$)
0.32×10^4 (no, $0.32 < 1$)
1.42×10^7 (yes)
2.0×10 (yes)

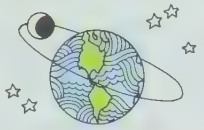
Using the Ideas

1. Find the number for n in each row of the table.

	Number		Number between 1 and 10		Power of 10
A	472.3	=	n 4.723	×	10^2
B	46	=	n 4.6	×	10^1
C	383	=	3.83	×	n 10^2
D	5378	=	n 5.378	×	10^3
E	4769	=	4.769	×	n 10^3
F	n 5900	=	5.9	×	10^3
G	n 649	=	6.49	×	10^2
H	65 000	=	6.5	×	n 10^4

2. Use scientific notation to represent each of these numbers.

- A Speed of sound in cool air in km/h: **1231** 1.231×10^3
- B 1971 school population, Northwest Territories: **10 006** 1.0006×10^4
- C Estimated age of Earth in millions of years: **4950** 4.95×10^3
- D Height in metres of tallest redwood tree: **113** 1.13×10^2
- E Orbital speed (km/h) of moon around Earth: **3792** 3.792×10^3
- F Equatorial diameter of Earth in kilometres: **12 753.77** 1.275377×10^4



3. Study the paper. Then use scientific notation to help you estimate these products.

To estimate the product **593.7 × 68.42**:

1. Write in scientific notation.
   $5.937 \times 10^2 \times 6.842 \times 10^1$
2. Round to the nearest whole number.
   $6 \times 10^2 \times 7 \times 10^1$
3. Find this product.
   $42 \times 10^2 \times 10 = \mathbf{42\,000}$

- A 5280×789 4 000 000
- B 317.5×59.38 18 000
- C 8926.52×314.16 2 700 000

4. Express each of these numbers in scientific notation.

- A Distance to sun: 149 600 000 kilometres 1.496×10^8
- B Speed of light: 9 174 000 000 000 000 metres per year 9.174×10^{15}
- C Sun's mass: 2 220 000 000 000 000 000 000 000 000 000 kg 2.22×10^{30}



253

Follow-up

Scientific notation makes it easy to handle very large numbers, like those used to measure distances in space. Supply children with data such as that below, and ask them to show the distances in scientific notation (for example, 36 000 000 = 3.6×10^7).

Planet	Average distance (kilometres) from the sun
Mercury	66 000 000
Venus	108 000 000
Earth	150 000 000
Mars	228 000 000
Jupiter	778 000 000
Saturn	1 427 000 000
Uranus	2 870 000 000
Neptune	4 500 000 000
Pluto	5 909 000 000

Using the Exercises

Assign the exercises on page 253 as independent work. Some children may benefit by working in groups. When they have finished, carefully check their work. Point out that in numerals as large as those in exercise 4C, they should simply count the number of zeros after the 4. Exercise 3 might need special discussion.

Assignments (page 253)*

Minimum: 1, 2. Average: 1–3.
Maximum: 1–4.

PAGES 254–255

Objective

The child will demonstrate his ability to work with the concepts presented in this chapter.

Preparation

Review any chapter topics which the children have found particularly difficult. If any children have had trouble with the algorithms, you might provide them with an exercise sequence such as the following:

A $325 \div 5$
B $32.5 \div 5$
C $3.25 \div 5$

The answer to each of these problems contains the same digits; the only difference is in the location of the decimal point. If you prefer, you might use the preparation time to stimulate interest in the topic treated on page 255—the speed of sound.

Reviewing the Ideas

1. 2 837 465.156427
For the decimal above, give the number of:
A tens 6 B tenths 1 C hundreds 4
D hundredths 5 E thousands 7
F thousandths 6 G ten thousands 3
H ten thousandths 4
I hundred thousands 8
J hundred thousandths 2
K millions 2 L millionths 7

think

Suppose you have 8 stamps left in this shape on a sheet. You are to tear off 3 connected stamps. How many choices do you have? 12 (See above.)



ABD, ABC, FGH, ACD, BCD, BDE, DEF, CDE, EGH, EFH, EFG, DEG

2. Give a decimal for each fractional number.
A $8\frac{9}{10}$ 8.9 B $67\frac{5}{100}$ 67.05 C $3\frac{76}{1000}$ 3.076 D $\frac{32}{10}$ 3.2 E $\frac{563}{100}$ 5.63 F $\frac{3}{5}$ 0.6 G $\frac{6}{250}$ 0.24

3. Give the correct sign ($<$, $>$, $=$) for each ◍.
A $34.06 \text{ ◍ } 34.60$ $<$ B $0.070 \text{ ◍ } 0.0700$ $=$ C $\frac{1}{7} \text{ ◍ } 0.142$ $>$ D $\frac{1}{3} \text{ ◍ } 0.3334$ $<$

4. Find the sums and differences.

A $\begin{array}{r} 5.47 \\ +7.63 \\ \hline 13.10 \end{array}$ B $\begin{array}{r} 27.34 \\ -9.27 \\ \hline 18.07 \end{array}$ C $\begin{array}{r} 407.6 \\ -29.83 \\ \hline 377.77 \end{array}$ D $9.7 + 0.35 + 0.867 + 17$ 27.917

E $354.6 + 98.67 + 3.704$ 456.974

5. Estimate each product. Then find the correct product.

A $\begin{array}{r} 4.5 \\ \times 6 \\ \hline 27.0 \end{array}$ B $\begin{array}{r} 0.79 \\ \times 9 \\ \hline 7.11 \end{array}$ C $\begin{array}{r} 0.063 \\ \times 32 \\ \hline 2.016 \end{array}$ D $\begin{array}{r} 3.14 \\ \times 6.7 \\ \hline 21.038 \end{array}$ E 0.057×8 0.456

F 5.6×7.3 40.88

6. Find the quotients.
A $5\overline{)32.5}$ 6.5 B $28\overline{)0.588}$ 0.021 C $0.6\overline{)3246}$ 5410 D $5.3\overline{)371}$ 70 E $8\overline{)7}$ 0.875

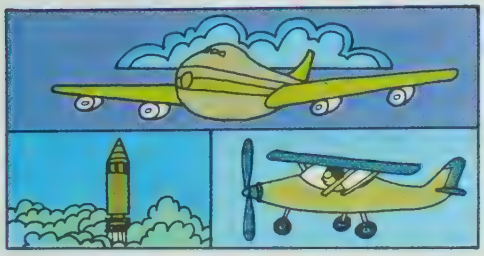
7. Represent $\frac{7}{9}$ as a repeating decimal. 0.777...
Then give this decimal rounded to the nearest hundredth. 0.78

★ **8.** Use scientific notation to represent each number.
A 736 7.36×10^2 B 8459 8.459×10^3 C 37.5 3.75×10
254

Discussion

Allow the children sufficient time to do the exercises on page 254. You might wish to make selective assignments for many of the children. Use the exercises with which the children still have difficulty for further review and discussion. It is particularly important that children understand how to change a fraction to a decimal, because this skill will be needed in the study of ratio.

SPEED AND SOUND



Aircraft	Speed	
Small plane	Mach 0.21	226.38
Propeller airliner	Mach 0.52	560.56
Jet airliner	Mach 0.9	970.2
Jet fighter	Mach 1.9	2048.2
Supersonic transport	Mach 2.2	2371.6
Rocket	Mach 5.75	6198.5

At sea level (0°C temperature) sound travels at a speed of about 1194 kilometres per hour. At higher altitudes, where the air is thinner and cooler, sound travels more slowly. The speeds of aircraft are often given according to the speed of sound. A speed of Mach 1 is the speed of sound, Mach 0.5 is half the speed of sound, Mach 2 is twice the speed of sound, Mach 2.2 is 2.2 times the speed of sound, and so on. Mach is the name of the scientist who made some important discoveries about sound.

For the exercises below use 1078 km/h for the speed of sound at the cruising altitude of the aircraft.

1. Give the speed in kilometres per hour of each aircraft listed in the table. See table above.
2. To find the Mach number for a certain aircraft speed, you divide its speed by the speed of sound. Find the Mach number, to the nearest tenth, for these speeds.
   A 2156 km/h 2 B 3234 km/h 3 C 2575 km/h 2.4 D 805 km/h 0.8

★ 3. The speed of sound at sea level is 1194 km/h.
Find this speed of sound to the nearest tenth in:
A kilometres per minute 19.9 B kilometres per second 0.33 C metres per second 331.7 mps

★ 4. On a very warm day, sound might travel 362.2 metres per second.
A How much more is this than the speed you found in exercise 3c? 30.5
B If you blow a whistle, how far will the sound travel in 5.4 seconds? 1955.88

255

Using the Exercises

The problems on page 255 may be a challenge for some children. Read the introductory paragraph together to help children acquire an understanding of the term *Mach 1*, the speed of sound. Assign the exercises either as group or independent work. Remind the children to work through each problem carefully. Not all of the children should be expected to do the starred exercises; these are intended for faster children.

Workbook, page 80

PAGES 256–257

Objective

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

You might review any topics from the text which previously caused difficulty. Or, you might review the special properties of the identity elements, zero and 1. The word problems on page 257 also provide topics suitable for preparation activity.

Keeping in Touch with

Computing
Measurement
Geometry
Functions
Principles
Fractional numbers

1. Solve the equations.

A $97 + 0 = n$ 97 C $97 \times 1 = n$ 97 E $97 \div 97 = n$ 1 G $0 + 0 = n$ 0
B $97 \times 0 = n$ 0 D $0 \div 97 = n$ 0 F $0 \times 83 = n$ 0 H $1 + 83 = n$ 84

2. Solve the equations.

A $5 \times 2\frac{1}{2} = (5 \times 2) + (5 \times n)$ $\frac{1}{2}$ B $\frac{1}{2} \times 7\frac{1}{5} = (n \times 7) + (\frac{1}{2} \times \frac{1}{5})$ $\frac{1}{2}$

3. Find the products in the blue screens in exercise 2 A $12\frac{1}{2}$, B $3\frac{3}{5}$

4. Find the sum and difference for each pair of numbers.

A $\frac{3}{4}, \frac{1}{6}$ $\frac{11}{12}, \frac{7}{12}$ B $\frac{2}{3}, \frac{1}{4}$ $\frac{11}{12}, \frac{5}{12}$ C $\frac{5}{4}, \frac{1}{10}$ $\frac{27}{20}, \frac{23}{20}$ D $\frac{5}{6}, \frac{1}{8}$ $\frac{23}{24}, \frac{17}{24}$ E $\frac{4}{9}, \frac{1}{6}$ $\frac{11}{18}, \frac{5}{18}$

5. Give each fraction in lowest terms.

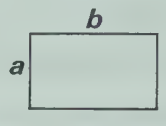
A $\frac{6}{8}$ Answer: $\frac{3}{4}$ B $\frac{12}{16}$ $\frac{3}{4}$ C $\frac{18}{24}$ $\frac{3}{4}$ D $\frac{75}{100}$ $\frac{3}{4}$ E $\frac{45}{100}$ $\frac{9}{20}$ F $\frac{27}{300}$ $\frac{9}{100}$ G $\frac{24}{800}$ $\frac{3}{100}$

6. Draw an angle. Use ruler and compass to copy the angle. Constructions will vary.

7. Find the missing numbers.

Function Rule: $(\frac{1}{3} \times n) + \frac{3}{4}$	
n	$f(n)$
$\frac{1}{4}$	A $\frac{5}{6}$
$\frac{1}{2}$	B $\frac{11}{12}$
9	C $3\frac{3}{4}$
8	D $3\frac{5}{12}$
★ E 0	$\frac{3}{4}$

★ 8. This table refers to rectangles. Give the number pair for each row.



	Area	Half the perimeter	
A	11	$11a$ +	b^1 12
B	27	$9a$ +	b^3 12
C	35	$7a$ +	b^5 12
D	36	$6a$ +	b^6 12
E	32	$4a$ +	b^8 12



You are invited to explore

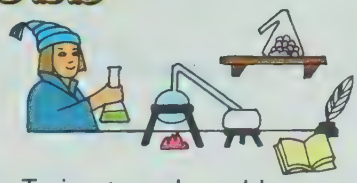
ACTIVITY CARD 12 Page 361

256

Discussion

Page 256 is intended to be done as independent work. However, the table given in starred exercise 8 is of particular interest, and you should call it to the attention of all the children during a discussion of the exercises. For a table of this type, with the product on one side and the sum of two numbers on the other, the children can think of a problem involving the area and half the perimeter of a rectangle.

GOLD



Trying to make gold



Trying to find gold

Because of its value, men have sought gold for thousands of years. It is valuable because it is rare, beautiful, and (to some extent) useful. There are at least 15 other elements that are less plentiful than gold, but gold has been accepted as a standard value for money. Some of the uses of gold, other than for money, are for decoration and dental work.

1. Gold was discovered in the Klondike in 1896. How many years ago was this ? (Depends on current year)
2. Gold is a soft metal. It can be hammered into sheets as thin as 0.0000102 cm. How many such sheets would make a pile 1 cm thick ? 98039
3. One gram of gold can be stretched into 3.28 km of fine wire. How many kilometres of wire could be made from 0.5 kg of gold ? 1640
4. Gold is 7.6 times as heavy as glass. A cubic centimetre of glass weighs 2.54 g. What is the weight of a cubic centimetre of gold ? 19.3
5. Gold is stored in standard-sized bars. Each bar is worth $14 000. The dimensions of a bar are 17.15 by 8.90 by 4.45 cm. Find the volume of a bar. 679.23 cm^3
6. One of the largest gold nuggets, the Welcome Nugget found in Australia, weighed 112.5 kg. About what was its volume ? 5892.02 cm^3

★7. All the gold mined in the world in the 469 years after the discovery of North America would form a cube with edges each about 14.9 m long.
   - A Between what two dates was the gold for this cube mined ? 1492-1961
   - B Estimate the volume of this cube. 3375 m^3

257

Using the Exercises

Since some of the problems on page 257 might be difficult for some children it would be helpful for them to work in groups of two or three, sharing ideas on how to solve each problem. Then, when they have finished, allow opportunity for them to explain their solutions to the rest of the children. Accept any valid reasoning which yields the correct answer. Remind them always to check their answers to see whether they are reasonable.

CHAPTER 11 | Geometry and Measurement II

Pages 258–281

General Objectives

To develop an understanding of the abstract concepts of a simple closed curve and a polygon
To review basic geometric concepts about circles
To introduce inscribed and circumscribed polygons
To introduce the concept of the circumference of a circle
To provide exercises leading to an estimation of the number π
To develop background leading to the formula for finding the area of a circle
To develop informally the formulas for finding the areas of triangles, rectangles, and parallelograms
To develop the formula for finding the volume of a rectangular prism
To study the concept of surface area

The chapter opens with an introduction to simple closed curves, of which polygons and circles are examples. The next two lessons deal with the measurement of the distance around such curves, namely, the perimeter of a polygon and the circumference of a circle. This latter concept is developed through some intuitive experiences which are designed to lead the children to estimate π. Following this, the concept of area is developed by studying progressively the area of a rectangle, a parallelogram, and a triangle. The Pythagorean Theorem for a right triangle is introduced through an application of the concept of area and leads into exponential notation for square numbers. The technique for finding the area of a circle is developed by leading the child from estimating the area of a circle to applying the formula $A = \pi r^2$.

The remaining lessons deal with rectangular prisms. Volume is studied, and then the concept of surface area and its relation to volume dimensions is introduced. Chapter and cumulative reviews conclude the chapter.

Mathematics

Much of the work in this chapter is based on investigations which, with the discussions, lead to intuitive, guided discovery of basic geometric formulas. The children approximate the circumference of a circle by rolling a circle along a surface and then measuring the distance the circle rolled. A sequence of exercises helps them estimate π and leads to the formulas for circumference (C) and area (A). (Diameter is denoted by d and radius by r.)

$$C = \pi \times d$$
$$A = \pi \times r^2$$

Just as linear measure is determined by counting a linear unit such as the centimetre, area measure is determined by counting a square unit such as the square centimetre. Exercises are provided to lead the child to see that multiplying the length times the width (or base times the height) is a shortcut method for finding the number of square units. Once the area of a rectangle and a parallelogram is established, the area of a triangle is easily developed, because the children can discover that the area of every triangle is one half the area of some rectangle or parallelogram.

The Pythagorean Theorem is introduced in such a way that children have an opportunity to conceptualize what the theorem actually says. The method of finding surface area of a rectangular solid is presented as an extension of the method for finding area of plane regions. Volume of rectangular solids is determined by the three dimensions (length, width, and height), as expressed in the formula

$$V = l \times w \times h.$$

Teaching the Chapter

Materials

Cardboard backing for compass tip
Chalkboard compass
Circular objects such as cans, coins, plastic lids, buttons
Compass, one per child (screw type if possible)
Cubes, one per child, if available
Dot paper
Geoboard, one per child (if possible)
Graph paper ($\frac{1}{2}$-cm grid)
Large sheets of paper
Rubber bands
Ruler, centimetre
String or yarn
Thin strips of paper

Vocabulary

area
base
circle
circumference
compass
cube
cubic unit
diameter
dimensions
edge
face
formula
height
hypotenuse
inscribed polygon
legs of a triangle
length
parallelogram
perimeter
polygon
pi (π)
prism
Pythagorean Theorem
radius
rectangle
rectangular prism
simple closed curve
square number
square unit
surface area
unit
vertex
volume
width

The chalkboard compass should be very helpful in making chalkboard illustrations to review radius and circumference of a circle. It is important for each child to have his own compass. Try to obtain sturdy compasses that will hold desired

settings. A cardboard backing is recommended to keep children from marring their desk tops while using their compasses.

The vocabulary list for this chapter is quite long. Many of the words given will already be familiar to the children, since they have occurred in measurement lessons in the earlier geometry and measurement chapter or in previous books.

Lesson Schedule

Plan to spend about three weeks on this chapter, but adjust this suggested schedule according to the needs, abilities, and interests of your children. Since some of them may find this an especially interesting chapter, you may want to allot extra time to encourage their interest.

Evaluation of Progress

In evaluating the children's achievement in this chapter, you should take care not to overemphasize the application of memorized formulas; rather, you should stress the importance of understanding a few general ideas. The children should grasp the idea of the circumference of a circle and be able to find the area of circles and rectangles, and the volume and surface area of rectangular prisms.

Although the children should have some intuitive ideas about the Pythagorean Theorem, they should not be expected to state it mathematically. Again, we emphasize that one of the chief objectives of the chapter is to give the children a general feeling for the study of geometry rather than to have them memorize specific facts and formulas.

Both the cumulative and the chapter reviews will help you evaluate the children's computational skills and their understanding of the formulas for area and volume.

Resources for Active Learning

General Activities

Experiments in Mathematics, Stage 2, pp. 46–47, Houghton Mifflin [Topology: twisting, stretching, and turning surfaces] (Available from Thomas Nelson & Sons Ltd.)

Franklin Series: *Patterns and Puzzles,* pp. 71–79, Lyons and Carnahan [Topology] (Available from McGraw-Hill Ryerson)

Freedom to Learn, pp. 84–85, Addison-Wesley [Investigating two- and three-dimensional shapes]

Geometry in the Classroom, pp. 128–130, Holt, Rinehart and Winston of Canada [Topology]

Mathematics Using String, Math Media

Math Activity Cards, D19, E7, 22, Macmillan [Area and perimeter]

Maths Mini-lab, Cards 98–105, 125, Selective Educational Equipment [Area and perimeter]

Measure and Find Out, Book 2, "Measuring Circles and Angles," Activity 1/4, Scott Foresman (Available from Gage Educational Publishing)

Notes on Mathematics in Primary Schools, pp. 161–189, Cambridge University Press [Geoboard activities] (Available from Macmillan)

Paper Folding for the Mathematics Class, NCTM [Topology]

Teaching Aids for Elementary Mathematics, pp. 116–117, Holt, Rinehart and Winston [Topology]

Manipulative Devices

Caliper (Responsive Environments Corp.; Selective Educational Equipment)

Clinometer (Mafex Associates; Selective Educational Equipment)

Cuisenaire Cubes, Squares, and Rods (Cuisenaire Co.)

Discovery Blocks (Educational Teaching Aids)

Geo Blocks (Selective Educational Equipment; Webster, McGraw-Hill)

Geoboards (Addison-Wesley)

Geometric String Pictures (Mafex Associates)

Lake and Island Board (Math Media; Responsive Environments Corp.)

Sigma Chips (Sigma, Scott Scientific)

Spirograph (Cuisenaire Co.)

Trundle Wheels (Educational Teaching Aids; Selective Educational Equipment)

Commercial Games

Beeline (Selective Educational Equipment)

Connect (local supplier)

Construct-a-Cube Puzzle (Selective Educational Equipment)

Madagascar Madness (Creative Publications; Math Media)

Psyche-paths (Cuisenaire Co.)

Soma and Wooden Puzzles (Creative Publications; Cuisenaire Co.; Edmund Scientific)

Twixt (Hammett; Selective Educational Equipment)

PAGES 258–259

Objective

Given a simple closed curve, the child will be able to give at least an approximate measure of its length.

Preparation

Materials

pieces of string; rulers; thin strips of paper; buttons

Investigation

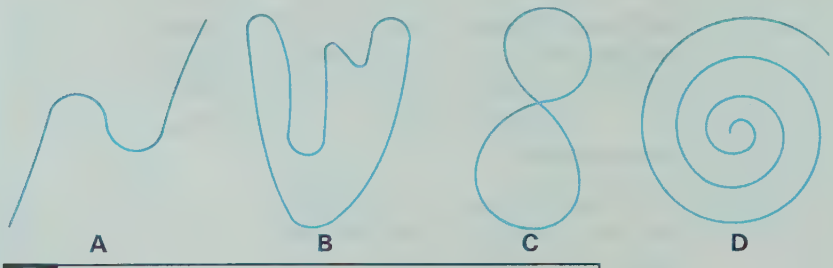
Suggest to the children that after they have guessed the length of each curve, they might want to use some of the materials listed above to check their guesses. A button might be used by marking a point on the circumference and then rolling it along the curve of the text, counting how many times it passes the mark. Then the button may be rolled the same number of rolls along a piece of paper and the resulting straight line segment measured. Thin strips of paper might be used in a manner similar to string, that is, by simply laying the strips to follow the path of the curves. The method discussed for exercise 2, page 259, is also applicable. Encourage children to try a variety of methods and to compare their findings. They might be surprised to discover that most observers guess the lengths of curves to be shorter than they actually are.

11 Geometry and Measurement II

Can you find approximate lengths of curves?

Investigating the Ideas

Guess the length of each curve.



? Can you find some way to measure each of the curves to check your guess?

See Investigation.

Discussing the Ideas

1. A What unit did you use to measure the curves? Answers will vary.
   B How accurate do you think your measurements are? How close were your lengths to those of your classmates? Answers will vary.

2. Can you think of any other way to measure the lengths of the curves? See Discussion.

3. You can think of a **simple closed curve** as a loop of string that is on a flat surface and does not cross itself. **Polygons** are special simple closed curves that are made up entirely of segments.



   A Which curve in the Investigation is a simple closed curve? B
   B Draw a simple closed curve on the chalkboard and estimate its length by looking. How can you get a closer estimate? Responses will vary.

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Discussion

Use discussion exercises 1 and 2 as an opportunity for children to share the results of their investigation. Stress the fact that, even though the lines they measured are not straight, the centimetre is an appropriate unit. For exercise 2, you might mention the use of a flexible plastic ruler which can be bent to follow the shape of the curve, or the use of a miniature trundle wheel similar to the kind used in sewing to mark material.

When you discuss exercise 3, point out not only that polygons are a special kind of simple closed curve but also that polygons may be concave or convex. It is not necessary that you stress or even use these terms, but since the children's study of polygons has for the most part been limited to convex polygons, it would be helpful to point out that figures such as the following are also called polygons:

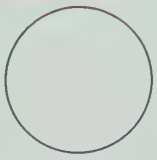


As children do exercise 3B, you might suggest that a few children use a piece of string at the chalkboard in order to get a closer estimate of the simple closed curve they have drawn.

Using the Ideas

1. The curves named below have no segments in them.
Find the length of each curve as accurately as you can. Answers may vary.

A (About 8 cm)



circle

B (About $9\frac{1}{2}$ cm)



cardioid

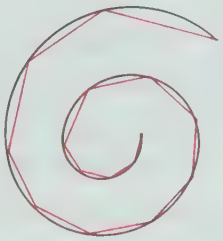
C (About $8\frac{3}{4}$ cm)



ellipse

2. A The approximate length of a curve can be found by measuring segments. Find the length of this red path to the nearest centimetre. About 15 cm

B Is it longer or shorter than the spiral? Shorter



3. Draw some curves of your own and find their lengths.
Drawings will vary.

think

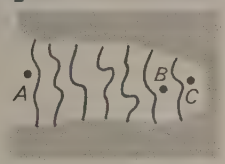
Only a small part of a simple closed curve is shown in each picture. Can you answer the questions?

A



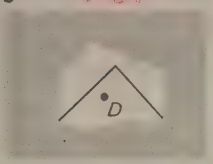
If the bug is outside the curve and crosses it an odd number of times, will he end up outside or inside?

B



If point *A* is outside the curve, where is *B*? Outside
Where is *C*? Inside

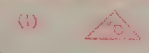
C



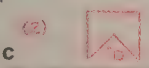
Draw a picture of how the polygon might look if
(1) *D* is inside.
(2) *D* is outside.

Possible drawings for C:

(1)



(2)





Using the Exercises

One of the most important techniques of measuring a curve is treated in exercise 2. After children have finished the exercises, it would be helpful to stress this idea. Point out that the smaller the segments, the closer the approximation will be to the actual length of the curve.

The *Think* problem provides material for a discussion of inside and outside a curve. You might use a loop of yarn on the overhead projector or on a flannelboard. Push it around in various ways to show a simple closed curve. Place a small object inside the curve and have children explain that if the object crosses the yarn once it is outside the curve; if it crosses twice, it is back inside; and so on. Try to let the children conclude that, if the object is inside the curve at the start, after crossing the curve an even number of times, it will be back inside at the finish; and after crossing the curve an odd number of times, it will be outside at the finish. Likewise, if the object is outside the curve at the start, it will be outside the curve after an even number of crossings, and inside after an odd number of crossings.

Assignments (page 259)

Minimum: 1–3. Average: 1–3. Maximum: 1–3.

Mathematics

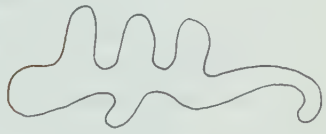
A curve is often described as "the path of a moving point." Curves, in geometry, can be parts of circles, ellipses, and other non-linear figures, but the broader use of the word curve includes "straight curves" such as segments, lines, and polygons. The important criterion for a curve is that it be continuous, that is, a path with no breaks in it.



Curve



Closed curve



Simple closed curve

A *closed curve* is such that one may begin at any point on the curve, trace around the curve and return to the starting point without lifting the pencil from the curve. A *simple closed curve* is a closed curve which does not intersect itself.

One important property of simple closed curves is that each simple closed curve separates the points in the plane into three sets: (1) the points in the interior of the curve, (2) the points in the exterior of the curve, and (3) the points on the curve.

Resources for Active Learning

Developmental Math Cards, L-15 (a game), L-18, Addison-Wesley.

Experiments in Mathematics, Stage 1, pp. 46-47, Houghton Mifflin. (Available from Thomas Nelson & Sons Ltd.)

Mathex: Geometry No. 9, pupil pages 49–55, Encyclopaedia Britannica Publications Ltd.

Modern Math Games, "Puzzle," p. 58, Fearon. (Available from Clarke, Irwin)

PAGES 260–261

Objective

Given a polygon, the child will be able to find its perimeter by finding the sum of its sides.

Preparation

Materials

geoboard and rubber bands (if available); dot paper (*Duplicator Masters*, page 64)

Investigation

You may need to explain to some children that each segment in picture A is congruent to one of the sides of the polygon in picture B. However, let them discover that in order to find the "distance around" the polygon they need only to use the lengths of the segments in picture A. They do not need a ruler or any other measuring device to find the distance around the polygon.

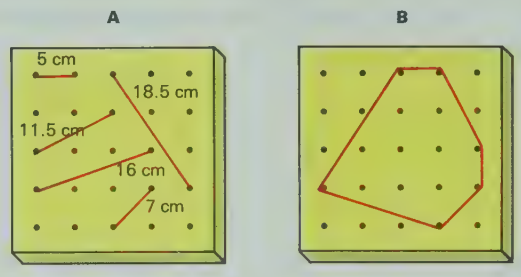
If children have difficulty matching the segments in picture A to those in the polygon, you might help them think of a segment as "2 over, 1 up" or as "2 over, 3 down" and so on.



How do you find the perimeter of a polygon?

Investigating the Ideas

Picture A shows approximate lengths of certain segments on the geoboard.



Can you find the distance around the polygon on geoboard B? 59 See Investigation.

Discussing the Ideas

1. The sum of the lengths of the sides of a polygon is called the **perimeter** of the polygon. Show each of these polygons on your geoboard or dot paper and explain how you would find its perimeter.

   A square B triangle C quadrilateral

   D pentagon E hexagon

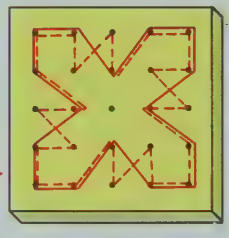
   Find the sum of the lengths of the sides. (In the case of regular polygons, multiplication also might be used.)

2. Here is a polygon with 16 sides.

   A What is its perimeter? $19\frac{1}{3}$ See Discussion.

   B Can you find a polygon with more sides and greater perimeter?

   Sample possibility indicated by dashed lines.



3. Show how you could use your ruler to find the perimeter of a polygon. Measure each side and add to find the sum of the lengths.

260

Discussion

As children discuss the results of the investigation, emphasize the meaning of *perimeter*. Point out that in the investigation their unit was determined by the distance of one nail from the next in a row. The measurements given for the segments in picture A of the investigation should be used when children discuss the investigation as well as when they try discussion exercise 2.

Be sure the children understand that the shorter the unit they use in measurement, the greater the number they will get for the perimeter, and the longer the unit, the smaller the number.

Using the Ideas

1. Show the largest right triangle you can on the geoboard or dot paper and find its perimeter in centimetres.
Answers will vary with dimensions of geoboard or dot paper.

2. Use your centimetre ruler to find each perimeter.

A



14 cm

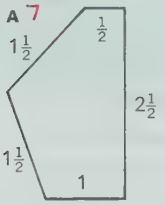
B



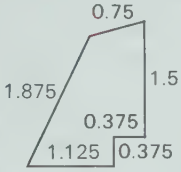
14 cm

3. Find the perimeter of each figure.

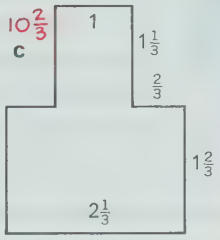
A 7



B 6



C $10\frac{2}{3}$



4. A A square has a perimeter of 36. What is the length of each side? 9

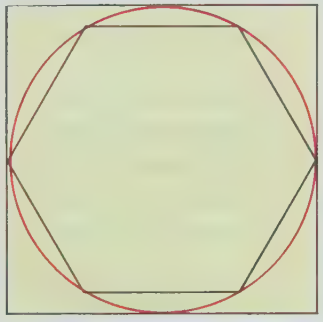
   B A rectangle has a perimeter of 36 and one side of 4. What is the length of each side? 14, 4, 14, 4

   C A triangle has two sides that have lengths of 17. It has a perimeter of 50. What is the length of the third side? 16

5. A What is the perimeter of the square in centimetres? 20.8cm

   B What is the perimeter of the hexagon? 15cm

   C Estimate the "perimeter" of the circle. Accept any answer between 15 and 18cm





Using the Exercises

Assign the exercises on page 261 as independent work. Note that the unit in exercise 3 is unimportant. The concept stressed here is that of perimeter as the sum of lengths.

The results of exercise 5 emphasize the fact that it is possible to arrive at approximations for the circumference (or as we say here, "perimeter") of the circle by measuring a polygon which has a perimeter less than that of the circle and a polygon which has a perimeter greater than that of the circle. The point to be made in this exercise is that we have, so to speak, "trapped" the perimeter of the circle between two numbers. You need not stress finding the circumference of the circle to any great degree of accuracy; that will be developed in a later lesson.

Assignments (page 261)

Minimum: 1–3. Average: 1–4. Maximum: 1–5.

Follow-up

Challenge the children to see how many polygons of different shapes they can find on the geoboard that have a perimeter of 4, 6, 8, and 10 using the unit distance on their geoboard. Suggest that they record their findings on dot paper.

Workbook, page 81

PAGES 262–263

Objective

Given the radius or diameter of a circle, the child will be able to find its circumference by applying the formula:

circumference = $\pi \times$ diameter.

Preparations

Materials

compass (1 per child); cardboard; centimetre ruler

To prepare for this lesson, it would be helpful to draw a circle on the chalkboard and review the terms *radius* and *diameter*. Stress the idea that the radius may be thought of as the segment from the centre of the circle to any point on the circle; and that the radius may be expressed as a number. Similarly, the diameter may be thought of as a segment which has its end points on the circle and passes through the centre. The diameter is also sometimes expressed as a number.

Investigation

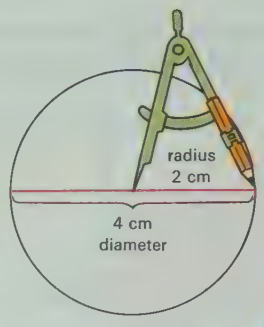
If necessary, help the children construct the circle with a radius of 2 centimetres. Unless the compass has a scale to indicate centimetres, children will need a centimetre ruler to adjust their compasses to the proper opening. As they continue to independently construct the other circles, make sure they realize that the opening of their compass, that is, the radius, is equal to half of the diameter. Thus, to construct a circle with diameter of 7 centimetres, the compass opening, or radius, should be $3\frac{1}{2}$ centimetres.

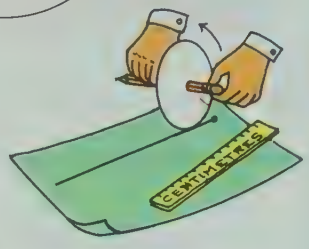
If the cardboard is fairly sturdy and children do not bend the circle when they put the pencil through the centre, this activity should not be too complicated. You might help them interpret the illustration and suggest that they draw a straight line 30 centimetres long on paper so that they can roll each circle along it, marking the place where the starting point touches the line again. Remind them to record their measurement for each circle.

How do you find the circumference of a circle?

Investigating the Ideas

If you open your compass to a **radius** of 2 centimetres, you can draw a circle with a **diameter** 4 centimetres. Draw circles with diameters 3, 4, 5, 6, 7, 8, and 9 centimetres on cardboard and cut them out.





? Can you use the idea suggested by this picture to find the "distance around" each circle to the nearest one-tenth centimetre ?
See Investigation.

Discussing the Ideas

1. The "distance around" a circle is called the **circumference** of the circle. Copy the table below on the chalkboard and write in the circumference you found for each circle. For each circle, which whole number times the diameter will give a number closest to the circumference ? 3

Answers may vary. See Discussion.

Special factor		Diameter (cm)		Circumference
	×	3	≈	9.4? cm
	×	4	≈	12.6? cm
	×	5	≈	15.7? cm
	×	6	≈	18.8? cm
	×	7	≈	22 ? cm
	×	8	≈	25.1? cm
	×	9	≈	28.3? cm

2. A special factor times the diameter gives the circumference. Do you think this special factor is more or less than the whole number you found in exercise 1 ? More



262

Discussion

Explain the meaning of the term *circumference*. Have the children give the circumferences they found for each circle. Explain that there is always some error of measurement made in such an activity, and give the children an opportunity to mention a few sources of error, such as inaccuracy in cutting or in using the compass. Complete the circumference column in the table with the measurements most children agree on. Explain that the wavy lines instead of the usual equals sign means "is approximately equal to" or "is almost the same as." When children examine the missing special factor, they should realize that the same number is the missing factor in each statement. If children's measurements were fairly accurate, the exact answer of the whole number, 3, times each diameter should yield a number slightly less than the circumferences. Therefore, children should conclude that the special factor is a little more than 3. Explain that this special factor is named by the Greek letter π (pi). For most work, the approximation 3.14 for π is quite adequate. Thus, a formula may be developed relating the circumference of any circle to its diameter:

circumference = $\pi \times$ diameter

Using the Ideas

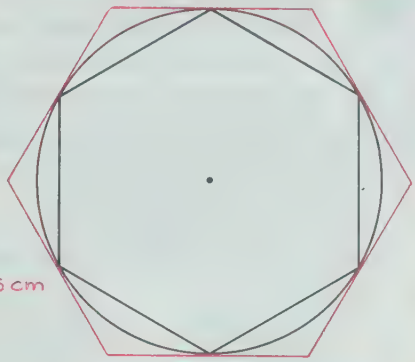
1. The experiments on page 262 suggest that the circumference of any circle is found by multiplying the diameter by a number slightly greater than 3. This number is the same for all circles. It is named by the Greek letter π (pronounced "pie").
We write:

> Circumference = π × diameter,
> where π = 3.14 (to the nearest hundredth).

Find the circumference of a circle with diameter:

A 3 cm (9.42 cm) B 7 cm (21.98 cm) C 10 cm (31.4 cm) D 2 cm (6.28 cm) E 100 cm (314 cm) F 12 cm (37.68 cm)

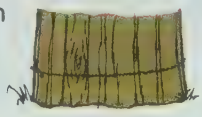
2. A Is the circumference of the circle greater or less than the perimeter of the polygon shown in black? in red?
(greater, less)

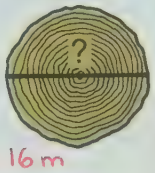


B Estimate the circumference of the circle by finding the average of the perimeters of the polygons. (Use centimetres.) (About 19.5 cm)

C Check your estimate by measuring to find the diameter (in centimetres) and multiplying to find the circumference. (18.21 cm)

★ **3.** A rope 49.682 m long was needed to encircle one of the world's largest trees. What was the diameter of a cross section of the tree trunk?

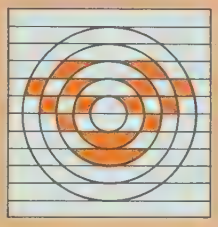


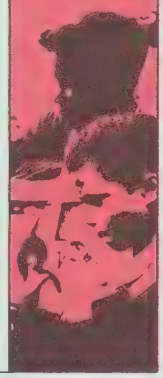


15.82 m or about 16 m

think

Use notebook paper to help you color a circle pattern.





More practice, page A-21, Set 42

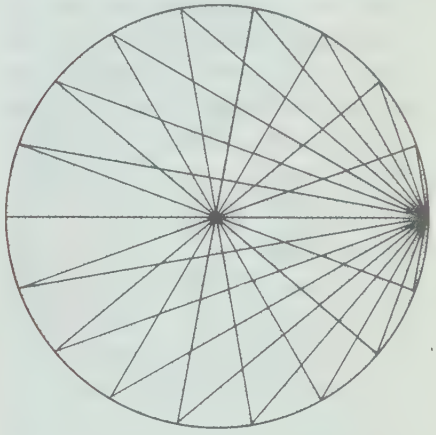
Using the Exercises

Assign the exercises on page 263 as independent work. If necessary, show children how to use the formula in exercise 1. When you have them check and discuss exercise 2, help them realize that they arrived at an approximation for the circumference of the circle which is a little more than three times the diameter of the circle. As you discuss starred exercise 3, point out to the children that since they know the circumference and they have an acceptable value for π (3.14), they can easily find the missing factor, which is the diameter.

Assignments (page 263)*
Minimum: 1. Average: 1, 2.
Maximum: 1–3.

Follow-up

Children might enjoy designing circle patterns like the one suggested in the *Think* problem. They might draw a circle with a 6-centimetre diameter and mark off intervals every 20 degrees on the circumference. Then they could draw diameters through the circle at each marked point on the circumference, and from the end point of one diameter, connect the end points of every other diameter to that point. Finally, they could color in a design such as that shown below.



Resources for Active Learning

Applied Mathematic Cards, "Measuring Circles," Group 3/16, Schofield and Sims. (Available from Mafex Associates)

Experiments in Mathematics, Stage 1, pp. 32–35, 38–39, Houghton Mifflin. [Circle patterns] (Available from Thomas Nelson & Sons Ltd.)

Nuffield Project: *Problems*—Green Set, No. 21, Wiley. [Circle patterns]

Duplicator Masters, page 52
Workbook, page 82

PAGES 264–265

Objective

Given the base and height of a rectangle or parallelogram, the child will be able to find the area.

Preparation

Materials

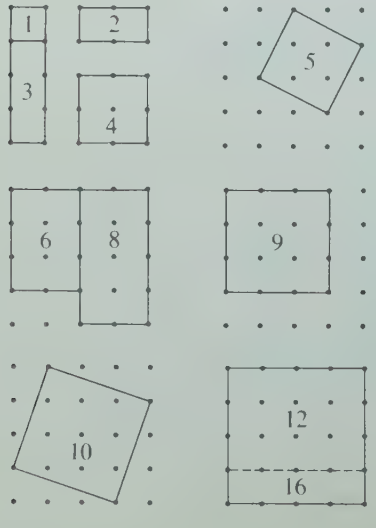
geoboards (if available); dot paper (*Duplicator Masters*, page 64)

To prepare for this lesson, you might ask the children to describe the unit they use in measuring length, leading them to recall that to measure length they simply count some small segment or unit of length. Ask them to describe area, and to try to recall the unit used to measure area. Point out that, just as in finding length they count a unit of length, to find area they count a unit of area.

Investigation

If it is not possible for each child to have a geoboard, supply a sufficient amount of dot paper to serve as a substitute. In either case, the child is asked to record his findings on dot paper. (If you duplicate your own dot paper, use a five-by-five array of dots.)

As children work through this investigation, you may need to remind them that they are to use only rectangles or squares. If a child uses some other region, help him to see that his region is not a rectangle or square. Do not be surprised if many children have difficulty finding the square with area 5 and the square with area 10. The possible areas are shown below.



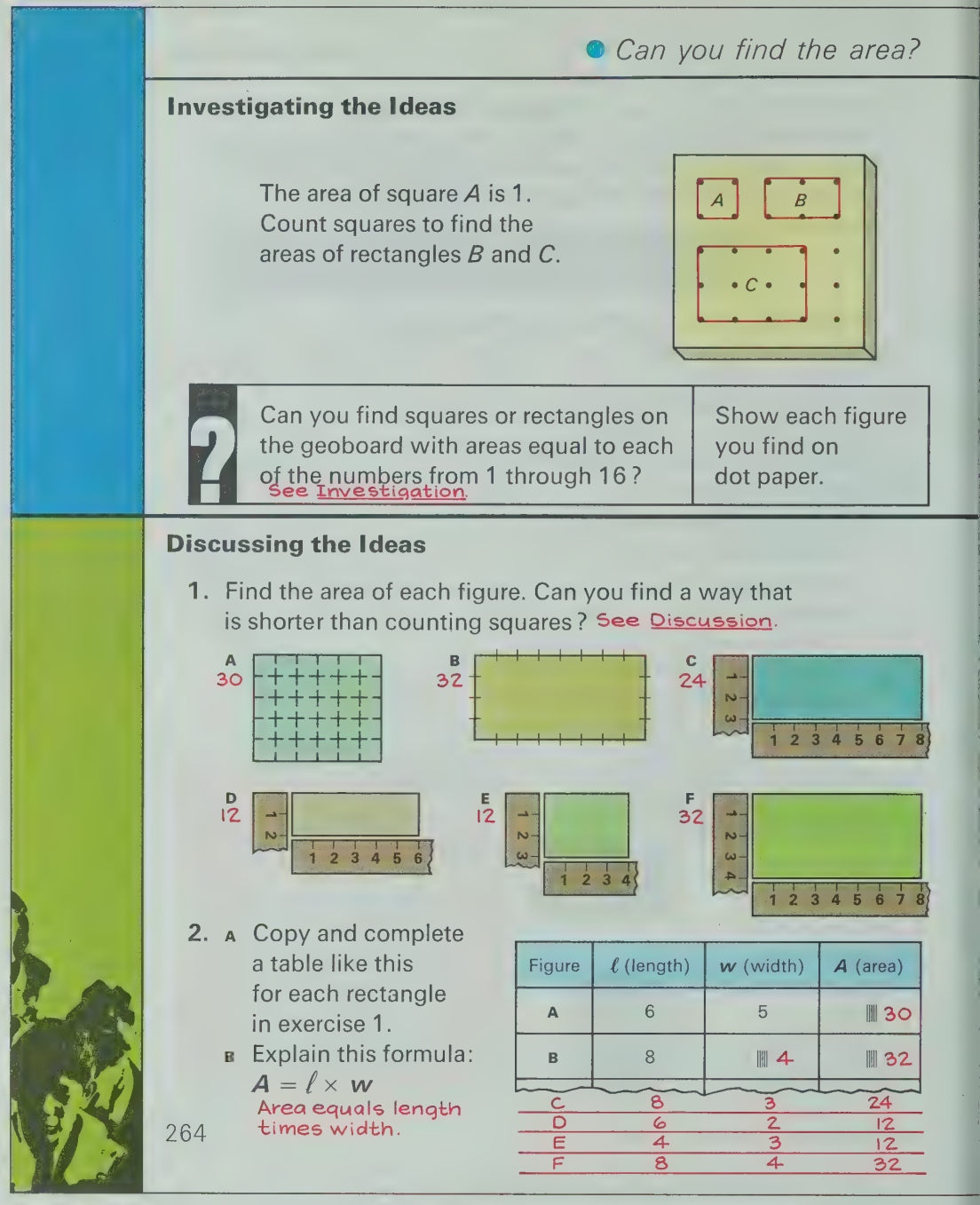


Figure	ℓ (length)	w (width)	A (area)
A	6	5	30
B	8	4	32
C	8	3	24
D	6	2	12
E	4	3	12
F	8	4	32

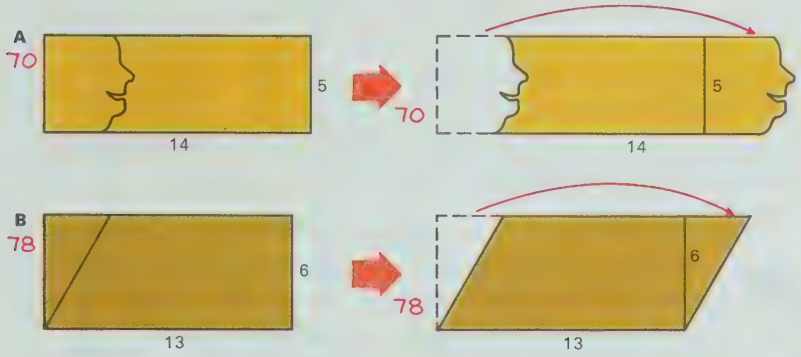
Discussion

Before discussing the exercises, have children share the results of the investigation. Ask them why they could not find squares or rectangles with areas 7, 11, 13, 14, and 15. Help them see that rectangular or square arrays cannot be formed on the five-by-five geoboard for these numbers.

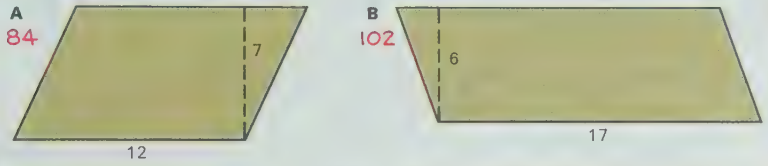
Lead the discussion into exercise 1 by helping children relate the number of squares in each row or column to factors of the number they were working with. As you discuss parts C, D, E, and F, use the terms *length* and *width* and guide children to a discovery of the formula for finding the area of a rectangle in exercise 2. Make sure children realize that this formula also applies to squares, in which the length and width are equal.

Using the Ideas

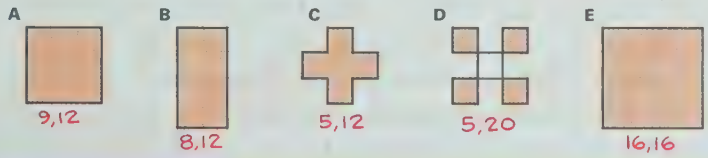
1. Find the area of each region.



2. Find the area of each of these **parallelogram** regions.

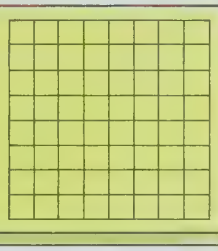


3. Find the area and perimeter of each figure.



think

Draw an 8-by-8 figure like this.
Color 8 small square regions so that no 2 colored regions lie in the same row (↔), column (↕), or diagonal (⤫).
There is more than one way to do this.
See Sample Solution, T.E. page 265.



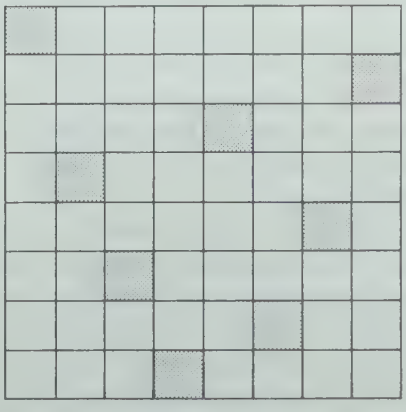


Using the Exercises

The exercises on page 265 are designed to lead the children to the technique for finding the area of a polygon without introducing a formula first. Let the children complete the exercises independently if they can. Allow time for discussion and questions after the children have had time to experiment.

The *Think* problem has several possible solutions. You may wish to allow the children to color 8-by-8-unit grids in the various patterns they create for a bulletin board display. (See *Sample solution* at the right.)

Sample solution, Think, page 265



Assignments (page 265)

Minimum: 1–3. Average: 1–3.
Maximum: 1–3.

Mathematics

Area is a measure or number associated with a bounded portion of a plane, that is, a region. In previous lessons, we have been careful to speak of areas of regions. However, it is common practice to speak of the area of a square, rectangle, or other polygons without referring to the fact that we mean the area of the region bounded by such polygons. Since there is little chance of misunderstanding what is meant by such terminology, we use the more familiar terminology in this lesson and in succeeding lessons. If you prefer, you may continue to use the "region" terminology when discussing area concepts.

Follow-up

You might have different groups of children measure the area of various surfaces. Some may cut out a square metre or square cm and count how many units of area are needed. Others may simply measure the length and width of the surface and use the formula to compute the area. This type of activity would be particularly worthwhile for any children who have difficulty relating the formula to the concept of measuring area.

Resources for Active Learning

Applied Mathematics Cards, Group 2/16; Group 3/5, Schofield and Sims. (Available from Gage Educational Publishing Ltd.)

Experiments in Mathematics, Stage 1, pp. 10-13, Houghton Mifflin, (Available from Thomas Nelson & Sons Ltd.)

Franklin Series: *Making and Using Graphs and Nomographs*, "Using Nomographs to Find Areas," pp. 73–74; "Finding Missing Dimensions," p. 82, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Madison Project: *Discovery in Mathematics* (Teacher's Text), pp. 229–236, Addison-Wesley.

Matrix: Measurement No. 10, pp. 9–17 (pupil pages 31–53), Encyclopaedia Britannica Publications Ltd.

Workbook, page 83

PAGES 266–267

Objective

Given the base and height of a triangle, the child will be able to find its area.

Preparation

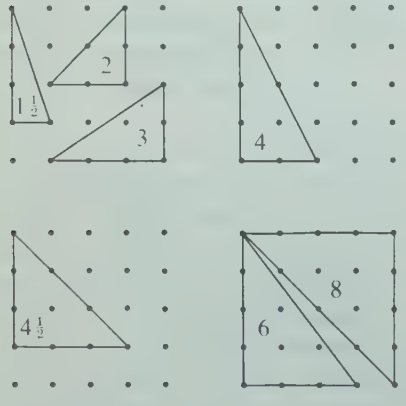
Material

geoboard; dot paper (*Duplicator Masters*, page 64)

To prepare for this lesson, briefly review the fact that a right triangle has one angle that is 90°. You might also review acute and obtuse angles as discussed on page 70. For example, draw an obtuse triangle similar to the one in exercise 3, page 267, and point out the two acute angles and the one obtuse angle. The main purpose of this preparation, however, should be simply to review triangles in general and right triangles in particular.

Investigation

Some children may notice that the areas of the triangles in this investigation are one half the areas of some of the squares and rectangles they found in the investigation in the preceding lesson. However, do not point this out; this investigation should lead the children to discover this relationship for themselves. Remind them to carefully record their results on dot paper by marking the area of each right triangle they find. Sample answers are shown below.



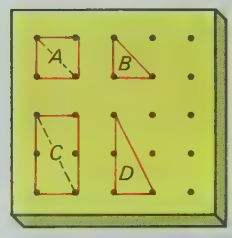


Can you find the area of any triangle?

Investigating the Ideas

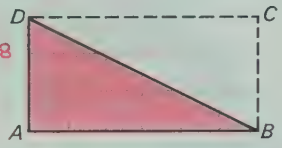
If the area of square *A* is 1, the area of right triangle *B* is __? __. $\frac{1}{2}$

If the area of rectangle *C* is 2, the area of right triangle *D* is __? __. 1

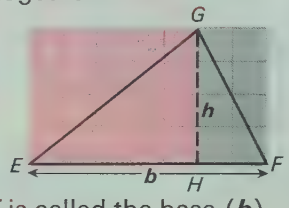


?	Can you find right triangles with areas $1\frac{1}{2}$, 2, 3, 4, $4\frac{1}{2}$, 6, and 8 on a geoboard?	Show each triangle you find on dot paper. See Investigation.

Discussing the Ideas



1. A What is the area of rectangle *ABCD*? 18
   B The area of triangle *ABD* is what fractional part of the area of *ABCD*? $\frac{1}{2}$
   C What is the area of triangle *ABD*? 9
   D Explain how to find the area of any right triangle. See Discussion.
2. A What is the area of the region shaded pink? 20
   B What is the area of the region shaded gray? 8
   C What is the area of the two regions together? 28
   D What is the area of triangle *EHG*? 10
   E What is the area of triangle *FHG*? 4
   F What is the area of triangle *EFG*? 14
   G The area of triangle *EFG* is what part of the entire shaded region? $\frac{1}{2}$



3. In exercise 2, the length of segment *EF* is called the base (***b***) of the triangle. The length of segment *GH* is called the height (***h***). Can you give a formula for finding the area of a triangle when you know its base and height?
   $A = \frac{1}{2} \times \text{base} \times \text{height}$

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Discussion

Use the results of the investigation to lead into a discussion of exercise 1. Emphasize that the diagonal in each rectangle divides it into two equal triangles. Thus, the area of each triangle is one half the area of the rectangle.

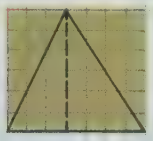
Help the children to see that for the triangles which they will be studying the base is the side of the triangle on which it appears to be resting and the height is the perpendicular distance from the vertex opposite the base to the base. Clarify these terms as children work through exercise 2. Also, stress the fact that the triangle *EHG* is one half the rectangle to the left of segment *GH* and that the area of triangle *FGH* is one half the rectangle to the right of segment *GH*. Since each of these triangles is one half of one of the two rectangles, the whole triangle, *EFG*, is one half the entire rectangle with base *EF*. Also, point out that segment *GH* is the height of the triangle *EFG*. Conclude by having children write the answer to exercise 3 in the general formula:

area of a triangle =
$\frac{1}{2} \times$ base $\times$ height

Using the Ideas

1. Find the area of each large triangle.

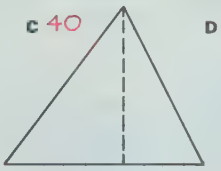
A 21



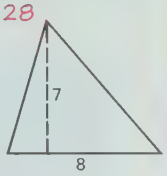
B 10



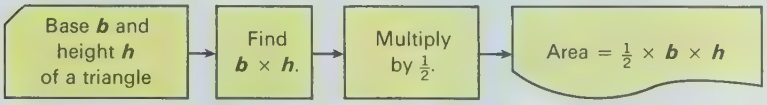
C 40



D 28



2. Use the flow chart to help you find the areas of the following triangles with the given bases and heights.



A $b = 12$, $h = 5$ 30

B $b = 10$, $h = 8$ 40

C $b = 14$, $h = 7$ 49

D $b = 27$, $h = 8$ 108

E $b = 54$, $h = 9$ 243

F $b = 8$ 18, $h = 4\frac{1}{2}$

G $b = 9$ $10\frac{1}{2}$, $h = 2\frac{1}{3}$

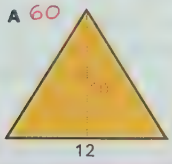
H $b = 12.4$, $h = 6.5$ 40.3

I $b = 16$, $h = 7\frac{1}{4}$ 58

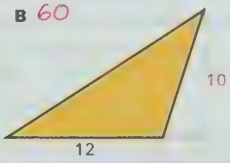
J $b = 128$, $h = 46\frac{3}{4}$ 2992

3. The height of each triangle is given in red. Find the area of each triangle.

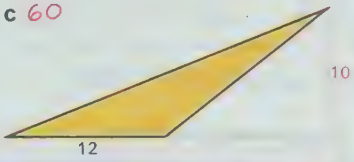
A 60



B 60

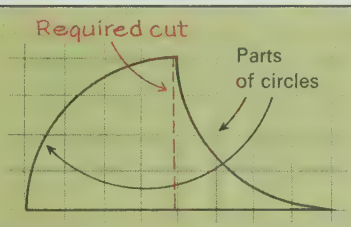


C 60



think

Make a larger figure like this on graph paper. Can you make one straight cut that separates the figure into two parts that will fit together to form a square?



More practice, page A-22, Set 43

Using the Exercises

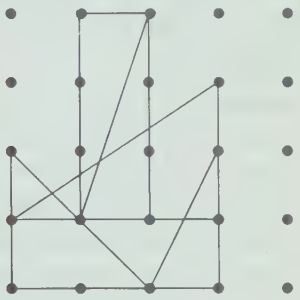
Assign the exercises on page 267 as independent work. Some children may need help in relating the formula $A = \frac{1}{2} \times b \times h$ to the problems and recognizing the base and the height in each triangle. Work through the exercises together with your children.

Assignments (page 267)

Minimum: 1, 2. Average: 1–3.
Maximum: 1–3.

Follow-up

Encourage children to make puzzles by forming overlapping triangles on the geoboard. Then other children can try to find as many "hidden" triangles as they can.



You might challenge capable children to estimate the area of the simpler designs.

Resources for Active Learning

Inquiry in Mathematics via the Geo-board, "Area of Triangles," Geo-cards 16–18, Walker. (Available from Fitzhenry & Whiteside Ltd.)

Mathematics in Modules, M8, Addison-Wesley.

Workbook, page 84

PAGES 268–269

Objective

Given the lengths of the sides of a right triangle, the child will show his understanding that the sum of the areas of the squares on the legs is equal to the area of the square on the hypotenuse.

Preparation

To prepare for this lesson, you might briefly review the various methods for finding the area of a region. Make sure children realize that sometimes the best way to find the area of a region is to think of it in parts and add the areas of the parts.

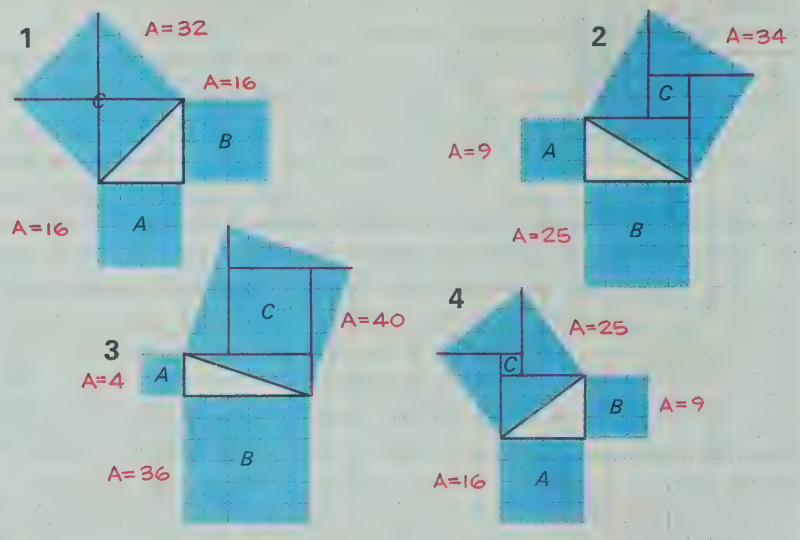
Investigation

Although the children should try to find the specified areas independently, it would be helpful for you to move around the room and check to see that they are correctly determining the areas, particularly for each of the *C* squares. Some children may find it helpful to use geoboards to find the area of these squares. Also, suggest that the children make a table to show the areas of the squares for each triangle.

Triangle	Area		
	Square *A*	Square *B*	Square *C*
1			
2			
3			
4			

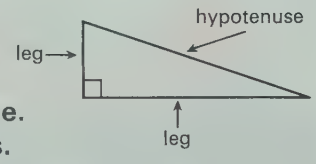
Let's explore the Pythagorean theorem.

Investigating the Ideas



Can you find the area of the smaller squares *A* and *B* and the larger square *C* for each triangle above?
See above and Investigation.

Discussing the Ideas



1. In a right triangle, the side opposite the right angle is called the **hypotenuse.** The other two sides are called the **legs.**
   - A What is the area of the square on the shortest leg of the right triangle in figure 4 above? What is the area of the square on the other leg? 16 9
   - B What is the area of the square on the hypotenuse in figure 4? 25
2. What pattern do you see in your answers to the Investigation question? Do you think this is true for all right triangles?
   See Discussion.

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Discussion

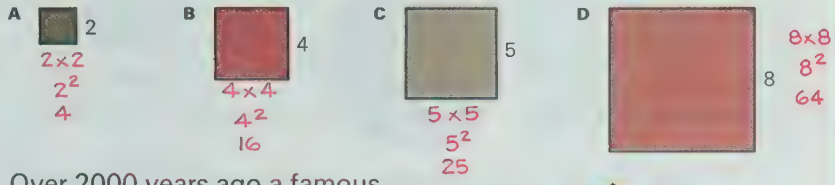
Introduce and explain the vocabulary terms *hypotenuse* and *legs*. As children discuss exercise 1, stress not only the use of the terms and the need for accuracy in finding the area but also ask the children if they see any relation between these numbers.

Use discussion exercise 2 to lead the children to discover the Pythagorean Theorem, namely, that the sum of the areas of the squares on the legs of a right triangle is equal to the area of the square on the hypotenuse. Using the labels of the squares, help the children to conclude $A + B = C$, where each letter represents the area of the square.

Using the Ideas

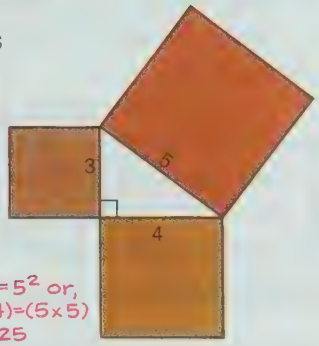
1. If square *A* has a side 3 units long, the **area of square *A*** is written **3 × 3** or 3^2 or simply **9.** Write the area of these squares in three ways.





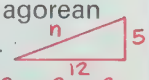
2. Over 2000 years ago a famous mathematician named Pythagoras proved this important theorem:

The sum of the areas of the squares on the legs of a right triangle is the same as the area of the square on the hypotenuse.



Write an equation to show that this theorem is true for this right triangle. $3^2+4^2=5^2$ or, $(3\times3)+(4\times4)=(5\times5)$ or, $9+16=25$

★ **3:** Draw a picture and write an equation to show the Pythagorean theorem for a right triangle with legs 5 and 12 cm long.



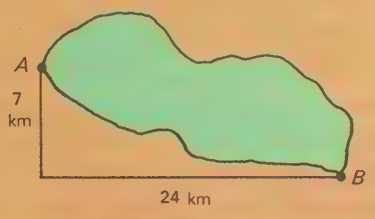
$5^2+12^2=n^2$

★ **4.** If a right triangle has legs 6 and 8 cm long, how long is the hypotenuse? 10 cm

$(25+144=169$
$n=13)$

think

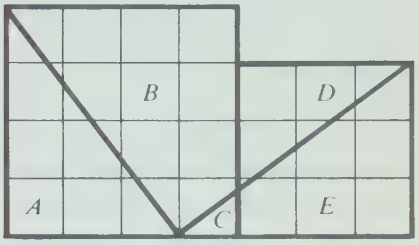
How far is it from point *A* on the shore to point *B*, straight across the lake? 25





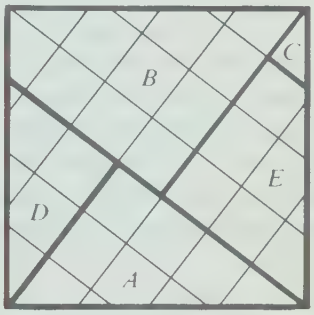
Follow-up

A challenging puzzle based on the Pythagorean Theorem is shown below.



Cut out the five pieces from graph paper. Then challenge children first to reconstruct the 3 x 3 and the 4 x 4 squares. After that, challenge them to use all five pieces to form one large square. It must, by the Pythagorean Theorem, have an area of $3^2 + 4^2$ or 25, so it will be a 5 x 5 square.

Solution



Resources for Active Learning

A Cloudburst, Vol. 2 & 3, No. 6800 series, Midwest Publications.

Experiments in Mathematics, Stage 2, pp. 12–13, Houghton Mifflin. (Available from Thomas Nelson & Sons Ltd.)

Inquiry in Mathematics via the Geo-board, Geo-cards 21/1–4, Walker. (Available from Fitzhenry & Whitehouse Ltd.)

Mathex: Geometry No. 9, pp. 56–60, Encyclopaedia Britannica Publications Ltd.

Workbook, page 85

Using the Exercises

You might find it necessary to discuss the concepts treated on page 269. For example, use other phrases to help children understand the notation for a squared number, $1^2 = 1 \times 1$; $10^2 = 10 \times 10$; $7^2 = 7 \times 7$; and so on. You might also point out that exercises 2, 3, and 4 treat the Pythagorean Theorem just as did the investigation. You might need to help children understand what type of equation is expected. If so, exercise 2 would be appropriate for discussion.

With a group of more capable children, you might have them work on these exercises together without your help, and then help them check their answers.

Assignments (page 269)*

Minimum: 1, 2. Average: 1, 2. Maximum: 1–4.

PAGES 270–271

Objective

Given the radius of a circle, the child will be able to find its area by applying the formula $A = \pi \times r^2$.

Preparation

Materials

circular objects such as cans, plastic lids, discs, and coins; graph paper, $\frac{1}{2}$-cm grid (*Duplicator Masters*, page 63)

To prepare for this lesson, you might review the technique of counting parts of squares when finding area from a grid. However, if children have had no difficulty with this method, you might want to begin immediately with the investigation.

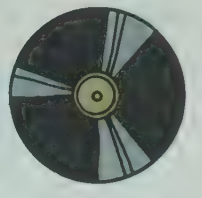
Investigation

If possible, have as many circular objects as you have children working on the investigation. An ample supply of graph paper will also be necessary. You might remind the children to carefully record their findings for each object that they use. Encourage them to try to find easy ways to count the squares.

Is there an easy way to find the area of a circle?

Investigating the Ideas

Choose one or more circular objects. Trace around each object on graph paper.

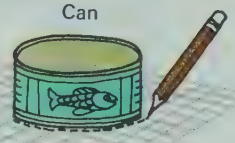


Record



Lid

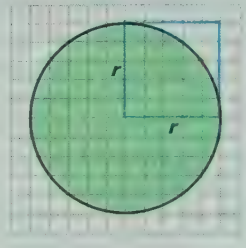
Can



? Can you find about how many squares each circular object covers?

Answers will vary. See Investigation.

Discussing the Ideas



1. Estimate the area of the circle shown at the right by counting squares.
   Estimates will vary. (Actual area: 78.5)
2. The radius (r) of the circle is the side of the blue square. What is the area of this square? 25
3. A Is your estimate for the area of the circle less than $4 \times r^2$ (four times the area of the square)? See Discussion.
   B Is your estimate more than $3 \times r^2$ (three times the area of the square)? See Discussion.
4. As you can see, the area of a circle is slightly more than $3 \times r^2$. More detailed mathematical methods have shown that the area of a circle (to the nearest hundredth) is $3.14 \times r^2$. Using A for **area**, we write: $A = \pi \times r^2$. Can you use this formula to check your estimates of the areas of the circles in the Investigation? See Discussion.



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Discussion

Before beginning the discussion exercises, have children share some of the results of their investigation.

Exercises 1 through 4 are designed to develop the background the children will need in order to understand the formula for finding the area of a circle.

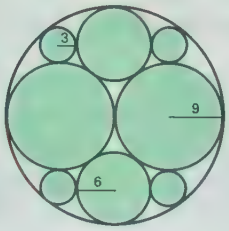
In discussing exercise 2, be sure the children see, first of all, that the area of the boldly outlined square region is $r \times r$, or r^2. They can find the area of the square both by counting the square units within it and by multiplying $r \times r$. For exercise 3, emphasize that the combined area of these four squares is larger than the area of the circle because there are a few extra square units at each corner. Thus, the area of the circle is less than $4 \times r^2$. If they use the given value for r they should be able to see that their estimate of the area is slightly more than $3 \times r^2$. After this background development, the children should find the formula $A = \pi \times r^2$ a reasonable one. Use several examples from the investigation to apply the formula. Be sure that the children understand that the radius is one half the diameter so that they will find the correct value on their graph paper.

Using the Ideas

1. A Find the area of each different-sized circle. (Use $\pi = 3.14$.) 28.26; 113.04; 254.34; 1017.36

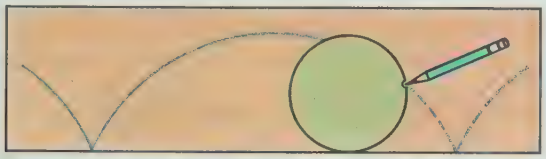
B Find the total area of the colored part. 847.8

C What is the area of the inside part not colored? 169.56

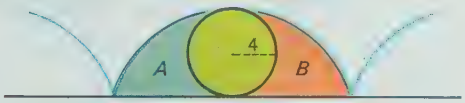


(The radius is 18.)

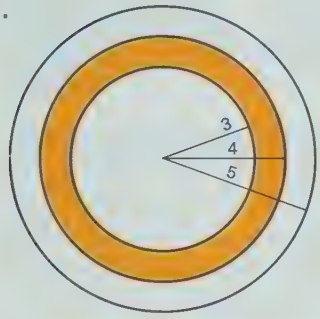
2. When a pencil fastened to the edge of a cardboard circle marks the wall as the circle rolls along, it makes a curve called a **cycloid.**



Mathematicians have proved that the area of region *A* (the area of region *B*, also) is the same as the area of the circle. What is the total area of the three colored regions under the cycloid? 150.72



★ **3.**



Is the area of the outside white ring less than, equal to, or more than the area of the inside white circle?
Equal to

More practice, page A-22, Set 44

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Using the Exercises

Assign the exercises on page 271 as independent work. However, for children who have shown difficulty using the formula, $A = \pi \times r^2$, you might work through these exercises together. For example, in exercise 1, children are not only given an opportunity to apply the formula but they must also realize which of the smaller circles have the same area, add areas of all the colored circles, and subtract this total from the area of the large outer circle. If they follow the exercise part by part, they should be able to do this one step at a time.

Exercise 2 deals with a very interesting curve. You might want to discuss it in connection with a follow-up project, as suggested below.

Starred exercise 3 should challenge the more capable children although the subtractive idea needed to solve it is similar to that used in exercise 1.

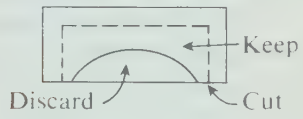
Assignments (page 271)
Minimum: 1. Average: 1, 2.
Maximum: 1–3.

Follow-up/Cycloid Curves

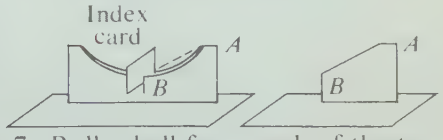
It is often thought that since a straight line is the shortest distance between two points it would also be the fastest path for an object to travel between two points. However a study of the cycloid proves that this is not so.

A project which might be of interest to the class would be the construction of a cycloid track.

1. Cut out a cardboard circle of radius 10 cm. Tape a pencil on its circumference and roll the circle on its side so the pencil leaves a mark on a large piece of rectangular cardboard (at least 70 cm).
2. Cut along this curve and retain the convex section.



3. Trace and cut out an identical section and place the two parts side by side separated by a thin strip of cardboard ($\frac{1}{2}$ cm) placed between the curves about $\frac{3}{4}$ cm from their edges, making a cycloid track.
4. Using a string, find the centre of the cycloid. Mark the centre and place an index card in a slit cut at that point.
5. Measure the straight-line distance from this centre point to the top of one side of the cycloid.
6. Make another track but make this one a straight track which has the length of the distance found in step 5 (distance from *A* to *B* in the diagram).



7. Roll a ball from each of the top points. Which one gets to point *B* first?

Resources for Active Learning

Franklin Series: *Patterns and Puzzles*, "A Circle Problem," pp. 69–70, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Duplicator Masters, page 53
Workbook, page 86

PAGES 272–273

Objective

Given the length, width, and height of a rectangular prism, the child will be able to find its volume.

Preparation

Materials

several small cubes (optional)

To prepare for this lesson, it would be helpful to review the term *rectangular prism*. Ask children if they can name any objects which could be called rectangular prisms. They might mention a box, the teacher's desk, a book, and so on.

Investigation

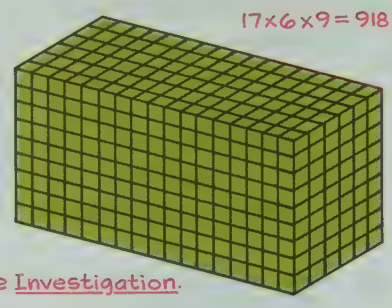
Although this investigation deals with three-dimensional objects, most children should be expected to explore the challenge question simply by studying the illustration in the text. In this way they are more apt to see that they can think in terms of how many cubes in one layer and then find the total number of layers. You might want them to work in pairs for this investigation.



How is the volume of a figure calculated?

Investigating the Ideas

Suppose this rectangular prism was made by gluing many small cubes together.



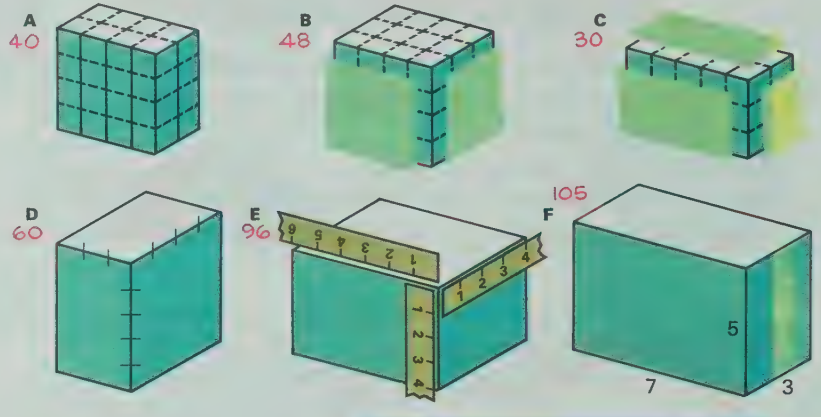
Can you think of an easy way to find how many cubes were used?

See Investigation.

Discussing the Ideas

1. The **volume** of a figure is the number of unit cubes it takes to "make" the figure. What is the volume of the figure above?

2. What is the volume of each of these figures?



3. A On the chalkboard, copy and complete a table like this for each figure in exercise 2.

Figure	ℓ (↔)	w (↗)	h (↕)	V (volume)
A	5	2	4	40
B	4	3	4	48

ℓ = length; w = width; h = height

B Can you give a formula for finding the volume of a rectangular prism? $V = \ell \times w \times h$

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Discussion

Give children an opportunity to discuss the methods they used to find the number of cubes in the investigation. Encourage use of the term *volume* when appropriate. Try to relate discussion exercise 2 to some of the children's suggestions.

Finally, have the children complete the table in exercise 3. Help them write and interpret the formula,

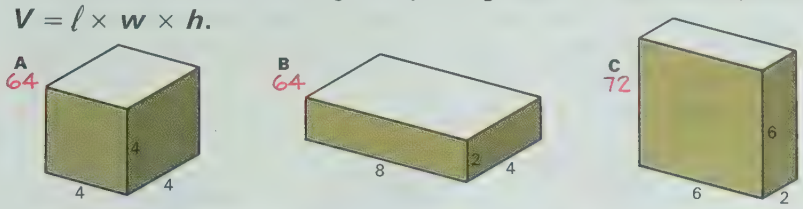
$$V = l \times w \times h,$$

making sure they know the meaning of each letter. Throughout the entire discussion, emphasize that volume, or cubic measure, is basically found by counting cubic units, just as area is found by finding the number of square units in a region.

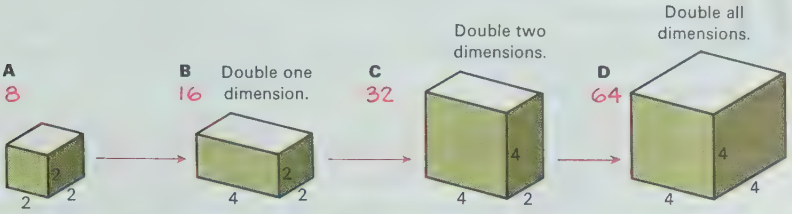
If you are working with an above-average group, you might discuss the formula for volume of a cube, that is, a rectangular prism which has equal length, width, and height. For example, the volume for the cube in exercise 1A on page 273 would be $4 \times 4 \times 4$. You might lead the children to generalize that, since 4×4 is written 4^2, $4 \times 4 \times 4$ may be written as 4^3 and read "four cubed," or "four to the third power."

Using the Ideas

1. Find the volume of each figure by using the volume formula, $V = \ell \times w \times h$.

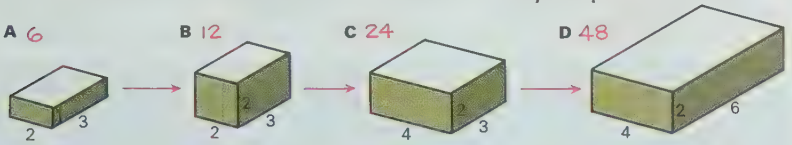


2. Length, width, and height are sometimes called the three **dimensions** of a figure. Find the volume of each figure.

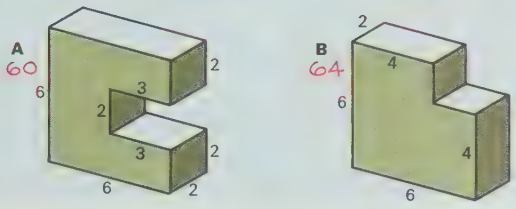


3. Use exercise 2 to help you complete each sentence.
   - A If two dimensions of a box are doubled, the volume is __? 4 times as large.
   - B If each dimension of a box is doubled, the volume is __? 8 times as large.

4. Find the volume of each box. Exercise 3 may help.



★ 5. Find the volume of each figure.



More practice, page A-23, Set 45

Using the Exercises

When you are sure children can apply the formula and find the volume of a cube, assign the exercises on page 273 as independent work. Exercises 2 and 3 are based on the interesting fact that by doubling one dimension the volume is doubled, and if all three dimensions are doubled, the volume of the resulting rectangular prism is eight times the original volume.

Assignments (page 273)

Minimum: 1, 2. Average: 1–4.
Maximum: 1–5.

Follow-up

To extend concepts of volume, construct a prism and a pyramid that are of equal height and have congruent bases. Make the space figures from cardboard and hinge one face with cloth tape, or remove one face so that the children can test the predictions they make regarding the figures' volumes.

Ask the children to predict which space figure has the greater volume; then encourage them to guess how many times greater. After they record their guesses, give the children a container of sand and direct one of them to experiment by filling the pyramid with sand and carefully pouring the sand into the prism. Suggest that various children repeat the experiment several times to validate their findings. The experiment can also be carried out with a cone and a cylinder of equal height and with congruent bases.

Resources for Active Learning

Applied Mathematics Cards, Group 2/5; Group 3/10, Schofield and Sims. (Available from Mafex Associates)

Math Activity Cards, "Volumes," E24, Macmillan.

Mathex: Measurement No. 10, pp. 18–21 (pupil pages 54–59), Encyclopaedia Britannica Publications Ltd.

Measure and Find Out, Book 2, "Measuring Volume," Activity 1/7, Scott Foresman. (Available from Gage Educational Publishing Ltd.)

Nuffield Project: *Problems*—Green Set, No. 27, Wiley.

PAGES 274–275

Objective

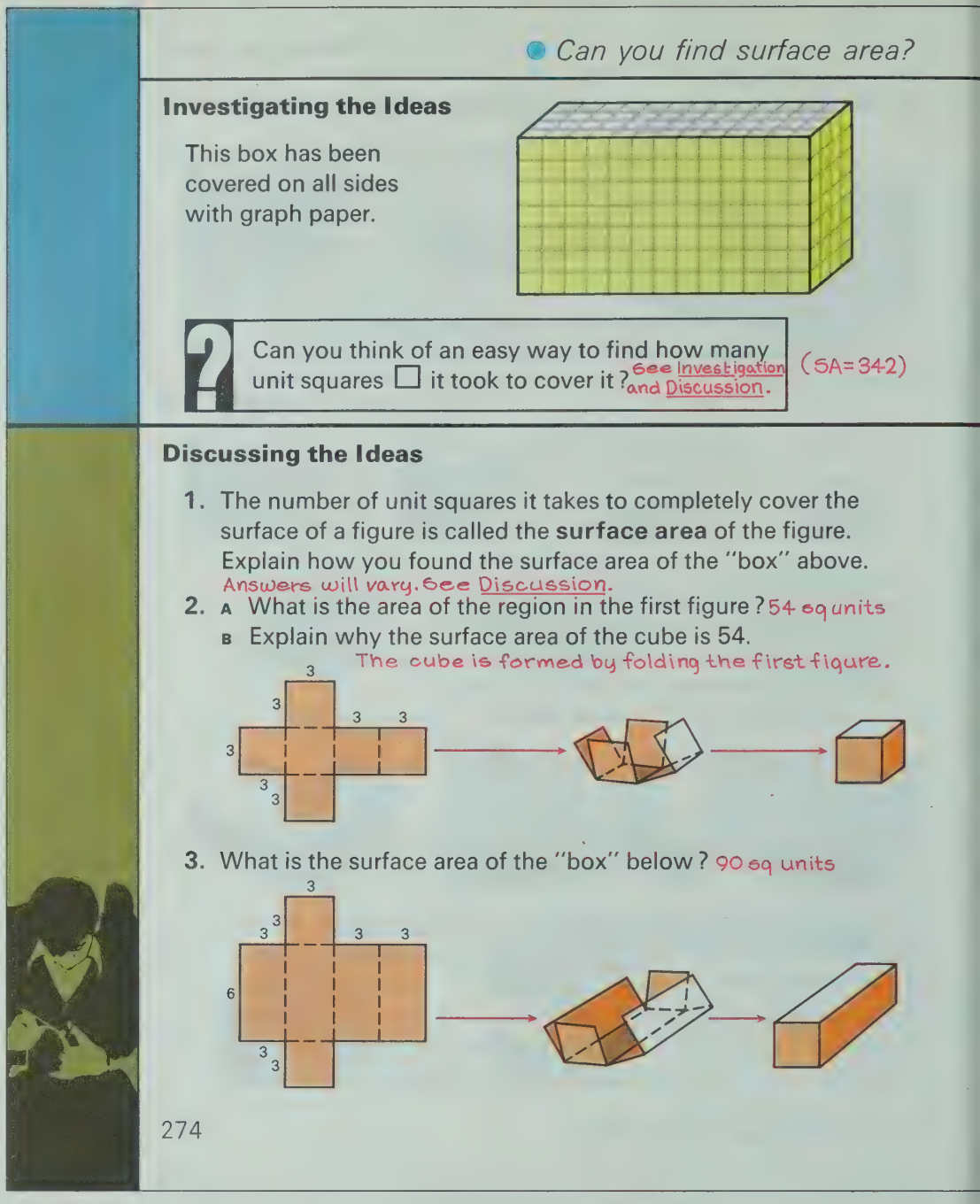
Given the length, width, and height of a rectangular prism, the child will be able to find its surface area.

Preparation

To prepare for this lesson, you might briefly review the fact that square units are used to measure area, and cubic units are used to measure volume. For example, name an object and ask children how they might measure it. Ask questions such as, "What unit would you use to measure carpeting to put on the floor? What unit would you use to measure how many boxes you could store in a large trunk? in any empty closet?" Keep such a review brief; its purpose should be simply to help children recall that area is a measure of surface in square units and volume is a measure of space in cubic units.

Investigation

Although a few children might think that in order to answer the challenge question they must cover a box with graph paper, it is intended that they answer the question simply by studying the illustration. However if any children do have difficulty interpreting the pictorial representation, you might give them a box to cover with graph paper. Most will realize that the pairs of opposite sides are the same and find the area of each visible side. Allow children to share ideas as they work, but encourage independent thinking by each child.



Can you find surface area?

Investigating the Ideas

This box has been covered on all sides with graph paper.

Can you think of an easy way to find how many unit squares ☐ it took to cover it? See Investigation and Discussion. (SA=342)

Discussing the Ideas

1. The number of unit squares it takes to completely cover the surface of a figure is called the **surface area** of the figure. Explain how you found the surface area of the "box" above. Answers will vary. See Discussion.
2. A What is the area of the region in the first figure? 54 sq units
   B Explain why the surface area of the cube is 54. The cube is formed by folding the first figure.

3. What is the surface area of the "box" below? 90 sq units

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Discussion

Use the results of the investigation, and discussion exercise 1 to explain the meaning of the term *surface area*. Help children realize that the surface area of a rectangular prism is the total of the areas of the faces of the prism. Exercises 2 and 3 are designed to lead the children step by step from finding the area of rectangular regions to finding surface area of space figures. If children seem to have difficulty with this concept, you might give them previously duplicated patterns for space figures and have them find the surface areas for each, both before and after the patterns have been cut out and made into space figures.

Using the Ideas

1. The two figures below have the same volume. Tell whether the first or the second figure has the greater surface area. Then tell how much greater. Use the units indicated.

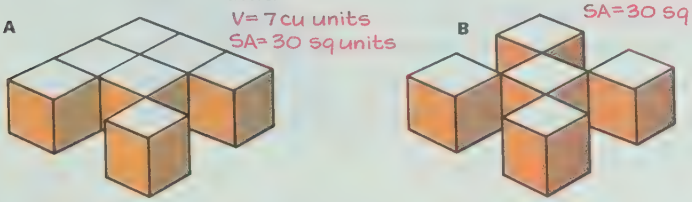
Second figure has 10 sq units more surface area.



2. Give the volume and surface area of each figure. Use the indicated unit.

A V= 7 cu units SA= 30 sq units

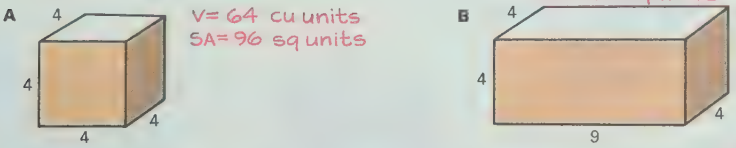
B V= 5 cu units SA= 30 sq units



3. Give the volume and surface area of each figure. Use the lengths given.

A V= 64 cu units SA= 96 sq units

B V= 144 cu units SA= 176 sq units



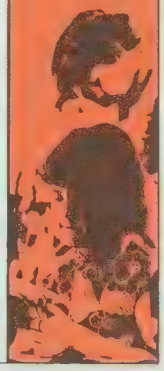
★ 4. Give the surface areas of the figures in exercise 5, page 273. A 120 B 112

think

FOR BASEBALL FANS ONLY:

It was near the end of the season. Jones and Smith were tied for the National League batting title. In the remaining games, Jones went 7 for 8 and Smith went 9 for 12. Who finished with the higher batting average? Smith. See Solution, T.E. page 175.





Using the Exercises

Exercise 1 on page 275 gives children an opportunity to compare the surface areas of figures that have the same volume but different shapes. Both exercise 1 and exercise 2 are relatively simple because each unit is marked so that the children can count the units if necessary. If children have understood the concept of surface area, most will do exercise 3 successfully.

Note, however, that exercise 4 is starred. Some of the children will see that each of the figures in exercise 5, page 273, may be thought of as two rectangular prisms together. More capable children may stimulate the rest of the class by explaining how to find the volume of these figures. You might give blocks to those children who wish to use them to work out figures like those in exercises 3 and 4, but urge most to work without actual models for the figures.

Assignments (page 275)

Minimum: 1, 3. Average: 1–3. Maximum: 1–4.

Follow-up

As a follow-up to this lesson, you might encourage children to guess the volume and surface area of rectangular prisms in the classroom and then try to check their guesses by making actual measurements. For example, instead of using graph paper, children might see how much newspaper is needed to cover the teacher's desk, a book shelf or cabinet area, and then figure out from the dimensions of the newspaper the surface area of the objects. If some of the cubes and polygons which children constructed for Chapter 4 are still available, they might try to find the surface area of these figures.

Solution, Think, page 275

Smith won. When their averages were tied, assume that both Smith and Jones had batted the same number of times. In Jones' remaining 8 times at bat, he got 7 hits. Smith got 9 hits out of his last 12 times at bat. Think of Smith's 9 for 12 in the following way. For the first 8 times at bat, he got 7 hits and was still tied with Jones, who also went 7 for 8. Then, for his next 4 times at bat, Smith got 2 more hits.

It is highly probable that their batting averages were less than 0.500. Therefore, the 2 for 4 will improve Smith's average and thus his batting average will be higher than Jones'.

Resources for Active Learning

Franklin Series: *Patterns and Puzzles*, "The Spider and the Fly," pp. 33–35, Lyons and Carnahan. [A puzzle] (Available from McGraw-Hill Ryerson)

Mathematics in Modules, M13, Addison-Wesley

Measure and Find Out, Book 1, "Length and Width and Height," Activity 4, Scott Foresman. (Available from Gage Educational Publishing Ltd.)

Duplicator Masters, page 54
Workbook, page 87

PAGES 276–277

Objectives

Given problems based on various properties of the cube, the child will be able to solve the problems by applying his understanding of the cube.

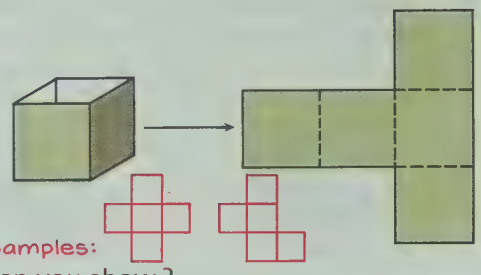
Given problems involving practical geometry, the child will be able to solve the problems by applying his understanding of geometric shapes.

Preparation

To prepare for this lesson, you might briefly review the fact that a rectangular prism which has equal length, width, and height is a cube.

Solving Cube *Problems*

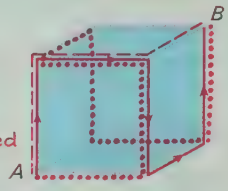
1. A cubical box without a lid was cut along certain edges and opened to make the pattern shown.



A Can you show another pattern that could have been made? Samples:

B How many different patterns can you show?

2. The picture shows a "5-edge trip" from *A* to *B*. Can you draw a picture of a cube to show



A a "3-edge trip" from *A* to *B*? (See dashed lines.)

B a "7-edge trip" from *A* to *B*? (See dotted lines.)

3. How many cubes are hidden (have no faces showing even if you could pick the stack up and look at all sides)

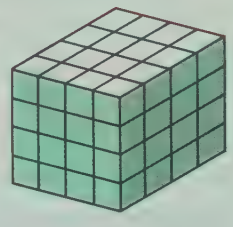
A in a 2 × 2 × 2 stack? 0

B in a 3 × 3 × 3 stack? 1

C in a 4 × 4 × 4 stack? 8



3 × 3 × 3 stack



4 × 4 × 4 stack

★ **4.** Here are 4 views of the same cube. Which designs are opposite each other on the cube? See Using the Exercises.



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Discussion

The problems on page 276 should be completed by the children before you discuss them. Encourage the children to check their answers by using actual materials to construct the box in exercise 1 and the stacks in exercise 3. However, in exercise 4, suggest that they try to figure out the answer first by examination of the illustrations. Then they might try to construct and label the cube shown to see if their conclusions about opposite designs are correct. Be sure the cube they construct follows the matchings given in the text, which are as illustrated in the next column.

■ is opposite ◆

♥ is opposite ✚

✱ is opposite ●

Assignments (page 276)*

Minimum: 1, 2. Average: 1–3. Maximum: 1–4.

Solving Practical Geometry Problems

1. Mrs. Alvarez saw a carpet that is 3.5 metres wide and 6.5 metres long on sale for $204.75.
   - A How many square metres? 22.75 m^2
   - B Does the carpet sell for more than ten dollars per square metre? No

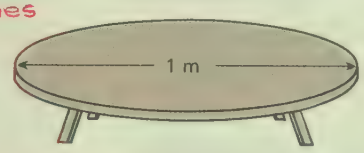


2. Bill measured his school classroom. It is 7.5 metres wide, 9 metres long, and 3.25 metres high.
   - A How many cubic metres of volume are in the room? 219.38 cm^3
   - B In Bill's class there are 26 children, one teacher, and a teacher's aide. How many cubic metres of room (to the nearest tenth) are there for each person? 7.8 cm^3

3. How much wire is needed to fence a rectangular field 80 metres wide and 120 metres long? 400 m



4. A bicycle wheel has a 66-cm diameter.
   - A What is the circumference of the wheel? 207.24 cm
   - B About how many metres will the bicycle travel in one turn of the wheel? About 2.07 m
   - C About how many times must the wheel turn in one kilometre? About 483 times



5. A circular table has a top that is 1 metre in diameter. What is the area of the table top? 785 000 cm^2

Resources for Active Learning

[The listed resources present cube puzzles.]

Franklin Series: *Patterns and Puzzles*, pp. 19–22, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Math Activity Cards, D13, Macmillan.

Nuffield Project: *Problems*—Green Set, No. 33, Wiley.

Using the Exercises

The problems on page 277 give the children an opportunity to apply their understanding of familiar geometric shapes with the ideas of area, volume, and circumference. Many would benefit from small-group discussions. If some have difficulty with a problem, suggest that they draw or make a model to represent the actual objects in the problem. For example, models of graph paper would be helpful for problems 1 and 3; and cubes for problem 2. Problems 4 and 5 might challenge some children enough that they would benefit from making actual circles of cardboard using the dimensions suggested. Models such as these, however, should be used only by children who cannot apply the formulas developed in previous lessons.

Assignments (page 277)*

Minimum: 1–5, oral. Average: 1–5. Maximum: 1–5.

PAGES 278–279

Objective

The child will demonstrate his ability to work with the concepts presented in this chapter.

Preparation

Materials

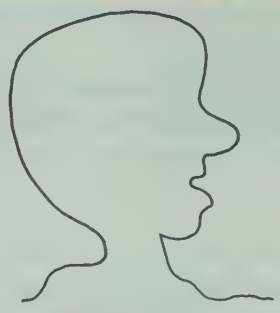
string; ruler

Review any topics from the chapter which seemed troublesome for the children. In particular, you might want to review the methods of finding the area of rectangles, parallelograms, and triangles. However, children should not be expected to remember the formulas. The emphasis should be on understanding the concepts.

Reviewing the Ideas

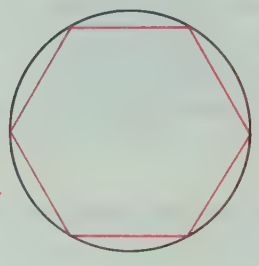
1. Guess the length of this curve in centimetres. Then measure it as accurately as you can.

Answers will vary.



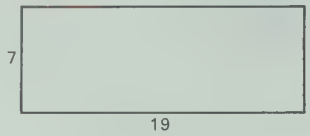
2. A Which is larger, the circumference of the circle or the perimeter of the polygon? (Answer before measuring.)

B Find the circumference of the circle and the perimeter of the polygon in centimetres. 12.56, 12

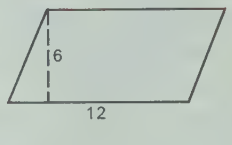


3. Find the area of these figures.

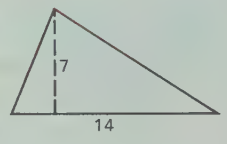
A 133



B 72

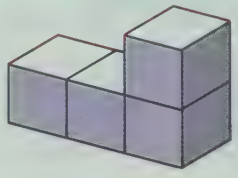


C 49

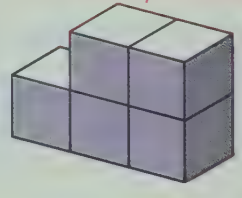


4. Find the volume and surface area of each figure. Use the units indicated by the figures.

A V = 4 cu units, SA = 18 sq units



B V = 5 cu units, SA = 20 sq units



278

Discussion

These pages would serve best as a review lesson in which you work through several exercises together with the children. However, they may also be used as an evaluative instrument. If you choose the latter method, either direct the children to refer to pages in the book for help in solving the problems or write the more difficult formulas and the Pythagorean Theorem on the chalkboard.

Area of parallelogram
= base × height

Area of triangle
= $\frac{1}{2}$ × base × height

Area of circle
= π × radius × radius

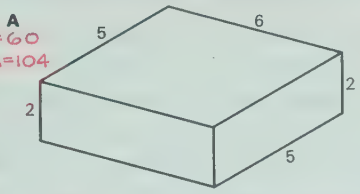
Pythagorean Theorem: In a right triangle, the sum of the areas of the squares on the sides is equal to the area of the square on the hypotenuse.

Most exercises of this review stress the understanding of basic geometric concepts and give children an opportunity to show their ability to think about the ideas which have been treated in this chapter. Whether you use these exercises as an evaluative instrument or as a review lesson, spend sufficient time checking each exer-

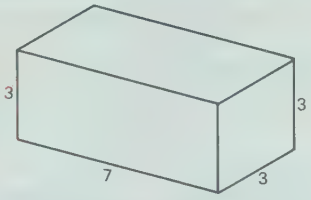


5. Find the volume and surface area of each box.

A V=60 SA=104

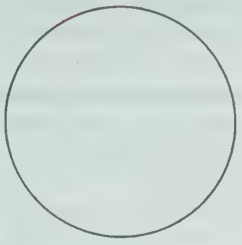


B V=63 SA=102

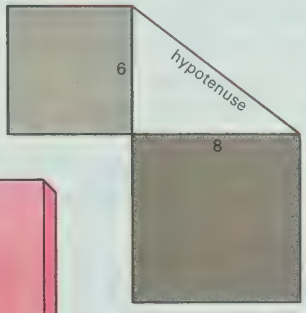


6. **A** Count squares to **estimate** the area of the circle. About 50 sq units

B The diameter of the circle in the figure is 8. Use 3.14 for π to find the area of the circle. 50.24 sq units



7. If a square were drawn on the hypotenuse of this right triangle, what would its area be? 100



think

The figures below show two different paths that start at the top and spell LEVEL. Show 7 other paths that start at the top and spell LEVEL.

L Start
LEL
LEVEL
LEL
L

L Start
LEL
LEVEL
LEL
L

See Solution, T.E. page 279.

8. **A** A cube has __? 8 vertices, 6 ? __ faces, and __? 12 edges.

B If the area of a face of a cube is 4 square units, what is the surface area of the cube? 24 sq units

cise and allow time for discussion of any questions the children might have.

Solution, Think, page 279

The seven other possible paths are as shown below.



Workbook, page 88

PAGES 280–281

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

Review any of the topics covered so far in the text which the children have found particularly difficult. You might find it especially helpful to review computation with fractional numbers in preparation for the work on ratio in Chapter 13. Exercises 2, 3, and 4 are designed to aid in reviewing skills that will again be employed in that chapter, so you should place special emphasis on them.

You might prefer to use the preparation time more explicitly for page 281. The paragraphs at the top of the page might serve this function.

Keeping in Touch with

Computing
Measurement
Inequalities
Fractional numbers
Number theory

1. A List the multiples of 20 (to 100). 20, 40, 60, 80, 100
   B List the multiples of 7 (to 35). 7, 14, 21, 28, 35
   C List five fractions equivalent to $\frac{7}{20}$. $\frac{14}{40}, \frac{21}{60}, \frac{28}{80}, \frac{35}{100}, \frac{42}{120}$

2. Solve these equations.

   A $\frac{7}{20} = \frac{n}{40}$ 14 C $\frac{1}{4} = \frac{n}{16}$ 4 E $\frac{2}{3} = \frac{n}{12}$ 8 G $\frac{1}{6} = \frac{5}{n}$ 30 I $\frac{3}{4} = \frac{n}{20}$ 15

   B $\frac{7}{20} = \frac{n}{100}$ 35 D $\frac{3}{5} = \frac{9}{n}$ 15 F $\frac{4}{9} = \frac{n}{18}$ 8 H $\frac{3}{8} = \frac{n}{24}$ 9 J $\frac{5}{4} = \frac{n}{100}$ 125

3. Give a set of five fractions equivalent to the given fraction.

   A $\frac{1}{2}$ $\frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}$ B $\frac{1}{3}$ $\frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \frac{5}{15}, \frac{6}{18}$ C $\frac{1}{4}$ $\frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \frac{5}{20}, \frac{6}{24}$ D $\frac{2}{3}$ $\frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15}, \frac{12}{18}$ E $\frac{3}{4}$ $\frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \frac{15}{20}, \frac{18}{24}$ F $\frac{1}{5}$ $\frac{2}{10}, \frac{3}{15}, \frac{4}{20}, \frac{5}{25}, \frac{6}{30}$ G $\frac{3}{5}$ $\frac{6}{10}, \frac{9}{15}, \frac{12}{20}, \frac{15}{25}, \frac{18}{30}$

4. The rings are shown to remind you of a check for equivalent fractions. Give the correct sign (= or ≠) for each ◍.

   A $\frac{3}{4} \neq \frac{7}{8}$ B $\frac{1}{3} \neq \frac{2}{7}$ C $\frac{3}{4} = \frac{75}{100}$ D $\frac{5}{9} \neq \frac{5}{8}$

5. Find the totals. Change each answer so that you have the greatest number of the larger unit.

   A 8 h 42 min + 7 h 21 min = 16 hr 3 min
   B 17 days 8 h + 9 days 16 h = 27 days
   C 4 mo 2 days + 5 mo 29 days = 10 mo 18 days
   D 4 wk 5 days + 2 wk 6 days = 7 wk 4 days
   E 5 min 19 sec + 3 min 18 sec + 2 min 29 sec = 11 min 6 sec
   F 4 h 35 min + 1 h 56 min + 8 h 49 min = 15 hr 20 min

think

Four sections of chain are shown. It costs 15¢ to cut a link open. It costs 35¢ to have a link welded together.



Show how you could form one long chain for $1.50. There is a cheaper way. What would it cost? $1.00

See Solutions, T.E. page 280.



You are invited to explore

ACTIVITY CARD 13
Page 361

280

Discussion

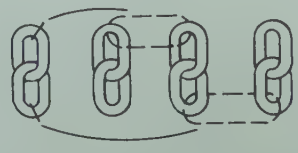
Page 280 is a review page, and may be assigned as independent work. When children finish, check the exercises carefully and help them clarify any misunderstandings.

Solutions, Think page 280

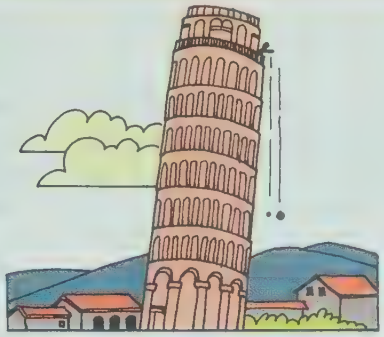
A chain costing $1.50 would require making 3 cuts and 3 welds, as shown below.



The chain can be made for $1.00, also, as shown below.



FALLING BODIES



The famous Leaning Tower of Pisa is located in Pisa, Italy. It has stood for hundreds of years, even though it leans so far to one side it looks as if it will fall. The tower is 54.56 m tall, and it is about 544 centimetres out of line. Galileo, a famous 17th century scientist, thought that two falling bodies would fall at the same speed, even if one were large and the other small. Galileo is said to have dropped objects from the Leaning Tower of Pisa to prove his theory. This theory, although unpopular in its day, is correct (assuming no air resistance).

1. The approximate distance (in metres) that an object falls when dropped to Earth can be found by the formula $d = 4.9 \times t^2$, where t is the time (in seconds). If an object is dropped, how far will it fall in
   A 1 second? 4.9 B 2 seconds? 19.6 C 4 seconds? 78.4 D 7 seconds? 240.1

2. If the objects Galileo dropped took 3 seconds to hit the ground, about how high was Galileo when he dropped them? 44.1

3. As objects fall, their speed increases. You can give the speed (in metres per second) of a falling object if you multiply 9.8 by the number of seconds the object has been falling.
   A An object dropped from a high place is going how fast after 3 seconds? 29.4 km/s
   ★ B One kilometre per hour is about 0.28 metres per second. What is the speed in kilometres per hour of the object in part A? 105

★ 4. If an object takes $5\frac{1}{2}$ seconds to fall to the ground, how far did it fall and how fast was it going when it hit? 148.2; 53.9

Using the Exercises

You might want to work through one or two of the problems on page 281 with the children. However, these problems might be used as a challenge for the children to try to apply a new formula on their own. In either case, help those who need it and allow capable children time to do some independent problem solving.

When children finish, carefully check their answers and give them time to explain their solutions. Starred exercise 4 uses a direct application of the formula, $d = 4.9 \times t^2$, but you might point out that here t^2 means $5\frac{1}{2} \times 5\frac{1}{2}$ or $\frac{11}{2} \times \frac{11}{2} = \frac{121}{4}$; so $d = 4.9 \times \frac{121}{4} = 148.2$ km. Remember that starred exercises are intended to be challenges for more interested or capable children.

Follow-up

After the word-problem set on page 281, the children might be interested in doing some research, not only on falling objects but also on the Leaning Tower of Pisa. In recent years, there has been some concern that the tower might eventually tip far enough to fall over. It has been calculated that the tower leans a little more each year, and attempts are being made to halt the leaning.

Galileo's theories on falling bodies also play a significant part in that phase of manned space flights in which the capsule must fall through the atmosphere. The increasing resistance to the acceleration of the space craft caused by the pull of gravity causes both extreme heating and a shock wave around the nose cone. These, in turn, create problems in shielding the men and the capsule from heat. Children who are interested can find references to these and other applications of arithmetic in NASA publications. (See Books to Explore, pages A25–A26, and More Books to Explore in the introductory section of this manual.)

Pages 282–303

General Objectives

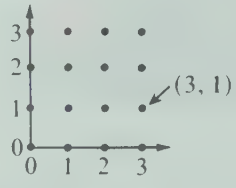
To review and extend graphing of number pairs
To relate graphed points to geometric figures
To develop the concept of reflection by means of co-ordinate geometry and relate reflection to line symmetry
To develop understanding of rotations, particularly 90°, 180°, and 270° rotations, and to introduce the concept of rotational symmetry
To develop understanding of, and ability to perform, translations
To explore tessellations of the plane
To introduce the concept of magnifications of a figure
To develop understanding of similar figures

The first lesson in this chapter relates co-ordinates with points graphed only in the positive quadrant of the plane; however, fractional numbers are considered, and their placement is found by estimation. Co-ordinates are then related to geometric figures and images of figures or parts of a figure which are symmetric with respect to a line. The use of a geoboard is recommended for a study of rotations and rotational symmetry. Translations are introduced by an enjoyable game with rules children can adapt or modify. Various figures are explored to see if they can be used to tessellate a plane. A study of the magnification of a figure leads into the development of similar figures. The chapter concludes with chapter review and cumulative review pages.

Mathematics

A major part of this chapter is devoted to an intuitive study of a kind of geometry that is known as *transformational geometry*. Certain transformations, such as reflections, rotations, and translations are called *rigid motions*, or *isometries* because such motions or transformations do not change the shape or size of geometric figures. Other transformations, such as similarity transformations, may change the size of a figure but not its shape. The approach to these topics in this chapter is by means of a co-ordinate system in the plane.

To set up a co-ordinate system, choose two perpendicular number lines that intersect at zero, and then associate points on the plane with number pairs. In the beginning, use number rays with the whole-number points labelled. Having chosen two rays, you can locate a point for any pair of whole numbers, provided you know which number in the pair refers to which ray. It is conventional to agree that the first number refers to the horizontal ray and the second number refers to the vertical ray.



When the integers are introduced in Chapter 15, the number rays become number lines labelled with positive and negative integers which intersect at the co-ordinates (0,0).

Teaching the Chapter

Materials

Cards, about 2-by-2 or 2-by-1½ centimetres
Compass
Cubes whose faces can be numbered or dice (1 pair for every two children)
Dot paper
Geoboard
Graph paper ¼-cm grid (large quantity) and 1-cm grid (1 sheet per child)
Newsprint or other large sheets of paper
Overhead projector
Protractor
Rubber bands
Ruler (centimetre)
Tracing paper
Transparent grid (if available)

Vocabulary

axis
centre of rotation
co-ordinate axis
co-ordinate plane
co-ordinates
graph
line of symmetry
magnification
reflection
reflection image
rotation
rotational symmetry
similar figures
symmetric figure
tessellation
translation

The most essential material for this chapter is graph paper. Have an abundant supply of commercial graph paper with ¼-cm grid or prepare your own from a duplicating master. A geoboard for each child is highly recommended; however, dot paper may be substituted if absolutely necessary. Dot paper should be available in any case, for the purpose of recording investigations on the geoboard. (See *Duplicator Masters* to accompany this book, pages 62–67.)

Lesson Schedule

Time allowance for this chapter may vary according to the interest of the children and the extension of activities you suggest, but plan to spend at least two and a half weeks on it.

Evaluation of Progress

Since one of the main purposes of this chapter is to provide children with stimulating and enjoyable geometric activities, you should not be overly concerned with the chil-

dren's proficiency in graphing or with their ability to use the new vocabulary easily. Instead concentrate on trying to develop correct conceptions of each topic treated. If you display an attitude of interest and enthusiasm, the children should enjoy the many activities of this chapter, even though some of them require a considerable amount of concentration.

Resources for Active Learning

GENERAL ACTIVITIES

[A unit to augment work in geometry, symmetry, and optics]

Mirror Cards, McGraw-Hill Ryerson

[Co-ordinates games]

Developmental Math Cards, K$^{1}_{2}$13, Addison-Wesley

Experiments in Mathematics, Stage 1, pp. 42–43, Houghton Mifflin (Available from Thomas Nelson & Sons Ltd.)

Inquiry in Mathematics via the Geo-board, Geo-cards 24/1–7, Walker (Available from Fitzhenry & Whiteside Ltd.)

Madison Project: *Discovery in Mathematics* (Teacher's Text), pp. 55–66, Addison-Wesley

Mathex: Graphing and Probability No. 6, pp. 3–7, Encyclopaedia Britannica Publications Ltd.

Notes on Mathematics in Primary Schools, pp. 229–241, Cambridge University Press. (Available from Macmillan).

[Graphing activities]

Applied Mathematics Cards, Group 2/27–30; Group 3/26–29, Schofield and Sims (Available from Mafex Associates)

Developmental Math Cards, K$^{2}_{2}$10, K$^{1}_{2}$12, L$^{2}_{2}$2, L$^{2}_{2}$7, L$^{1}_{2}$21, Addison-Wesley

Experiments in Mathematics, Stage 3, pp. 16–17, Houghton Mifflin (Available from Thomas Nelson & Sons Ltd.)

Math Activity Cards, D4–6, 9, Macmillan

Mathex: Graphing and Probability No. 6, pp. 1–3, Encyclopaedia Britannica Publications Ltd.

Measure and Find Out, Book 2, Activity 1/9, Scott Foresman (Available from Gage Educational Publishing Ltd.)

Independent Exploration: Activity Cards, Concept Co.

Madison Project: *Independent Exploration Material*, Math Media

MANIPULATIVE DEVICES

Pantograph (Hammett; Lano)

Pattern Blocks (Selective Educational Equipment; Mc-Graw-Hill Ryerson)

COMMERCIAL GAMES

Battleship (buy locally)

Go (Gomoku) (Math Media; World Wide Games)

Hindu Pyramid Puzzle (Tower of Hanoi) (Creative Publications; World Wide Games)

Independent Exploration: Games and Puzzle Kit (Concept Co.)

PAGES 282–283

Objective

Given a point on the first quadrant of the co-ordinate plane, the child will be able to give the corresponding number pair for the point.

Prepartion

Materials

paper suitable for use in making cards approximately 2-by-2 or 2-by- $1\frac{1}{2}$ centimetres; graph paper or grids *(Duplicator Masters*, page 62 or 66)

One of the main purposes of the investigation is to review with the children the correspondence between a point on a graph and a number pair. You may want to review with some children how to name a point with a number pair, before they start the investigation. Be sure they remember that the first number listed in the pair is the number marked by the horizontal axis.

Investigation

As children study the directions and make their cards, move around the room to be sure they end up with a complete set of number pairs. The children need only one set of cards for every two players since this game should be played in pairs. Provide graph paper or duplicated grids for children to use for the game. Encourage the children to try to figure out the game rules from the text. However, you might point out that tie games are possible and players should take turns being first at the start of a game.

12 Geometry and Graphing

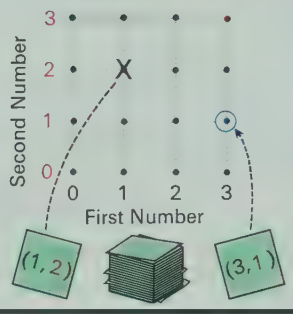
How do you graph number pairs?

Investigating the Ideas

Can you play the Tic-Tac-Toe game described below with one of your classmates?

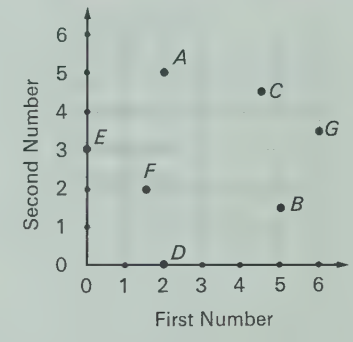
See Investigation.

On small cards, write the 16 pairs of whole numbers for the points shown on the grid. Shuffle the cards and turn them face down. Each player in turn takes a card from the stack and marks an ○ or an X on the grid at the point for his number pair. The first player to get 4 of his marks in a row, column, or diagonal wins the game.



Discussing the Ideas

1. The two numbers which give the location of a point on a grid like this are called the **co-ordinates** of the point. Each of the two number lines is a **co-ordinate axis.** What number pair designates the point where the co-ordinate axes intersect? (0,0)



2. To describe the location of some points, you may need to use fractional numbers. For example, in the figure for question 1, the co-ordinates of *B* are $(5, 1\frac{1}{2})$.
   - A Which points on the grid have whole-numbered co-ordinates? A, D, E
   - B Give the co-ordinates of those lettered points which are described by fractional numbers.

     B $(5, 1\frac{1}{2})$ C $(4\frac{1}{2}, 4\frac{1}{2})$ F $(1\frac{1}{2}, 2)$ G $(6, 3\frac{1}{2})$

282

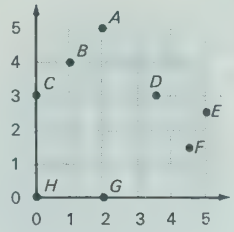
Discussion

Remind children how the co-ordinate plane they are using is formed by the intersection of a horizontal and vertical number line or axis. Use discussion exercise 1 to explain the terms co-ordinates and co-ordinate axis.

Besides reviewing whole-number pairs as co-ordinates, exercise 2 introduces the use of fractional numbers. Point out that graphing fractional-number co-ordinates is much the same as graphing whole-number co-ordinates. However, it is often necessary to estimate locations when marking points for fractional numbers, or to estimate the fractional number when given the point.

Using the Ideas

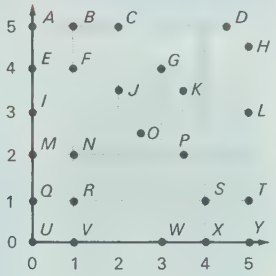
1. Give the co-ordinates for each point.



A (2,5)
B (1,4)
C (0,3)
D ($3\frac{1}{2}$,3)
E (5,$2\frac{1}{2}$)
F ($4\frac{1}{2}$,$1\frac{1}{2}$)
G (2,0)
H (0,0)

2. From the graph below select the letter of the point for each pair of co-ordinates.

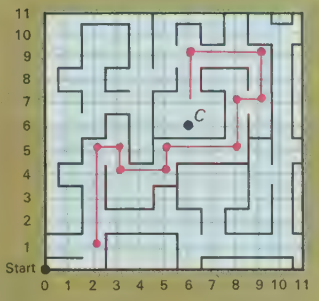
($2,3\frac{1}{2}$) J	(3,4) G	(1,2) N
(4,0) X	(0,5) A	($3\frac{1}{2},3\frac{1}{2}$) K
($4\frac{1}{2}$,5) D	(2,5) C	(5,0) Y
(5,$4\frac{1}{2}$) H	(0,1) Q	(1,0) V



3. A All the points directly above the point (3,0) have co-ordinates with what first number?

B All the points directly to the right of the point (0,4) have co-ordinates with what second number?

think



Give the shortest path along blue lines to the center of the maze by writing, in order, the coordinates of the points where you turn.

(2,1),(2,5),(3,5),(3,4),(5,4),(5,5),(8,5),(8,7),(9,7),(9,9),(6,9)

4. What figure is formed when these points are graphed and connected in order? Pentagon

(2,2) → (4,7) → (6,2) → (1,5) → (7,5) → (2,2)

5. On a 10-by-10 grid, mark at least 10 points in which the second co-ordinate is twice the first. Examples: (1,2), ($2\frac{1}{2}$,5) What do you notice about all of these points?

They are on a straight line.

6. Invent a picture and give the co-ordinates, in order, to a classmate. Ask your classmate to draw the picture.

Answers will vary.



Using the Exercises

Assign the exercises on page 283 as independent work. Since much of the other work in this chapter requires graphing skills, children should be able to graph number pairs as well as give a number pair for a particular marked point. Review any aspect of this skill which children find difficult.

Assignments (page 283)

Minimum: 1, 2. Average: 1–4.
Maximum: 1–6.

Follow-up

Encourage children to create adaptations of the game described in the investigation. They might use a graph on which each axis is numbered to 4. In this case, they would need 25 cards. Or, they might change the scoring so that the winner is the first player who can connect four of his points in the form of a square.

Resources for Active Learning

Franklin Series: *Making and Using Graphs and Nomographs*, pp. 49–70, Lyons and Carnahan. [Co-ordinate graphing] (Available from McGraw-Hill Ryerson)

Madison Project: *Explorations in Mathematics* (Teacher's Text), "The Cartesian Product of Two Sets," pp. 30–45, Addison-Wesley.

Nuffield Project: *Graphs Leading to Algebra* 2, "Introduction to Co-ordinates," pp. 2–11, Wiley.

Workbook, page 89

PAGES 284–285

Objective

Given geometric figures in a co-ordinate plane, the child will be able to locate specific points of the figures and give their co-ordinates.

Preparation

Materials

graph paper or 10 × 10 co-ordinate grids (*Duplicator Masters*, page 62 or 66); overhead projector and transparent grid (if available)

If possible, use a grid on the overhead projector (or draw one on the chalkboard) and review with the children how to locate number pairs on the co-ordinate plane. For example, show a pair of labelled axes and place a point on the graph. Ask children to explain what number pair corresponds to the point. Briefly work through other, similar examples.

Investigation

Have sufficient graph paper available so that children can draw a pair of co-ordinate axes for each point if they consider it necessary. Many should be able to find the required points by using the same co-ordinate grid. Have children look at the points mentioned in each case they are asked to think about. Help them see that they should number both their axes from 0 to 8. Note that the conditions for some of these "points" may be met by more than one point. For example, any point which has 5 as the first number in its co-ordinate pair, except (5, 3), satisfies the condition for point *C*: (0, 0), ($1\frac{1}{2}$, $1\frac{1}{2}$), or (3, 3) satisfies the condition for point *E*, and (8, 6) or (4, 2) may be called point *F*. This fact will be discussed in the discussion exercises, so simply encourage children to record the co-ordinates of all the pairs which they think satisfy the requirements.

Let's look at geometric figures on a graph.

Investigating the Ideas

Think about these points:

Point *A* is the midpoint of the segment from (2,2) to (6,6). (4,4)

Point *B* is a point on the line through (2,4) and (4,8). May vary

Point *C* is a point that goes with (2,3) and (8,3) to form the corners of an isosceles triangle. May vary

Point *D* goes with (2,3), (2,8), and (7,8) to form the corners of a square. (7,3)

Point *E* is the center of a circle that goes through (3,0) and (0,3). May vary

Point *F* is the vertex of a right angle whose sides go through (8,2) and (4,6). May vary

?	How many of the points described above can you find? Answers may vary. See Investigation.	Show your findings on graph paper.



Discussing the Ideas

1. For which of the descriptions above is more than one choice for the point possible? Points B, C, E, and F
2. Choose one of the situations you mentioned in exercise 1 and give some other possibilities for the point. See Investigation.
3. Can you make up a description of a point of your choice and see if your classmates can give its co-ordinates? Answers will vary.

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Discussion

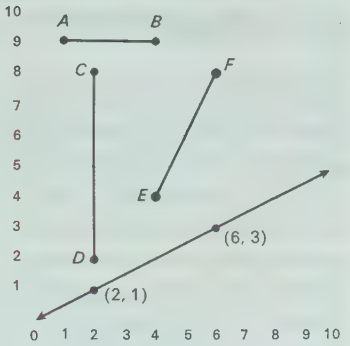
Use the discussion exercises to have children report on the points they found for the investigation. You might have children respond orally to exercise 3. Or you might give them some time to work out a description of a point they choose, as a small-group activity.

Notice how this lesson develops the skills of associating co-ordinates and points by relating graphing to geometric figures. It is important that the children be able to use co-ordinates to locate points on a grid and be able to give the co-ordinates of points related to the graphed geometric figures.

Using the Ideas

1. A Give the co-ordinates of the midpoints of each segment.
$\overline{AB}$ $(2\frac{1}{2}, 9)$
$\overline{CD}$ $(2, 5)$
$\overline{EF}$ $(5, 6)$

B Give the co-ordinates of two more points on the line. Answers may vary.
Samples: $\overline{AB}$ (1,9), (4,9)
$\overline{CD}$ (2,2), (2,8)
$\overline{EF}$ (4,4), (6,8)



2. What geometric figure is formed when these points are graphed and connected in order? Hexagon
(3,4) → (3,7) → (6,9) → (9,7) → (9,4) → (6,2) → (3,4)

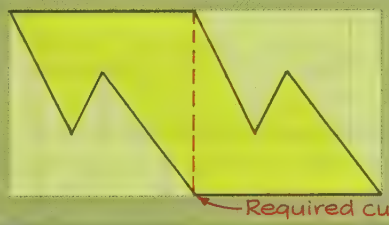
3. Give the co-ordinates of the vertices of: Answers will vary. Sample answers:
A a rectangle with area 8 (0,0), (2,0), (0,4), (2,4)
B a right triangle with area 10 (0,0), (0,5), (4,0)

4. Give the co-ordinates of the fourth vertex of a rectangle with vertices at $(2\frac{1}{2}, 3\frac{1}{2})$, $(2\frac{1}{2}, 7\frac{1}{2})$, and $(8\frac{1}{2}, 3\frac{1}{2})$. $(8\frac{1}{2}, 7\frac{1}{2})$

★ **5.** (1,3), (6,3), and (3,6) are the co-ordinates of three vertices of a parallelogram. What are possible co-ordinates of the other vertex? (8,6) or (4,0)

think

Draw a figure like this on your graph paper and cut it out.



Make one straight cut so that the two parts can be placed together to form a square.



Resources for Active Learning

Applied Mathematics Cards, "Map-Fixing Position," Group 2/23–24; Group 3/22–23, Schofield and Sims. (Available from Mafex Associates)

Franklin Series: *Making and Using Graphs and Nomographs*, "Arrays of Points and Quadrilaterals," pp. 63–66, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Using the Exercises

Assign the exercises on page 285 as independent work. Before children begin, you might point out that exercise 1A, when asking for co-ordinates of the midpoints of each segment, does not include just any segment on the line, but only the segments labelled with letters.

Assignments (page 285)
Minimum: 1, 2. Average: 1–4.
Maximum: 1–5.

PAGES 286–287

Objective

Given a point and a line in the co-ordinate plane, the child will be able to find the reflection of the point in the line.

Preparation

Materials

graph paper (*Duplicator Masters*, page 62); compass (optional); tracing paper (optional)

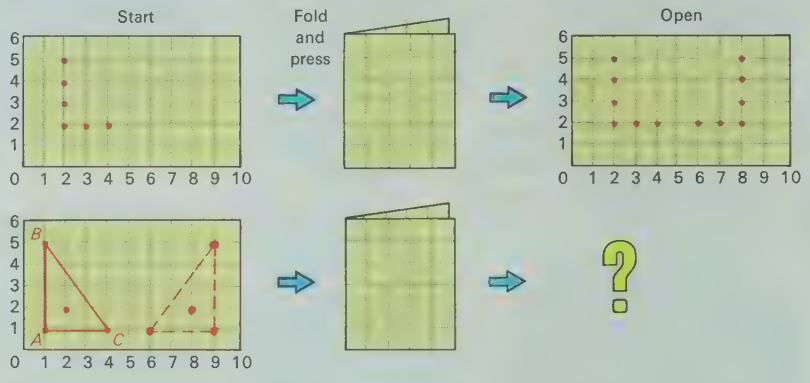
Investigation

Many children should be able to show on graph paper the reflection of the investigation figure by studying it and counting the grid lines. If you think it necessary, however, provide tracing paper so that a child can copy the figure and flip it over the line of symmetry. Or, if convenient, you might suggest that a child use the tip of his compass to mark the image points. The degree of activity involved in this investigation should be based on the ability of the children. The goal is for children to see the relationship between the original figure and its image.

Let's explore reflections and line symmetry.

Investigating the Ideas

Imagine "magic" paper that works like this: When the two halves are folded together, anything printed in red on one half is automatically printed on the other half.



? Can you show on graph paper what this paper will look like when it is opened? See above.

Discussing the Ideas

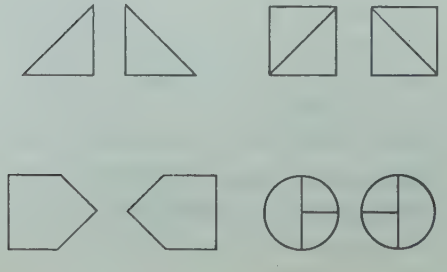
1. The dot pattern made up of two "L's" above is **symmetrical** because it can be folded so that one half exactly matches the other half. The fold line is called a **line of symmetry.** Name a geometrical figure that is symmetrical, and describe a line of symmetry of the figure. Answers will vary. See Discussion.

2. In the triangle that will be printed on the right above, the point that matches point *A* is called the **reflection image** of point *A*. Can you give the co-ordinates of the reflection image of
   - A point *A*? (9,1)
   - B point *B*? (9,5)
   - C point *C*? (6,1)
   - D the point inside the triangle? (8,2)

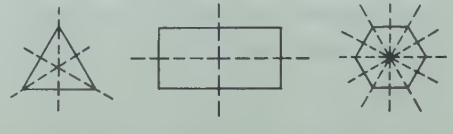
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Discussion

The terms *symmetrical*, *line of symmetry*, and *image* should not be new to children who have studied from other books of this series. However, it would be helpful to have on display some pairs of symmetrical figures such as the following:



Have the children identify the line of symmetry for each pair. Then have them name and show some figures which are symmetrical within themselves and find the line or lines of symmetry for each.

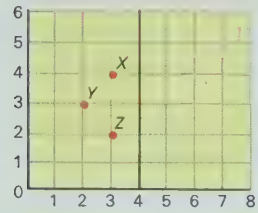


As you discuss exercise 2, observe with the children that a reflection image of a figure may be thought of as the mirror image of the figure. Also, point out that each point on the left of the original fig-

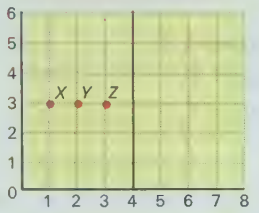
Using the Ideas

1. Give the co-ordinates of the reflection images of points *X*, *Y*, and *Z*.

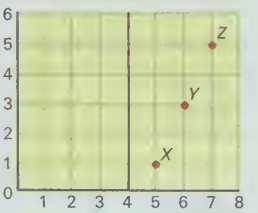
A X(5,4), Y(6,3), Z(5,2)



B X(7,3), Y(6,3), Z(5,3)

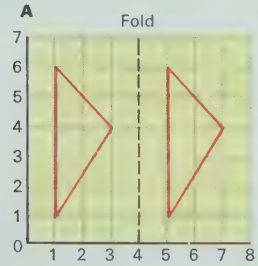


C X(3,1), Y(2,3), Z(1,5)

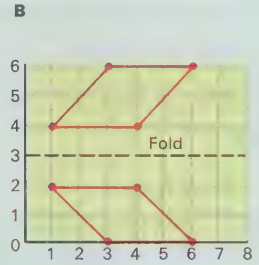


2. Which pairs of figures are reflections of each other? B and C

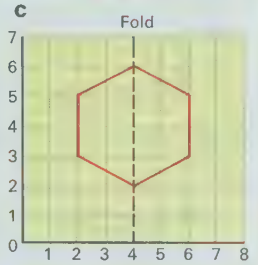
A



B



C



3. Use your graph paper to draw three symmetrical figures. Show a line of symmetry for each one. Figures will vary.

think

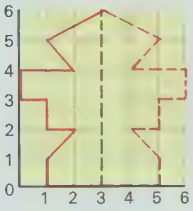
Make a "mirror" by taping plastic or cellophane to a cardboard frame.



Can you use your "mirror" to find out what this message says?

Write a "mirror message" to a friend. What's today's date?

4. Copy this figure on your graph paper. Then draw the other half to make a symmetrical figure.



ure is the same distance from the line of symmetry as each corresponding point on the reflection image is to the right of the line of symmetry.

Using the Exercises

Assign the exercises on page 287 as independent work. Be sure enough graph paper is available for children to do exercises 3 and 4. They might like to display some of their figures on a bulletin board.

Many children will find the *Think* problem an enjoyable activity. They might also enjoy putting their "mirror messages" on a bulletin board for display.

Assignments (page 287)

Minimum: 1, 2. Average: 1–4. Maximum: 1–4.

Mathematics



Suppose that line 1 is the perpendicular bisector of $\overline{PQ}$. Then point Q is the *reflection* of point P in line 1. Each point is the *reflection image* of the other. Intuitively, we may think of the line as a plane mirror which is silvered on both sides so that each point is the image of the other.

A line in a plane is the *line of symmetry* of a given geometric figure if all the points of the figure on one side of the line are reflection images of the points of the figure



on the other sides of the line. Many geometric figures have more than one line of symmetry. When children draw geometric figures on paper and want to locate lines of symmetry, the definition above justifies the folding process to see if the two sets of points coincide when the figure is folded on a presumed line of symmetry.

Follow-up

The study of symmetrical figures suggests a variety of art projects. For example, you might have children paint a design with water colors on one half of a large sheet of art paper. While the paint is still wet, they should fold and press the painted half against the blank half to form an imprint. The new image will be the reflection of the original.

Resources for Active Learning

Experiments in Mathematics, Stage 1, "Symmetry," pp. 22–23, Houghton Mifflin. (Available from Thomas Nelson & Sons Ltd.)

Geometry in the Classroom, "Symmetry," pp. 103–106, Holt, Rinehart and Winston of Canada.

Nuffield Project: *Shape and Size* 3, ". . . Symmetry," pp. 19–26, Wiley.

SMSG: *Puzzle Problems and Games Project*, "Symmetry," pp. 45–58, Stanford University.

Workbook, page 90

PAGES 288–289

Objective

Given a geometric figure and its centre of rotation, the child will be able to show the figure after rotations of 90°, 180°, or 270°.

Preparation

Materials

dot paper for recording (*Duplicator Masters*, page 64 or 65); geoboard (1 per child, if possible); large sheets of paper (newsprint)

The geoboard recommended for this lesson is one in which the nails are arranged in a 5-by-5 square. Dot paper may be used as a substitute, if necessary. This may be duplicated from a master containing sets of dots in a 5-by-5 square array.

Investigation

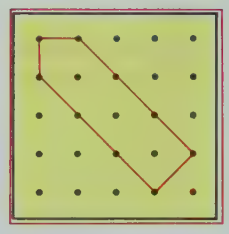
If each child has a geoboard, each will need a large sheet of paper on which to draw the red square. Even ordinary newspaper or blank newsprint would be sufficient. The children should find that, since they cannot turn the geoboard over, there are only four positions in which the figure may be placed. Each of these should be recorded on dot paper.

If children do not have a geoboard, have them cut out one 5-by-5 square of dots from the dot paper and copy the figure from the text. Then they should make a red outline around this small square rather than around a full piece of dot paper. The purpose of the outlined red square is to give children a framework so that their recorded rotations will actually be rotations of 90°, 180°, and 270°.

● *Let's explore rotations.*

Investigating the Ideas

Draw a red square around a geoboard (or a piece of dot paper) with this figure on it.

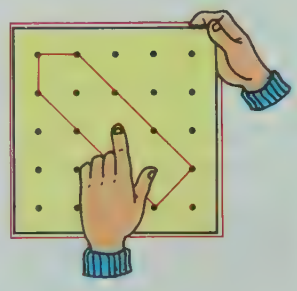


? If you pick up the board and turn it, in how many different ways can you put it back inside the square? 4 See Investigation.

Show each way on dot paper.

Discussing the Ideas

1. A motion that turns the board about a single fixed point is called a **rotation** of the board. The fixed point is called the **centre** of rotation. Rotations are often described in degrees. Which of the positions you drew in the Investigation do you think is a result of



A a rotation of 90°? 

B a rotation of 180°? 



C a rotation of 270°? 



2. You perform a rotation when you turn a doorknob to open a door. Give three other examples of situations when rotations are performed. See Discussion.

3. Through how many degrees does the minute hand of a clock rotate in

A 30 minutes? 180° B 15 minutes? 90° C 45 minutes? 270° D 2 hours? 720°

Discussion

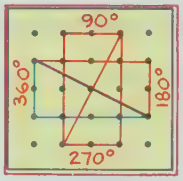
If possible, use an overhead projector to have children point out the centre of rotation and to show the positions they drew in the investigation. A transparent plastic protractor would also be helpful to aid children in relating the rotations to degree measure. Note that all rotations are made in a clockwise direction.

As the class discusses exercise 2, you might suggest such examples of rotation as moving a hand from shoulder to writing position, or turning your head from west to north, or pivoting on a basketball court and so on.

Other clock problems might be used to expand exercise 3. For example: Where will the minute hand be if it started at 12 and rotated 270°? 90°? 180°? If it started at 6 and rotated 270°? 180°?

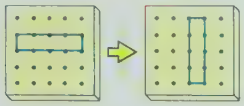
Using the Ideas

1. Draw a picture on dot or graph paper to show how this geoboard figure will look after: See figure at right.
   - A a 90° rotation
   - B a 180° rotation
   - C a 270° rotation
   - D a 360° rotation

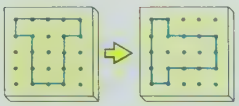


2. Each part shows a geoboard at the start and end of a rotation. What rotation was made?

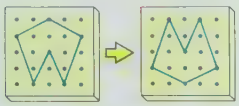
A Start 90° End



B Start 270° End



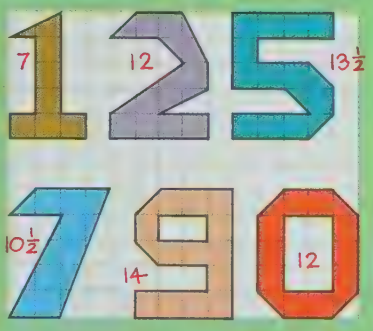
C Start 180° End



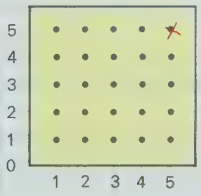
3. Make a figure on your geoboard or on dot paper and show the "start" and "end" of a rotation of your figure. Ask a classmate to tell what rotation was performed. Answers will vary.

think

Find the exact area of each digit.



4. If the geoboard were placed on graph paper, each "nail" would have co-ordinates
   - A If the board were turned 90° about the centre, what would be the new co-ordinates for the "nail" now at (3,5)? (5,3)
   - B Give the **90° rotation image** of the point (1,5). (5,5)



Using the Exercises

Before children begin to work on the exercises on page 289, you might discuss how graph paper may be thought of as dot paper if only the points of intersection are considered. This interpretation of graph paper is useful for both exercises 1 and 4. In exercise 4, be sure children understand the phrase "90° rotation image." Use this phrase in other examples, such as, "The 90° rotation image of point (3, 5) is (5, 3);" or, "The 90° rotation image of point (3, 3) is (3, 3)." You might ask some children to list all the points on the 5-by-5 grid and give their 90° rotation image.

When children have finished the exercises, check their work, discussing any further questions. You might have several children show their rotation for exercise 3 by using the overhead projector.

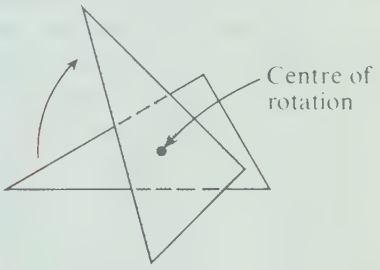
Assignments (page 289)

Minimum: 1, 2. Average: 1–4. Maximum: 1–4.

Mathematics

A simple model which can be used to demonstrate rotations of figures can be constructed as follows. Cut out a geometric figure from tagboard and pin it to a piece of thick cardboard. Mark around the region. Now rotate the region using the pin point as the centre of rotation and mark around the region again in a new position. Each figure is the rotation image of the other with respect to the centre of rotation (the pin point).

Using squares, equilateral triangles, and other regular figures, one may show how some rotation images fall again on themselves if the proper centre of rotation is chosen.



Resources for Active Learning

A Cloudburst, Vol. 2, No. 8453, 8473, Midwest Publications.

PAGES 290–291

Objective

Given a figure on a geoboard, the child will be able to recognize whether the figure has rotational symmetry.

Preparation

Materials

geoboard, 1 per child (optional)

To prepare for this lesson, you might review the concept of line symmetry. Emphasize that in line symmetry, the image reflected in the line is the mirror image of the original figure. Explain that in this lesson they will study another kind of symmetry.

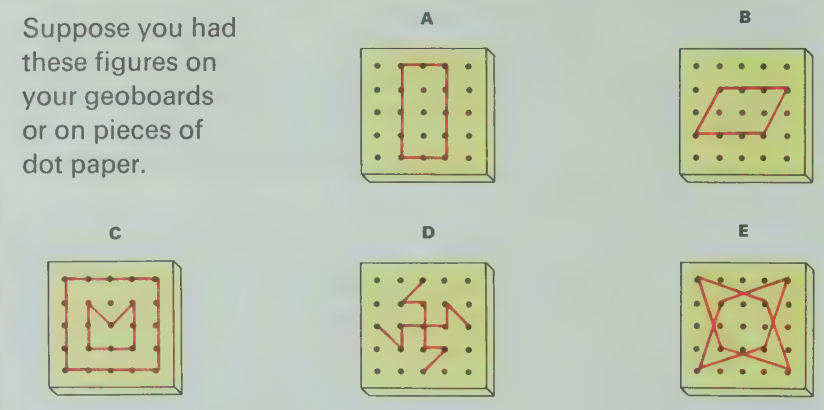
Investigation

Although children may work solely from the illustrations in the text, many would benefit from using a geoboard or the small squares cut from dot paper. For most, this investigation will require use of a trial-and-error approach. Each figure shown should be examined in its original position and then rotated 180° and compared to the original figure. Suggest to those who have difficulty deciding the answer to the question that they copy the original figure, rotate the original figure 180°, and compare the rotated figure with their copy of the original. For many, such a procedure might be necessary only for the more difficult figures such as figure D.

What is rotational symmetry?

Investigating the Ideas

Suppose you had these figures on your geoboards or on pieces of dot paper.



Can you tell which geoboards would look exactly the same if each one were rotated 180°? A, B, D, E

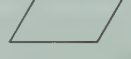
Discussing the Ideas

1. If the board looks exactly the same at the end of a rotation (other than 360°) as it did at the beginning, the figure on the board is said to have **rotational symmetry.**
   - A Do you think every rectangle has 180° rotational symmetry? Yes
   - B Name some other quadrilaterals that have 180° rotational symmetry. (The pictures above may help you.) See Discussion.
   - C Can you think of a quadrilateral that has 90° rotational symmetry? Square
2. A rectangle has both a line of symmetry and 180° rotational symmetry. Which quadrilateral has only rotational symmetry? Parallelogram
3. A Can you think of any polygons, other than quadrilaterals, that have rotational symmetry? Explain. Regular hexagon, octagon, etc.
   - B Invent a figure that has rotational symmetry. Figures will vary.

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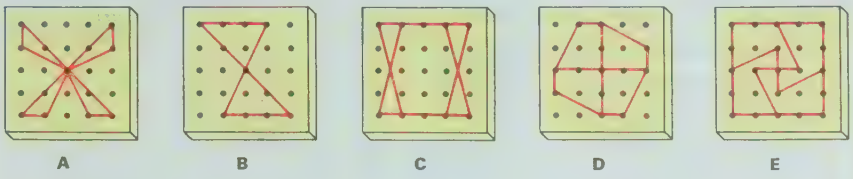
Discussion

As the children share their investigation results, use examples A, B, D, and E to describe rotational symmetry. Also, be sure the children realize how figures return to their original position after a rotation of 360°. Use a demonstration geoboard (or transparencies on the overhead projector) to discuss 90° rotational symmetry and 180° rotational symmetry. Help the children observe that figures A and B have 180° rotational symmetry. If children are interested, you might have them complete a chart such as the one illustrated in the column to the right.

Figure	Line of symmetry	Rotational symmetry
	4	90°, 180°, 270°
	2	180°
	0	180°
	2	180°
	0	none

Using the Ideas

1. Which of these geoboard designs have 180° rotational symmetry? B, C, D, E



2. Which figures in exercise 1 have 90° rotational symmetry? D and E

3. Make up a figure on your geoboard or on dot paper that has 90° rotational symmetry; 180° rotational symmetry. Answers will vary.

4. Think about polygons that are outlines of the 26 letters of the alphabet. Which of these polygons have 180° rotational symmetry? H, I, N, O, S, X, Z



think

A pentomino is a figure made from 5 squares. Each square can touch another only on a complete common side. Find as many pentominoes that have 180° rotational symmetry as you can and draw them on graph paper.



See Solution, T.E. page 291

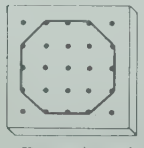
★ 5. Draw a picture of a figure that contains no straight lines and has 180° rotational symmetry.
Sample: S

★ 6. Use only squares and circles to make a design that has 90° rotational symmetry.

Using the Exercises

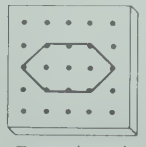
The exercises on page 291 may be assigned as independent work, although some children might benefit from working in small groups or with partners. If children need help in finding figures with rotational symmetry, suggest that they investigate an octagon or a hexagon on the geoboard.

An octagon



Rotational symmetry of 90°, 180°, 270°

A hexagon



Rotational symmetry of 180°

In order to help children solve the *Think* problem, suggest that they first find the 12 pentominoes and then identify those which have rotational symmetry.

Assignments (page 291)*
Minimum: 1, 2. Average: 1–4.
Maximum: 1–6.

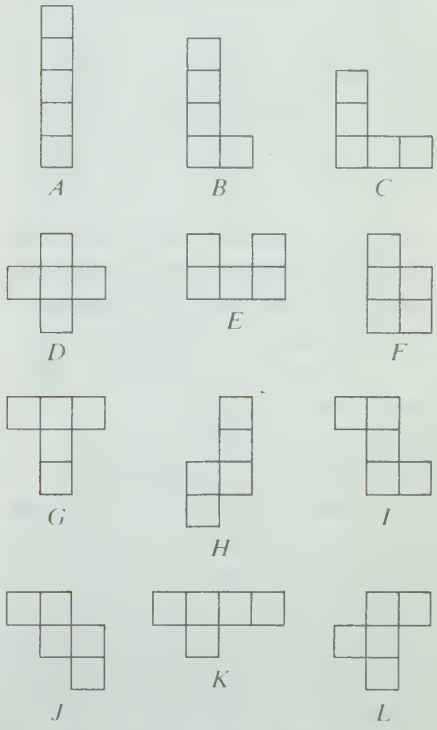
Mathematics

A figure is said to have rotational symmetry with respect to its centre of rotation if it appears not to have been moved by the rotation. That is, all the points of the figure fall again on the figure after the rotation. A square has 90°, 180°, and 270° rotational symmetry. Every rectangle has 180° rotational symmetry with respect to its centre.

If the centre of gravity (centroid) of an equilateral triangle is chosen as the centre of rotation then the triangle has 120° rotational symmetry. A circle has rotational symmetry for any rotation if the centre of rotation is the centre of the circle.

Solution, Think, page 291

The 12 pentominoes are shown. Figures *A*, *D*, and *I* are the only ones with 180° rotational symmetry.



Resources for Active Learning

Inquiry in Mathematics via the Geo-board, "Symmetries of a Rectangle," Geo-cards 34/1–15, Walker. (Available from Fitzhenry & Whiteside Ltd.)

PAGES 292–293

Objective

Given a point or set of points on a graph, the child will be able to find its image for a given translation.

Preparation

Materials

cubes (1 pair for every 2 children); graph paper or 10-by-10 grids (*Duplicator Masters*, page 62, 63, or 66)

The cubes recommended for this lesson may be made from a variety of materials. Pairs of different-colored dice might be used and the color could indicate "over" or "up." One-cm cubes could be cut from light wood or Styrofoam and marked by the children. If no other materials are available, children can make a cube out of cardboard or stiff paper. In this case, provide a pattern of a cube for them to use.

Investigation

Encourage children to study and discuss with their partner the explanation of the game in the text. They might record their moves simply by using an X or 0. Since the probability of throwing pairs which would move the player directly to (10, 10) is quite small, children will discover that they have to make up their own rules about arriving at the end point. For example, if a player's tosses are 1) O4, U5; and 2) O2, U6, he will be above the 10 mark and no toss can get him back. So children might make a rule which says that he must always toss an exact number to get him on either the horizontal or vertical 10 line and then he should use only one die (the "over" or "up" die) and wait for an exact toss to move him to (10, 10). They could also use a scoring method whereby when a player reaches either 10 line the game is over and each player adds the co-ordinates of the point he is on, with the winner being the player with the highest score.

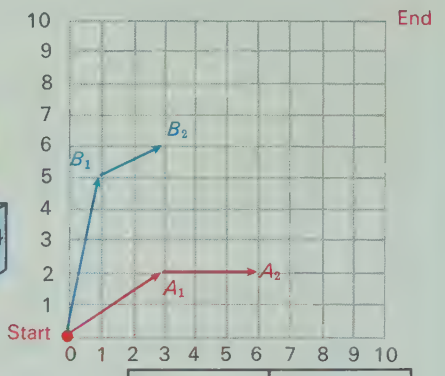
● *Let's explore translations.*

Investigating the Ideas

To play the game "Translations," each player in turn rolls two cubes with faces marked **O0** through **O5** and **U0** through **U5**. Then he draws an arrow on the graph to show the "slide" made by his point. The chart shows the record of the start of a game played by Ann and Bob. The object of the game is to reach the point (10, 10) first.







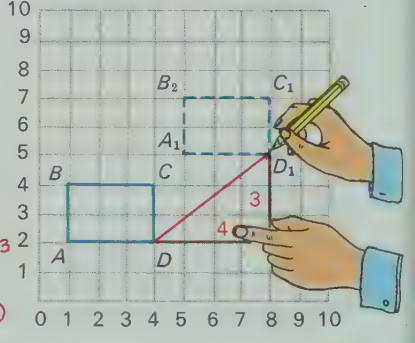
	Ann	Bob
1st roll	A_1 (03, U2)	B_1 (01, U5)
2nd roll	A_2 (03, U0)	B_2 (02, U1)

? Can you play the game with a classmate? | Make up your own rules for the game.

See Investigation.

Discussing the Ideas

1. What are some rules you would use for the game?
   Answers will vary. See Discussion.
2. How could you win the game in two moves?
   Roll (05,U5) twice in a row.
3. A student used an "over 4, up 3" point slider to "slide" each point of a figure to a new position. The red arrow shows the distance and direction of this move, called a **translation** of the figure.
   How far was the figure moved? O4,U3
   Can you give the co-ordinates of the **slide image** of point *D*? (8,5)



292

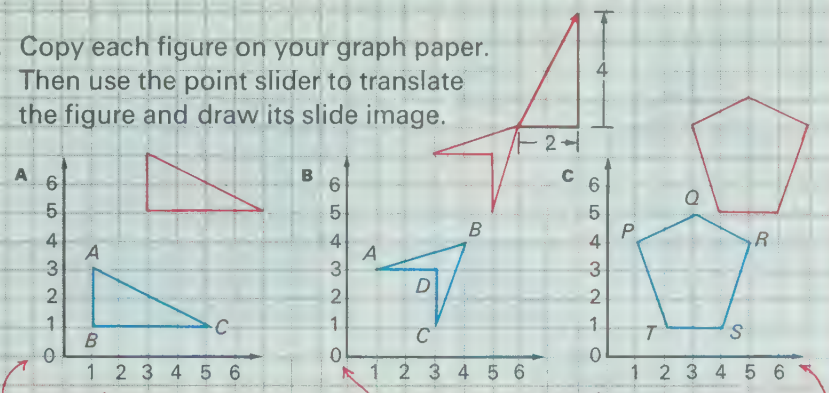
Discussion

Use exercises 1 and 2 as a basis for discussing both the various moves possible to win the game in two moves and the modifications in the rules which children made. Then point out how, in order to make a move that is both horizontal and vertical, they needed an over and up direction. Unless both dice come up zero, their move was a slide (hence, the term *slide image*, in exercise 3). You may want to emphasize with children the fact that, in translations, as well as in the other motions previously studied, a point is not relocated or actually moved. Rather, a second point is designated as an image of the original point according to a particular motion.

You might find the overhead projector particularly helpful for displaying translations on a grid.

Using the Ideas

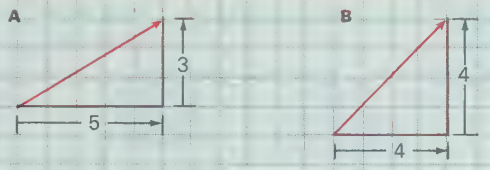
1. Copy each figure on your graph paper. Then use the point slider to translate the figure and draw its slide image.



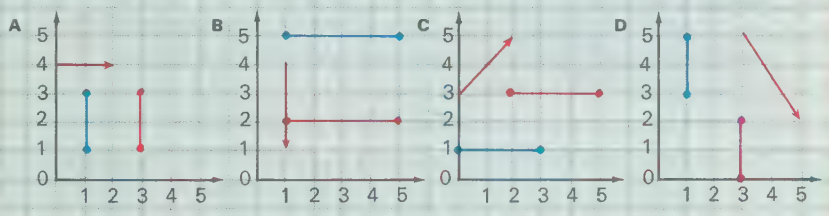
2A: A→(6,6); B→(6,4); C→(10,4). 2A: A→(6,7); B→(9,7); C→(8,4); D→(8,6).
2B: A→(5,7); B→(5,5)→(9,5). 2B: A→(5,7); B→(8,8); C→(7,5); D→(7,7).

2. Complete exercise 1 for each of these translations. New coordinates for each figure are given above.

2A: P→(6,7); Q→(8,8); R→(10,7); S→(9,4); T→(7,4).
2B: P→(5,8); Q→(7,9); R→(9,8); S→(8,5); T→(6,5)



3. The red arrow tells the distance and direction of a translation. Copy each segment on your graph paper and draw its image after the translation.



4. Draw a figure on your graph paper and show its new position after these translations. Figures will vary.

A right 1, up 6 B right 3, down 2 C left 4, down 3

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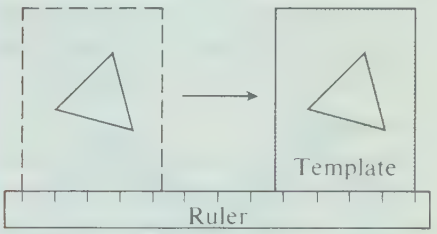
Using the Exercises

Supply the children with sufficient graph paper and assign the exercises on page 293 as independent work. Some children might need guidance with exercise 3. Point out that the red arrow simply appears on the same grid as the figure. It should be interpreted in the same manner as the arrows in exercises 1 and 2.

Assignments (page 293)
Minimum: 1, 2. Average: 1–4.
Maximum: 1–4.

Mathematics

A simple model which will show translations can be made as follows. Cut out a triangular region from a piece of cardboard to form a template. Place one edge of the template along a ruler and draw a triangle using the template. Slide the template along the edge of the ruler to a new position and draw another triangle. The second triangle is a translation image of the first triangle.



When a figure is given in the coordinate plane, a translation may be made by "moving" every point of the figure "over and up" by the same amount. This has the effect of sliding every point the same direction and the same distance. This is the basis for the feasibility of the "point finder" described in the lesson.

Resources for Active Learning

Developmental Math Cards, "Rotations, Reflections, Translations," J-16. Addison-Wesley.

PAGES 294–295

Objective

Given a geometric region, the child will be able to determine whether or not it can be used to tessellate the plane.

Preparation

Materials

graph paper, $\frac{1}{2}$-cm grid (*Duplicator Masters*, page 63)

Introduce the concept of tessellations by referring to tiling on the floor or in some part of the building and have the children begin immediately with the investigation.

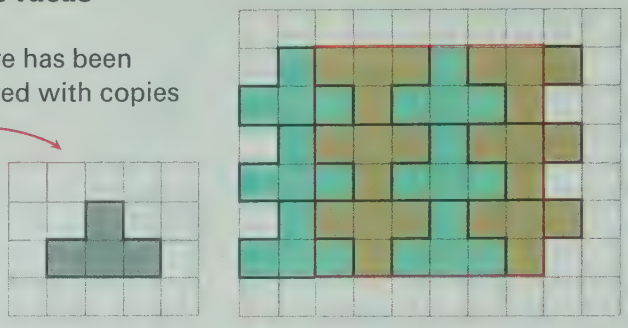
Investigation

Supply the children with graph paper and encourage them to draw a tiling of each figure within a 6 × 6 square. Some may want to use tracing paper and cut out the shapes A, B, C, and D so that they can use them as tiles. Either method is acceptable.

Which figures will tessellate a plane?

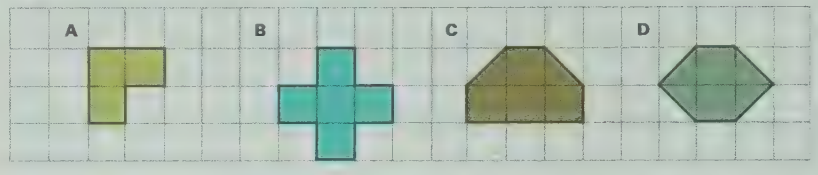
Investigating the Ideas

A 6 × 6 red square has been completely covered with copies of this shape. (Some squares fell outside.)



? Which of these shapes can be used to completely cover a 6 × 6 red square? A,B,D

Draw the covering for those that can.



Discussing the Ideas

1. You may remember that a pattern that can "cover a plane" is called a **tessellation** of the plane. If a particular shape is used to form a tessellation, we sometimes say that the shape will **tessellate** the plane.
   - A What patterns do you see in the tessellations in the Investigation? See Discussion.
   - B How can you color the tessellations to make the patterns stand out? Many variations are possible.

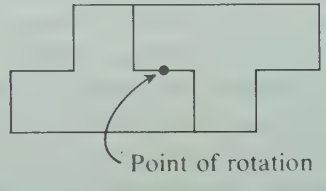
2. Is there more than one way to tessellate a plane with shape A above? Yes with shape D? NO

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Discussion

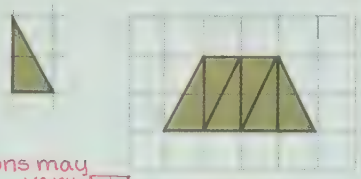
As children give their responses to the investigation question, help them to see that since figures A, B, and D completely cover the 6 × 6 square, we say that these figures will *tessellate* the plane. Point out the similarity to tiling patterns on a floor or other tiled surface. With some children, it would be helpful to discuss various tessellation patterns in terms of translations, rotations, and reflections. For example, the pattern in the illustration at the top of page 294 is a rotational pattern, since a 180° rotation about the point shown here takes the figure at the left to the same position as the figure on the right.



An example of a tessellation described in terms of a translation may be seen with figure D. An ascending row of the figures may be formed by applying the translation over 2, up 1. Other translations may also be found to describe the tessellation, such as over 2 down 1, as shown to the right.

Using the Ideas

1. Six of these right triangle shapes have been put together to form a trapezoid. On your graph paper show how to put copies of this shape together to form: Constructions may vary.



A an isosceles triangle 

B a square 

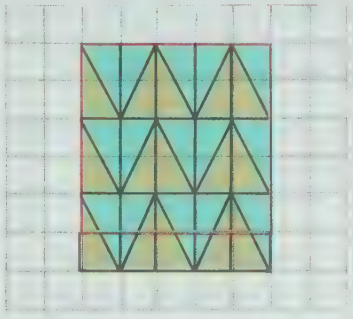
C a rectangle 

D a parallelogram 

E a rhombus 

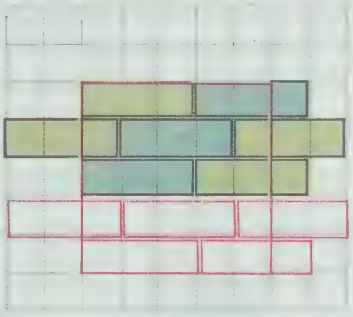
F a hexagon 

2. Here is a tessellation with the right triangle as the basic shape.



On your graph paper draw and color two other tessellations with this shape. Constructions will vary.

3. A Copy and complete this covering of the red square by using the suggested pattern. See figure →

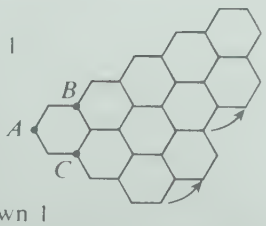


B Use this same basic shape and show a different tessellation.

C Color your tessellation so the pattern stands out.

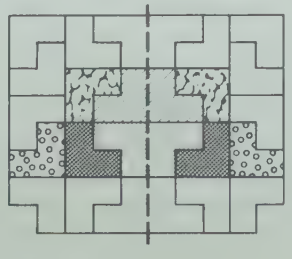
4. Invent a figure of your own that will tessellate a plane. Figures will vary.

$A \rightarrow B$
over 2, up 1



$A \rightarrow C$
over 2, down 1

Other patterns may be thought of as reflections. (The shaded figures show patterns of reflection.)



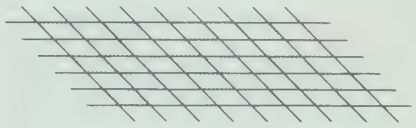
Using the Exercises

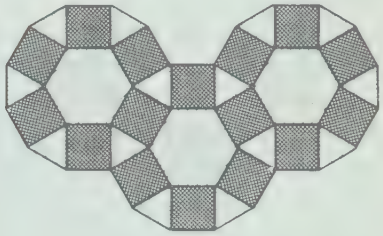
One of the main purposes of the exercises on page 295 is for children to recognize and create various patterns. Encourage as much creativity as possible, and display the patterns children invent. As children are discussing their work, point out how they could continue their patterns indefinitely. The phrase "tessellate the plane" means that the tessellation can continue indefinitely due to the concept of plane.

Assignments (page 295)
Minimum: 1, 2. Average: 1–4.
Maximum: 1–4.

Follow-up

As previously mentioned, much of the children's work would be suitable for display. Encourage children to extend any of the exercises. You might also suggest that they discover as many patterns as they can in a tessellation of parallelograms, or with a combination of shapes such as triangles, squares, and hexagons.





Resources for Active Learning

(See the resources listed on page 77 for tessellation activities.)

PAGES 296–297

Objective

Given a triangle on a grid or geoboard, the child will be able to construct a similar triangle and compare corresponding parts.

Preparation

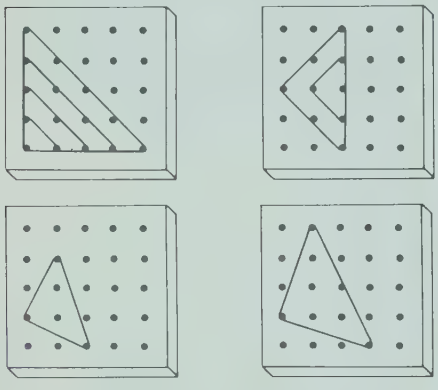
Materials

geoboard; dot paper for recording (*Duplicator Masters*, page 64); graph paper, 1-cm grid, if possible; tracing paper

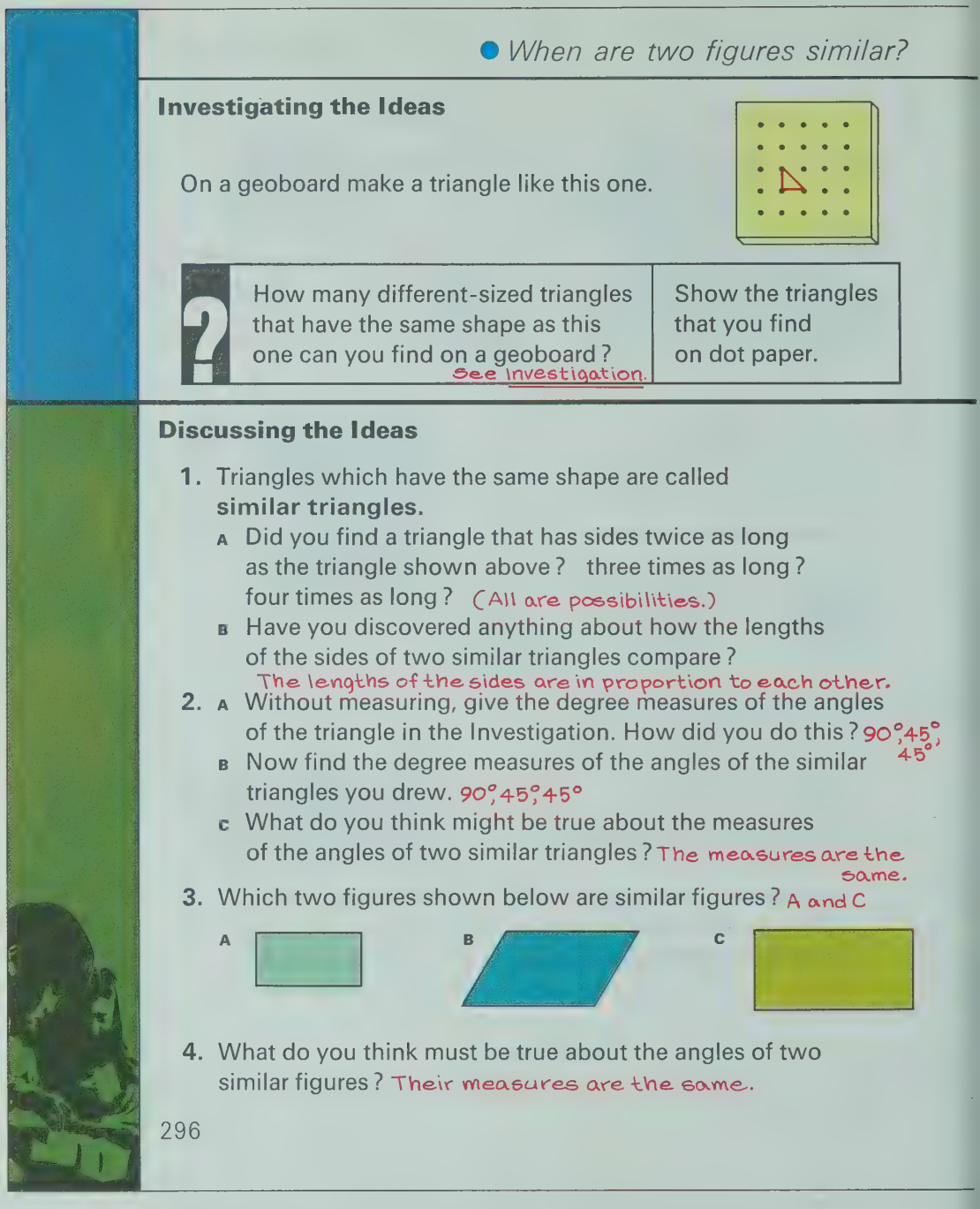
The investigation in this lesson may be undertaken without specific preparation.

Investigation

If children do not each have a geoboard, they may use dot paper as a substitute but a geoboard is much more conducive to discovery. There are 8 different-sized right isosceles triangles that can be formed on a 5-by-5 geoboard:



Do not expect all children to find all of these; some are quite difficult to discover. It may be helpful to observe that each of them is half of a square which can be formed on the geoboard.



● *When are two figures similar?*

Investigating the Ideas

On a geoboard make a triangle like this one.

? How many different-sized triangles that have the same shape as this one can you find on a geoboard? See Investigation.

Show the triangles that you find on dot paper.

Discussing the Ideas

1. Triangles which have the same shape are called **similar triangles.**
   - A Did you find a triangle that has sides twice as long as the triangle shown above? three times as long? four times as long? (All are possibilities.)
   - B Have you discovered anything about how the lengths of the sides of two similar triangles compare? The lengths of the sides are in proportion to each other.
2. A Without measuring, give the degree measures of the angles of the triangle in the Investigation. How did you do this? 90°, 45°, 45°
   - B Now find the degree measures of the angles of the similar triangles you drew. 90°, 45°, 45°
   - C What do you think might be true about the measures of the angles of two similar triangles? The measures are the same.
3. Which two figures shown below are similar figures? A and C

A B C

4. What do you think must be true about the angles of two similar figures? Their measures are the same.

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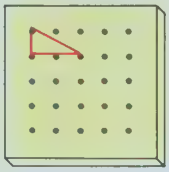
Discussion

If possible, use a transparent geoboard or dot paper transparencies on the overhead projector during this discussion. Encourage children to show the triangles they found and use them in discussion of exercise 1.

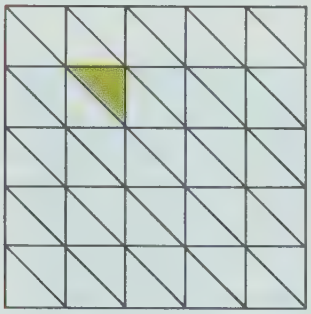
For exercise 2, children must recall that the sum of the angles of a triangle is 180°. Then, since they know that one angle is 90° and the other two angles are equal, these latter must both be 45°. It would be helpful in exercise 2C, to have the children cut out from dot paper a large pair of similar triangles and compare angles and sides.

Using the Ideas

1. On your geoboard find a triangle similar to this triangle.
Draw it on dot paper.
See Answers, T.E. page 297.

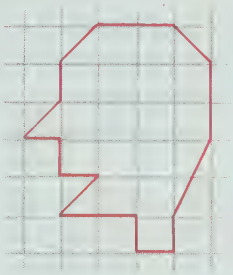


2. A Place tracing paper over this grid and trace as many different-sized triangles that are similar to this triangle as you can. You may include only those triangles that can be traced directly from this grid. See Answers, T.E. page 297.



B Find another familiar figure on the grid and repeat part A.
Answers will vary. Similar squares, parallelograms, and trapezoids can be found.

3. Draw a figure like this one on graph paper. Then make a larger figure that is similar to the one shown.
Sizes may vary.



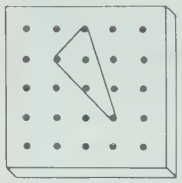
★ 4. Trace a favorite cartoon character or picture on a small grid and reproduce it on a larger grid to make a larger figure similar to it. Figures will vary.

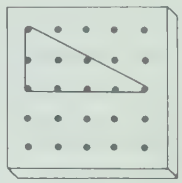
297



Answers, exercises 1 and 2A, page 297

1. Possible answers:





Exercise 2

2.A 4 possible similar triangles:



Resources for Active Learning

Freedom to Learn, "Patterns in Shapes," pp. 94–99; "Congruence and Symmetry" and "Similarity," pp. 153–157, Addison-Wesley.

Geometry in the Classroom, "Similarity," pp. 95–98, Holt, Rinehart and Winston of Canada.

Inquiry in Mathematics via the Geo-board, "Similar Triangles," Geo-cards 23/1-3, Walker. (Available from Fitzhenry & Whiteside Ltd.)

Using the Exercises

Assign the exercises on page 297 as independent work, but encourage children to share their ideas. These exercises give children an opportunity to apply their understanding of similarity in a variety of ways. Guide children individually, and only as needed. In order for them to do exercise 2B, they must recognize the square, parallelogram, or the trapezoid as a familiar figure on the grid. Have a variety of comic strips available for those who have not brought their favorite cartoon character from home. If you have no 1-cm graph paper, children can carefully mark art paper and lightly draw in the grid lines themselves.

Notice that exercise 4 is intended as a special challenge. However, demonstrations by those who worked it would be of benefit to all.

Assignments (page 297)

Minimum: 1, 2. Average: 1–3. Maximum: 1–4.

PAGES 298–299

Objective

Given a figure in the co-ordinate plane, the child will be able to graph its magnification from (0, 0) according to a given scale factor.

Preparation

Materials

graph paper (*Duplicator Masters*, page 62); overhead projector and transparent grid (optional)

To prepare for this lesson, teach the children how to read the letters with a subscript. A_1 is read "A sub-one"; A_2 is read "A sub-two"; B_1 is read "B sub-one"; and so on.

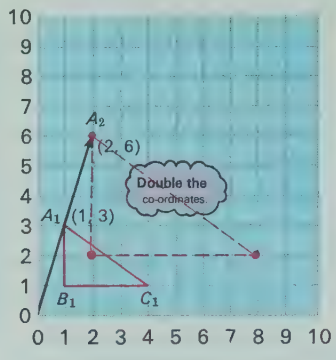
Investigation

It is important that children understand the directions for what they are to graph. The value of each co-ordinate should be doubled, then the new value graphed. Thus, since B_1 has co-ordinates (1, 1), the new point B_2 will have co-ordinates (2, 2). Similarly, co-ordinates (4, 1) of point C_1 will correspond to co-ordinates (8, 2) for the new point C_2. The new figure will be a triangle similar to triangle ABC.

★ *Let's explore magnifications.*

Investigating the Ideas

Copy the red triangle on your graph paper.



? Can you draw the figures that would result if the co-ordinates of A_1, B_1, and C_1 were doubled, graphed, and connected? See above.

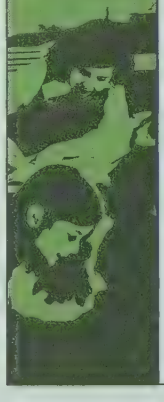
Discussing the Ideas

1. A What kind of figure do you get when you double the co-ordinates of A_1, B_1, and C_1 and connect them? Triangle similar to $A_1B_1C_1$
   B How do the sides of the new figure compare with the sides of $\triangle A_1B_1C_1$? They are twice the length.
   C If we call the points with the doubled co-ordinates A_2, B_2, and C_2, how does the distance from (0,0) to A_2 compare with the distance from (0,0) to A_1? Twice as far

2. What are the answers to the questions in exercise 1 when
   A the co-ordinates are tripled? Sides are three times the length.
   B the co-ordinates are quadrupled? Sides are four times the length.

3. We say that $\triangle A_2B_2C_2$ is a **magnification** from (0,0) of $\triangle A_1B_1C_1$, using scale factor 2. Can you describe the magnification from (0,0) of $\triangle A_1B_1C_1$, using scale factor 3?
   Lengths of sides are tripled.

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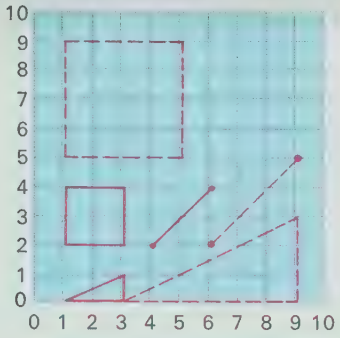


Discussion

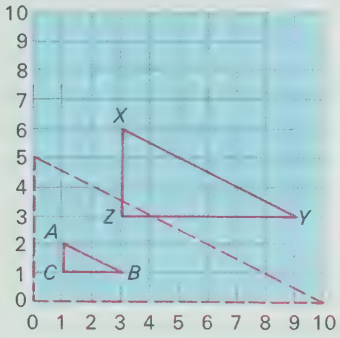
An overhead projector would be very useful for this lesson. First discuss the new triangle the children formed for the investigation. Then, for exercise 2, ask them to triple and then quadruple the co-ordinates. Triangle $A_3B_3C_3$ will have co-ordinates (3, 9), (3, 3), and (12, 3) respectively; triangle $A_4B_4C_4$ will have co-ordinates (4, 12), (4, 4), and (16, 4) respectively. As children graph these triangles, help them compare the lengths of corresponding sides. If children are able, you might suggest that they measure the angles of each triangle to see that they are congruent although the sides are not. However, keep the emphasis of this lesson on the fact that the triangles resulting from the magnifications are similar.

Using the Ideas

1. On your graph paper, copy these figures and show the magnification from (0,0) of:
   - A the square, using scale factor 2
   - B the segment, using scale factor $1\frac{1}{2}$
   - C the triangle, using scale factor 3

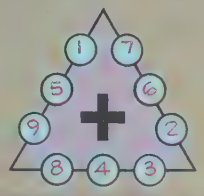


2. $\triangle XYZ$ is a magnification from (0,0) of $\triangle ABC$, using scale factor 3.
   - A The length of $\overline{XZ}$ is __? 3 times the length of $\overline{AC}$.
   - B The length of $\overline{ZY}$ is __? 3 times the length of $\overline{CB}$.
   - C The length of $\overline{XY}$ is __? 3 times the length of $\overline{AB}$.



think

Can you copy the figure and write the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 in the circles so that the sum "on each side of the triangle" is the same?



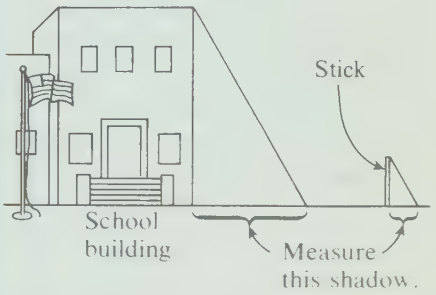
3. In exercise 2, $\triangle ABC$ is similar to (the same shape as) $\triangle XYZ$. Show on a large sheet of graph paper a triangle similar to $\triangle ABC$ with matching sides 5 times as long. Indicated by dashed lines above.

4. Draw some interesting figures on your graph paper and use magnification to make figures similar to them. Figures will vary.



Follow-up

An excellent follow-up may be developed by having the children determine the height of the school building, the flagpole, or another tall object nearby. On a sunny day, you might go into the school yard with the children, have them hold a stick perpendicular to the ground, and measure the length of its shadow. Then measure the length of the shadow of the school building to establish the relationship between the triangle associated with the stick and its shadow and the triangle associated with the side of the school building and its shadow. With this relationship, you can find the height of the school building fairly accurately.



Resources for Active Learning

Developmental Math Cards, "Projections," L320, Addison-Wesley.

Notes on Mathematics in Primary Schools, "Enlargement," pp. 154–156, Cambridge University Press. (Available from Macmillan)

Duplicator Masters, page 55
Workbook, page 91

Using the Exercises

Assign the exercises on page 299 as independent work. Again, an overhead projector would be very helpful when checking and discussing the children's work. Stress how the lengths of the sides of similar figures are related, and corresponding sides of similar figures are related by a constant.

The original figures and their magnifications drawn for exercise 4 should make fine display material.

Assignments (page 299)
Minimum: 1, 2. Average: 1–4.
Maximum: 1–4.

PAGES 300–301

Objective

The child will demonstrate his ability to work with the concepts presented in this chapter.

Preparation

Materials

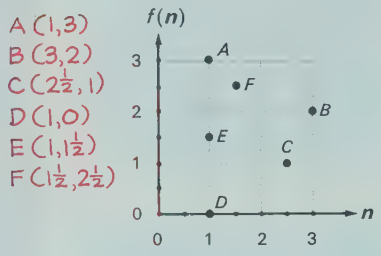
graph paper (*Duplicator Masters*, page 62)

To prepare for this lesson, review any of the topics which the children found troublesome, or have the class play a short game of co-ordinate tick-tac-toe. Divide the class into two groups using symbols such as X and 0 for each. Use a grid with axes labelled from 0 to 8. Have a member of each team mark his team's choices of where to place the X or 0, indicated by a pair of co-ordinates given him by one of his team members. The first team which gets four X's or four 0's in a vertical, horizontal, or diagonal row wins.

Reviewing the Ideas

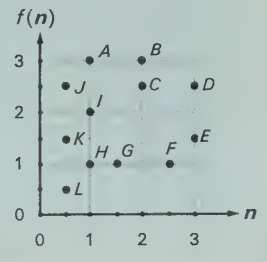
1. Give the number pair for each letter in the graph.

A (1,3)
B (3,2)
C (2½, 1)
D (1,0)
E (1, 1½)
F (1½, 2½)



2. Give the letter for each number pair.

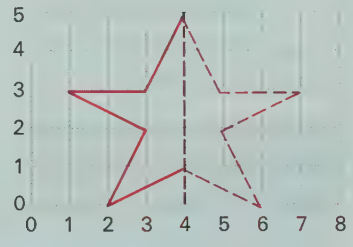
B (2, 3)
G (1½, 1)
E (3, 1½)
K (½, 1½)



3. A Give the co-ordinates of the midpoint of the segment whose endpoints have co-ordinates (1,1) and (7,7). (4,4)

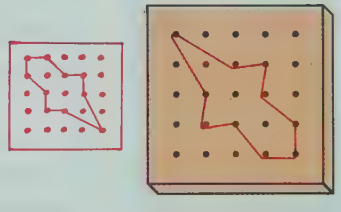
B Three corners of a rectangle are at (2,3), (2,6), and (8,6.) Give the co-ordinates of the fourth corner. (8,3)

4. Copy this picture and complete the other half so that it will be symmetrical.



5. How many different lines of symmetry does a rectangle have? 2

6. Draw a picture on dot paper to show how this geoboard will look after it has been rotated 180°.

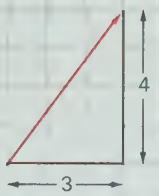


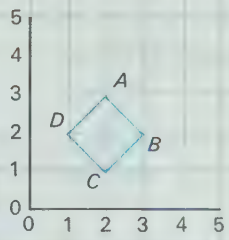
7. Draw a figure that has 180° rotational symmetry on dot paper.
Figures will vary.

Discussion

The exercises on these two pages may be used indirectly for evaluation purposes. For example, you may simply assign them and give guidance as the children need it, noticing which children are able to work independently. Be sure to have available a sufficient supply of graph paper and dot paper. When children have completed the exercises, an overhead projector would be very useful for discussion of their work. Stress the importance of developing the ability to correctly relate the points with co-ordinates.

8. Copy figure *ABCD* on your graph paper and draw the figure in its final position after this translation.

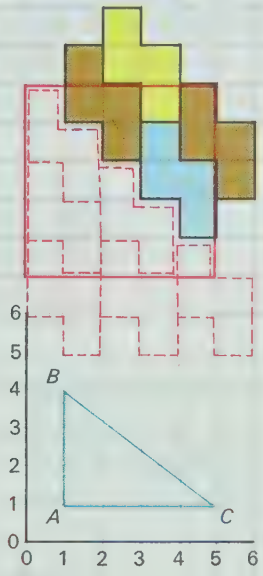




New coordinates:
A(5,7); B(6,6); C(5,5); D(4,6)

9. A Copy and complete the tessellation of the red square by using the suggested pattern.

B Color your tessellation so that the pattern stands out.



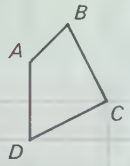
10. A Draw the magnification from (0,0) of $\triangle ABC$, using scale factor 3.

$\overline{AB}$: 3
$\overline{AC}$: 4
$\overline{BC}$: 5

B How long are the sides of $\triangle ABC$? How long are the sides of the "magnified" triangle?

Three times as long

11. Draw a figure like *ABCD* on graph paper. Then draw a larger, similar figure.



Sizes may vary.

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Workbook, page 92

PAGES 302–303

Objective

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

Depending on the needs and ability of the children, review any topics treated in this review lesson with which they had previous difficulty. Or, you might simply have some warm-up activities such as a "Buzz" game with multiples (see page 135, follow-up section) or a "What's My Rule" game (see page 42, discussion section) or some mental chain games, such as:

"Start with $\frac{1}{2}$. . . Add 1. . . Multiply by 2. . . What's your answer?" (3)

"Start with 56. . . Divide by 7. . . Multiply by $\frac{1}{4}$. . . What's your answer?" (2)

"Start with $\frac{1}{2}$ of 12. . . Multiply by 7. . . Subtract 2. . . Divide by 8. . . Multiply by $\frac{1}{5}$. . . What's your answer?" (1)

Keeping in Touch with

Computing
Place value
Principles
Fractional numbers
Decimals

1. For the decimal 5 407 368.159283, give the number of:

A tens 6 C hundreds 3 E thousands 7 G hundred thousands 4
B tenths 1 D hundredths 5 F thousandths 9 H hundred thousandths 8

2. Round 3654.563 to the nearest:

A hundredth 3654.56 B tenth 3654.6 C whole number 3655 D ten 3650 E hundred 3700 F thousand 4000

3. Solve the equations.

A $19 \times 1 = a$ 19 D $(8 \times 7) \div 7 = t$ 8 G $5 \times 23 = (5 \times n) + (5 \times 3)$ 20
B $28 + 0 = p$ 28 E $63 - (n \times 7) = 0$ 9 H $976 + 843 = n + 976$ 843
C $0 \div 5 = n$ 0 F $10^2 \times 2^2 = b$ 400 I $34 + (17 + 19) = (n + 17) + 19$ 34

4. Compute the following.

A $3872 + 5799 = 9671$ B $537 - 69 = 468$ C $4007 - 2768 = 1239$ D $356 \times 47 = 16732$ E $89\overline{)5976}$ 67 R13 F $16\overline{)35\,472}$ 2217

5. A List all the factors of 16. 1, 2, 4, 8, 16 B List the multiples of 4 (to 28). 0, 4, 8, 12, 16, 20, 24, 28

6. Which of these are prime numbers? (5), (7), 9, 21, (29), 33

7. List three fractions equivalent to $\frac{3}{5}$. $\frac{6}{10}, \frac{9}{15}, \frac{12}{20}$ (Answers may vary.)

8. Give the lowest-terms fraction for each. A $\frac{9}{12}$ $\frac{3}{4}$ B $\frac{24}{100}$ $\frac{6}{25}$ C $\frac{20}{48}$ $\frac{5}{12}$

9. Find the sums and differences.

A $\frac{3}{8} + \frac{1}{4}$ $\frac{5}{8}$ B $\frac{5}{8} + \frac{1}{6}$ $\frac{19}{24}$ C $\frac{4}{5} + \frac{2}{7}$ $1\frac{3}{35}$ D $\frac{5}{6} - \frac{1}{4}$ $\frac{7}{12}$ E $\frac{5}{8} - \frac{3}{10}$ $\frac{13}{40}$

F $17\frac{3}{8} + 15\frac{1}{2} = 32\frac{7}{8}$ G $34\frac{1}{6} + 59\frac{1}{9} = 93\frac{5}{18}$ H $54\frac{1}{4} - 21\frac{3}{8} = 32\frac{7}{8}$ I $37\frac{1}{6} - 26\frac{2}{3} = 10\frac{1}{2}$ J $51\frac{1}{2} - 39\frac{3}{5} = 11\frac{9}{10}$

10. Find the products and quotients.

A $\frac{3}{5} \times \frac{2}{3}$ $\frac{2}{5}$ C $5 \times 4\frac{1}{2}$ $22\frac{1}{2}$ E $\frac{20}{24} \times \frac{6}{5}$ 1 G $\frac{2}{3} \div \frac{1}{4}$ $\frac{8}{3}$ I $\frac{8}{5} \div \frac{2}{3}$ $\frac{12}{5}$
B $\frac{1}{4} \times \frac{2}{5}$ $\frac{1}{10}$ D $3\frac{1}{3} \times 6$ 20 F $1 \div \frac{5}{6}$ $\frac{6}{5}$ H $3 \div 27\frac{1}{9}$ J $2\frac{1}{4} \times \frac{1}{3}$ $\frac{3}{4}$

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Discussion

Assign the exercises as independent work. When children finish, discuss in particular any problems which caused difficulty. You might stress the place-value concepts with decimals, particularly with hundredths, since the following chapter introduces percent. Multiplication and division with decimals should also be given special review.

11. Give the sign ($<$, $>$, $=$) for each ◍.

A $347 \text{ ◍ } 357$ ($<$) B $4 \div 2 \text{ ◍ } 4 \div \frac{1}{2}$ ($<$) C $5.763 \text{ ◍ } 5.367$ ($>$)

12. Find the average of the numbers in the set {9, 12, 17, 8}. $11\frac{1}{2}$

13. Find the total amounts.

A
$18.95
7.28
11.07
28.36
$65.66

B
$728.46
819.72
728.55
$2276.73

14. Find the differences in the amounts.

A
$5076.42
2748.59
$2327.83

B
$600.00
129.95
$470.05

15. Find the products and quotients.

A $3.14 \times 8 = 25.12$

B $1.783 \times 0.24 = 0.42792$

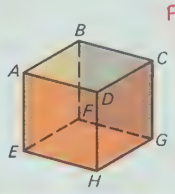
C $5\overline{)154.5} = 30.9$

D $0.64\overline{)7.68} = 12$

16. A Name a pair of parallel edges.

B Each edge is 2 units. What is the surface area? 24 the volume? 8

C What is the perimeter of square *ABCD*? 8



Possible answers, A:

$\overline{AD}, \overline{BC}$ $\overline{DC}, \overline{HG}$
$\overline{AD}, \overline{EH}$ $\overline{HG}, \overline{EF}$
$\overline{EH}, \overline{FG}$ $\overline{AE}, \overline{BF}$
$\overline{FG}, \overline{BC}$ $\overline{AE}, \overline{DH}$
$\overline{AB}, \overline{EF}$ $\overline{BF}, \overline{CG}$
$\overline{AB}, \overline{DC}$ $\overline{CG}, \overline{DH}$

17. Mr. Price can buy an airline ticket to Chicago for $36.50. If he uses a family plan, he can take his child for $\frac{1}{2}$ price and his wife for $\frac{1}{4}$ off the regular price.

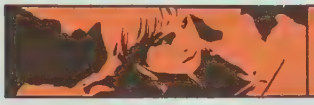
$18.25 A How much would it cost for his child?

$27.38 B How much would it cost for his wife?

$82.13 C How much would it cost for all three?

think

Can you place a mathematical symbol between the digits 5 and 6 so that you have a name for a number that is between 5 and 6? 5.6



You are invited to explore

ACTIVITY CARD 14 Page 362

303

CHAPTER 13 | Ratio and Percent

Pages 304–329

General Objectives

To introduce the term ratio
To provide experience with ratios
To provide a variety of word problems
To provide experience in working with ratios in scale drawings
To introduce the notation for percent
To provide experience in working with percent notation
To provide a variety of experiences with percent notation in word problems
To introduce the concepts of discount and interest

The initial lesson in this chapter uses the strips to introduce the language of ratio in a comparison of lengths. The following lesson defines equal ratios and relates the concept of ratio to some concepts of fractional numbers. This is followed by a set of word problems in which the children interpret and use the ratios.

Percent is introduced as a special kind of ratio. Percent notation for numbers is compared with fraction and decimal notation for numbers. Emphasis is placed upon the fact that the percent notation is simply another way of naming fractional numbers. Word problems provide a variety of additional work in computing with percent notation. These include problems about percent discount, interest, and circle graphs. An optional lesson in the chapter provides a variety of experiences in working with percent in equations. That lesson is designed primarily for more capable children but, in a carefully controlled situation, it may be assigned to the whole class.

The usual chapter and cumulative reviews complete the chapter.

Mathematics

A ratio is a comparison between two numbers. This chapter introduces applications of ratio and the useful language associated with such number pairs. Ratio is not an integral part of the mathematical development of the set of fractional numbers because it deals primarily with applications of arithmetic. In this section, therefore, we simply explain that the material on ratio given in the text has to do primarily with physical concepts. To help the children acquire some skill in computing with ratios, the idea of ratio is related to fractions and to fractional numbers. By identifying ratios with fractional numbers, the children should see that in certain situations they can find missing numbers in ratio sentences in much the same way that they found missing numerators and denominators in fractional-number sentences.

Percent is introduced as a special kind of ratio. The percent notation merely involves a new symbol, devised to represent certain fractional numbers. Since the set of numbers is the same, the mathematical concepts are all the same, and the same basic principles apply. The best evidence of this is that much of the work with percent is devoted to word problems which illustrate that using percent notation is primarily a matter of convenience rather than a matter of using new mathematical concepts.

Teaching the Chapter

Materials

Advertisements of sales, discount prices
Centimetre strips
Construction paper or cut-out red circles and blue squares
Crayons or colored pencils
Graph paper, no smaller than $\frac{1}{4}$-cm grid
Rulers, centimetre scales
Scale for weighing children (if available)

Vocabulary

discount
interest
percent
ratio
scale drawing

Although few materials are listed for this chapter, you will want to use familiar objects around the classroom as well as ideas associated with measurement to explain the idea of a ratio to the children. The centimetre strips which accompany this series are appropriate for developing concepts of ratio. It is important that each child feel familiar with the strips before they use them here.

Scale drawing is a familiar application of the concept of ratio. A ratio is set up between the physical model and the drawing so that the drawing can easily be interpreted. Constructing maps and drawings to scale reinforces for the children the idea of ratio as a comparison. They may enjoy making scale drawings or maps of their neighborhoods or school grounds. We recommend that you use diagrams involving 100 sections to help illustrate concepts of percent. Transparencies on the overhead projector are very helpful for this. It may also be helpful to exhibit a set of 100 objects and have the children select various subsets from this to illustrate concepts of percent. For example, you might exhibit 100 objects, represented by nine 10-strips and 10 units, so that they can easily be counted, and then call attention to 34 of these objects and observe that this represents 34% of the set. In presenting such demonstrations, however, take care that the children do not conclude that percent is associated only with $\frac{1}{100}$ of a region or with a set of 100 objects. The purpose of such demonstrations is to emphasize the basic meaning of percent and the fact that percent can be applied to a variety of physical concepts.

Lesson Schedule

This chapter is designed to be covered in about three weeks. However, since the school year is nearing an end, you may want to vary this schedule according to the time available. Because of the wide variety of percent problems available, you could easily spend considerably more than 3 weeks on the material in this chapter and on similar problems. On the other hand, if time is short, you may choose to treat only the basic concepts of ratio and percent so that you can move on and expose the children to integers and graphing, the topics covered in the last two chapters of the book.

Evaluation of Progress

While working with the ideas of ratio, the children should rely heavily on their experience with fractions and fractional numbers. In fact, ratio and the comparison of two numbers is so closely related to fractions and fractional numbers that the children may even wonder why it is considered a new topic. It is important for you to note that, although ratio concepts have been presented many times prior to this, the language of ratio was not used. Certainly, much of the work involving fractions and fractional numbers could be interpreted in terms of the ratio language, but we believe that it should be employed primarily when it can be useful for describing certain situations.

Since much of the material on percent involves applications and word problems, your task of evaluating children's grasp of this concept should not be difficult. It is desirable for the children to understand the idea of percent, and you can usually find out whether they do by giving them problems. Much of your evaluation for this chapter can be based on a written test.

Although percent is associated with fractional-number concepts, most of the work with percent should be centred on practical, everyday problem-solving situations, because our use of percent is almost always associated with computing interest, discounts, and so on. Throughout your work in this chapter, supply a wide variety of physical situations involving the use of percent.

The chapter review on pages 326 and 327 will help you evaluate the children's understanding of the concepts developed. Maintenance of skills and concepts previously introduced can be checked on pages 328 and 329.

Resources for Active Learning

GENERAL ACTIVITIES

Developmental Math Cards, J#7 and 10, Addison-Wesley [Rate problems]

Experiences in Mathematical Ideas, Volume 2, "Ratio," pp. 69–95, NCTM

Freedom to Learn, "Ratio and Proportion," p. 147, Addison-Wesley

MANIPULATIVE DEVICES

Map Measurer (Edmund Scientific; Math Media)

COMMERCIAL GAMES

Stocks and Bonds (Selective Educational Equipment)

PAGES 304–305

Objective

Given two numbers, the child will be able to compare them using the language of ratio, such as 2 to 4 or 3 to 5.

Preparation

Materials

colored strips

Although the language of ratio was introduced in Book 5 of *Investigating School Mathematics*, you will want to treat this concept as new material. To prepare for this lesson, review the number names of the strips *when the number one has been assigned to the white strip.* You might also review relationships between certain strips, since the investigation uses these relationships to introduce the concept of ratio. For example, review the fact that 2 purple strips match a brown strip, or that 3 light green strips match a blue strip, and so on.

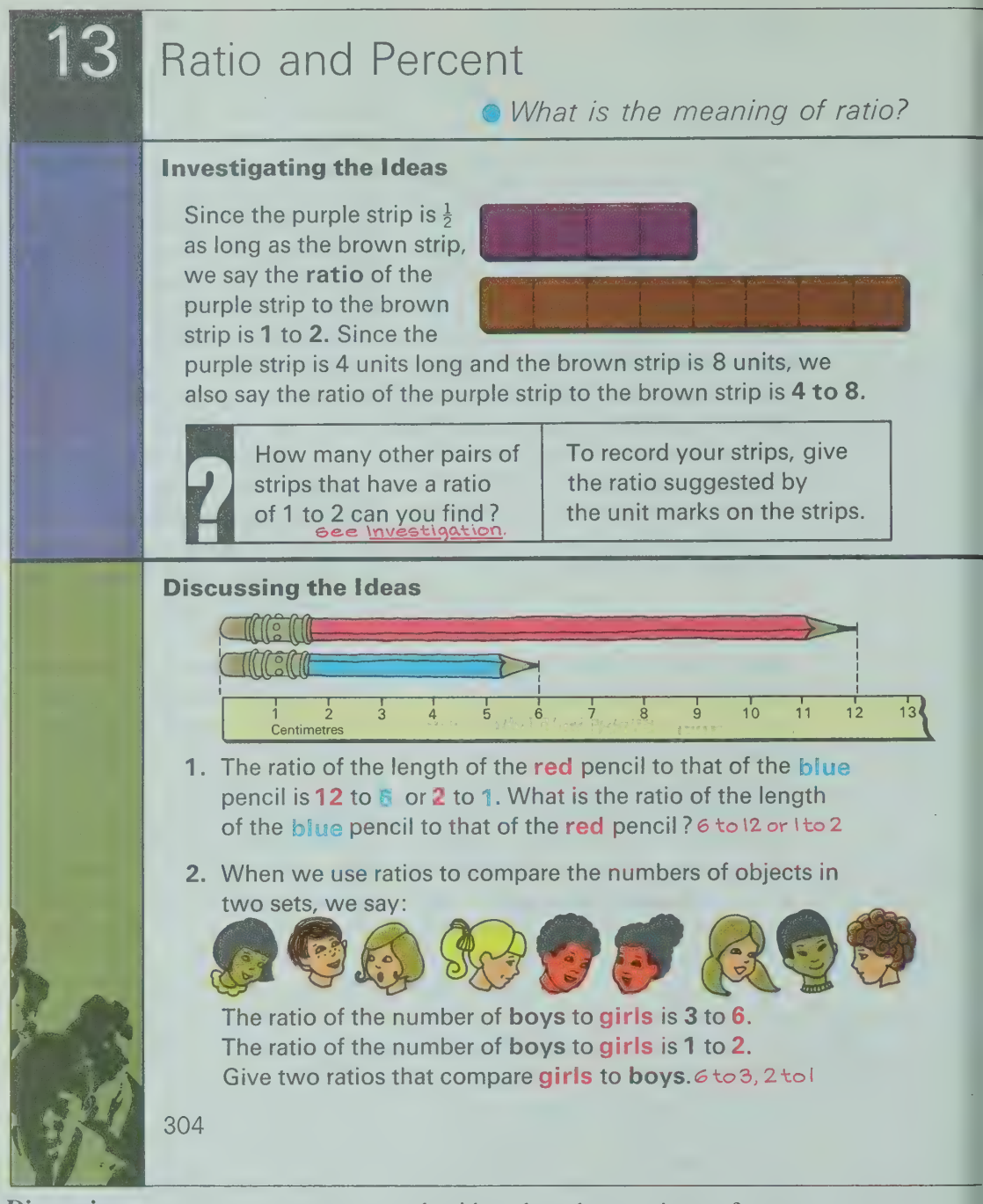
Investigation

Read the introductory material with the children. Point out that a ratio may be thought of simply as another way of comparing two numbers. When the children see that the ratio of the purple strip compared to the brown strip is 1 to 2, encourage them to continue with the investigation independently. You might suggest that they use the following format to record their findings:

Strips Compared	Ratio
yellow to orange	*(5 to 10)*
red to purple	*(2 to 4)*
lt. green to dark green	*(3 to 6)*
white to red	*(1 to 2)*

(Answers are shown for your convenience.)

You might suggest that children who finish quickly combine some of their strips and compare one strip with a train of two strips; for instance, they might compare the black strip to the orange and purple, or the blue strip to the orange and brown.



13 Ratio and Percent

What is the meaning of ratio?

Investigating the Ideas

Since the purple strip is $\frac{1}{2}$ as long as the brown strip, we say the **ratio** of the purple strip to the brown strip is **1** to **2.** Since the purple strip is 4 units long and the brown strip is 8 units, we also say the ratio of the purple strip to the brown strip is **4 to 8.**

?	How many other pairs of strips that have a ratio of 1 to 2 can you find? See Investigation.	To record your strips, give the ratio suggested by the unit marks on the strips.

Discussing the Ideas

1. The ratio of the length of the **red** pencil to that of the **blue** pencil is **12** to **6** or **2** to **1.** What is the ratio of the length of the **blue** pencil to that of the **red** pencil? 6 to 12 or 1 to 2

2. When we use ratios to compare the numbers of objects in two sets, we say:

The ratio of the number of **boys** to **girls** is **3** to **6.**
The ratio of the number of **boys** to **girls** is **1** to **2.**
Give two ratios that compare **girls** to **boys.** 6 to 3, 2 to 1

304

Discussion

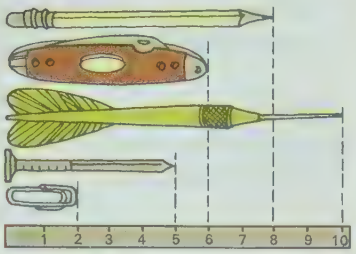
Before discussing the exercises in the text, it would be helpful to discuss the investigation material. When children have given the ratios they found for the strips, it would be helpful to show that for each pair, different ratios may be considered. When we consider the length of the light green strip in comparison to the dark green strip, we say the ratio is 3 to 6, or 1 to 2. However, we might consider the dark green strip in comparison to the light green strip, in which case we say the ratio of their lengths is 6 to 3, or 2 to 1.

Discussion exercise 1 stresses the idea that the numbers of two sets may be compared to each other in two different ways: the ratio of the length of the red pencil to that of the blue pencil is 12 to 6; the ratio of the length of the blue pencil to that of the red pencil is 6 to 12. Also, emphasize the fact that either of these ratios may be expressed in different ways: we may say the ratio of the length of the red pencil to that of the blue pencil is 12 to 6, or we may say that it is 2 to 1.

Emphasize these same points in exercise 2. Be sure children see why we can say that the ratio of the number of boys to girls is 3 to 6 as well as calling it 1 to 2.

Using the Ideas

1. Give the ratio of these lengths.
   - A paper clip to nail 2 to 5
   - B nail to pencil 5 to 8
   - C knife to nail 6 to 5
   - D nail to paper clip 5 to 2
   - E pencil to nail 8 to 5



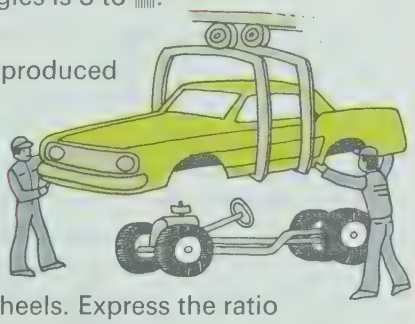
2. For each exercise, give the ratio of the lengths. Use two different pairs of numbers.
   - A dart to nail
     Answer: 10 to 5, 2 to 1
   - B paper clip to knife 2 to 6, 1 to 3
   - C knife to pencil 6 to 8, 3 to 4
   - D dart to knife 10 to 6, 5 to 3
   - E knife to dart 6 to 10, 3 to 5
   - F pencil to paper clip 8 to 2, 4 to 1
   - G dart to paper clip 10 to 2, 5 to 1

3. Give the missing numbers.



   - A The ratio of triangles to squares is 4 to ▥. 6
   - B The ratio of triangles to squares is 2 to ▥. 3
   - C The ratio of squares to triangles is 6 to ▥. 4
   - D The ratio of squares to triangles is 3 to ▥. 2

4. A In an auto factory, each car produced requires 5 tires. Give the ratio of cars to tires. 1 to 5



   - B Each car requires 2 headlights. Give the ratio of tires to headlights. 5 to 2
   - C Each 8-cylinder car has 4 wheels. Express the ratio of wheels to cylinders in two different ways. 4 to 8, 1 to 2
   - D A machine makes 24 parts for an automobile in 8 hours. Give the ratio that compares parts with hours. 24 to 8
     How many parts does the machine make per hour? 3

Resources for Active Learning

Teaching Aids for Elementary Mathematics, "Correspondence and Mapping," pp. 126–127, Holt, Rinehart and Winston.

Workbook, page 93

During this discussion, mention the fact that ideas concerning ratio are often much like those involving fractions and fractional numbers. For example, some of the children may notice that 3 to 6, and 1 to 2 suggest $\frac{1}{2} = \frac{3}{6}$. However, this concept will be more fully developed in subsequent lessons.

Encourage the children to use ratio terminology when discussing the problems on this page and the next. It is important for them to become familiar with the language of ratio that is used throughout this chapter.

Using the Exercises

Ask the children to do the exercises on page 305. When they have finished, allow time for discussion and checking papers.

Again point out that the ratio of a first number to a second is different from the ratio of that second number to the first. Thus, the ratio of the length of the paper clip to the nail (2 to 5) is different from the ratio of the length of the nail to the paper clip (5 to 2).

Assignments (page 305)

Minimum: 1–3. Average: 1–4.
Maximum: 1–4.

PAGES 306–307

Objective

Given two ratios, the child will be able to determine whether they are equal and will be able to express equal ratios in an equation.

Preparation

Materials (optional)

red circles (12 per child) and blue squares (8 per child) or construction paper and scissors to make these

To prepare for this lesson, you might think of sets of objects in the classroom and ask children to give certain ratios. For example, ask them to compare the number of desks to the number of children; or the number of windows to the number of doors; or the number of children to the number of teachers and teacher aides; or the number of erasers at the chalkboard to the number of children sitting in a row or at a table.

Investigation

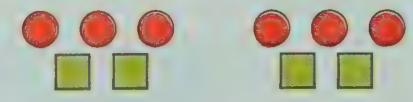
Although children may do this investigation simply by drawing the groupings on paper, you might prefer to have some children use actual circles and squares. Point out that, with each grouping they form, they should always have 12 circles and 8 squares. Remind them to express the ratio of their grouping numerically, as they did in the previous lesson. Since the sets of objects remain constant here, the ratios for this investigation are equal ratios. The ratio of the number of circles to the number of squares may be expressed as 3 to 2, 6 to 4, 9 to 6, and 12 to 8.



● *What are equal ratios?*

Investigating the Ideas

There are 3 circles for every 2 squares.



The ratio of circles to squares is 3 to 2.



Can you show some other groupings of these figures that suggest other ratios? See Investigation.

Discussing the Ideas

1. The Investigation shows that different pairs of numbers can be used to express the same ratio. Give other ratios that are the same as 3 to 2. Answers may vary. 6 to 4, 9 to 6, 12 to 8, etc.



2. To express the ratio of the number of **books** to **children**, you can write **2** to **4** or you can write **1** to **2**.

We often use a colon (:) to write a ratio.
The ratio of **books** to **children** is **2** : **4**.
The ratio of **books** to **children** is **1** : **2**.
Since 2 : 4 and 1 : 2 are the same ratio, we write: 2 : 4 = 1 : 2
Give three more ratios that are equal to 1 : 2.
Answers may vary. 2:4, 3:6, 4:8, etc.

3. Work with equal ratios is much like work with equivalent fractions. Explain how to find the missing numbers in A and B.

A $\frac{3}{4} = \frac{\square}{12}$ 9

B $3 : 4 = \square : 12$ 9

306

Discussion

Ask children to show their groupings on the overhead projector or chalkboard, writing the ratio for each.

One important point to continue to stress is that work with equal ratios is much like work with equivalent fractions and fractional numbers. Use exercises 2 and 3 to discuss this point and to help children understand that ratios may be expressed in different ways. The ratio of books to children in exercise 2 may be shown as 2 to 4, as 2:4, or as $\frac{2}{4}$. Similarly, equal ratios may be shown as 2 to 4 = 1 to 2; as 2:4 = 1:2; or as $\frac{2}{4} = \frac{1}{2}$—all of which are read "2 is to 4 as 1 is to 2." Use exercise 3 to show how the problem of finding a missing number in a ratio may be thought of as finding a missing term in a statement with equivalent fractions.

It would be helpful to create a variety of comparisons to give children an opportunity to work with equal ratios. For example, you might construct several problems similar to the Short Stories on page 307. Or, you might distribute a duplicated sheet like the one suggested in the follow-up section. You might also use tables on the chalkboard, such as those in exercise 1 on page 307.

Using the Ideas

1. Each table gives several number pairs for the same ratio. Give the missing numbers.

A

2 : 1	
4 : 2	
6 : ▒	3
8 : ▒	4
10	▒ : 5

B

3 : 2	
6 : 4	
9 : ▒	6
12 : ▒	8
15	▒ : 10

C

4 : 5	
8 : ▒	10
12 : ▒	15
16	▒ : 20
20 : ▒	25

D

3 : 4	
6 : ▒	8
9	▒ : 12
12 : ▒	16
15	▒ : 20

2. For the first part of each exercise, think about fractional numbers. For the second part, think about ratios. Find the missing numbers.

A $\frac{1}{2}=\frac{▒}{8}$ 4 — 1:2 = ▒:8 4

B $\frac{3}{10}=\frac{9}{▒}$ 30 — 3:10 = 9:▒ 30

C $\frac{3}{4}=\frac{6}{▒}$ 8 — 3:4 = 6:▒ 8

D $\frac{▒}{3}=\frac{8}{6}$ 4 — ▒:3 = 8:6 4

E $\frac{▒}{8}=\frac{15}{24}$ 5 — ▒:8 = 15:24 5

F $\frac{2}{▒}=\frac{14}{21}$ 3 — 2:▒ = 14:21 3

G $\frac{3}{▒}=\frac{12}{4}$ 1 — 3:▒ = 12:4 1

H $\frac{▒}{2}=\frac{30}{20}$ 3 — ▒:2 = 30:20 3

I $\frac{1}{2}=\frac{▒}{100}$ 50 — 1:2 = ▒:100 50

J $\frac{5}{2}=\frac{25}{▒}$ 10 — 5:2 = 25:▒ 10

K $\frac{2}{▒}=\frac{10}{15}$ 3 — 2:▒ = 10:15 3

L $\frac{10}{4}=\frac{▒}{40}$ 100 — 10:4 = ▒:40 100

Short Stories RATIO

Write a fractional-number equation and solve it to answer these ratio problems.

1 Ratio of men to women, 4:5. 20 women. How many men? (Solution: $\frac{4}{5}=\frac{n}{20}$, $n = 16$)

2 Ratio of batteries to flashlights, 2:1. 9 flashlights. How many batteries? $\frac{2}{1}=\frac{n}{9}$, $n=18$





3 Ratio of scouts to tents, 3:1. 4 tents. How many scouts? $\frac{3}{1}=\frac{n}{4}$, $n=12$

4 Ratio of candy bars to cents, 1:5. 3 candy bars. How many cents? $\frac{1}{5}=\frac{3}{n}$, $n=15$

5 Ratio of length to width, 3:2. Length, 12. What is the width? $\frac{3}{2}=\frac{12}{n}$, $n=8$

6 Ratio, kilometres to litres, 15:1. 9 litres. How many kilometres? $\frac{15}{1}=\frac{n}{9}$, $n=135$



More practice, page A-23, Set 46



Follow-up

A worksheet such as the following may be useful during the discussion or as a follow-up.

Give at least two equal ratios for each of these situations.

A ✱ ✱ ✱ ✱ ○ ○ ○ ○ ○ ○

B △ △ △ △ □ □ □

C 

D 2 cars 8 people

E 12 times at bat 4 hits

Resources for Active Learning

Math Activity Cards, "Ratios," E38, Macmillan.

Workbook, page 94

Using the Exercises

Ask the children to do the exercises on page 307 independently. If necessary, review the method of finding a missing term in a pair of equivalent fractions. Note that the directions for the Short Stories call for writing fractional-number equations to solve them. When children finish and as you are helping them check their work, have children write several of these equations and their solutions on the chalkboard. Remind children that when statements such as $\frac{4}{5}=\frac{n}{20}$ are used with ratio problems, they should be read in the language of ratio: "4 is to 5 as n is to 20."

Assignments (page 307)

Minimum: 1, 2. Average: 1, 2, Short Stories. Maximum: 1, 2, Short Stories.

PAGES 308–309

Objective

Given problems whose solutions may be found by using ratios, the child will be able to solve the problems.

Preparation

Materials

scale for weighing children (if available)

Children would benefit from a review of writing equations with ratios and finding missing terms. You might display on the overhead projector or chalkboard a few tables of equivalent ratios such as those in exercise 1 on page 307 and have children find the missing numbers.

3 : 1	3 : 5
6 : 2	? : 10
? : 9	? : 45
? : 12	15 : ?
72 : ?	? : 30

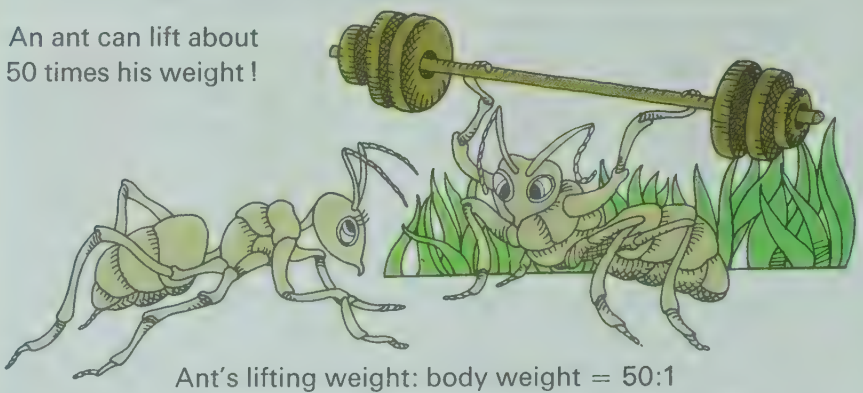
Investigation

Since many children might not know how much they weigh, try to have a scale available. (Otherwise, each child can simply estimate his weight.) Even though each child has to use his own weight or an estimate of his own weight, some may benefit from sharing ideas about the solution. Children will have to understand that the ratio for their lifting weight to their body weight must follow the same order of the ratio for the ant's lifting weight to its body weight. Suggest that they study the problem and set up a statement of equal ratios. If a child's weight is 45 kg, his statement should be $50:1 = n:45$, or $\frac{50}{1} = \frac{n}{45}$, so his lifting weight would be 2250 kg! If children find the answer to the question by straightforward reasoning, without actually setting up an equation, that is acceptable; such a child already has a firm grasp of the relationships involved in equal ratios.

How can you use ratios in problem solving?

Investigating the Ideas

An ant can lift about 50 times his weight!



Ant's lifting weight: body weight = 50:1

? If you could lift as much for your weight as an ant can for his weight, how many kilograms could you lift? See Investigation

Discussing the Ideas

1. Explain how you found the weight you could lift if the ant ratio were true for you. See Discussion.
2. Is the weight you found more than one tonne? yes
3. Suppose 20 000 ants weigh a total of one kilogram. If they could all lift together, how much weight could they lift? 50 kg
4. Suppose that a man can safely lift twice his weight.
   - A What is the ratio of his "lifting weight" to his weight? 2:1
   - B How many kilograms should such a person weigh to lift 150 kilograms? 75 kg 200 kilograms? 100 kg 90 kilograms? 45 kg
5. Suppose you could lift only 18 kilograms. What is the ratio of this weight to your body weight? Answers will vary.

308



Discussion

Have children compare results and explain how they found them. Point out the balance in their statement of equal ratios.

ant's lifting weight: its body weight = their lifting weight : their body weight

This may be read as "The ratio of the ant's lifting weight to its body weight is the same as the ratio of their lifting weight to their body weight." However, be sure children realize that this statement represents a supposition, or an imaginary situation. Use some of the remaining discussion questions to make it clear that in actuality the ratio of a man's lifting weight to his body weight is far from 50 to 1.

Ratio Fantasy

Using the Ideas

1. Assuming that for a man, as for an ant, the ratio of the weight lifted to body weight is 50 to 1, how much could a 90-kilogram man lift? How many tonnes is this? 4500; 4.5



2. The ratio of the weight a bee can pull (on wheels) to the bee's body weight is 300:1.
   - A If the ratio were the same for a man, how many kilograms could a 90-kg man pull? 27000
   - B How many tonnes is this? 27
   - C How many kilograms could you pull at this ratio? Answers will vary.

3. The ratio of a grasshopper's length to the distance he can jump is about 1:20. Use this ratio to find how many metres a 2-metre man could jump. 40 m



4. The ratio of a flea's body length to the distance he can jump is about 1:200. How far could you jump if you could jump in the 1:200 ratio? Answers will vary.

5. The ratio of the number of metres travelled to the number of minutes to travel that distance is 5:1 for the very slow giant tortoise of the Mauritius.
   - A How long would it take the tortoise to "run" the 100-metre dash? 20 min
   - B How long would it take the tortoise to "run" the 400-metre dash? 80 min

★6. The ratio of a champion athlete's height to the distance he can broad jump is 2:9.
   - A Given the same ratio, how far could a 2-cm cricket jump? 9 cm
   - B A cricket can actually jump about 60 cm. How many of the "short jumps" must he make to pass the 60-cm mark? 7 cm

★7. In Diane's math class the ratio of girls to boys was 5 to 6. 3 new girls came into the class. Now there are as many girls as boys. How many students are now in the class? 36



309

Resources for Active Learning

A Cloudburst, Vol. 2, No. 3713-53, Midwest Publications.

Developmental Math Cards, "How much do your feet suffer?" L 4, Addison-Wesley.

Math Activity Cards, "Inclined Plane," E30, Macmillan.

Workbook, page 95

Using the Exercises

Before assigning the exercises on page 309, be sure children realize that the statements are purely hypothetical. Allow the children considerable freedom in the solution of these problems. Some may be able to set up equations involving equal ratio (proportions) and use the ideas of equivalent fractions to find the solution. Others may use more intuitive reasoning techniques. For example, in exercise 1, a child might reason: For every 1 kilogram of his own weight, a man can lift 50 kilograms, so a 90-kilogram man could lift 90 × 50 kilograms, or 4500 kilograms. When the children have finished, ask volunteers to explain the solution for each problem. Although exercises 6 and 7 are starred, all might benefit from an explanation of them.

If children have difficulty with exercise 7, they might use a trial-and-error method, finding equal ratios until the difference of the two numbers is 3. For example: 5 to 6, 10 to 12, 15 to 18. Since 18 − 15 = 3, the original count must have been 15 girls and 18 boys, so 3 more girls gives a total of 36 students.

Assignments (page 309)

Minimum: 1–5, oral. Average: 1–5. Maximum: 1–7.

PAGES 310–311

Objective

Given a scale drawing and the scale used, the child will be able to find the actual distances represented by the drawing.

Preparation

Materials

centimetre ruler (1 per child)

If you think children need a review of equal ratios, present a few problems like those below and ask children to find the missing term.

$$\frac{2}{5} = \frac{n}{15}$$

$$\frac{3}{7} = \frac{n}{28}$$

$$\frac{4}{1} = \frac{n}{5}$$

Depending on the time you wish to spend on this preparation and the ability of the children, you might include a few ratios in which the missing term is a mixed numeral:

$$\frac{7}{10} = \frac{n}{15} \rightarrow 10 \times n = 7 \times 15 \rightarrow n = 10\frac{1}{2}$$

$$\frac{8}{3} = \frac{14}{n} \rightarrow 8 \times n = 3 \times 14 \rightarrow n = 5\frac{1}{4}$$

If you prefer a less formal preparation, you might simply discuss scale drawing.

Investigation

Many children will find the distances requested by the investigation question without thinking of equal ratios. For example, since the distance on the map from Lakeview to Seashore is $5\frac{1}{4}$ cm, they can readily see that the actual distance is $5\frac{1}{4}$ times 5 km, or $26\frac{1}{4}$ km. This is the expected approach. However, if children have done very well with the ratio concept or if they themselves think of applying the ratio, encourage them to attempt to express the problems in the form of a proportion:

Lakeview to Seashore
1 cm : $5\frac{1}{4}$ cm = 5 km : n
Seashore to Crusoe =
1 cm : $3\frac{3}{4}$ cm = 5 km : n

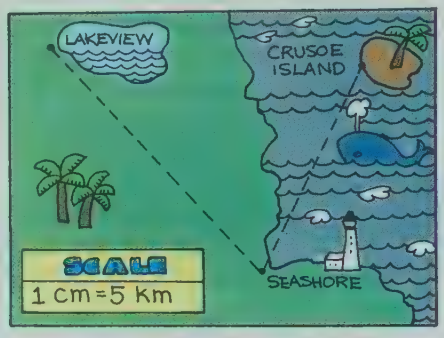


● *How can you use scale drawings?*

Investigating the Ideas

Maps that are drawn to scale give information about distances. 1 cm on this map represents an actual distance of 5 kilometres. We say the scale is:

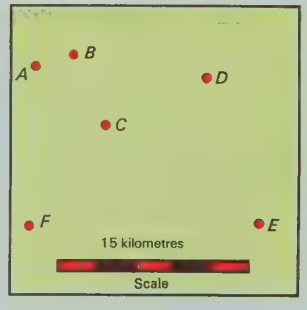
1 centimetre = 5 kilometres



? Can you use your ruler and the map to find the distance from Lakeview to Seashore and the distance from Seashore to Crusoe Island? $26\frac{1}{4}$ km; $18\frac{3}{4}$ km See Investigation

Discussing the Ideas

1. Explain how you used your ruler on the map to find distances. See Discussion.
2. How can you think of the scale on the map as a ratio? See Discussion.
3. Give the two distances for the map above if the scale is:
   - A $\frac{1}{4}$ cm = 8 km 168, 120
   - B 1 mm = 4 km 210; 150
   - C $\frac{2}{5}$ cm = 10 km $134\frac{1}{4}$, $93\frac{1}{4}$
   - D $\frac{1}{2}$ cm = 20 km 210; 150
4. Sometimes the scale for a map is shown as a segment that represents a given number of kilometres. On this map the segment is 30 mm and represents 15 kilometres. Explain how to find the following distances: See Discussion.



$3\frac{1}{2}$ A City *A* to City *B* $12\frac{1}{2}$ D City *D* to City *E*
$6\frac{1}{2}$ B City *B* to City *C* 20 E City *E* to City *F*
10 C City *C* to City *D* 20 F City *F* to City *D*

310

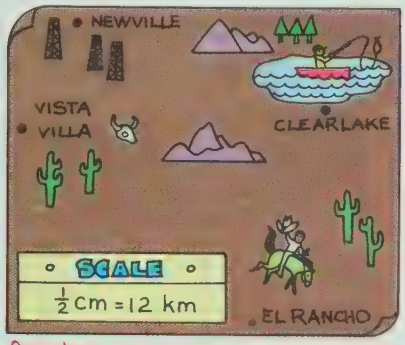
Discussion

Give children ample opportunity to discuss exercises 1 and 2 and share their findings from the investigation. If the use of ratio and ratio statements arises naturally, develop it with the examples from the investigation. However, this lesson stresses the concept of scale drawing, so ratio statements need not be emphasized.

In exercise 3, the children are to find the same two distances, Lakeview to Seashore and Seashore to Crusoe, with various scales. Observe with them that the smaller the scale, the more miles are represented.

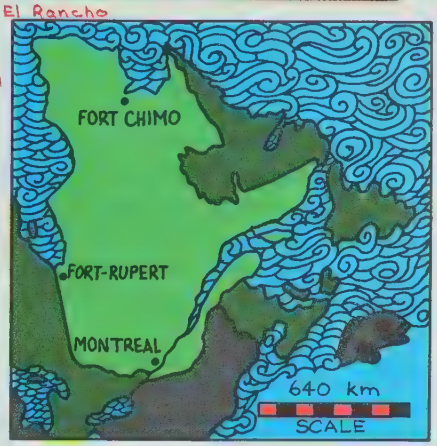
Using the Ideas

1. Use this map and scale to give the following distances:
   - A Newville to Clearlake 108 km
   - B Vista Villa to Clearlake 126 km
   - C El Rancho to Newville 138 km
   - D Vista Villa to El Rancho 120 km
   - E Clearlake to El Rancho 90 km



2. Where on the map would you show a city that is 18 km from Newville and between Newville and El Rancho?
   ¾ cm from Newville, between Newville and El Rancho

3. Use the map and the scale to give the following distances:
   - A Fort Chimo—Fort Rupert 793.6 km
   - B Fort Rupert—Montreal 512 km
   - C Montreal—Fort Chimo 1126.4 km

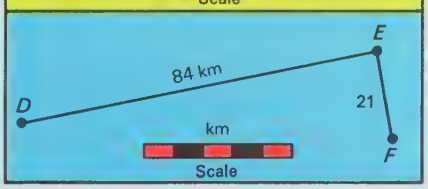


4. Toronto is about 560 km from Montreal. On a map where 1 cm = 60 km, how many centimetres would Toronto be from Montreal? 9.33 cm

★ 5. Study the maps carefully. [A]
   - A Use your ruler to find the scale for map A. 1 cm = 33 km
   - B Use your ruler to find the scale for map B. 1 cm = 14 km



★ 6. Draw your own map, using the scale 1 cm = 10 km. Show 3 cities, *A*, *B*, and *C*, and tell the distance between each pair. [B] Answers will vary.



311



Using the Exercises

It is not essential to encourage children to write statements with equal ratios for the problems on page 311. Many will prefer to work directly for the answer using their understanding of scale drawing. When children finish, have volunteers explain their solutions.

Assignments (page 311)
Minimum: 1, 3. Average: 1–4.
Maximum: 1–6.

Follow-up

Encourage children to make a scale drawing of their route from home to school. They should choose their scale of centimetres to kilometres according to the distances they travel.

Resources for Active Learning

(See the resources listed on page 199 for maps and scales.)

Workbook, page 96

PAGES 312–313

Objective

Given a fraction with a denominator which is a factor of 100, the child will be able to express it as a percent and as a decimal.

Preparation

Materials

graph paper, no smaller than $\frac{1}{2}$-cm grid (*Duplicator Masters*, pages 62, 63); crayons or colored pencils; demonstration 10-by-10 square grid

Children would benefit from a review of changing lowest-terms fractions first to fractions that have a denominator of 100, and then to decimals. For example, display the following chart on the chalkboard and ask them to complete it.

Fractions		Decimals
$\frac{3}{10}$	$\frac{?}{100}$	?
?	$\frac{?}{100}$	.75
?	$\frac{40}{100}$	?
$\frac{1}{4}$	$\frac{?}{100}$	?

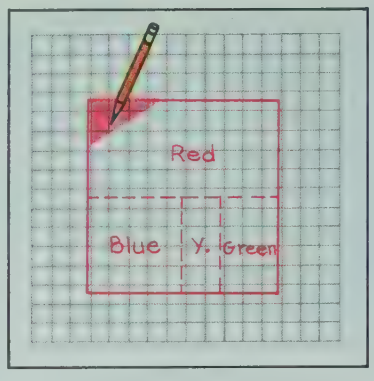
Investigation

In the investigation, children simply identify and color fractional parts of a 10-by-10 square region. Since the square is a 10-by-10 grid, the children are working with fractional parts of 100. It is important that they identify and color each fractional region correctly, since this 10-by-10 square provides background for introducing the concept of percent.

● *What is the meaning of percent?*

Investigating the Ideas

Outline a 10-by-10 square region on graph paper.



? Can you color $\frac{1}{2}$ of the square red, $\frac{1}{4}$ blue, $\frac{1}{10}$ yellow, and the rest green? See above and Investigation.

Discussing the Ideas

1. A How many small squares make up the region in the Investigation? 100
   B How many did you color red? 50 blue? 25 yellow? 10 green? 15
   C What is the ratio of red squares to all squares? 50:100 or 1:2

2. A special ratio which compares a number to 100 is called a **percent**. The symbol for percent is %.
   Examples: 50% means 50:100, $\frac{50}{100}$, or .50.
   13% means 13:100, $\frac{13}{100}$, or .13.
   A In the Investigation, what percent of the square region was colored blue? 25%
   B One-tenth of the square region was colored yellow. What percent was colored yellow? 10%
   C What percent of the square region did you color green? 15%



312

Discussion

If possible, use a large demonstration grid or a 10-by-10 grid on the overhead projector for this discussion.

As you work through discussion exercises 1 A and B, relate the fractions, $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{10}$ to equivalent fractions with a denominator of 100. Also, ask the children to express the green region as a fractional part of the 100 small squares. Use part C of exercise 1 to point out how $\frac{50}{100}$, $\frac{25}{100}$, $\frac{10}{100}$, and $\frac{15}{100}$ may also be thought of as ratios.

Call particular attention to the explanation of *percent* in exercise 2. Children might be interested in the fact that the word *percent* is derived from the Latin words; *per* meaning "for," and *centum* meaning "one hundred." As you work through exercise 2, stress that percent is simply a special case of the ratio concept and the percent notation is simply a new way of representing fractional numbers. Again, use the regions of the 10-by-10 grid to relate the various ways of indicating a particular area.

$\frac{1}{4} = \frac{25}{100}$ or 25:100

$\frac{25}{100} = 0.25 = 25\%$

Using the Ideas

1. Study the first row of the chart to see how the percent symbol (%) is used. Give the missing symbols for the rest of the table.

Region	Fraction		Decimal	Percent
	Lowest terms	Hundredths		
	$\frac{3}{4}$	$\frac{75}{100}$	0.75	75%
	$\frac{1}{4}$	$\frac{25}{100}$	A 0.25	25%
	$\frac{1}{5}$	$\frac{20}{100}$	0.20	B 20%
	$\frac{3}{5}$	$\frac{60}{100}$	C 0.60	D 60%
	$\frac{7}{10}$	E $\frac{70}{100}$	F 0.70	G 70%

2. Give the missing numerators.

A $25\% = \frac{\text{|||}}{100}\ 25$ B $20\% = \frac{\text{|||}}{100}\ 20$ C $80\% = \frac{\text{|||}}{100}\ 80$ D $10\% = \frac{\text{|||}}{100}\ 10$

3. Give the correct percent.

A $\frac{25}{100} =$ ||| 25% C $\frac{15}{100} =$ ||| 15%

B $\frac{75}{100} =$ ||| 75% D $\frac{18}{100} =$ ||| 18%

4. Give the percent for each exercise.

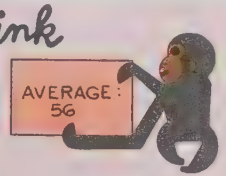
A Since $\frac{3}{5} = \frac{60}{100}$, we know $\frac{3}{5} =$ ||| 60%.

B Since $\frac{9}{10} = \frac{90}{100}$, we know $\frac{9}{10} =$ ||| 90%.

C Since $\frac{1}{4} = \frac{25}{100}$, we know $\frac{1}{4} =$ ||| 25%.

D Since $\frac{1}{20} = \frac{5}{100}$, we know $\frac{1}{20} =$ ||| 5%.

think



The average of 8 numbers is 56. Three of the numbers are 72, 57, and 49. What is the average of the other 5 numbers? 54

Using the Exercises

Encourage the children to work independently on the exercises on page 313. As they work, you might emphasize for some that they are simply dealing with various ways of expressing one fractional number. In particular, stress that in exercise 1, in each of the four columns to the right of the column labelled "Region," there is a different name for one fractional number.

Assignments (page 313)

Minimum: 1–3. Average: 1–4.
Maximum: 1–4.

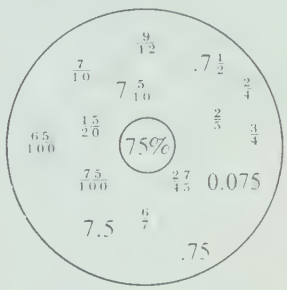
Follow-up

To give children practice in converting fractional notation to decimal and percent notation, and back again, provide them with a worksheet like one of the following.

Names for Fractional Numbers

Complete the table.							
Fraction			$\frac{2}{5}$				$\frac{7}{5}$
Decimal		0.38			0.05		
Percent	16			2		90	

Cross out the expressions which do not name 75%.



Resources for Active Learning

Developmental Math Cards, "Percentage," K¹12. Addison-Wesley.

Workbook, page 97

PAGES 314–315

Objective

Given decimals such as $0.12\frac{1}{2}$, $0.33\frac{1}{3}$, *and* 0.005, *the child will be able to express them by using percent notation.*

Preparation

Materials

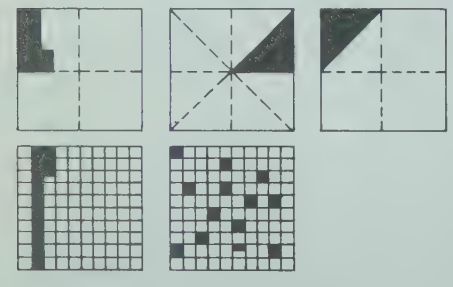
graph paper (*Duplicator Masters*, page 62)

Children would benefit from a review of changing fractions to decimals by using the dividing process. You might list some fractions on the chalkboard and ask children to give the decimal equivalent for each. Remind them that $\frac{1}{4}$ may be thought of as 1 divided by 4. When they consider a fraction such as $\frac{1}{3}$ remind them that it may be written as a mixed decimal numeral: $0.33\frac{1}{3}$.

Fraction	Decimal
$\frac{1}{2}$	0.50
$\frac{1}{4}$	0.25
$\frac{1}{3}$	$0.33\frac{1}{3}$
$\frac{1}{5}$	0.20
$\frac{2}{5}$	0.40

Investigation

For the first part of the investigation, children may simply count the shaded squares to find that $\frac{1}{8}$ of the large square consists of $12\frac{1}{2}$ small squares. They should then be able to think of a variety of ways to shade $\frac{1}{8}$ of the large square. Some of the possible shadings are:

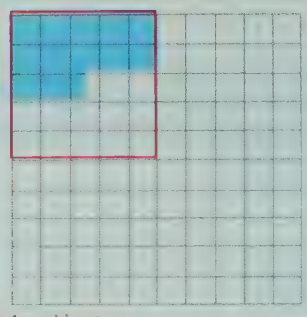


While the children work, it would be helpful for you to move around the classroom and pose questions for them to consider: "What fractional part or percent of the 10-by-10 square is each small square?" ($\frac{1}{100}$ or 1%) "What fractional part or percent of the 10-by-10 square is the medium-size square?" ($\frac{25}{100}$ or 25%) "How much of the square have you been shading?" ($\frac{1}{8}$)

Let's explore more about percent notation.

Investigating the Ideas

One eighth of the large square region is shaded. How many small squares make up $\frac{1}{8}$ of the large square? $12\frac{1}{2}$



Can you show some other ways to shade $\frac{1}{8}$ of a 10-by-10 square?

See Investigation.

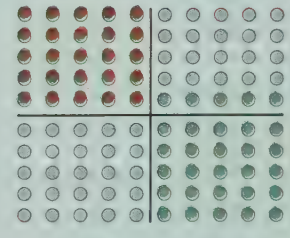
Discussing the Ideas

1. The shaded region above is $12\frac{1}{2}$ small squares. What percent of the large square is shaded? $12\frac{1}{2}\%$

2. A Since $\frac{1}{3} = 0.33\frac{1}{3}$, how many small squares would you have to shade if you wanted to shade $\frac{1}{3}$ of a 10-by-10 square? $33\frac{1}{3}$
   B What percent of the large square would be shaded? $33\frac{1}{3}\%$

3. A Each small square is what percent of the 10-by-10 square? 1%
   B How would you shade $\frac{1}{2}\%$ of the large square? $\frac{1}{2}$ of 1 small square

4. A What percent of the coins are copper? 25%
   B What percent are lead? 30%
   C What percent are silver? 45%
   D What percent are **not** lead? 70%



copper silver lead



Discussion

Give the children an opportunity to show their shadings on a demonstration 10-by-10 square or on the overhead projector. Use the various shadings to discuss some of the questions suggested in the investigation. Then use the discussion exercises to stress that, if they have shaded $12\frac{1}{2}$ squares, then they have shaded $12\frac{1}{2}\%$ of the large square. Point out this relation:

$$\frac{1}{8} = 0.12\frac{1}{2} = 12\frac{1}{2}\%$$

If necessary, for exercise 2, again review how the dividing process may be used to express $\frac{1}{3}$ as a decimal. You might ask these same questions for such other fractions as $\frac{1}{5}$, $\frac{3}{4}$, and $\frac{2}{3}$, being mindful that this lesson particularly emphasizes percents which may be written as mixed decimals. Continue to stress how a fractional number such as $\frac{3}{4}$ may be expressed as a fraction, a decimal, or a percent. Thus $\frac{3}{4}$, 0.75, and 75% are simply different ways of expressing one fractional number.

As you work through exercise 4, you might ask other questions, such as: "What percent of items illustrated are coins?" (100%) "What percent of the coins are iron?" (zero percent) Questions involving 100% and 0% will aid the children in future work with percent.

Using the Ideas

1. Find the numbers for ***a***, ***b***, and ***c***.

- A Since $\frac{2}{8} = \frac{1}{4} = 25\%$, we know that $\frac{1}{8} = a\%$. $12\frac{1}{2}$
- B Since $\frac{1}{8} = a\%$ and $\frac{2}{8} = b\%$, we know that $\frac{3}{8} = c\%$. $12\frac{1}{2}, 25, 37\frac{1}{2}$
- C Since $\frac{4}{8} = a\%$ and $\frac{1}{8} = b\%$, we know that $\frac{5}{8} = c\%$. $50, 12\frac{1}{2}, 62\frac{1}{2}$
- D Since $\frac{8}{8} = a\%$ and $\frac{1}{8} = b\%$, we know that $\frac{7}{8} = c\%$. $100, 12\frac{1}{2}, 87\frac{1}{2}$
- E Since $\frac{3}{3} = a\%$, we know that $\frac{1}{3} = b\%$. $100, 33\frac{1}{3}$
- F Since $\frac{1}{3} = a\%$, we know that $\frac{2}{3} = b\%$. $33\frac{1}{3}, 66\frac{2}{3}$
- ★ G Since $1.25 = \frac{125}{100}$, we know that $1.25 = a\%$. 125
- ★ H Since $2 = \frac{200}{100}$, we know that $2 = a\%$. 200
- ★ I Since $\frac{1}{6} = 0.16\frac{2}{3}$, we know that $\frac{1}{6} = a$ $16\frac{2}{3}$
- ★ J Since $\frac{1}{8} = 12\frac{1}{2}\%$, we know that $\frac{1}{16} = a\%$. $6\frac{1}{4}$

2. Copy each table and give the missing percents.

Fraction	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{0}{5}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{5}{5}$
Percent	25% A	50% B	75% C	0% D	20% E	40% F	60% G	80% H	100% I

Fraction	$\frac{0}{8}$	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{3}{8}$	$\frac{4}{8}$	$\frac{5}{8}$	$\frac{6}{8}$	$\frac{7}{8}$	$\frac{8}{8}$
Percent	0% J	$12\frac{1}{2}\%$ K	25% L	$37\frac{1}{2}\%$ M	50% N	$62\frac{1}{2}\%$ O	75% P	$87\frac{1}{2}\%$ Q	100% R

Fraction	$\frac{1}{3}$	$\frac{2}{3}$
Percent	$33\frac{1}{3}\%$ S	$66\frac{2}{3}\%$ T

$\frac{1}{6}$	$\frac{5}{6}$
★ $16\frac{2}{3}\%$ U	★ $83\frac{1}{3}\%$ V

think

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1. What percent of the whole numbers from 1 through 100
- A have 2 as a factor? 50%
- B have 5 as a factor? 20%
- C are prime numbers? 25%

2. What percent of these numbers are prime?
- A 1 through 50 30%
- B 51 through 100 20%



Using the Exercises

Some children may need some guidance regarding the kind of reasoning they must use in exercise 1 on page 315. Help them see that the first missing numbers they find for each statement should be used to find the remaining missing numbers. In exercise 2, be sure children realize that the dividing process may be used first to write a fraction as a decimal and then to write the decimal as a percent.

Even though the *Think* problem is challenging, you will probably find that all the children will understand and enjoy a discussion of it. They should all be able to count the number of squares that show 2 as a factor or 5 as a factor and then arrive at the correct percentage for that particular part of the problem, since there are exactly 100 squares.

Assignments (page 315)
Minimum: 1A–F, 2A–T.
Average: 1, 2. Maximum: 1, 2.

Duplicator Masters, page 56
Workbook, page 98
Skill Masters, page 56

PAGES 316–317

Objective

Given word problems involving a percent of a number, the child will be able to solve the problem by using either the decimal or fraction equivalent of the percent.

Preparation

Since the method of solving most of the problems in this lesson will be multiplication with either decimals or fractions, you might want to provide a review of multiplication similar to the following:

$$428 \times 0.38 = n$$
$$637 \times 0.04 = n$$
$$905 \times 0.67 = n$$
$$217 \times 0.42 = n$$
$$64 \times \tfrac{1}{4} = n$$
$$762 \times \tfrac{1}{2} = n$$
$$522 \times \tfrac{2}{3} = n$$
$$81 \times \tfrac{4}{5} = n$$

Investigation

This investigation presents the child with two problems: First, he must change $66\frac{2}{3}\%$ to a fraction or to a decimal; then he must determine how much of his body weight is water. There are a variety of ways in which a child might reason to solve this problem. For example, he might use the fact that $66\frac{2}{3}\% = \frac{2}{3}$ and find $\frac{2}{3} \times$ body weight = water. Or, he might round the decimal equivalent of $66\frac{2}{3}\%$ to 0.67 and use this equation: water = 0.67 × body weight.

Encourage the children to share ideas since this investigation should prove challenging to many. If some do not know how to begin, help them change $66\frac{2}{3}\%$ to the fraction $\frac{2}{3}$. Then give them an opportunity to try the rest of the problem.

● *Let's solve some percent problems.*

Investigating the Ideas

Your body is about $66\frac{2}{3}\%$ water.



? Can you change this percent to a decimal or a fraction and then find out how many pounds of your body are water? See Investigation.

Discussing the Ideas

When you work with percent problems, you use either fractions or decimals to do the computing. You will find that sometimes it is easier to use a decimal, while other times it is easier to use a fraction. In each problem below a decimal and a fraction are given for the percent. Tell which one you would use to solve the problem and explain why.

1. A man saved 25% of $848. How much did he save?
   25% = 0.25 = (1/4) $212
2. 44% of the people in a small town are adults. There are 725 people. How many adults? 44% = (0.44) = 11/25 319
3. In a basketball game, Tom made exactly $37\frac{1}{2}\%$ of the shots he attempted. He attempted 24 shots. How many shots did he make? $37\frac{1}{2}\% = 0.375 = (\frac{3}{8})$ 9
4. About 68% of 97 square kilometres is covered by water. How much of the area is covered with water? 68% = (0.68) = 17/25 65.96
5. A man paid 24% of his yearly income in taxes. His income was $8357. How much tax did he pay? 24% = (0.24) = 6/25 $2005.68

316



Discussion

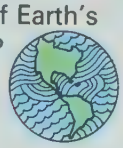
Ask children to explain the methods they used to solve the investigation problem. Help them compare methods and discuss which one seems easiest. Point out that sometimes it is easier to use a decimal and sometimes it is easier to use a fraction. In some cases, it is not obvious which method is easier and you might encourage children to then use the method, fraction or decimal, with which they are most comfortable. In discussion exercise 1, it is easier to use the fraction $\frac{1}{4}$. In exercise 2, multiplying by 0.44 involves fewer steps than multiplying by $\frac{11}{25}$. The use of $\frac{3}{8}$ clearly is easier than 0.375, in exercise 3. For exercises 4 and 5, most will probably agree that it is easier to use the decimals: $0.68 \times 97 = n$ and $0.24 \times 8357 = n$.

Short Stories Water

Using the Ideas



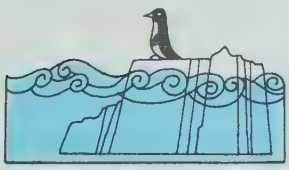
1. Earth's surface: 75% water.
   - A What fraction is this? $\frac{3}{4}$
   - B What percent of Earth's surface is land? 25%



2. Ocean: water and dissolved minerals. Minerals: $3\frac{1}{2}$%. What percent water? $96\frac{1}{2}$%

3. A certain type of rock is 3% water. How much water in a 45-kg rock of this type? 1.35 kg

4. Blood: 90% water.
   - A Adult: 4.75 litres of blood. How much water? 4.28 litres
   - B Child: 2.38 litres of blood. How much water? 2.14 litres



5. Floating ice: $12\frac{1}{2}$% above the surface.
   - A What percent is below the surface? $87\frac{1}{2}$%
   - B What fraction is above the surface? $\frac{1}{8}$

6. When water freezes, its volume increases by about 9%. Freeze 500 cubic centimetres of water.
   - A How much does the volume increase? 45 cm^3
   - B What is the volume of the ice? 545 cm^3

7. A Milk: $87\frac{1}{2}$% water. What fraction is this? $\frac{7}{8}$
   - B Boiled potato: 80% water. What fraction is this? $\frac{4}{5}$
   - C Fried chicken: 52% water. Give the lowest-terms fraction for this percent. $\frac{13}{25}$
   - D Egg: about $\frac{3}{4}$ water. What percent is this? 75%
   - E Bread: 36% water. How many grams of water in a 480-gram loaf? 172.8 g
   - F Apple: 85% water. How many kilograms of water in 10 kilograms of apples?

★8. About $\frac{1}{30}$% of all water on Earth evaporates each year. Give a fraction for this percent. $\frac{1}{3000}$

★9. When water changes to steam, its volume increases by 167 000%. What is the volume of the steam from 1 cubic centimetre of water? 1670 cm^3



More practice, page A-24, Set 47

Resources for Active Learning

A Cloudburst, Vol. 3, No. 3515-45, Midwest Publications.

Using the Exercises

With some children you will want to work through several of the problems on page 317 together. Continue to relate the percent given in each problem to its fraction or decimal equivalent. Many children will find a problem easier to interpret when thought of as a problem involving fractions. Others may prefer to interpret most of the problems in terms of decimals.

When they have finished, encourage discussion and comparison of solutions. Notice that problems 8 and 9 are particularly challenging. You might also note that the theme of the problems on this page is water, and the children may become interested in the various facts presented about water. Therefore, you should allow time for discussion of the word-problem theme as well as for the arithmetic involved in finding the solutions for the problems.

Assignments (page 317)

Minimum: 1–6. Average: 1–7. Maximum: 1–9.

PAGES 318–319

Objective

Given the regular price and the discount rate, the child will be able to find the amount of discount and the sale price.

Preparation

Material

advertisements for sales (optional)

To prepare for this lesson, ask the children the meaning of the phrases *20% off* and *save 25%*. Discuss sales and how stores advertise for sales. You might read from an advertisement a variety of expressions used to indicate sales. Do not give them explanations such as those suggested for the discussion section. Simply be sure they have a general familiarity with sales and the terminology used.

Investigation

With some children, you may want to distribute samples of newspaper advertisements of sales to provide a source of practical problems, but be sure they do not involve difficult arithmetic. The three examples in the investigation deal purposefully with the easier percents that are related to familiar fractions. Encourage children to work together, possibly in assigned groups, and share their ideas for solving one of the problems. If some children finish quickly, suggest that they solve the other two problems or that they use advertisements you give them.

How can you find the amount of discount and the sale price?

Investigating the Ideas



Save $15 Save $2.00 Save $12

Can you find how much you would save by buying one of the items on sale? See above and Investigation.

Discussing the Ideas

1. The amount you save by buying on sale is called the **amount of discount.** The percent off the regular price is the **discount rate.**
   - A Which of the **discount rates** above is largest? $33\frac{1}{3}$ %
   - B Which item has the largest amount of discount, the bicycle or the record player? Bicycle

2. What fraction is the discount rate for the record player? $\frac{1}{3}$

3. Explain how you would find the **sale price** of the bicycle.
   $75 − 15 = $60

Reg. Price	$75	Sale Price
Discount Rate	×0.20	?
Discount	$15.00	

4. How would you find the sale price of a tennis racket?
   $8 − ($8 × .25) = $8 − $2 = $6

Discussion

Use discussion exercise 1 as a basis for explaining the meaning of the phrases *amount of discount* and *discount rate*. Be sure the children can identify the discount rate in each of the investigation problems. Discuss the procedures they suggest for finding solutions. You might find it helpful to express correct solutions in equation form, as in this example, for the bicycle:

$$20\% \text{ of } \$75.00 = n$$
$$0.20 \times 75 = n$$
$$\tfrac{1}{5} \times 75 = n$$

Once children understand how to find the amount of discount, stress that this number must be subtracted from the regular price to find the sale price. Thus, each problem in the investigation has two steps.

(1) discount rate × regular price = amount of discount

(2) regular price − amount of discount = sale price

Using the Ideas

1. Study the first two rows of the table. Then give the missing entries.

Original price	Percent discount	Amount of discount	Sale price
$10.00	10%	$1.00	$9.00
$10.00	25%	$2.50	$7.50
$ 5.00	10%	$0.50	A $4.50
$ 5.00	25%	$1.25	B $3.75
$12.00	$33\frac{1}{3}$%	C $4.00	D $8.00
$ 8.00	20%	E $1.60	F $6.40
$16.00	$12\frac{1}{2}$%	G $2.00	H $14.00
$30.00	15%	I $4.50	J $25.50

2. Each of the following is an original price. Give the sale price if the discount is 10%.

$9.00 A $10.00 $45.00 C $50.00 $5.76 E $6.40

$3.60 B $4.00 D $25.00 $22.50 F $1.80 $1.62

3. Give the sale price for each part of exercise 2 if the discount is 25%.
See Answers, T.E. page 319.

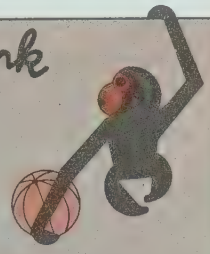
4. Use the given price and percent discount to find the sale price.

$9.50 A $10.00; 5% $12.50 D $25.00; 50%

$4.00 B $5.00; 20% $30.00 E $50.00; 40%

$7.20 C $9.00; 20% $24.00 F $36.00; $33\frac{1}{3}$%

think



A basketball team won 4 more games than it lost. If it won $\frac{3}{5}$ of its games, how many games did it play? 20

★ **5.** Which store will sell the dress at the lowest price? What is the sale price?
Store B; $18.00





Follow-up

Suggest that children clip advertisements of sales from newspapers and then make up and solve some problems these advertisements suggest. The problems could be presented in the form of a chart, as shown below.

Inventory Sale			
Item	Original Price	% Off	Sale Price
Record	$4.79	22%	?
Sandals	$4.94	20%	?
Guitar	$31.00	?	$18.60
TV set	$75.00	12%	?

Solution, Think, page 319

Organizing guesses in a table may help the children solve the *Think* problem as a class activity.

Games	Guess 1	Guess 2	Guess 3
Lost	2	4	6
Won	2 + 4 6	4 + 4 8	6 + 4 ?
Total	2 + 6 8	4 + 8 12	?
% of total won	$\frac{6}{8} = 75\%$	$\frac{8}{12} = ?$	?
Check	Too big	?	?

Workbook, page 99

Using the Exercises

Depending on how well children understand the concepts of discount and sale price, you might prefer to use several of the exercises on page 319 as a basis of discussion. In exercise 2, point out that, since $10\% = \frac{10}{100} = \frac{1}{10}$, using a 10% discount rate is the same as dividing by 10. Dividing by 10 may be done mentally, and indicated by moving the decimal point one place to the left. Similarly, in exercise 3, using a discount rate of 25% is the same as using the fraction $\frac{1}{4}$, or simply dividing by 4. Properly used, relationships such as these simplify the work.

Answers, exercise 3, page 319

2. A $7.50
   B $3.00
   C $37.50
   D $18.75
   E $4.80
   F $1.35

Assignments (page 319)*

Minimum: 1, 2. Average: 1–4.
Maximum: 1–5.

PAGES 320–321

Objective

Given word problems involving the use of percents, the child will be able to solve the problems by using his understanding of percent.

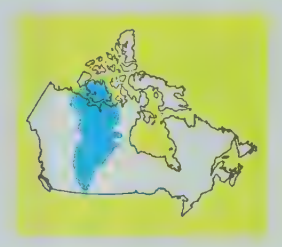
Preparation

Review any concepts pertaining to percent which have been troublesome for the children. In particular, you might review the fact that a percent is a special kind of ratio.

Solving Story Problems

GREENLAND

Greenland is the world's largest island. It is a county of Denmark, even though it is separated from Denmark by 2080 km of ocean.



Greenland is about 20% of the size of Canada.



1. Greenland is about 20% of the size of Canada. The area of Canada is 9 975 800 square kilometres. About what is the area of Greenland? 1 995 160 km^2
2. The area of Denmark is about 2% of the area of Greenland. About what is the area of Denmark? 39 903.2 km^2
3. The ratio of the population of Greenland to the population of Denmark is 1:150. If the population of Denmark is 4 581 000, about what is the population of Greenland? 30 540
4. About 85% of Greenland is covered with ice. About how many square kilometres is that? 1 695 886 km^2
5. Greenland is 2670 kilometres from north to south. The northern tip of Greenland is just 716 kilometres from the North Pole. It is the world's northernmost land. About how far is the southern tip of Greenland from the North Pole? 3386 km
6. Most of Greenland lies above the Arctic Circle. Look at the map above and estimate the percent of Greenland that lies above the Arctic Circle. 80%

320

Discussion

Since many children will find the problems on pages 320 and 321 challenging, you might want them to work in groups of three or four, or you might want to use some of the problems in a teacher-guided discussion. However, if you think the children have a sufficient understanding of the concept of percent, you probably will want to encourage them to work through the exercises individually.

Point out to them that the information needed to solve some of the problems must be found from a previous problem. For example, the answer to exercise 4, page 320, depends on the answer obtained for exercise 1.

Also stress how fractions may sometimes replace percents, as in problem 1 of page 320. It is easier to multiply 3 615 000 by $\frac{1}{4}$ than by 0.25.

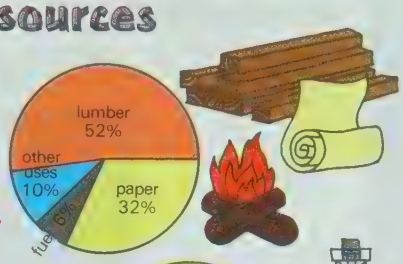
Assignments (page 320)

Minimum: 1–6, oral. Average: 1–6. Maximum: 1–6.

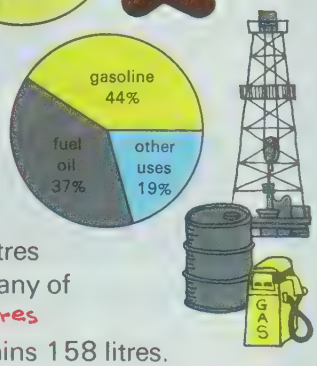
Solving Story Problems

Using Our Natural Resources

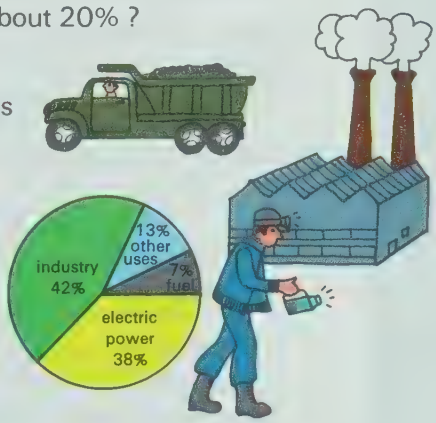
1. The circle graph shows approximate percents of three major uses of wood.
   - A The amount of wood used for lumber is about how many times the amount used for paper? 1½ times
   - B If wood were measured by the kilogram, how many kilograms of each tonne of wood are used for fuel? 60 kg



2. This graph shows the approximate percent of petroleum used for two products.
   - A An average oil well produces about 2160 litres of oil a day. According to the graph, how many of those litres are used for gasoline? 950 2/5 litres
   - B Oil is measured by the barrel. A barrel contains 158 litres. According to the graph, about how much petroleum from each barrel is used for fuel oil? 58.46 litres
   - C About 4 litres of each barrel of petroleum is used for jet fuel. Is this less than 1%, about 2%, or about 20%?



3. The graph shows approximate percents for some common uses of coal.
   - A Industry and production of electric power together use what percent of the total amount of coal produced? 80%
   - B Of each tonne of coal, how many kilograms are needed for fuel? 70kg
   - C Railroads use about 10 kilograms of each tonne of coal mined. What percent is that? 1%



321

Using the Exercises

Do not give the entire class any rules for solving the problems on page 321; encourage them to reason through each problem on their own, using their knowledge of percent. If any children do need help with the problems, you might remind them of certain facts about percent. For example, percent expresses a special kind of ratio, so in problem 2, they might think of 44% as 44 in relation to 100, and set up the ratio statement $44 : 100 = n : 2160$. Or, they may realize that 44% of 2160, or 0.44×2160, is the number they are looking for.

Do not expect children to work these problems quickly. Allow them time to think through each problem carefully, and discuss their solutions with each other.

Assignments (page 321)

Minimum: 1, 2, oral. Average: 1–3. Maximum: 1–3.

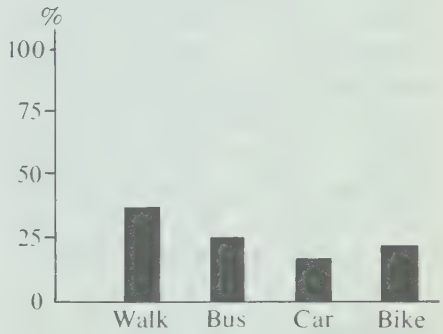
Follow-up

Children would benefit from applying their knowledge of percents to interesting topics in the classroom. For example, they might collect data on various means of transportation the class members use to get to school. Suggest that they set up a graph showing in percent notation the number of children who use each means of transportation. Here is sample data and a graph for a class of 32.

Data

Means of Transport	No.	Percent of Class
Walk	12	$\frac{12}{32} = 37\frac{1}{2}\%$
Bus	8	$\frac{8}{32} = 25\%$
Car	5	$\frac{5}{32} = 0.156 \approx 16\%$
Bike	7	$\frac{7}{32} = 0.219 \approx 22\%$
Other	0	$\frac{0}{32} = 0\%$

Graph



Resources for Active Learning

Nuffield Project: *Problems* – Red Set, No. 24, Wiley.

PAGES 322–323

Objectives

The child will understand the meaning of the terms savings, interest rate, *and* amount of interest.

Given the amount of savings and the interest rate, the child will be able to find the simple interest saved in one year.

Preparation

To prepare for this lesson, give children an opportunity to discuss savings accounts. If any children have accounts, ask them to describe what they know about them. Depending on the familiarity children have with bank services, spend as much time as necessary to help them understand the terms *deposit, borrow, savings, loans*. Also be sure they realize that to borrow money, they must pay for it according to a certain rate, and when they deposit savings, they earn money according to a rate because the bank "pays" the depositor for letting them use his savings. The more familiar children are with these banking processes, the better they will understand the concepts regarding interest that are treated in this lesson.

Investigation

This investigation gives children an opportunity to participate in the role of banker, borrower, or depositor. Suggest that the children divide themselves into these three groups. Then, members of each group can take their part in acting out the investigation. Children might also make play money to use in their transactions. Encourage children to try to figure out among themselves how to apply the 5% interest rate and the 10% rate for loans. It would also be helpful to write other, similar problems on the chalkboard, for those who already understand this banking process. For example, ask children what the bank would earn on these transactions:

Deposits (5%)	Loans (10%)
$250	$300
$300	$200
$ 50	$250

What is the meaning of interest?

Investigating the Ideas



? Can you find how much the bank would earn all together in these two transactions? $5 See Investigation.

Discussing the Ideas

1. When you put your money in a savings account, the money the bank pays you for the use of your money is called **interest**. How much interest would $100 earn at a 5% yearly **interest rate**? $5
2. When you borrow you must pay interest for using the bank's money. How much interest would you pay for $100 if the yearly interest rate for loans were 10%? $10
3. How would you find the interest on these savings?

Save	Yearly interest rate	Amount of interest
 for one year.	4%	? $12

4. How much money would you have at the end of one year? $312

322

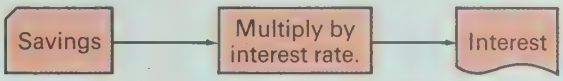


Discussion

Use discussion exercises 1 and 2 to explain the terms used in the investigation and to encourage children to share their results for the investigation. Work exercise 3 together. Note that exercise 4 refers to the problem in exercise 3. Help children realize that the amount of interest should be added to the amount saved in order to find the total amount referred to in exercise 4.

Using the Ideas

1. Follow the interest flow chart to find the amount of interest for each amount of savings and rate of interest.



A Savings: $200 Rate: 4% $8.00

B Savings: $500 Rate: 5% $25.00

C Savings: $1000 Rate: 6% $60.00

D Savings: $700 Rate: 3% $21.00

2. Study the first row of the table. Then give the missing entries for the rest of the table.

Savings	Interest rate (per year)	Amount of interest in one year	Savings + interest
$100.00	2%	$2.00	$102.00
$ 10.00	4%	A $0.40	B $10.40
$100.00	3%	C $3.00	D $103.00
$ 50.00	2%	E $1.00	F $51.00
$ 25.00	5%	G $1.25	H $26.25
$ 76.00	4%	I $3.04	J $79.04
$124.00	$3\frac{1}{2}$%	★ K $4.34	L $128.34

★ **3.** Suppose a man invests $1000 at 5 percent per year. At the end of each year, he leaves his interest in the account to draw interest the next year. How much would he have in his account at the end of 3 years? $1157.63

More practice, page A-24, Set 48



Using the Exercises

According to the need of the children, work through several of the exercises on page 325 together. Many children may still need help with the computation processes. For example, be sure no one confuses 4% and the fraction representation of 25%, $\frac{1}{4}$. Point out how with percents such as 4%, 5%, 6%, and 3%, they may simply multiply by the single digit 4, 5, 6, or 3 and then place the decimal point according to the rule for hundredths.

In exercise 2, you will probably want to explain the column headed "Savings + Interest." Be sure the children understand that, since interest on savings is an earning for the saver, it is added to the original savings amount deposited.

The starred exercise is intended for children who have understood the concepts of the lesson quite well. With such children, you might want to be sure they understand the idea of compound interest described in the problem.

Assignments (page 323)*

Minimum: 1, 2. Average: 1, 2. Maximum: 1–3.

Follow-up

If children's understanding of saving and borrowing money is sufficiently well developed, and they can find amounts of interest without too much guidance, suggest that they "simulate" what they would do if they had a certain amount of money. You might "give" most children a starting amount, such as $200 or $300, and call them "Spenders" or "Savers." Also, appoint bankers to whom you "give" about $2000 or $5000. Then suggest that the "Spenders" and "Savers" can deposit their money in the bank or borrow for some particular purchases you suggest:

Guitar:	$385	Bike:	$ 75
Skis:	170	Stereo set:	350
Ski boots:	80		

You might also set a fixed interest rate for savings and loans:

Savings: 5%, every six months
Loans: 7% of amount owed to be added to monthly payments

This type of project would best be extended so that each day of school is equivalent to a month in the project. Thus, after two days, children who borrowed money would have made two payments. After 6 days, any amount they have saved would have earned 5% interest. You might also give them a monthly "salary" of $15.00 or $20.00.

A project such as this must be adapted to the ability of the children. However, even if it takes a while for them to learn the procedures involved, a continuing project which simulates an aspect of actual money transactions can have great educational value.

Workbook, page 100

PAGES 324–325

Objective

Given an equation which involves percents and which has an unknown factor or product, the child will be able to find the missing term.

Preparation

To prepare for this lesson, you might simply review equations with missing terms, like those in discussion exercise 1. For example, write the following equations on the chalkboard and ask children to find the missing terms.

$4 \times n = 20$
$8 \times 6 = n$
$n \times 9 = 54$
$7 \times n = 21$
$12 \times n = 144$
$n \times 8 = 56$

Stress how division may be used to find a missing factor. You might use more difficult numbers also.



★Can you solve percent problems?

Discussing the Ideas

1. Three types of simple equations are given in the examples. To solve example A, you must find the product of the two factors. In examples B and C, the products are given and you must find the missing factors.

A	B	C
$5 \times 7 = n$	$5 \times n = 35$	$n \times 7 = 35$

Complete the sentences.
- A You can solve A if you multiply 5 by ▮. 7
- B You can solve B if you divide 35 by ▮. 5
- C You can solve C if you __? __ 35 by 7. divide

2. Problems using percents are solved much the same as the examples in 1 above. Give the missing numbers.
- A To solve $8 \times 25\% = n$, you multiply 8 by ▮. .25
- B To solve $n \times 25\% = 2$, you divide 2 by ▮. .25
- C To solve $8 \times n = 2$, you divide ▮ by 8. 2

3. How can you express the number for n in 2C as a percent? 25%

A 10% of what number is 7?

10% × n = 7
n = 7 ÷ 10%
n = 7 ÷ $\frac{1}{10}$
n = ▮ 70

4. It is often helpful to write an equation before attempting to solve a percent problem. Study example A, then give the number for n.

B What percent of 12 is 9?

n × 12 = 9
n = $\frac{9}{12}$ = $\frac{3}{4}$
n = ▮% 75%

5. Study example B. Then give the missing percent. Note that you must change the fraction to a percent.

324

Discussion

Note that this entire page is starred, because it is designed primarily for the more capable children. However, you can use it for all the children in a carefully supervised situation.

Work through the exercises, stressing the method of finding missing factors by dividing.

When the children work percent problems which require finding the missing factor, it is important for them to recognize automatically that the process of division will yield the missing factor. Such equations often contain division problems involving decimals, and naturally, the work is more difficult than it is for these problems. Therefore, keep in mind that the most important thing for the children to gain from this discussion is the ability to apply the division operation automatically in the missing-factor problems. In exercises 4 and 5, note how the percent being thought about may be expressed as a fraction. It would be helpful to present examples in which the percent is expressed as a decimal.

Using the Ideas

1. Give the missing words and numbers.
   - A To solve $6 \times 7 = n$, you multiply 6 by ▮. 7
   - B To solve $9 \times n = 54$, you divide 54 by ▮. 9
   - C To solve $n \times 4 = 48$, you __? __ 48 by 4. divide
   - D To solve $30 \times 10\% = n$, you __? __ 30 by 10%. multiply
   - E To solve $n \times 10\% = 3$, you __? __ 3 by 10%. divide
   - F To solve $n \times 50\% = 4$, you divide 4 by ▮. 50

2. Solve each of the equations in exercise 1.
   A 42, B 6, C 12, D 3, E 30, F 8

3. Solve the equations. When your solution is a decimal or a fraction, give a percent for the solution.
   - A $10 \times n = 2$ Answer: $n = 20\%$
   - B $12 \times n = 3$ 25%
   - C $n \times 10\% = 12$ 120
   - D $60 \times n = 15$ 25%
   - E $20 \times n = 10$ 50%
   - F $n \times 60\% = 12$ 20
   - G $n \times 25\% = 7$ 28
   - H $16 \times n = 2$ $12\frac{1}{2}\%$

For each exercise below, write and solve an equation. Be sure to give a percent when you are asked to.

4. 25% of what number is 8?
   $n \times 25\% = 8$, $n = 32$
5. What percent of 25 is 5?
   $25 \times n = 5$, $n = 20\%$
6. 20% of what number equals 9? $n \times 20\% = 9$, $n = 45$
7. What percent of 50 is 40?
   $50 \times n = 40$, $n = 80\%$
8. 10% of what number is 15?
   $n \times 10\% = 15$, $n = 150$
9. What percent of 60 is 45?
   $n \times 60 = 45$, $n = 75\%$
10. $12\frac{1}{2}\%$ of what number is 6?
    $n \times 12\frac{1}{2}\% = 6$, $n = 48$
11. What percent of 8 is 1?
    $n \times 8 = 1$, $n = 12\frac{1}{2}\%$
12. 50% of what number is $7\frac{1}{2}$?
    $n \times 50\% = 7\frac{1}{2}$, $n = 15$
13. What percent of 4 is 3?
    $n \times 4 = 3$, $n = 75\%$
14. The sign in the store window said 20% off on all items. Tom figured that this sale would save him $2 on the football he wanted. What was the original cost of the football? $n \times 20\% = \$2.00$, $n = \$10.00$
15. Jane bought a $12 mini skirt. It was on sale for $9. What was the percent of discount?
    $12 \times n = 9$, $n = 25\%$
16. Tim had $20. If he spent $2, what percent of his money did he spend?
    $20 \times n = 2$, $n = 10\%$



Using the Exercises

According to the ability of the children, assign the problems on page 325 as independent work or use them as a basis for further discussion. Although children should know how to work with percents expressed as decimals, note that many of the problems involve percents for which children may use a fraction equivalent. Problems 14, 15, and 16 might stimulate worthwhile discussion.

Assignments (page 325)*
Minimum: 1–3. Average: 1–16.
Maximum: 1–16.

Duplicator Masters, page 57
Workbook, page 101
Skill Masters, page 57

PAGES 326–327

Objective

The child will demonstrate his ability to work with the concepts presented in this chapter.

Preparation

Review any topics from the chapter which the children found difficult. Concentrate your efforts on problems like those in exercises 4, 5, and 6, and on a variety of word problems.

Reviewing the Ideas

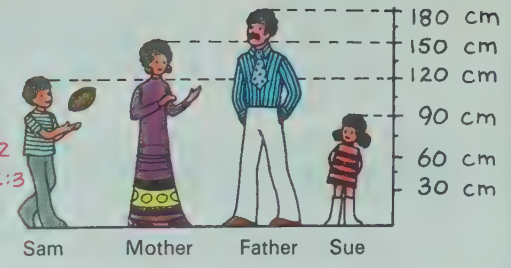
1. Give the following ratios.

A Sam's height to Sue's 4:3

B Father's height to Mother's 6:5

C Sue's height to Father's 3:6 or 1:2

D Sam's height to Father's 4:6 or 2:3



2. Copy the table and give the missing numbers. Each pair should give the same ratio.

4:3	
8:[6]	
[12]:9	
16:[12]	
[40]:30	

3. Give the missing numbers.

A 2:1 = 4:[2] D 5:2 = 10:[4]

B 2:[3] = 8:12 E 3:[10] = 30:40 4

C 4:1 = [12]:3 F [56]:8 = 77:88 7

4. Give the correct percent for each decimal.

A 0.75 75% B 0.70 70% C 0.07 7% D 0.23 23% E 1.00 100%

5. Give the lowest-terms fraction for each percent.

A 90% $\frac{9}{10}$ B 80% $\frac{4}{5}$ C $87\frac{1}{2}\%$ $\frac{7}{8}$ D 36% $\frac{9}{25}$ E 45% $\frac{9}{20}$

6. Compute.

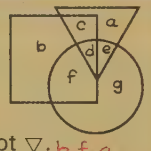
A 10% of 70 7 E $33\frac{1}{3}\%$ of 36 12 I 40% of 10 4 J 10% of 40 4

B 25% of 20 5 F 75% of 4 3

C 30% of 40 12 G 20% of 50 10

D 25% of 100 25 H 50% of 20 10

think



Give the letter or letters that are:

A in □ or ○ but not ▽. b, f, g

B in both □ and ○ but not ▽. f

C in □ and ○ and ▽. d

D in both □ and ▽ but not ○. c

★ **7.** Solve the equations.

100 A $n \times 10\% = 10$ E $n \times 33\frac{1}{3}\% = 12$ 36

10 B $n \times 10\% = 1$ F $n \times 50\% = 18$ 36

20 C $n \times 10\% = 2$ G $n \times 20\% = 20$ 100

32 D $n \times 25\% = 8$ H $n \times 40\% = 80$ 200

326

Discussion

Depending on the level of understanding children reached with these concepts, you might use pages 326 and 327 for evaluation or you might use them as a review. Continue to stress that percent is a special kind of ratio. Help children notice that in exercise 6, rather than multiply with the decimal form for each percent, they might find it easier to replace the percents with their fractional equivalents. For exercises 9 and 10, suggest that the children turn back to the lessons on discount and interest (pages 318–319 and 322–323). Even if children do these exercises independently, spend ample time in checking the solutions and discussing the problems.

★ **8.** Give the percent for n.

25% A $8 \times n = 2$ 10% D $80 \times n = 8$

50% B $8 \times n = 4$ 5% E $80 \times n = 4$

$12\frac{1}{2}\%$ C $8 \times n = 1$ $12\frac{1}{2}\%$ F $80 \times n = 10$

9. If the discount is 20%, give the sale price for each of the following.

A \$10.00 \$8.00 E \$15.00 \$12.00

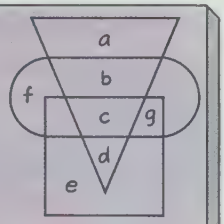
B \$5.00 \$4.00 F \$100.00 \$80.00

C \$1.00 \$.80 G \$25.00 \$20.00

D \$4.00 \$3.20 H \$17.00 \$13.60

think

Example:
d is in ▽ and □ but not ⬭.



A Describe the location of b. in ⬭ and ▽ but not □

B Describe the location of c. in ⬭ and ▽ and □

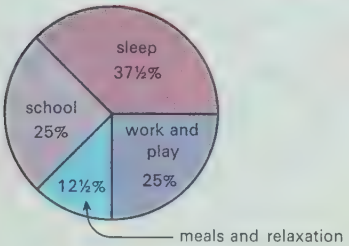
C Describe the location of e. in □ but not in ▽ or ⬭

D Describe the location of g. in □ and ⬭ but not in ▽

10. For the following amounts, give the amount of interest earned in one year at 4% per year.

A \$100 \$4 B \$10 \$0.40 C \$50 \$2 D \$25 \$1 E \$15 \$6

11.



The circle graph shows how Bill might spend an average 24-hour period. Give the numbers of hours for each of the four activities.
School, 6; sleep, 9; work and play, 6; meals, 3

12. In Alice's class, $33\frac{1}{3}\%$ of the children ride the bus. If there are 36 children in the class, how many ride the bus? 12

13. Robert said, "$\frac{1}{8}$ of the children in this school are in my class." What percent of the children are in Robert's class? $12\frac{1}{2}\%$

14. Pam said, "I spend 9 out of 12 months in school." What percent of the months does Pam spend in school? 75%

★ **15.** Tim brought home 6 boxes of strawberries. He said, "This is 10% of what I picked." How many boxes of strawberries did Tim pick? 60

Follow-up

A worksheet similar to the following may be used to help evaluate the children's understanding of the relationship between fractions, decimals, and percents, or simply as a reinforcement activity.

Write >, <, or = in each ○.	
1. 50%	○ $\frac{1}{2}$
2. $\frac{1}{2}^0$	○ 100×0.5
3. 1	○ 100%
4. $15 \div 0.5$	○ $0.5 \div 15$
5. 37.5	○ 3.75×10
6. 0.4	○ 4%
7. 3	○ 300%
8. $\frac{2}{3}$	○ 75%
9. 7×0.75	○ 7.5×0.7
10. 5280	○ 5.28×10
11. $\frac{3}{8}$	○ 40%
12. 10%	○ 10
13. $\frac{3}{7}$	○ $\frac{3}{7}\%$
14. 0.035	○ 4%
15. $12\frac{1}{2}\%$	○ $\frac{1}{9}$
16. 80%	○ $\frac{4}{5}$

Workbook, page 102

PAGES 328–329

Objective

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

Review any topics covered so far in the text which the children have found particularly difficult. It might be especially helpful at this time to review the basic principles for addition and multiplication of whole numbers and of fractional numbers.

Keeping in Touch with Computing Measurement Geometry Fractional numbers

1. Give the correct sign (<, >, =) for each ◍.

A $\frac{1}{3}$ ◍ 0.334 (<)

B $\frac{1}{2}$ ◍ $\frac{1}{4}$ (>)

C 56 754 ◍ 57 654 (<)

D 3.07 ◍ 3.007 (>)

E $3\frac{1}{8}$ ◍ 3.25 (<)

F $2\frac{5}{8}$ ◍ 2.50 (>)

2. Solve the equations.

A $587+395=n$ (982)

B $803-695=n$ (108)

C $324\times32=n$ (10 368)

D $1421\div53=n$ ($26\frac{43}{53}$)

E $617+n=1076$ (459)

F $n-374=297$ (671)

G $21\times n=1827$ (87)

H $n\div45=19$ (855)

3. Solve the equations.

A $\frac{3}{4}+\frac{1}{6}=n$ ($\frac{11}{12}$)

B $2\frac{1}{3}+4\frac{1}{2}=n$ ($6\frac{5}{6}$)

C $\frac{7}{8}-\frac{2}{5}=n$ ($\frac{19}{40}$)

D $3\frac{1}{2}-1\frac{3}{4}=n$ ($1\frac{3}{4}$)

E $\frac{1}{3}\times\frac{3}{5}=n$ ($\frac{1}{5}$)

F $\frac{3}{8}\times\frac{2}{3}=n$ ($\frac{1}{4}$)

G $5\times3\frac{1}{2}=n$ ($17\frac{1}{2}$)

H $2\frac{1}{3}\times1\frac{2}{5}=n$ ($3\frac{4}{15}$)

I $1\div\frac{3}{5}=n$ ($1\frac{2}{3}$)

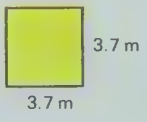
J $2\div\frac{3}{5}=n$ ($3\frac{1}{3}$)

K $\frac{1}{2}\div\frac{3}{5}=n$ ($\frac{5}{6}$)

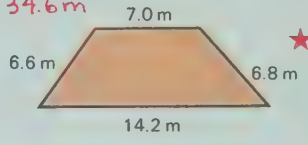
L $\frac{3}{8}\div\frac{5}{8}=n$ ($\frac{3}{5}$)

4. Find the perimeter of each figure. Write the answer by using the greatest possible number of the larger unit.

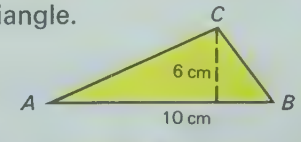
A 14.8 m



B 34.6 m



5. Find the area of this triangle. 30 cm²



6. Find the volume and surface area of this figure. V= 60 cu units SA= 94 sq units



★ **7.** This table is about squares. Find the missing numbers.

Perimeter	Area
8	A 4
4	B 1
C 16	16
D 28	49
10	E $6\frac{1}{4}$



ACTIVITY CARD 15 Page 362

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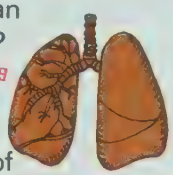
Discussion

With most children, assign the exercises on page 328 as independent work. Note that exercise 7 is intended as a special challenge. The column for area involves numbers that are perfect squares of whole numbers. You should encourage the children to explain how they reasoned in order to obtain the various answers. For exercise 7A, they should think, "If the perimeter is 8, each side would be 2; hence, I multiply 2 × 2 to get the area, which is 4."

THE HUMAN BODY

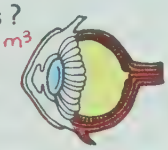
1. Your body contains 206 bones. About $\frac{1}{7}$ of these bones are in your head. About how many bones are there in your head? 29

2. Muscles make up about 0.4 of a man's body weight. What do the muscles of an 80-kg man weigh?



32 kg

3. In one day, a man breathes in about 11.4 cubic metres of air. About 0.05 of this is oxygen that is absorbed into the blood stream. How many cubic metres of oxygen is this?



0.57 m^3

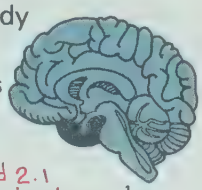
4. An eye blinks about 25 times each minute. About how many times does it blink in a day (24 hours)?

36 000

5. Your intestines (large and small) are about 7.6 m long. Your small intestine is about $\frac{4}{5}$ of this length. How long is your small intestine? 6.08 m



6. A man's brain is between 0.02 and 0.03 of his body weight. Between what two numbers is the brain weight of a 70-kg man?

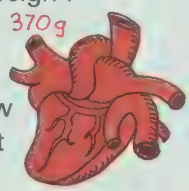


1.4 and 2.1

7. The brain of a baby is about $\frac{1}{10}$ its body weight. About how many grams does the brain of a 3.7-kg baby weigh?

370 g

8. Your heart beats about 80 times a minute. About how many times does it beat in a day?



115 200

9. The body of an adult contains about 4.7 litres of blood. A blood donor usually gives 470 ml of blood. What fraction of his blood does the donor give? $\frac{1}{10}$

10. Your body contains about 160 900 km of blood vessels. Write this number as a power of ten.

1.609×10^5

11. The liver is the largest gland in your body. In a 66.5-kg person, the liver is about $\frac{1}{35}$ his body weight. How many grams would that be? 1900 g



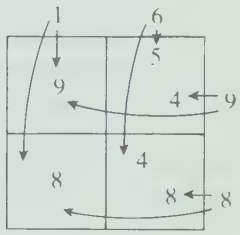
Using the Exercises

You might assign the problems on page 329 as a group activity, encouraging children to discuss their ideas for solving each. Each group of 4 or 5 children might work on three or four problems and then report on their work to the whole class.

Follow-up

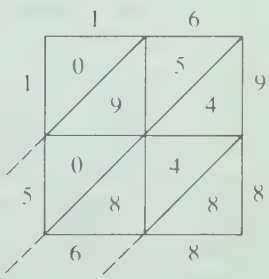
Children may enjoy learning an old method of multiplication known variously as "lattice," "grating," or "gelosia" multiplication (because it looks like a medieval window grating called "gelosia" in Italian).

In this method of multiplication, the factors are written across the top and down the right side of a lattice and multiplied, as shown in Figure A. The products are then written within bisected rectangles, and the diagonal columns are added (beginning in the lower right corner), using the usual regrouping process, as shown in Figure B. Study the first example, and then try a problem such as 231×45. See Figure C for the solution.



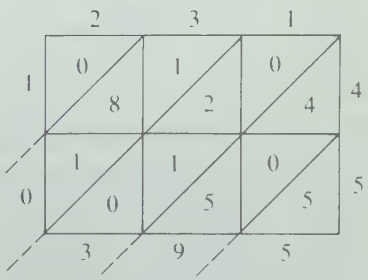
16×98

Figure A



$16 \times 98 = 1568$

Figure B



$231 \times 45 = 10,395$

Figure C

Pages 330–339

General Objectives

To provide experiences with simple concepts of probability
To develop an intuitive feeling for some of the basic ideas of probability such as outcomes, equally likely outcomes, and simple probability
To provide experience in conducting probability experiments and tabulating the results
To give children the opportunity to draw conclusions from experimental data

This chapter is designed to introduce the children to some concepts of probability by means of simple experiments. To achieve the most effective results with this material, encourage all the children to participate fully in the activities.

In the first lesson, children perform a simple experiment in which there are only two possible outcomes. Following this lesson, experiments are suggested in which the number of possible outcomes is increased and then extended to include the possible outcomes of pairs of objects. Other equally likely outcomes are explored, and finally, the last lesson of the chapter provides material that involves more difficult reasoning and basic concepts of probability.

Mathematics

Although the study of probability involves an extensive body of mathematics, the mathematical concepts needed to present the material in this chapter are already available to you. Knowledge of the arithmetic of the fractional numbers together with the simple definitions presented in this chapter should provide adequate background for an effective presentation. For example, the probability of a particular event is expressed in fractional form according to the following rule.

$$\left.\begin{array}{l}\text{Probability}\\ \text{of an}\\ \text{outcome}\end{array}\right\} = \frac{\begin{array}{c}\text{Number of ways}\\ \text{outcome can occur}\end{array}}{\begin{array}{c}\text{Total number of}\\ \text{equally likely}\\ \text{outcomes}\end{array}}$$

The probability of getting a 1 or a 5 with a toss of a die would be calculated as follows:

$$\left.\begin{array}{l}\text{Number of ways outcome can}\\ \quad\text{occur (either a 1 or a 5)}\\ \text{Total possible outcomes}\\ \quad\text{(6 faces on a die)}\end{array}\right\} \frac{2}{6}$$

Hence, the probability of throwing a 1 or a 5 in a single toss of a die is $\frac{2}{6}$, or $\frac{1}{3}$. From this conclusion, we may infer that over a large number of tosses we will get either a 1 or a 5 about $\frac{1}{3}$ of the time. If we toss the die 1000 times, we would expect to get a 1 or a 5 a little more than 300 times, since $\frac{1}{3} \times 1000 = 333\frac{1}{3}$.

Teaching the Chapter

Materials

Almanac or encyclopaedia
Brass paper fasteners
Cardboard, oaktag, or tagboard
Felt-tip pens or colored pencils
Gummed labels
Hexagonal pencils, one per child
Index cards
Paper clips
Pennies (one per child)
Rods, one 1-by-2-centimetre rod per child, and two 1-by-4-centimetre rods per child (if available)
Scissors
Student dictionaries
Tracing paper
Transparent tape
Wood, cardboard, or plastic cubes (one per child)

Vocabulary

chance
equally likely outcomes
experiment
outcome
probability

The key to a successful experience with material in this chapter is the full and active participation of all of the children. Notice that no specific concepts or skills to be learned or mastered are listed among the objectives. Although the children are expected to acquire certain skills in conducting an experiment and recording results, they should not be required to master particular facts or concepts. Treat the chapter with a light touch so that the children will enjoy it and not be burdened by having to memorize these concepts and facts. Plan to have enough materials to permit all the children to participate in some of the activities. Even if full participation is impossible for some activities, every child should have a chance to experiment with coins. When there is a shortage of materials, group the children and divide the tasks among them; that is, let some tally, let others take turns conducting events, and let still others summarize and announce the results of the events as they occur.

Lesson Schedule

The amount of time you spend on this chapter will be determined by the time left in the school year. Even a very few days is better than none. Notice, however, that there is probably enough extra material provided to occupy the children for more than two weeks.

Evaluation of Progress

The child's attitude toward this material and his enthusiasm for it are far more important factors in his achievement than particular test scores. Evaluate participation in day-to-day activities rather than relying on a chapter test.

Resources for Active Learning

General Activities

Franklin Series: *Probability*, Lyons and Carnahan (Available from McGraw-Hill Ryerson)

Nuffield Project: *Probability and Statistics*, Wiley

SMSG: *Probability for Intermediate Grades*, Stanford University

[The following activities relate to Pascal's Triangle.]

Inquiry in Mathematics via the Geo-board, Geo-cards 6/1–4, Walker (Available from Fitzhenry & Whiteside)

Mathex: Numeration No. 7, pp. 12–16 (pupil pages 11, 12), Encyclopaedia Britannica Publications Ltd.

Notes on Mathematics in Primary Schools, pp. 204–223, Cambridge University Press. (Available from Macmillan)

Nuffield Project: *Problems*—Green Set, No. 4, 9; Red Set, No. 9-9B, Wiley

Manipulative Devices

Blank dice and blank gummed labels (Creative Publications)

Blank playing cards (Creative Publications)

Hexstat (Gamco; Lano)

Commercial Games

Probability and Statistics Lab Kit (Gamco; Lano)

Yahtzee (local supplier: Math Media)

PAGES 330–331

Objective

Given an experiment which will produce only two possible outcomes, the child will be able to perform the experiment and record his results.

Preparation

Materials

coins: one penny per child

To prepare for this and subsequent lessons, you might briefly review with the children how important it is to record results of experiments, whether in science or mathematics. Much of their work in this chapter depends on the results of their experiments.

Investigation

This investigation is actually a very simple experiment. Point out how a tally can be kept to record not only the number of heads or the number of tails but also the number of tosses, so that the total number of tosses can be calculated.

14 Probability

Let's explore some probability experiments.

Investigating the Ideas

Here is a probability experiment for you to try.

Experiment	Possible Outcomes	Tally
 Toss a penny.	 Heads Tails	H |||| T |||| Keep a tally of the outcomes.

? How many times do you think you will have to flip a penny before you get 50 heads? | Try it and record your results.

See Investigation and Discussion.

Discussing the Ideas

1. A What fractional part of the tosses of a coin should come up "heads"? $\frac{1}{2}$
   B What part should come up "tails"? $\frac{1}{2}$
   C Do you think you will always get the same number of heads as tails when you toss a coin several times? Probably not

2. Predict, as closely as you can, the number of heads and tails that you think will appear after
   A 10 tosses. B 20 tosses. C 50 tosses. D 1000 tosses.
   E your classmates each toss a penny once, and the results are combined.

   Give reasons, if any, for your predictions. See Discussion.



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Discussion

Have children explain the results of their experiment. You might make a chart of the various results so that children can compare their tallies.

As children respond to discussion exercise 1, have them estimate the average number of tosses everyone needed in order to get 50 heads. This example should help them see that about $\frac{1}{2}$ of the tosses come up heads. Emphasize the meaning of the word *outcome* and begin to use the phrase *equally likely outcomes*. Help them see that a question such as 1C may be answered as "probably not" because you will not always get the same number of heads as tails even after several tosses.

Discuss the effects of greater and greater numbers of tosses on the outcomes. Some children may feel that one or the other side of the coin is biased (i.e., that the outcome is prejudiced toward one side or the other) so that the outcomes, a head or a tail, are not equally likely. The applications of probability theory will often not be seen until a great number of tosses are made, even though for every toss there is 1 chance out of 2 that heads will occur.

Using the Ideas

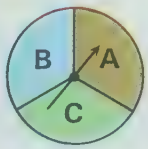
1. Complete a table like the one below for the coin-tossing experiment. Use the combined results of your class.

Total number of heads	Total number of tails	Total number of tosses

Results will vary.

2. Use the table in exercise 1.
   - A Write the ratio: $\frac{\text{total number of heads}}{\text{total number of tosses}}$ Results will vary.
   - B Is this ratio more than $\frac{1}{2}$ or less than $\frac{1}{2}$? Answers will vary.

3. The spinner is divided into 3 parts of equal size. If you spin the arrow several times, for what fractional part of the spins will the arrow probably stop in region **B**? $\frac{1}{3}$



4. How many times do you think you would have to spin the arrow to get 10 "**B**" outcomes? 30

5. Suppose a cube has the letters **A**, **B**, **C**, **D**, **E**, and **F** on its faces. If you toss the cube several times, for what fractional part of the tosses should the cube land with the face lettered **C** on top? $\frac{1}{6}$





6. Predict, as closely as you can, the number of times each letter on the cube in exercise 5 should appear after
   - A 24 tosses.
   - B 48 tosses.
   - C 72 tosses.
   - D 600 tosses.

Each letter should appear about 1 in every 6 tosses.

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Using the Exercises

Before assigning the exercises on page 331, stress again the importance of accurate record-keeping. You might want to treat exercise 1 as a class activity, figuring out together the results of exercise 2E, on page 330.

Exercises 3 and 5 may also be used as the basis for additional probability experiments. Children might each take a turn with the spinner and the cube and record their results on a class chart. These combined results should be fairly close to theoretical expectations.

Note in exercise 6 that, since each letter has a chance of occurring 1 out of 6 times, this ratio can be expressed as 4 out of 24, or as 8 out of 48 tosses, and so on. Children may also think of the chance of appearance of one particular letter as $\frac{1}{6}$ of the total number of tosses. Continue to stress that the discussions here have to do with expected outcomes.

Assignments (page 331)

Minimum: 1–4. Average: 1–6. Maximum: 1–6.

Mathematics

In a simple probability experiment, there is a definite result from each trial of the experiment. Each result of the experiment is called an *outcome* of the experiment. The set of all the possible outcomes for a probability experiment is called the *sample space* for the experiment.

If a die is tossed, exactly one of the numbers 1, 2, 3, 4, 5, or 6 must come up on the top face. Thus, the sample space for the experiment of tossing a die is the set of outcomes $\{1, 2, 3, 4, 5, 6\}$. If the die is true, that is, it is an evenly balanced die, each of the six outcomes is equally likely to occur on a given toss of the die. Hence the set of outcomes are called *equally likely outcomes*.

Not all probability experiments have equally likely outcomes. If an experiment consisted of tossing a conical paper cup on the floor and noting whether it came to rest on its circular base or on its side, we would probably find that these two outcomes were not equally likely.

Follow-up

Discuss what effect a large number of tosses will have on the results of a probability experiment (for example, 10 000 tosses in the penny experiment or 6000 tosses in the cube experiment). Urge the students to devise a method of getting a great many tosses. (One way is to have each child carry out the experiment in his free time for a few days until 200 tosses have been recorded, and then combine all the results.)

Collecting results from the whole class will help the children develop some intuition about the equal likelihood that certain events will occur. In addition, they should glean some notion of the laws of large numbers, that is, the evening-out effect of a large number of tosses on early differences in the number of heads and tails.

Resources for Active Learning

Developmental Math Cards, "What are the chances?" L13, Addison-Wesley.

Workbook, page 103

PAGES 332–333

Objective

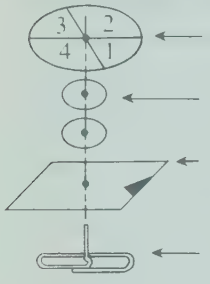
Given a description of a probability experiment with outcomes which are ordered pairs, the child will be able to perform the experiment and record his results.

Preparation

Materials

2 spinners of different colors for every 1 or 2 children; paper clips; cardboard

Instead of the arrow spinner shown in the text, children might find this disc spinner easier to construct and use:



4-cm cardboard disc spinner

Cardboard washers

5-by-5-cm square cardboard base

Paper clip with one end bent up (Tape clip to bottom of the square.)

Investigation

Direct the children to follow the instructions in the text. Be sure they realize that the first number should always come from the same spinner, and the second number from the other spinner. The coordinate system need not be written on graph paper; children might find it easier to make their own, similar to the one in the text. You might have children work in pairs and have each record the results of 50 trials.

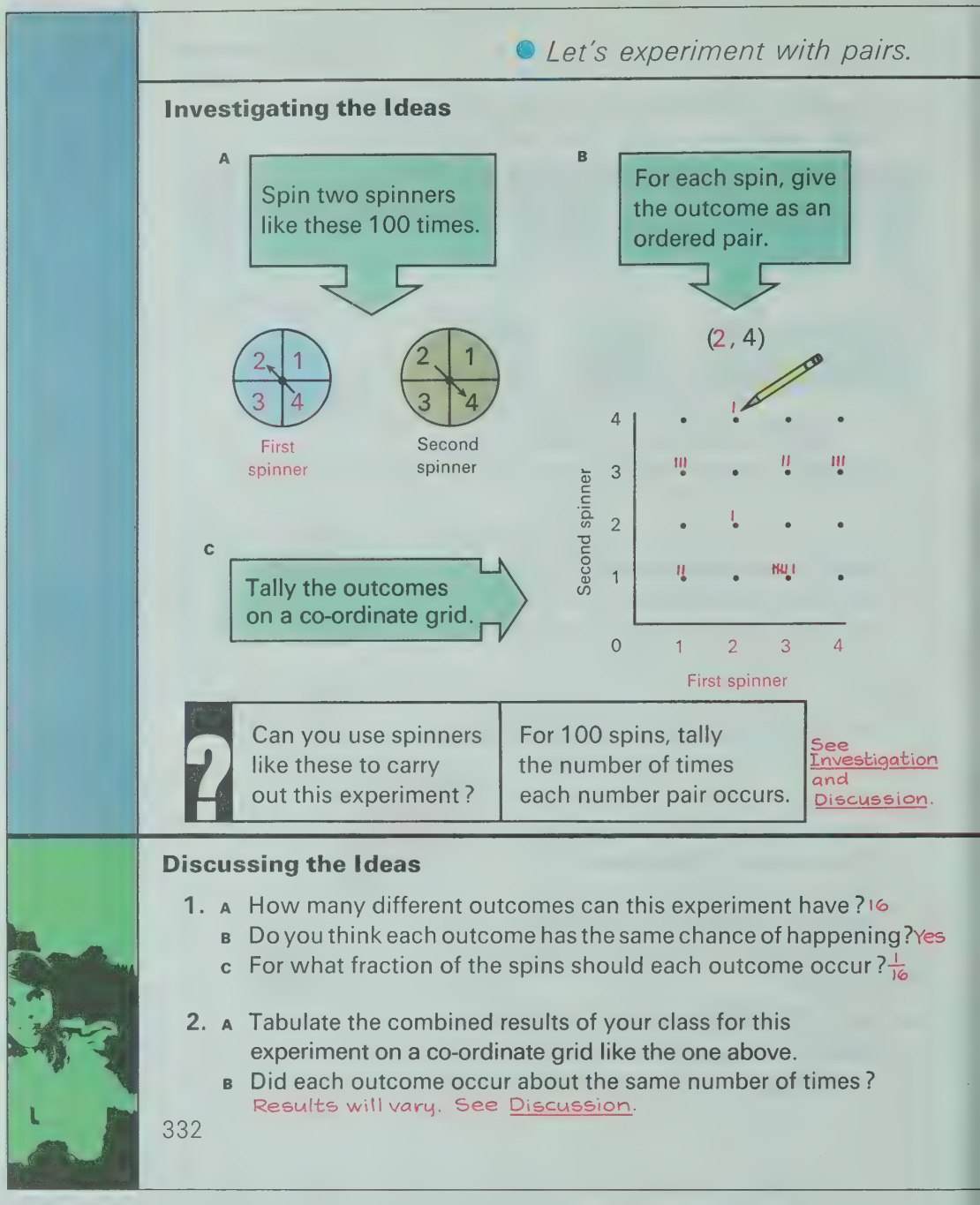
Discussion

Before asking children to share their results for the investigation, which may be done in exercise 2, you might ask the children to explain why (2, 4) and (4, 2) represent different outcomes. Help them see that, for the pair (2, 4), the first spinner showed a 2, and for (4, 2), the first spinner showed a 4. Thus, both would be recorded differently in the coordinate system. Emphasize that, since there are 16 possible pairs or outcomes, each pair "should" occur $\frac{1}{16}$ of the number of spins. However, the fact that any of the sixteen possible outcomes is equally likely is not obvious. In fact, a typical result of the spinner experiment of 100 spins might yield the following:

Number of Occurrences of Each Pair

2 – (4, 1)
3 – (2, 2)
4 – (2, 1), (3, 3), (3, 4)
5 – (3, 1), (1, 4), (4, 2)
6 – (1, 1)
7 – (4, 1), (2, 4)
8 – (4, 3)
9 – (3, 1)
10 – (1, 3), (4, 4)
11 – (2, 3)

Since there were 100 spins, theoretically, each possible outcome, that is, each pair, should occur $\frac{1}{16}$ of 100, or between 6 and 7 times. However the results show great divergence from this: only the pair (1, 1) occurred 6 times and (4, 1) and (2, 4) occurred 7 times. Furthermore, one pair, (2, 3), occurred as many as 11 times and one pair, (4, 1), occurred as few as 2 times. Work through a sample such as this. Be sure the children realize that even though each pair is equally likely to occur this is often not obvious. If possible, have children combine and chart their results to see if a greater number of spins gives a result nearer to $\frac{1}{16}$ of the number of spins. They should find



Using the Ideas

1. A Write the ordered pair that represents the outcome shown here. (4,3)



B How many times did this outcome appear in your class experiment? Results will vary.

C Did this outcome appear for more than, less than, or exexactly $\frac{1}{16}$ of the number of spins? Results will vary.

2. An experiment consists of spinning the pointers on both spinners and recording the outcomes. The outcome shown is (2, 3).



A List all of the possible outcomes for this experiment.

B For what fraction of the spins should outcome (2, 3) occur? $\frac{1}{9}$

(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)

3. Suppose a penny and a nickel are tossed at the same time.

A Complete the table of outcomes.

Outcomes	
Nickel	Penny
Heads	Heads
Heads	Tails
Tails	Tails
Tails	Heads

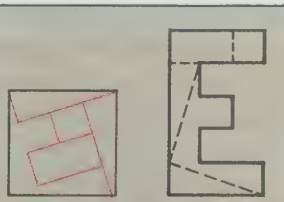
B How many outcomes are possible? 4

C If both coins are tossed several times, for what part of the time would you expect to get heads on both coins?

D For what part of the time would you expect the coins not to "match"? $\frac{1}{2}$

think

Trace the letter E and cut along the dotted lines. Then paste these pieces so that they exactly cover a tracing of the square.





that a greater number of spins tends to smooth out the data, because the differences in occurrences seem to be less extreme. You might ask the children what effect they think increasing the number of spins to 1 000 000 would have on the outcome of this experiment.

Using the Exercises

The exercises on page 333 provide additional probability experiments. Encourage children to combine results and chart them, discussing any conclusions they think are valid. One of the main purposes of these activities is simply to provide experiences in probability experiments, so children should not be expected to make conclusions or try to give reasons for the results. Encourage them simply to "think aloud" with each other.

Assignments (page 333)

Minimum: 1, 2. Average: 1–3. Maximum: 1–3.

Mathematics

This lesson introduces a sample space in which each outcome is an ordered pair. The notion that the outcomes of this experiment are equally likely is not readily apparent. Some children may feel that pairs in the centre of the co-ordinated quadrant are more likely to occur than those at the edges, or that the "middle" numbers (2 and 3) occur more often than the "extreme" numbers (1 and 4).

Experiences with ordered pairs and in keeping a record on the co-ordinate system extends concepts and skills of graphing. This lesson should also deepen the children's understanding of the outcome of an experiment and prepare them for the vocabulary and ideas introduced in the next lesson.

Follow-up

More capable children may wish to extend their experimentation with two-dimensional spaces by using lettered cubes to work out predictions for two cubes. Suggest that they apply exercises 1 and 2 on page 333 to the rolling of the two lettered cubes.

Workbook, page 104

PAGES 334–335

Objective

Given a probability experiment, the child will be able to state whether or not it has equally likely outcomes.

Preparation

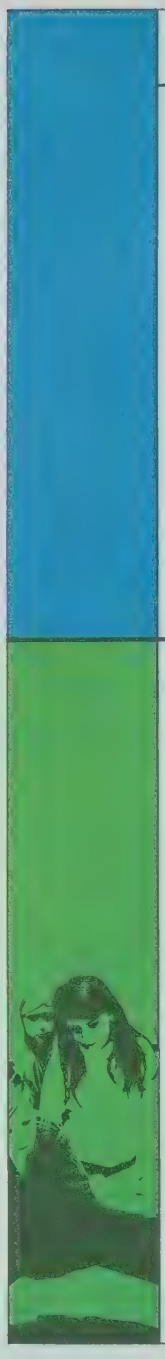
Materials

hexagonal pencils (preferably, unused; 1 per child); cubes; paper fasteners

Review any experiments which children found interesting. If some children worked out the suggestions in the follow-up section of the preceding lesson, have them share their results.

Investigation

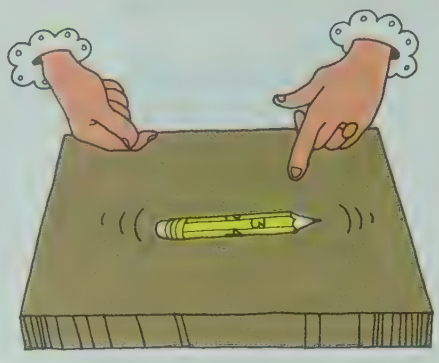
Most pencils which the children use are hexagonal, so simply be sure each child has a six-sided pencil which he can label. (This investigation might equally well be carried out with cubes or dice instead of pencils, if you prefer.) Remind the children how important it is to keep an accurate record of the outcome of every roll. Notice that the investigation question calls for a prediction, so be sure the children make their prediction before undertaking the experiment.



What are equally likely outcomes?

Investigating the Ideas

Label each side of a six-sided pencil with one of the digits 1, 2, 3, 4, 5, 6. Roll the pencil along a table and record the top number when it stops.



? How close can you come to predicting the number of rolls you must make before recording ten 2's? About 60 rolls

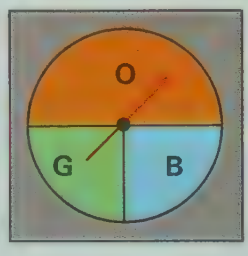
See Investigation.

Discussing the Ideas

1. If all possible outcomes of an experiment have the same chance of occurring, we say these outcomes are **equally likely.** Do you think the outcomes in the Investigation are equally likely? See Discussion.

2. Explain how you made your prediction in the Investigation. Answers will vary.

3. A What are the possible outcomes for the spinner if "liners" are not counted? D,G,B
   B Do you think the outcomes are equally likely? No
   C Which outcome do you think will occur most often? O



Discussion

The main point of this lesson is for children to compare probability experiments which have equally likely outcomes with those which do not. The investigation experiment is designed to help children understand the idea of *equally likely outcomes*. If the pencil used is a *fair* pencil, then each of the six sides has an equal chance of appearing on top when the pencil comes to rest. This does not mean, of course, that each side will come up exactly the same number of times in a given number of rolls, but, if the experiment is carried out a great many times, there would not be a large difference in the number of times any two of its sides come up. On the other hand, a pencil (or other object) which is bent, scarred, or irregular in shape or weight is likely to be *biased;* that is, one or more sides are likely to come up much more frequently than others.

In probability experiments, an attempt is made to achieve *randomness* by imposing conditions under which each outcome is equally likely to occur. Thus, we try to use cubes which are as nearly perfect as possible.

In exercise 3, the unequal division of the dial suggests that the outcomes are not equally likely.

Using the Ideas

For each probability experiment below, tell whether or not the outcomes are equally likely. If the outcomes are not equally likely, tell which outcome is most likely to occur.

1. Toss a cube with faces numbered 1 to 6. Record the number on the top face.



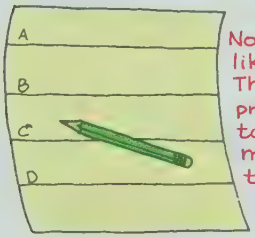
Equally likely

2. Open a dictionary at random and choose the last digit of the page numeral.



Equally likely

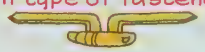
3. Drop a 20-cm pencil on a paper with lines drawn 7.5 cm apart. Record whether or not the pencil touches a line.



Not equally likely. The pencil probably will touch a line more than half the drops.

4. Open a paper fastener as shown here, and toss it onto your desk. Record which way it lands.

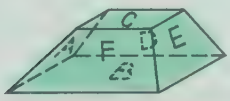
Results may vary depending on type of fastener.



(Encourage children to experiment with different types.)

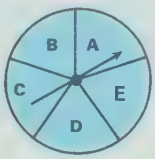


5. Toss an object like this and record the letter on the top face.



Not equally likely. "C" will probably be on top most often.

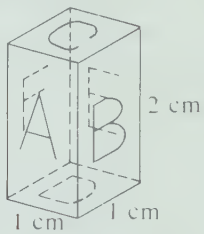
6. Spin the arrow and record the letter of the region in which it stops.



Equally likely

Follow-up

If number rods, such as the Cuisenaire rods, are available, you might have children choose the 1-by-1-by-2 centimetre rods, and label each face:



Then direct the children to toss the rods on their desks and answer questions like these:

1. What are the possible outcomes for this experiment?
2. Do you think the outcomes are equally likely?
3. If the "rod" were 1 centimetre on each edge, would the number of possible outcomes change? Would the outcomes be random?
4. What if the rod were $1\frac{1}{2}$ centimetres long rather than 2 centimetres long?
5. Discuss the effect of using rods of different lengths in this experiment.

Encourage some children to make predictions and then design experiments to verify or deny their guesses.

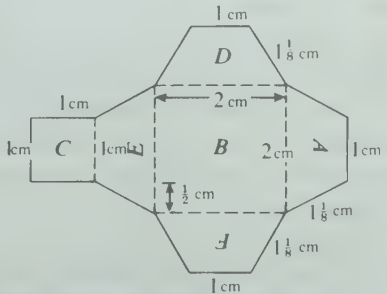
Resources for Active Learning

Nuffield Project: *Problems*—Red Set, No. 15–15A, Wiley.

Using the Exercises

It is recommended that children perform as many as possible of the experiments on page 335. If feasible, set up stations around the classroom and have pairs of children carry out each experiment with at least 100 trials for each experiment. By combining results, a large number of trials can be obtained, thus making it easier to decide whether each experiment has equally likely outcomes.

If cubes are not available, children can make a pattern for a 1-centimetre cube. Tagboard would be best for both objects; the patterns may be cut out of paper and then pasted onto the tagboard to be cut and folded into the object. The object in exercise 5 may be made from the following pattern:



Make faces *A*, *D*, and *F* exactly the same as face *E*.

Assignments (page 335)

Minimum: 1–6. Average: 1–6. Maximum: 1–6.

PAGES 336–337

Objective

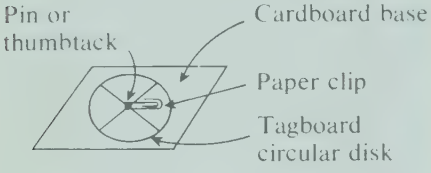
Given a simple probability experiment with equally likely outcomes, the child will be able to give the probability of each outcome.

Preparation

Materials

spinners like those recommended on page 332 or like the one illustrated in the investigation section of this lesson; coins; cubes; rods (if available)

For this lesson, each child needs a spinner. The spinner used for page 332 may be used here simply by replacing the top circular disc. Or children may make one like that pictured at the top of page 336.



Investigation

Remind the children that they should first write their prediction and then tally the outcome of each of 30 spins to test their prediction. As children finish, you might have them write their results in columns on the chalkboard so that they can compare results.

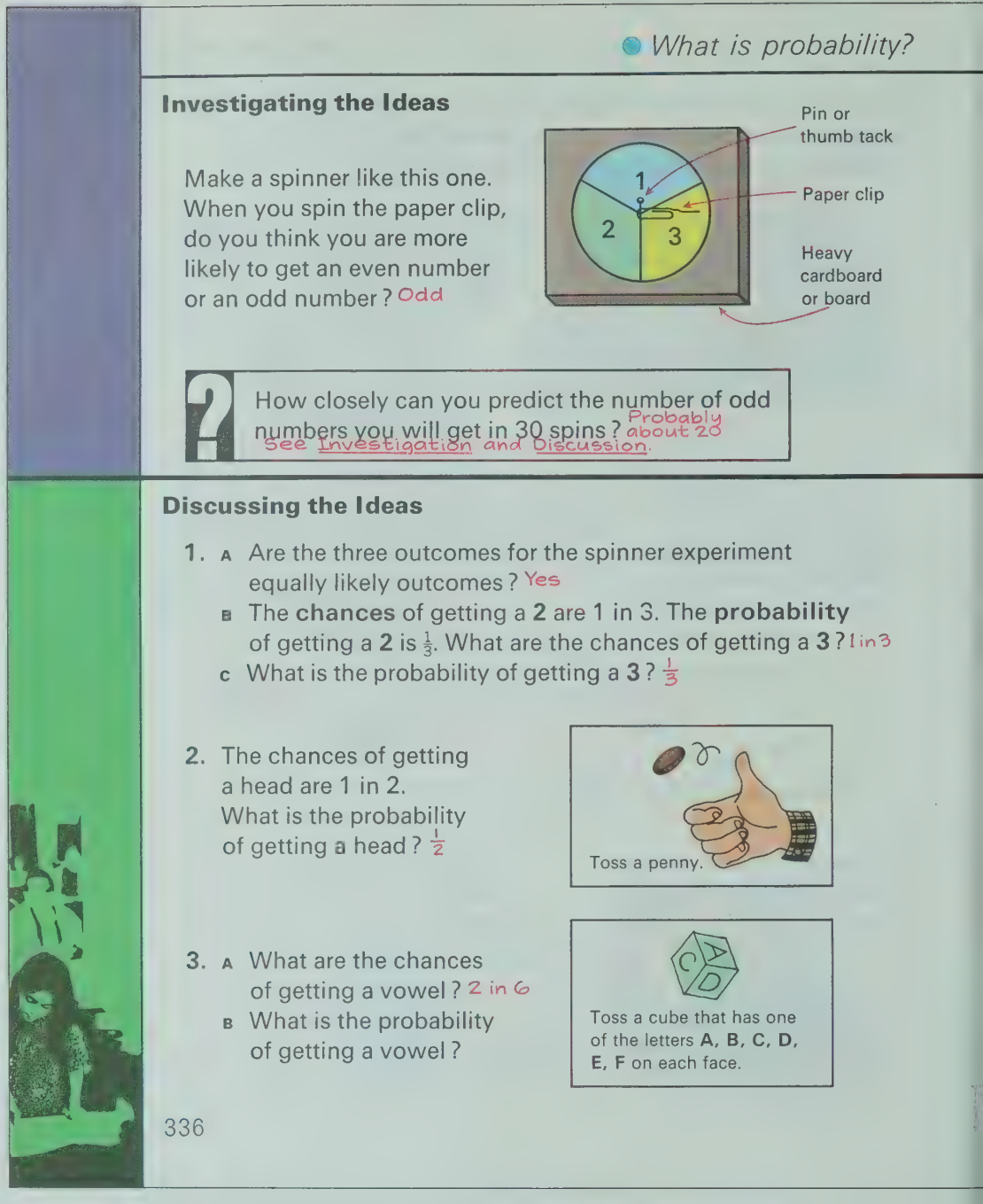
Child's Name	Even Number	Odd Number

Discussion

Lead children in a discussion of their results from the investigation. Then compare them to the responses children give for discussion exercise 1. The children should see that we can determine the probability of an outcome, even though the results of the experiment may not exactly agree with the results predicted by probability theory. Because there is one number out of three that is even and because each of the three numbers is equally likely to occur as an outcome, we say, "There is 1 *chance* in 3 of getting an even number." Another way of expressing the same idea is to say, "The probability of getting an even number is $\frac{1}{3}$." The probability of getting an odd number is $\frac{2}{3}$, because 2 out of the 3 numbers are odd. Note with them that the sum of both probabilities is 1; that is, the probability of getting either an odd or an even number is $\frac{3}{3}$ or 1. Thus, the probability of an event that is certain to happen is 1.

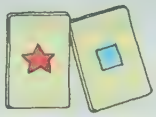
Similarly, in exercise 2, since there are only two equally likely outcomes, the probability of getting a head is $\frac{1}{2}$ and the probability of getting tails is $\frac{1}{2}$. The probability of getting either heads or tails is 1. The probability of getting neither heads nor tails is 0. The probability of an event that is certain not to happen is 0.

In exercise 3, since there are six equally likely outcomes, the probability of each outcome is $\frac{1}{6}$. Since there are 2 vowels, A and E, the probability of getting a vowel is $\frac{1}{6} + \frac{1}{6}$ or $\frac{2}{6}$, which may be expressed as $\frac{1}{3}$. According to the ability of the children, develop this discussion with other examples and include ideas from the mathematics section of this lesson. However, with most children, you will want to emphasize particular experiments rather than stress any generalizations.



Using the Ideas

1. These two cards are turned over and shuffled. If one card is then turned up, two equally likely outcomes are possible.

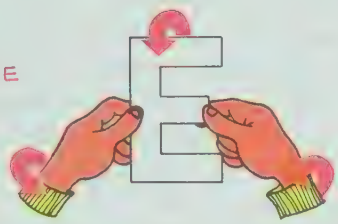


   A What is the probability that a star will be turned up? $\frac{1}{2}$
   B What is the probability that a square will be turned up? $\frac{1}{2}$

2. A cube with letters **A, B, C, D, E, F** is tossed.



   A What is the probability that the letter **A** will appear on top? $\frac{1}{6}$
   B Think of the 6 equally likely outcomes. What are the chances that a consonant will appear on top? 4 in 6
   C What is the probability that a consonant will appear on top? $\frac{4}{6}$ or $\frac{2}{3}$
   D Certain letters, like **E**, look about the same after they have been turned upside down, as in this picture. Which of the letters **A, B, C, D, E, F** have this property? B,C,D,E Think of the 6 equally likely outcomes. What are the chances that a letter with this property will appear on the top face? 4 in 6



   E What is the probability of getting a letter with this property? $\frac{4}{6}$ or $\frac{2}{3}$

3. Two spinners like these are spun.



   A What is the probability that the pair (2,2) will appear? $\frac{1}{9}$
   B In the 9 possible outcomes, how many chances are there of a pair appearing with both numerals the same? 3
   C What is the probability of getting a pair with both numerals the same? $\frac{3}{9}$ or $\frac{1}{3}$
★ D What is the probability of getting a pair that has a sum less than 5? $\frac{6}{9}$ or $\frac{2}{3}$

337

Mathematics

In this lesson one of the basic definitions in probability theory is developed. The probability of a particular outcome may be expressed as follows:

$$\text{Probability of an outcome} = \frac{\text{Number of ways outcome can occur}}{\text{Total number of equally likely outcomes}}$$

In other words, if, in a given experiment, there are n possible outcomes, all of which are equally likely, and x of these are regarded as favorable, then the probability of a favorable outcome is simply the number $\frac{x}{n}$. The *chances* of a favorable outcome are "x in n." If all the n outcomes are favorable, then the probability of a favorable outcome is 1. If none of the n outcomes is favorable, then the probability of a favorable outcome is zero. Thus, every probability will be zero, 1, or some number between zero and 1.

Duplicator Masters, page 58
Workbook, page 105

Using the Exercises

Although some children will be able to think through each experiment on page 337 without actually working with the objects, others will want to make the materials and perform each experiment.

After children have completed them, these exercises would provide excellent material for further discussion. Help the children see how the ratio of outcomes, that is the probability of each single outcome, is related to the chance of occurrence of particular outcomes to probability of an event. For example, in exercise 2, the probability for the occurrence of an equally likely outcome is $\frac{1}{6}$; each letter has 1 chance in 6 to appear on top. Since 4 out of 6 letters are consonants, the chances of getting a consonant are 4 in 6, or 4 out of 6. This ratio may be expressed as the fractional number $\frac{4}{6}$ and we say the probability of a consonant appearing is $\frac{4}{6}$.

Assignments (page 337)
Minimum: 1, 2. Average: 1–3C. Maximum: 1–3.

PAGES 338–339

Objectives

The child will demonstrate his ability to work with the concepts presented in this chapter.

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

Since evaluation of children's grasp of probability concepts should not be determined by a test, you might briefly discuss the various probability experiments made throughout the chapter. If you prefer, you might review odd and even, and prime and composite numbers in preparation for page 339.

1. A If you do not count spins that end on a line, how many outcomes are there for this spinner? 4



   B Do you think these outcomes are equally likely? Yes
   C What are the chances of spinning a 2 in one spin? 1 in 4
   D What is the probability of spinning a 2? $\frac{1}{4}$
   E Of the 4 equally likely outcomes for the experiment, how many are even numbers? 2
   F What are the chances of spinning an even number in one spin? 2 in 4
   G What is the probability of spinning an even number? $\frac{2}{4}$ or $\frac{1}{2}$

2. The outcome shown in the picture represents the pair (3, 4).

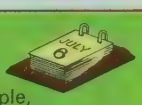


   A List the 16 outcomes. Are they equally likely? Yes. See Answers.
   B (2, 1) is one of the 16 outcomes. What are the chances of spinning a (2, 1) in one trial? 1 in 16
   C What is the probability of spinning a (2, 1)? $\frac{1}{16}$
   D Of the 16 equally likely outcomes for the experiment, how many have both numbers the same, like (1, 1)? 4



   E What are the chances of spinning the same two numbers? 4 in 16
   F What is the probability of spinning the same two numbers? $\frac{4}{16}$ or $\frac{1}{4}$

think



Mathematical calculations have shown that if you choose a group of 25 people, the chances are about 1 in 2 that two of the people will have the same birthday. If there are 35 people, the chances are about 3 in 4 that this will happen.

A Find a list of the birthdays of famous Canadians. Did any of the famous Canadians have the same birthday?

B Did any members of the group you chose die on the same day?

C Do any members of your family have the same birthday?

D Make a survey of the classes in your school to see if any two people in a class have a birthday on the same day, and report your results.

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Discussion

The experiments on page 338 will help the children summarize what they have learned about probability. As you discuss the experiments, remember that the most important goal of this chapter has been to create a positive attitude toward the idea of probability.

Answers, exercise 2A, page 338

(1, 1), (1, 2), (1, 3), (1, 4)
(2, 1), (2, 2), (2, 3), (2, 4)
(3, 1), (3, 2), (3, 3), (3, 4)
(4, 1), (4, 2), (4, 3), (4, 4)

Keeping in Touch with

Computing
Measurement
Principles

Fractional numbers
Number theory

1. Remember that a prime number has exactly 2 factors.

- A Name an odd number that is not prime. Many possible answers: 9, 15, 21, etc.
- B Name a prime number that is not odd. 2
- C List the prime numbers between 20 and 40. 23, 29, 31, 37

2. Compute the following.

A
$$\begin{array}{r} 7428 \\ 6917 \\ +3844 \\ \hline 18\,189 \end{array}$$

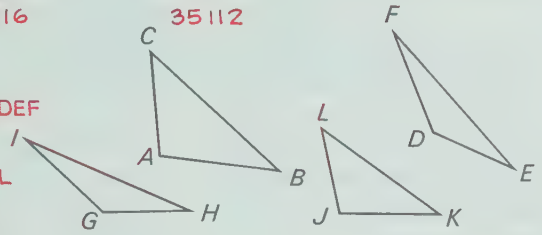
B
$$\begin{array}{r} 7090 \\ -3774 \\ \hline 3316 \end{array}$$

C
$$\begin{array}{r} 924 \\ \times 38 \\ \hline 35\,112 \end{array}$$

D $2052 \div 38$ 54

3. A Which two triangles are congruent? $\triangle GHI \cong \triangle DEF$

B Which two are similar triangles? $\triangle ABC$ and $\triangle JKL$



4. Find the sum or difference.

A $\frac{3}{4} + \frac{1}{2} = 1\frac{1}{4}$

B $\frac{3}{8} + \frac{3}{4} = 1\frac{1}{8}$

C $\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$

D $\frac{7}{8} - \frac{1}{2} = \frac{3}{8}$

E $\frac{5}{6} - \frac{1}{3} = \frac{1}{2}$

F $1\frac{1}{2} - \frac{1}{4} = 1\frac{1}{4}$

G $7\frac{1}{3} + \frac{2}{3} = 8$

H $6\frac{3}{4} - 5\frac{1}{8} = 1\frac{5}{8}$

5. Change each fraction to lowest terms.

A $\frac{12}{16} = \frac{3}{4}$

B $\frac{14}{20} = \frac{7}{10}$

C $\frac{9}{15} = \frac{3}{5}$

D $\frac{40}{30} = \frac{4}{3}$

E $\frac{27}{36} = \frac{3}{4}$

6. Express as an improper fraction.

A $3\frac{1}{2} = \frac{7}{2}$

B $1\frac{2}{3} = \frac{5}{3}$

C $4\frac{3}{4} = \frac{19}{4}$

D $7\frac{1}{2} = \frac{15}{2}$

E $3\frac{1}{10} = \frac{31}{10}$

7. Express as a mixed numeral.

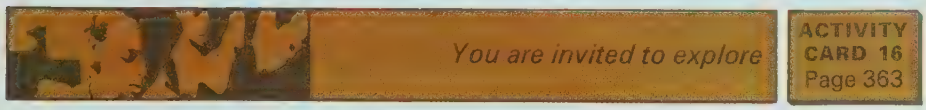
A $\frac{15}{8} = 1\frac{7}{8}$

B $\frac{23}{10} = 2\frac{3}{10}$

C $\frac{45}{7} = 6\frac{3}{7}$

D $\frac{33}{9} = 3\frac{2}{3}$

E $\frac{27}{4} = 6\frac{3}{4}$



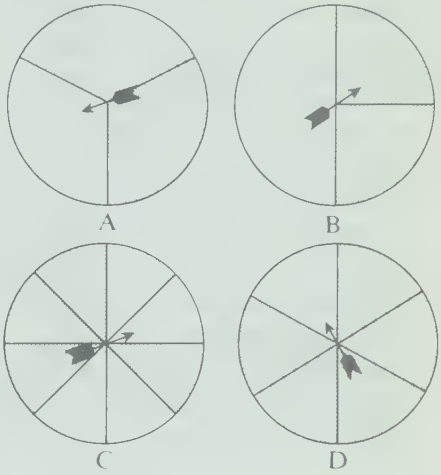
339

Using the Exercises

Page 339 may be used as an evaluation page for basic skills, particularly with whole and fractional numbers. Help any child who has difficulty with the basic concepts of exercises 2, 4, 5, 6, and 7; these are fundamental to continued work in junior high school mathematics.

Follow-up

More capable children may wish to carry their probability investigations further. Suggest they make spinners like those shown below.



The children might investigate questions like these:

1. On which spinners will the outcomes be equally likely?
2. Can you predict the probability of each possible outcome for each spinner?

Or, they might design and carry out experiments to test their ideas. If you wish, ask them to try to figure out the possible outcomes and then the probability for a given event or combination of events. For example, using spinner C:

1. What is the probability of your first spin landing on gray?
2. What is the probability of your second spin landing on gray?
3. What is the probability of both spins landing on gray?

The children in your class may enjoy reading "Fires, Coins and Pinball Machines" from Irving Adler's *Giant Golden Book of Mathematics* (New York: Golden Press, 1960) or "Chance and Chanceability" in Kasner and Newman's *Mathematics and the Imagination* (New York: Simon and Schuster, 1940).

See also "Books to Explore" at the back of this manual and "More Books to Explore" in the resource section at the front of this manual.

Workbook, page 106

Pages 340–354

General Objectives

To introduce negative numbers
To introduce the set of integers
To provide additional experience in using the basic principles
To introduce addition and subtraction of integers
To relate the integers to familiar physical situations
To show how the number line can be extended to the left and labelled with the negative integers
To provide experience in graphing with integers

The first investigation in this chapter introduces the integers in a variety of interesting situations which require the children to think about physical concepts. The integers are then shown on the number line and related to the practical applications. Next, basic principles are assumed for the integers, and work is presented in which these principles are used to find sums of pairs of integers. Following this, the children find sums of this type by using color-coded counters representing negative and positive integers.

Subsequently, the inverse relation between addition and subtraction is used to present subtraction of integers. Material is then provided to develop problem-solving skills for addition and subtraction of integers. The last lesson in this chapter provides experiences in graphing integers and relates function rules to number pairs with integers.

Mathematics

In certain ways the set of whole numbers is inadequate. Subtraction, for example, is not always possible in the system of whole numbers, since there are no whole-number solutions for equations such as

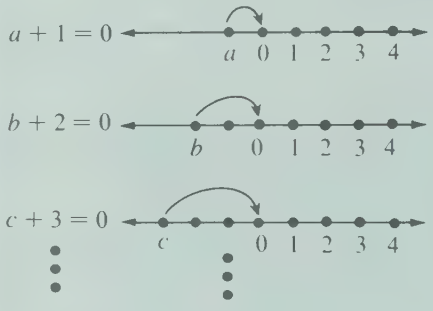
$0 - 1 = a$, $0 - 2 = b$, $0 - 3 = c$, $0 - 4 = d$, and $0 - 5 = n$.

These considerations motivate an extension of the whole-number system.

For each subtraction equation above, there is a corresponding addition equation with the same solution:

$a + 1 = 0$, $b + 2 = 0$, $c + 3 = 0$, $d + 4 = 0$, and $n + 5 = 0$.

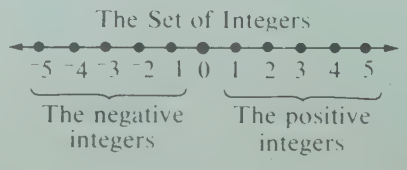
The following figure shows the suggested position on the number line of the points for the solution of each of these equations.



With these ideas in mind, we accept the following principle, sometimes called the *opposites* principle.

> For each whole number a there is a corresponding number ^{-}a, which has the property
>
> $$^{-}a + a = 0.$$

The numbers indicated by ^{-}a are called the *negative integers*. These are opposite the whole numbers on the number line. The whole numbers are also called the *positive integers*. For ^{-}a, we read "negative a" or, sometimes, "the opposite of a." The set containing the positive integers, the negative integers, and zero is called the set of *integers*.



The following basic principles, which hold true for the addition and multiplication of whole numbers, are now assumed to hold true for the integers. For any integers a, b, and c:

Commutative Principle

Add. $a + b = b + a$; $^{-}2 + 3 = 3 + {}^{-}2$

Mult. $a \times b = b \times a$; $4 \times {}^{-}3 = {}^{-}3 \times 4$

Associative Principle

Add.

$a + (b + c) = (a + b) + c$;
$5 + (^{-}3 + {}^{-}2) = (5 + {}^{-}3) + {}^{-}2$

Mult.

$a \times (b \times c) = (a \times b) \times c$;
$^{-}3 \times (2 \times {}^{-}4) = ({}^{-}3 \times 2) \times {}^{-}4$

Distributive Principle

$a \times (b + c) = (a \times b) + (a \times c)$;
$^{-}5 \times (4 + 3) = (^{-}5 \times 4) + (^{-}5 \times 3)$

Zero Principle

$a + 0 = a$; $a \times 0 = 0$;
$^{-}3 + 0 = {}^{-}3$; $^{-}2 \times 0 = 0$

One Principle

$a \times 1 = a$;
$^{-}2 \times 1 = {}^{-}2$

These principles and the opposites principle help us to establish techniques for adding and multiplying integers.

Teaching the Chapter

Materials

Counters, 10 red and 10 blue for each child
Demonstration number line showing integers
Graph paper ($\frac{1}{4}$-cm grid)
Overhead projector (if available) and integer number-line transparency with overlay
Thermometers
World map showing time zones (if available)

Vocabulary

integer — negative integer
negative — opposite
positive integer

Although only a few objects are listed for use in connection with this chapter, you are encouraged to look for a variety of other items to use with the physical situations the children suggest in the first lesson. Certainly, any materials that help demonstrate the ideas will show why these new numbers – the negative integers – needed to be devised.

The words *opposite* and *negative* are both introduced. The word opposite is used to help the children understand that each integer other than zero has an opposite and that the sum of an integer and its opposite is zero. Opposite is used both to promote understanding and as a convenience, but more often, the terms positive and negative integer will be used.

Lesson Schedule

This chapter is designed to be covered in a little more than a week. This schedule can be adjusted according to the children's needs and the amount of time left in the school year. You might find your class motivated sufficiently to extend this chapter to two, or even three, weeks' work if your time schedule permits. Since the material in this chapter, as well as that in Chapter 14, will be covered again in junior high school, you should feel free to be selective in the remaining part of the year, if time is limited.

Evaluation of Progress

Evaluate the children's achievement in this chapter with a light touch. Do not demand a great deal of skill in computing with integers. Attempt to evaluate the children's progress on a day-to-day basis rather than on cumulative achievement of skill at the completion of the chapter. Ideally, many of the children will gain some skill and facility with integers, but this should not be your primary objective for the chapter.

The major emphasis in this chapter should be on helping the children develop their intuition about integers and on using the basic principles to develop computational rules for performing the basic operations. It is not easy to tell from their work whether or not the children have attained these goals. However, a careful, day-to-day evaluation of their work, both written and oral, will be very helpful.

Maintain a spirit of exploration and discovery in this chapter. Do not allow the children to become frustrated by efforts to remember particular details. Rather, create the air of adventure and enjoyment that comes from studying something new and interesting. Minimize the importance of mastering specific skills or techniques. The attitude the children develop toward negative numbers is more important than any specific skill with integers. The review pages are intended for summarizing rather than evaluating skills and concepts.

Resources for Active Learning

General Activities

Franklin Series: *Making and Using Graphs and Nomographs*, "Graphs: Zero and the Positive Real Numbers," pp. 93–96, Lyons and Carnahan (Available from McGraw-Hill Ryerson)

Madison Project: *Discovery in Mathematics* (Teacher's Text), "Can you add or subtract?" pp. 35–37; "The Matrix Game," pp. 43–44; "The Point-Set Game" (Go), pp. 55–66; "Postman Stories," pp. 95–98; *Explorations in Mathematics* (Teacher's Text), "Signed Numbers," pp. 55–68; "Postman Stories," pp. 69–79, Addison-Wesley

Mathex: Graphing and Probability No. 6, "Positive and Negative Numbers," pp. 12–15, Encyclopaedia Britannica Publications Ltd.

Nuffield Project: *Computation and Structure* 4, "The Integers," pp. 22–39, Wiley

Manipulative Devices

"Invicta" Integers Kit (Mafex Associates)

Thermometer (Edmund Scientific: Nasco)

Commercial Games

Go (Gomoku) (Math Media; World Wide Games)

Vectors (General Learning Corp.; Selective Educational Equipment)

PAGES 340–341

Objectives

Given negative integers, the child will show his understanding of their application by describing a physical situation in which they can be used.

Given an integer, the child will be able to find its opposite.

Preparation

Since the investigation in this lesson serves as an introduction to negative integers, have the children begin immediately with it.

Investigation

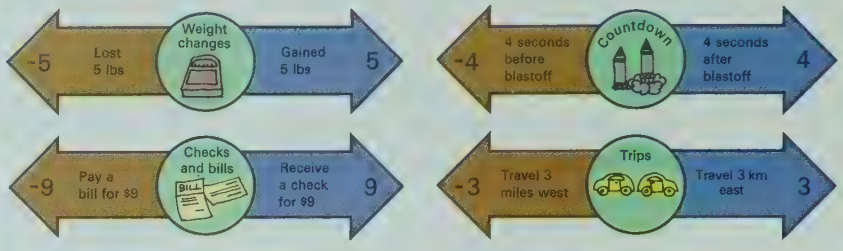
It would be helpful for children to work in groups of two or three, and together try to describe the other situations requested. If children have difficulty, even after considerable thought, you might suggest one of the following: stock market, up and down; thermometer, above and below freezing; ammeter, positive and negative charge; elevators, above and below ground level; game scores "in the hole" and "out of the hole"; above and below sea level. The important thing here is for children to see how many familiar situations can be thought of in terms of positive and negative numbers, that is, in terms of opposites.

15 Integers

Let's explore positive and negative numbers.

Investigating the Ideas

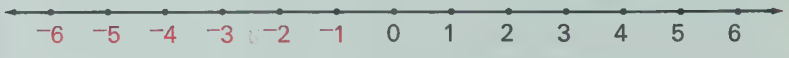
Here are some situations that can be described by using positive and negative numbers.



Can you show some other situations like these?
See Investigation.

Discussing the Ideas

Situations such as those above suggest the invention of numbers for the points **opposite** the whole-number points on the number line. The symbols for these numbers are given in red on the number line below. These numbers are called **negative numbers.** The non-zero whole numbers are often called **positive numbers.**



1. Why do you think pairs of positive and negative numbers, such as 4 and -4 are sometimes called **opposites?**
   See Discussion.
2. Choose some positive and negative numbers and give the opposite of each one.
   Answers will vary.
3. How could you locate the point for $^{-}27$ on the number line?
   Count 27 places to the left of zero, or find the opposite of 27.



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Discussion

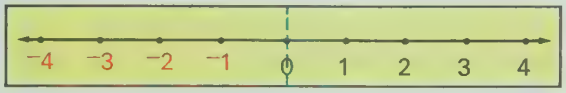
Use the situations children thought of in the investigation to expand the explanation given in the text. Stress the idea of opposites. When you refer to the number line, be sure the children understand that in relation to the centre-point 0, $^{-}1$ is opposite 1, $^{-}2$ is opposite 2, and so on. Ask the children to read the negative integers and observe with them the distinguishing symbol for these new numbers, the small dash, or negative sign, at the upper left-hand corner of the numeral. Do not call this a minus sign. Also, point out that a positive integer is sometimes written with a positive sign, such as $^{+}3$, or $^{+}6$, but this sign usually is not used. It would also be helpful to point out that the positive and negative integers, together with zero, form a set of numbers called *the integers* and are often written as: The set of integers = {0, 1, $^{-}1$, 2, $^{-}2$, 3, $^{-}3$, . . .}.

Help the children observe that opposites, such as 4 and $^{-}4$, are equally distant from 0 on the number line, but in opposite directions.

Note that when the set of integers is listed, the numbers need not be put in ordered form, as on the number line. The set of integers is sometimes listed {0, 1, $^{-}1$, 2, $^{-}2$, 3, $^{-}3$, 4, $^{-}4$, 5, $^{-}5$, . . .}; and at other times they are listed {. . . $^{-}3$, $^{-}2$, $^{-}1$, 0, 1, 2, 3, . . .}. Also, have the children observe that zero is an integer even though zero is neither positive nor negative, because zero has no opposite.

Using the Ideas

1. Imagine cutting out a number-line strip and folding it along the blue line.



- A The point for 3 would fall on the point for what number? -3
- B The point for −4 would match with the point for what number? 4
- C If the strip were long enough, −62 would match with what number? 62
- D What number would match 103? -103

2. Give the opposite of each integer.

A 2 -2 C -2 2 E 9 -9 G 56 -56 I -18 18 K -268 268 M -786 786
B 6 -6 D -6 6 F -20 20 H 100 -100 J -59 59 L 597 -597 N -260 260

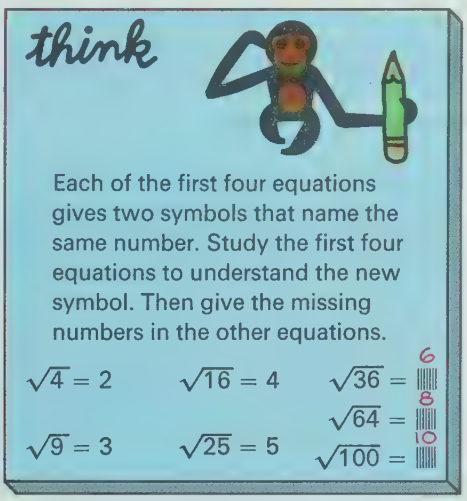
3. Copy and complete this number-line picture.



4. The planet Pluto has an estimated surface temperature of 194°C below zero.

- A How could you represent this temperature? -194°C
- B Give the opposite of this temperature. 194°C

★ **5.** What is the opposite of the opposite of the opposite of 9? -9





Using the Exercises

Assign the exercises on page 341 as independent work. If children have understood the general notion of integers as opposites they should not have difficulty with these exercises. Note that exercise 5 is intended as a special challenge, but all the children would benefit from a discussion of it. The best way to think through the problem is to begin with the last phrase and find the opposite of 9, which is −9. You might point out that the whole question may be expressed with parentheses: −(−(−9)).

The *Think* problem gives children an opportunity to discover the meaning of the square root symbol, but discuss this with the children after they have worked on it, to make sure they realize that $\sqrt{4} = 2$ may be read "the square root of 4 equals 2."

Assignments (page 341)

Minimum: 1–3. Average: 1–4. Maximum: 1–5.

Resources for Active Learning

Developmental Math Cards, "Adding with Arrows, "L¹16; "Subtracting with Arrows, "L¹17, Addison-Wesley.

Franklin Series: *Making and Using Graphs and Nomographs*, "Graphs: The Number Line for Integers," and "Comparing Integers," pp. 39–44, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Nuffield Project: *Computation and Structure* 4, "Application of the Integers," pp. 40–49, Wiley.

Workbook, page 107

PAGES 342–343

Objective

Given any two or three integers, the child will be able to apply the opposites principle or the basic principles of addition to them.

Preparation

To prepare for this lesson, you might review how the number line is used to show negative and positive integers. Or, you might give children practice in finding the opposites of integers, giving them phrases such as the following to interpret.

What is the opposite of positive 4? ($^-4$)

What is the opposite of negative 3? (3 or positive 3)

What is the opposite of the opposite of negative 7? (Since the opposite of $^-7$ is positive 7, the opposite of the opposite of $^-7$ is $^-7$.)

What is the opposite of 5? ($^-5$)

What is the opposite of the opposite of $^-5$? ($^-5$)

Children may need to think through questions such as these very carefully. If you think such a review is too difficult for your children, recall with them how addition on a number line is thought of as a move to the right and subtraction a move to the left, as a specific preparation for the investigation.

Investigation

Encourage children to work on this investigation independently. Suggest that they try other examples of addition before they try to write a rule. You might write the following on the chalkboard:

$^-4 + 4 = ?$ $\quad$ $^-5 + 5 = ?$

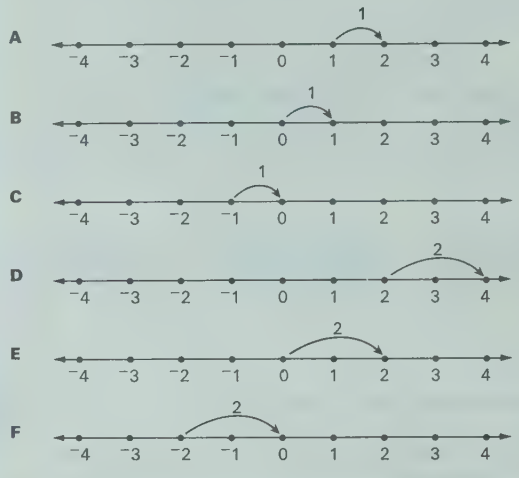
$^-3 + 3 = ?$ $\quad$ $^-7 + 7 = ?$

Suggest that they check their answers by using a number line. Finally, they should summarize their conclusion in a rule.

● *Let's explore basic principles for addition of integers.*

Investigating the Ideas

Study the patterns.



$1 + 1 = 2$

$0 + 1 = 1$

$^-1 + 1 = 0$

$2 + 2 = 4$

$0 + 2 = 2$

$^-2 + 2 = 0$

? Can you find each of these sums and then write a rule for adding any negative number and its opposite ?

See Investigation and Discussion.

Discussing the Ideas

Use positive and negative numbers to give examples. Examples will vary.

Opposites principle	The sum of any negative number and its opposite is zero. $2+^-2=0$
0 principle (+)	If you choose a number and add 0, the sum is the number you chose. $-2+0=-2$
Commutative principle (+)	When finding the sum of two numbers, you can change the order of the addends and the sum is the same. $-2+2=2+^-2$
Associative principle (+)	When finding the sum of three numbers, you can change the grouping of the addends and the sum is the same. $2+(^-2+3)=(2+^-2)+3$

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Discussion

Have children compare the rule they wrote in the investigation to the opposites principle given in the text. Point out that by assuming that the basic principles for the addition of whole numbers and the addition of fractional numbers hold for the addition of integers, they should soon be ready to learn to add integers. Note that, when children are asked to give an example of each principle, they need not be able to find the sum of the integers they use. Thus, $^-6 + 8 = 8 + ^-6$ is an acceptable example of the commutative principle even though a child may not be able to find the sum $^-6 + 8$. However, it would be best to discuss examples in which the opposites principle can be applied, such as

$$^-6 + (6 + 2) = (^-6 + 6) + 2 = 2.$$

Using the Ideas

1. The number line suggests that we can solve the equation $-1 + 1 = n$ by thinking about a jump 1 unit to the right from -1. What is this sum? Solve each equation. It may help to think about the number line.



A $-2+2=n$ 0 C $-4+4=n$ 0 E $-10+10=n$ 0 G $-97+97=n$ 0
B $-3+3=n$ 0 D $-5+5=n$ 0 F $-53+53=n$ 0 H $-508+508=n$ 0

2. Find the sums. Because of the **commutative** and **associative principles**, you can first find the sum of the addends given in red.

A $1 + 2 + -2$ 1 C $4 + 1 + -14$ E $6 + 5 + -56$ G $24 + 6 + -6$ 24
B $2 + 3 + -3$ 2 D $-4 + 4 + 6$ 6 F $-7 + 7 + 3$ 3 H $-8 + 8 + 4$ 4

3. The **basic principles** will help you complete each exercise. Give the number for ***a***. Then give the number for ***b***.

A Since $-25 + 25 = a$, we know that $25 + -25 = b$. 0, 0
B Since $-3 + 0 = a$, we know that $0 + -3 = b$. -3, -3
C Since $10 + -7 = a$, we know that $-7 + 10 = b$. 3, 3
D Since $-5 + (-4 + 4) = a$, we know that $(-5 + -4) + 4 = b$. -5, -5

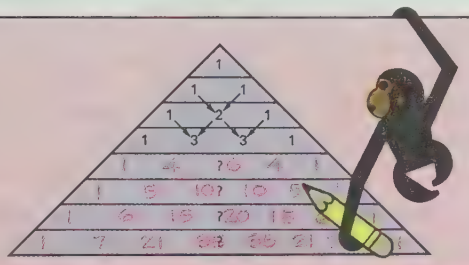
4. These temperatures were recorded on board a trawler in the North Atlantic. Tell whether the temperature rose or fell and give the number of degrees of change for each morning.

	9 a.m.	noon	
1st day	-5°C	8°C	+13°
2nd day	-6°C	3°C	+9°

	9 a.m.	noon	
3rd day	-2°C	-4°C	-2°
4th day	-3°C	11°C	+14°

think

Here are 4 rows of a "triangle." Can you discover the pattern and make 4 more rows?





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Mathematics

The following basic principles, which hold true for the addition and multiplication of whole numbers, are now assumed to hold true for the integers. For any integers a, b, and c:

Opposites Principle: The sum of any integer and its opposite is zero.

$$6 + {}^-6 = 0$$

Associative Principle:

Add.

$$a + (b + c) = (a + b) + c;$$
$$5 + ({}^-3 + {}^-2) = (5 + {}^-3) + {}^-2$$

Mult.

$$a \times (b \times c) = (a \times b) \times c$$
$$3 \times (2 \times {}^-4) = (3 \times 2) \times {}^-4$$

Distributive Principle:

$$a \times (b + c) = (a \times b) + (a \times c);$$
$${}^-5 \times (4 + 3) = ({}^-5 \times 4) + ({}^-5 \times 3)$$

Zero Principle:

$$a + 0 = a; \quad a \times 0 = 0;$$
$${}^-3 + 0 = {}^-3; \quad {}^-2 \times 0 = 0$$

One Principle:

$$a \times 1 = a;$$
$${}^-2 \times 1 = {}^-2$$

Commutative Principle:

Add. $a + b = b + a;$
$${}^-2 + 3 = 3 + {}^-2$$

Mult. $a \times b = b \times a$
$$4 \times {}^-3 = {}^-3 \times 4$$

Workbook, page 108

Using the Exercises

On page 343, exercises 1 and 2 give the child an opportunity to apply the rule from the investigation—the opposites principle. The parts of exercise 3 can be solved by reasoning with the basic principles.

When children finish these exercises, help them check their answers and clarify any troublesome points. Continue to stress the usefulness of the opposites principle and the regrouping which can be done because of the commutative and associative principles.

Assignments (page 343)
Minimum: 1, 2. Average: 1–3.
Maximum: 1–3.

PAGES 344–345

Objective

Given two integers, the child will be able to find their sum.

Preparation

Materials

10 red and 10 blue checkers, discs, or counters for each child

To prepare for this lesson, you might recall with the children how various objects can be used to symbolize different mathematical ideas. For example, the colored centimetre strips were used for many topics, such as fractions, whole numbers, and so on. In this investigation, colored counters are used with a particular meaning given to each color.

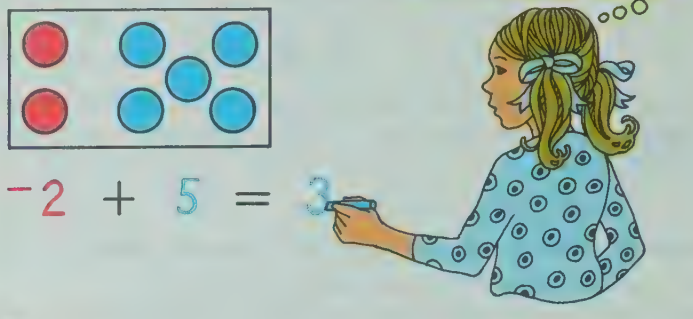
Investigation

If necessary, children can simply cut out and color small squares or circles to be used in place of counters. It is essential, however, that there be at least 8 red and 8 blue. Then a study of the text should help the child relate the red counters to negative integers and the blue counters to positive integers. Encourage them to think carefully as they work out each sum with the counters. If a few children have difficulty relating the red and blue colors to negative and positive integers, help them understand how one red counter and one blue counter cancel each other. For those who grasp the ideas quickly, you might suggest other sums such as $6 + {}^-7 = ?$; $3 + {}^-6 = ?$; $5 + {}^-1 = ?$; $7 + {}^-3 = ?$

Can you add positive and negative numbers?

Investigating the Ideas

Use counters or checkers. Think of the two colors as opposites.



$^-2 + 5 = 3$

Can you use this idea and your counters to find these sums?

A $^-3 + 7 = ?$ 4 C $^-2 + 2 = ?$ 0

B $8 + {}^-5 = ?$ 3 D $^-4 + {}^-3 = ?$ $^-7$

See Investigation.

Discussing the Ideas

1. What key ideas did you need to find the sums by using the counters? See Discussion.

2. A One student found the sum of $^-2$ and 5 by finding the sum shown in the box. How is this method like using the counters?

$^-2 + 2 + 3$

$^-2 + 5 = n$

Equal numbers of opposites are "canceled out"

B What basic principles were used to find this sum?

Opposites principle and grouping principle

3. A How would you find the sum $5 + {}^-8$ by using counters? By using the method in exercise 2?



$5 + {}^-5 + {}^-3 = {}^-3$

B Can you think of another way to find this sum?

See Discussion.



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Discussion

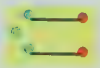
As children respond to discussion exercise 1, emphasize that negative numbers are represented by red counters; and positive numbers, by blue counters; also, stress how opposites "destroy each other" or eliminate each other. A red and a blue counter may be thought of as combining to make zero. For exercise 2, use other examples to explain how the opposites principle may be used along with the grouping principle to find the sum of a positive and a negative integer. Point out how these principles may be used to help them find sums without the number line or physical models. That is, by rearranging and breaking numbers apart by using the opposites, commutative, and associative principles (which means merely rearranging addends), they can arrive at correct sums. For example, $11 + {}^-4$ may be thought of as "$^-4$ cancels out 4 of the 11, leaving 7 as the answer."

As you discuss exercise 3B, be sure children observe the way a number line may be used to find sums of integers. For example, $5 + {}^-8$ may be thought of as $^-8 + 5$, in this manner:



Using the Ideas

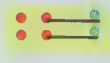
1. Find the sums. It may help you to think about the counters.

A 

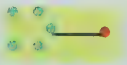
$3 + {}^{-}2 = n$ 1

B 

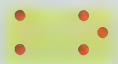
${}^{-}3 + 5 = n$ 2

C 

${}^{-}4 + 2 = n$ ${}^{-}2$

D 

$5 + {}^{-}1 = n$ 4

E 

${}^{-}2 + {}^{-}3 = n$ ${}^{-}5$

F 

${}^{-}5 + 4 = n$ ${}^{-}1$

2. Find the sums. The red and blue numerals may help you think of the counters.

A 6 + 5 + ${}^{-}5$

$11 + {}^{-}5 = n$ 6

B ${}^{-}9 + 9 + 11$

${}^{-}9 + 20 = n$ 11

C ${}^{-}4 + {}^{-}3 + 3$

${}^{-}7 + 3 = n$ ${}^{-}4$

D 5 + ${}^{-}5$ + ${}^{-}13$

$5 + {}^{-}18 = n$ ${}^{-}13$

E 24 + 6 + ${}^{-}6$

$30 + {}^{-}6 = n$ 24

F ${}^{-}8 + {}^{-}4 + 4$

${}^{-}12 + 4 = n$ ${}^{-}8$

3. Find these sums.

A $4 + {}^{-}2$ 2 B ${}^{-}8 + 5$ ${}^{-}3$ C ${}^{-}2 + {}^{-}3$ ${}^{-}5$ D $10 + {}^{-}6$ 4 E ${}^{-}9 + 5$ ${}^{-}4$ F $12 + {}^{-}2$ 10 G $8 + {}^{-}17$ ${}^{-}9$ H $56 + {}^{-}5$ 51

4. Think about jumps on the number line, **starting at 0.**

A

Jump 5 units right (+).	Jump 3 units left (−).	Left or right of 0? How far?
5	${}^{-}3$	n 2

B

Jump 4 units left.	Jump 2 units left.	How far left or right of 0?
${}^{-}4$	${}^{-}2$	n ${}^{-}6$

C

Jump 7 units left.	Jump 5 units right.	How far left or right of 0?
${}^{-}7$	5	n ${}^{-}2$

D

Jump 4 units right.	Jump 9 units left.	How far left or right of 0?
4	${}^{-}9$	n ${}^{-}5$

5. Draw number lines and show the jumps for each sum. Write an equation for each number-line picture.

A $4 + 3$ B ${}^{-}4 + {}^{-}3$ C $3 + {}^{-}3$ D $6 + {}^{-}3$ E ${}^{-}6 + 3$

See Answers, T.E. page 345.



Using the Exercises

Encourage children to work the exercises on page 345 independently. If children find the counters helpful, they might use them, but suggest that they try to work without them if possible.

Answers, exercise 5, page 345

A



$4 + 3 = 7$

B



${}^{-}4 + 3 = {}^{-}7$

C



$3 + {}^{-}3 = 0$

D



$6 + {}^{-}3 = 3$

E



$6 + 3 = {}^{-}3$

Assignments (page 345)*

Minimum: 1–5. Average: 1–5. Maximum: 1–5.

Mathematics

The sum of two integers can be found by using the basic principles. It is also helpful for children to see physical examples that suggest what the sum of two integers should be. In the following examples, a situation from the physical world illustrates finding the sum of two integers first. Then the sum is found mathematically by using the basic principles.

Example 1

Using a physical situation:

Spend $4.00	Spend $2.00 more	Richer or poorer? ($6.00 poorer)
${}^{-}4$	${}^{-}2$	${}^{-}6$

Using basic principles:

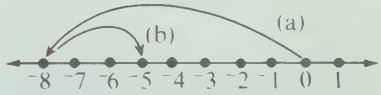
$$\begin{aligned}({}^{-}4 + {}^{-}2) + 6 &= ({}^{-}4 + {}^{-}2) + (4 + 2) \\ &= ({}^{-}4 + 4) + ({}^{-}2 + 2) \quad \text{Comm. and Assoc. Principles} \\ &= 0 + 0 \quad \text{Opposites Principle} \\ &= 0\end{aligned}$$

Now, ${}^{-}6$ is the only number that gives 0 when added to 6. Since ${}^{-}4 + {}^{-}2$ gives 0 when added to 6, we have ${}^{-}4 + {}^{-}2 = {}^{-}6$.

Example 2

Using a physical situation:

(a) Jump 8 units left.	(b) Jump 3 units right.	Finish to the right or left of starting point? (5 units to left)
${}^{-}8$	3	${}^{-}5$



Using basic principles:

$$\begin{aligned}{}^{-}8 + 3 &= ({}^{-}5 + {}^{-}3) + 3 \\ &= {}^{-}5 + ({}^{-}3 + 3) \quad \text{Associative} \\ &= {}^{-}5 + 0 \quad \text{Opposites} \\ &= {}^{-}5 \quad \text{Zero}\end{aligned}$$

These two examples may be generalized to provide a technique for finding the sum of any pair of integers.

Resources for Active Learning

Franklin Series: *Making and Using Graphs and Nomographs*, "Nomographs: Addition with Integers," pp. 45–46, Lyons and Carnahan. (Available from McGraw-Hill Ryerson)

Duplicator Masters, page 59
Workbook, page 109

PAGES 346–347

Objective

Given problems which include the notion of opposites in familiar situations, the child will be able to solve the problems by applying his understanding of positive and negative integers.

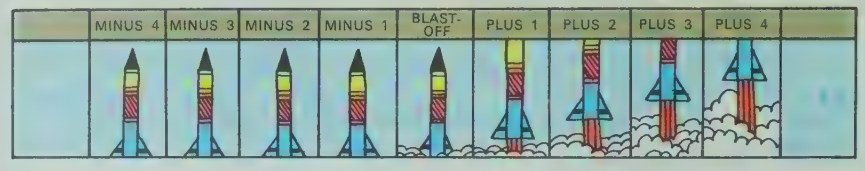
Preparation

To prepare for this lesson, you might review some simple addition and subtraction equations. Or, you might ask the children to suggest any applications of positive and negative integers they can think of. Also, point out how various illustrations, counters, and number lines may be used to aid in thinking of physical situations in terms of integers.

Solving Integer Story Problems

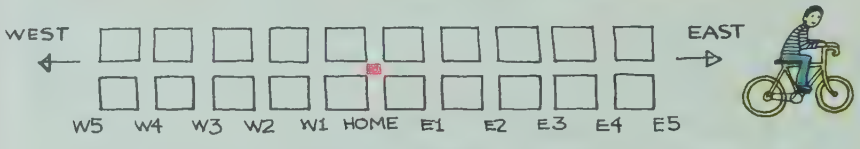
Write and solve an equation to answer the question in each problem.

1. These are pictures shown on television during the countdown and launching of a rocket.



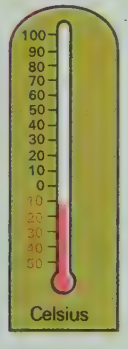
If you started watching at "minus 4," which screen would you be watching after 7 seconds? $-4+7=3$

2. Jerry keeps track of some of his bike trips on a map like this:



If Jerry starts at home, how far from home is he after a trip 4 blocks west, followed by a trip 9 blocks east? $-4+9=5$ (5 blocks east)

3. Water freezes at the temperature of zero degrees Celsius. We often speak of temperatures **above** and **below** zero. If the temperature was 12° below zero and rose 24°, what was the new temperature? $-12°+24°=12°$



4. A If you spend \$5 and earn \$8, how much richer or poorer are you? $-\$5+\$8=\$3$ (richer)
   B If you earn \$6 and spend \$9, how much richer or poorer are you? $\$6-\$9=\$-3$ (poorer)

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Discussion

Unless children have had difficulty in developing their understanding of addition and subtraction of positive and negative integers, assign the problems as independent work. Some children may find it difficult to give an equation for each of the problems on page 346 even though they can find the answer. Do not insist that such children write an equation. The equations may be helpful while discussing interpretations of the problems, but it is not essential that children be able to give the equations at this level.

Assignments (page 346)

Minimum: 1–4, oral. Average: 1–4. Maximum: 1–4.

Solving Inequality Problems

This number line may help you complete some of the exercises below.



1. Complete each sentence. Then give the correct sign ($<$, $>$) for each ▒.

A A temperature **above** zero is higher than (higher than, lower than) a temperature **below** zero.

10 > −24

B When the temperature is 5° **below** zero, it is warmer than when the temperature is 6° **below** zero.

−5 > −6

2. Jed and Sandy are playing a game in which they can get scores less than zero. They put rings around these below-zero scores. Who has the higher score for this game? Jed

Score	
Jed	Sandy
(6)	(8)

3. -7 is less than the number you get when you add 1 to -7. What number is this? Write an inequality statement about this.

-6 $\quad -7 < -6$

4. Use "greater than" or "less than" to complete each sentence.

A Any positive integer is greater than a negative integer.

B An integer is greater than an integer that is to the left of it on the number line.

Using the Exercises

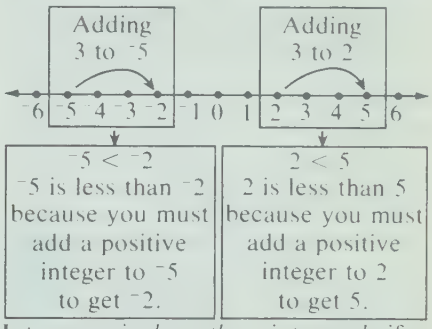
The problems on page 347 deal with inequality. You will find these suitable for discussion, either as you work through them with the children or when discussing the children's completed work. Help children realize that as the numerical value of negative integers increases, the value of the number decreases. Thus, $^-7 < ^-3$, and $^-25 < ^-1$. Point out that $^-25$ is further in a negative direction from zero than $^-1$. You might discuss how much better it is to be one dollar in debt ($^-1$) than 25 dollars in debt ($^-25$); or how much colder 25° below zero is than 1° below zero.

Assignments (page 347)

Minimum: 1, 2. Average: 1–4. Maximum: 1–4.

Mathematics

The basis of the definition of the order relations for the integers is illustrated in the following diagram.



Integer a is *less than* integer b if a positive integer added to a produces b.

Notice that, to show the addition of a positive integer on the number line, a "jump" to the right is shown. Hence, using the number line and the preceding definition, we arrive at the following interpretation of *greater than* and *less than*:

For all integers a and b,
if $a < b$, then a is to the left of b on the number line, and
if $b > a$, then b is to the right of a on the number line.

Follow-up

Illustrate the need for negative numbers with a problem like this one.

> Two boys ran a lemonade stand for 6 days. Find the balance for each day, and tell whether the boys made a profit.

Day	Spent	Received	Balance
			Starting $1.50
Mon.	$1.40	$0.75	
Tue.		$2.20	
Wed.	$2.15	$1.70	
Thu.	$1.80	$1.20	
Fri.		$1.80	
Sat.	$0.50	$1.10	Final

Final Balance − Starting Balance = Profit or Loss?

Resources for Active Learning

Nuffield Project: *Problems*—Red Set, No. 8, Wiley.

PAGES 348–349

Objective

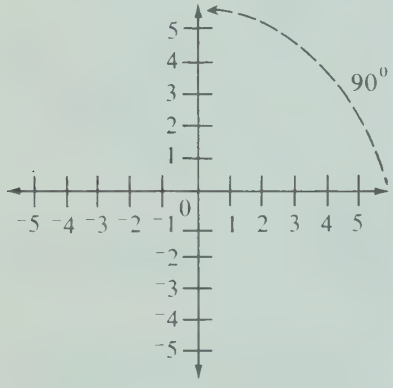
Given the co-ordinate plane, the child will be able to graph points whose co-ordinates include positive and negative integers.

Preparation

Materials

graph paper, $\frac{1}{4}$-cm grid (*Duplicator Masters*, page 63)

It would be helpful to work through the development of constructing the co-ordinate plane. Begin with a horizontal number line showing positive and negative integers. Then discuss how children might think of this line rotating 90° into a vertical position. The grids are included as guides to identify locations of points on the plane so that every point may be matched with a pair of co-ordinates.



Investigation

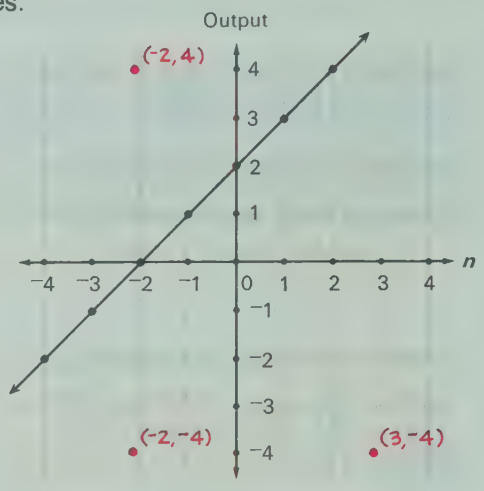
Direct the children to study the material presented in the text, noticing in particular the placement of points whose co-ordinates are negative integers. Be sure they understand that they are to make up a rule first, before graphing any points. Since multiplication with integers has not yet been developed, children should limit their rules to those involving addition or subtraction.

Can you graph functions?

Investigating the Ideas

The input-output pairs of numbers produced by a function machine can be used as co-ordinates.

Function Rule $n + 2$		
n	Output	
−4	−2	→ (−4, −2)
−3	−1	→ (−3, −1)
−2	0	→ (−2, 0)
−1	1	→ (−1, 1)
0	2	→ (0, 2)
1	3	→ (1, 3)
2	4	→ (2, 4)



When you draw the graph for the set of co-ordinates produced by this function rule, the points all lie on a straight line.

? Can you choose another function rule and draw a graph for the co-ordinates of points produced by your rule? See Investigation.

Discussing the Ideas

1. Explain how to graph the points for (3, −4), (−2, −4), and (−2, 4). Are any of these input-output pairs from the function rule used above? No. See graph above.

2. If you graphed the number pairs produced by the function rule

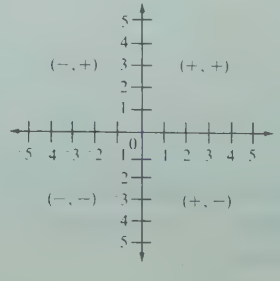
$$\text{output} = \begin{cases} 0 \text{ if input is even} \\ 1 \text{ if input is odd} \end{cases}$$

would the points all fall on one straight line? No. See Discussion.

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Discussion

If possible, have several children demonstrate on the overhead projector the rules and graphs they developed in the investigation. Help them see how co-ordinates with positive and negative integers are placed. As they discuss their graphs and exercise 1, you might use this:



You might point out that each of the four sections resulting in this diagram is called a *quadrant*.

In order for children to understand discussion exercise 2, have them graph some number pairs for the function, such as (2, 0); (3, 1); (−4, 0); (−5, 1). Clearly, this function could not result in one straight line.

Using the Ideas

For graphs, see Answers, T.E. page 351.

Find the missing numbers. Graph the number pairs.

1. Function Rule: $n + 3$

n	Output
-4	-1
-3	A 0
-2	B 1
-1	C 2
0	D 3
1	E 4

2. Function Rule: $n + -2$

n	Output
-2	-4
-1	A -3
0	B -2
1	C -1
2	D 0
3	E 1

3. Function Rule: $0 - n$

n	Output
-3	3
-2	A 2
-1	B 1
0	C 0
1	D -1
2	E -2

4. Function Rule: $(2 \times n) + 1$

n	Output
$\frac{1}{2}$	A 2
$1\frac{1}{2}$	B 4
$2\frac{1}{2}$	C 6
$3\frac{1}{2}$	D 8
$4\frac{1}{2}$	E 10

5. Function Rule: $(12 \div n) - 1$

n	Output
1	A 11
2	B 5
3	C 3
6	D 1
12	E 0

6. Function Rule: $n \times n$

n	Output
0	A 0
1	B 1
2	C 4
3	D 9
4	E 16

think

Give the missing numbers so that this will be a magic square.

7	-7	-6	4
-4	2	1	-1
0	-2	-3	3
-5	5	6	-8



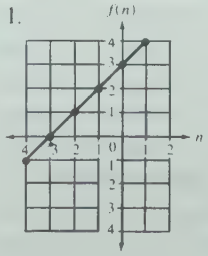
Using the Exercises

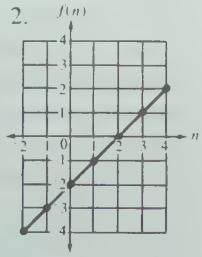
Supply a sufficient quantity of graph paper and assign the exercises on page 349 as independent work. Suggest to the children that they figure out what the number pairs are before they construct a graph for each exercise. For example, in exercises 4, 5, and 6, only the positive number quadrant is needed.

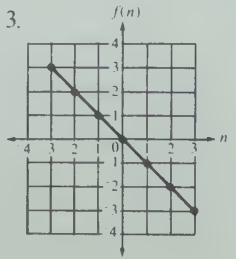
Assignments (page 349)*

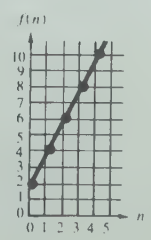
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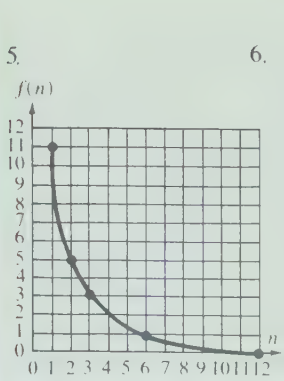
Answers, exercises 1–6, page 349

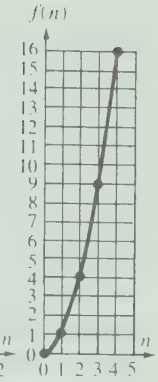
1. 

2. 

3. 

4. 

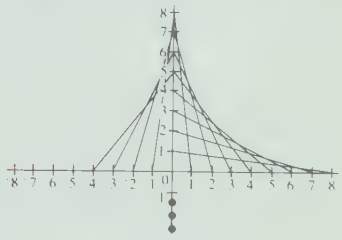
5. 

6. 

Follow-up/Curve Stitching

Children may enjoy curve stitching. One of the simplest examples is included here; others can be found in the resources listed below.

On stiff card or paper, have children label a pair of axes like this:



Use a needle to prick a hole at each labelled point. Then connect the points of each pair of axes that form a 90° angle, connecting the points farthest from 0 to one closest to zero: 8 to 1, 7 to 2, 6 to 3, 5 to 4, and so on. In the second quadrant, the order would be 8 to $^{-}1$, 7 to $^{-}2$, 6 to $^{-}3$, 5 to $^{-}4$, and so on. In the third quadrant, $^{-}8$ to $^{-}1$, $^{-}7$ to $^{-}2$, $^{-}6$ to $^{-}3$, and so on. Finally, in the fourth quadrant, $^{-}8$ to 1, $^{-}7$ to 2, $^{-}6$ to 3, $^{-}5$ to 4, and so on.

Resources for Active Learning

Experiments in Mathematics, Stage 1, pp. 30–31, Houghton Mifflin. [Curve stitching] (Available from Mafex Associates)

Mathex: Geometry No. 9, pp. 37–43 (pupil pages 56–59), Encyclopaedia Britannica Publications Ltd. [Curve stitching]

Mathex: Graphing and Probability No. 6, pp. 10–11 (pupil pages 26–27), Encyclopaedia Britannica Publications Ltd.

Nuffield Project: *Graphs Leading to Albegra* 2, pp. 37–43, Wiley.

Teaching Aids for Elementary Mathematics, "Graphing Pictures," pp. 132–137, Holt, Rinehart and Winston.

Workbook, page 111

PAGES 350–351

Objectives

The child will demonstrate his ability to work with the concepts presented in this chapter.

The child will demonstrate his ability to work with the concepts indicated for cumulative review.

Preparation

Most children would benefit from a review of addition and subtraction of integers. Keep the numbers in the examples small so that children can work them out on the number line, or if they prefer, with the red and blue counters.

Reviewing the Ideas

1. List the positive and negative whole numbers from −15 to 15.
−15,−14,−13,−12,−11,−10,−9,−8,−7,−6,−5,−4,−3,−2,−1,0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15

2. Give the opposite of each:
A 3 −3 B −3 3 C 9 −9 D −7 7 E 75 −75 F −75 75 G −183 183

3. Give the sign (<, >, =) for each ◍.
A 6 ◍ −6 B 0 ◍ −6 C −30 ◍ 7 D −9 ◍ −7

4. List the whole numbers greater than −10 and less than −5.
−9,−8,−7,−6

5. Find the sums.
A 9 + −5 4 C −9 + −8 −17 E 11 + −11 0 G 17 + −4 13 I −4 + 9 5
B −8 + 3 −5 D −8 + 12 4 F −7 + 12 5 H −12 + −5 −17 J 8 + −15 −7

6. Find the missing addends.
A 4 − −3 = *n* 7 B −4 − 6 = *n* −10 C −16 − −7 = *n* −9 D 0 − −8 = *n* 8

7. Use integers to write an addition equation for this problem.
You lose $9 and then find $12. How much do you have? −9+12=3
$3

8. Complete each sentence.
A A negative number is __ ? __ than a positive number. less
B __ ? __ is the only number that is zero never positive or negative.
C A positive number is __ ? __ than another positive number that is to the right of it on the number line. Would this also be true for two negative numbers? less

★ 9. Solve the equations.
A 2 − −3 = *n* + 3 2 D 4 − −2 = 4 + *n* 2
B −3 − 5 = *n* + −5 −3 E −2 − 7 = −2 + *n* −7
C −1 − −3 = *n* + 3 −1 F −4 − −6 = −4 + *n* 6

★ 10. Find this sum.
5 + −3 + 7 + −9 + −5 + −6 + −1 + 9 = *n* −3

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Discussion

Page 350 may be used as a review page worked together or as an evaluation page. However, children should not be expected to be skillful in adding and subtracting integers. It is more important for them to develop some intuitive notions about integers than to be able to perform the operations quickly.

Keeping in Touch with Computing Fractional numbers

1. Find the sums and differences.

A	B	C	D	E	F
34.3 + 47.7 = 82.0	29.6 + 54.8 = 84.4	73.7 − 26.5 = 47.2	56.2 − 37.5 = 18.7	9.643 + 8.27 = 17.913	58.3 − 9.764 = 48.536

2. Find the products and quotients.

A	B	C	D	E	F
9.3 × 0.7 = 6.51	5.65 × 3 = 16.95	4.26 × 5.3 = 22.578	91 / 0.6)54.6	76.9 / 0.04)3.076	.57 / 2.4)1.368

3. Find the missing percent or fraction.

Fraction	$\frac{1}{2}$	$\frac{1}{4}$ B	$\frac{3}{4}$	$\frac{1}{5}$ D	$\frac{3}{5}$	$\frac{7}{10}$ F	$\frac{1}{3}$	$\frac{1}{8}$ H	$\frac{3}{8}$	J
Percent	50% A	25%	75% C	20%	60% E	70%	33⅓% G	12½%	37½% I	66⅔%

4. Copy and complete the function table.

Function Rule: $n + -3$

n	Output
4	1
3	A 0
2	B −1
1	C −2
0	D −3
−1	E −4

5. Old Faithful, a geyser in Yellowstone National Park, erupts on the average of once every 64.5 minutes.

A About how many times does it erupt in a 24-hour day? About 22

B Starting right after an eruption, how many minutes pass while Old Faithful erupts 56 times? 3612 min



You are invited to explore

ACTIVITY CARD 17 Page 363

Using the Exercises

Page 351 reviews various concepts covered in the year's work. Again, this page may be used to evaluate children's understanding or to clarify certain points which the children found troublesome.

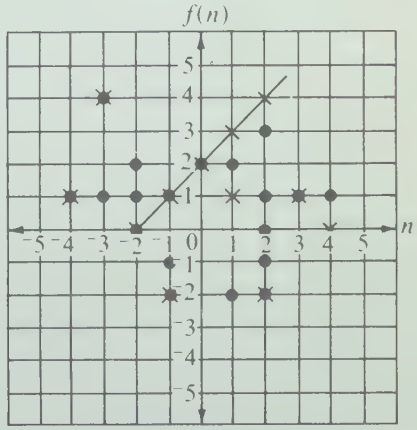
Most children will enjoy the approach used on page 352. Encourage children to try first to work the problems independently. Then provide opportunity for them to discuss their methods for solving them. Encourage acceptance of any reasonable method which yields the correct answer.

Follow-up/Tic-Tac-Toe

The children can have fun playing Tic-Tac-Toe, adapted to a co-ordinate system, as a class activity. Prepare a co-ordinate system using the integers −5 through 5, either on a large chart that can be covered with acetate or on a separate transparency that can be covered with acetate on an overhead projector.

Explain that the object of the game is to get five ×'s or ●'s in a row, column, or diagonal by naming the co-ordinates of each × or ●. Then divide the class into two closely matched teams. The members of each team take turns naming any number pair they wish. (It is usually wise to insist that each child make his own choice, whether it is the best choice or not, without help from anyone else on his team.)

As they play, the children will learn strategies for blocking the other team while they continue to build a string of ×'s or ●'s. Eventually, they should become skillful enough to make every contest a draw. (A draw is traditionally called "the cat's game.") You can make the game more difficult by extending the game-board to a larger grid and by requiring six ×'s or ●'s in a row.



A Sample Winning Game

Workbook, page 112 (Use with text page 352.)

PAGES 352–354

Objective

The child will demonstrate his ability to work with the concepts presented in this chapter.

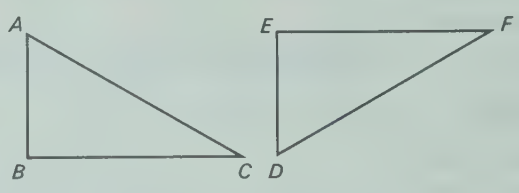
Preparation

You might review the 'congruency' of triangles, the properties of a cube, and the phrase '180° rotation image'.

Keeping in Touch with Geometry

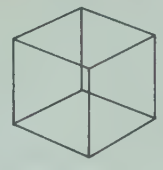


1. Assume that triangle *ABC* is congruent to triangle *DEF*. Complete the table.



	△ABC	△DEF
A	$\overline{AC}$ ≅	▒ D F
B	▒ A B ≅	$\overline{ED}$
C	$\overline{BC}$ ≅	▒ E F
D	▒ ∠ABC ≅	∠DEF
E	∠BAC ≅	▒ ∠EDF
F	∠BCA ≅	▒ ∠EFD

2. Look at a cube.



A How many vertices has it ? 8

B How many edges has it ? 12

C What can you say about the lengths of the edges ?

They are all equal

3. Using centimetre squared paper, copy each figure and mark the point P. Then rotate each figure through 180° about P and draw its image. Use tracing paper if you wish.



4. Draw a figure of your own on centimetre squared paper. Rotate the figure through 180° and draw its image.

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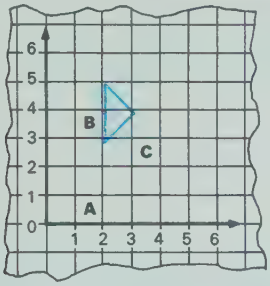
Discussion

As this is a review lesson, the child should not come upon undue difficulties. However it would be a good idea to compare the results achieved. The children could be reminded that all rotations are made in a clockwise direction, and that a plastic ruler would be a definite asset to aid them in relating the rotations to degree measure.

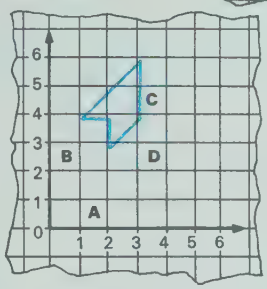
5. Copy each figure on your graph paper. Then use the point slider to translate the figure and draw its slide image.



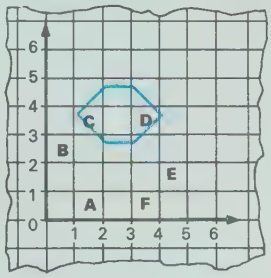
A



B



C

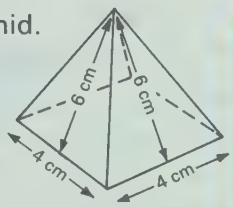


6. How many different lines of symmetry does a square have ? 4

7. A Give the co-ordinates of the midpoint of the segment whose endpoints have co-ordinates (1,1) and (5,5). (3,3)

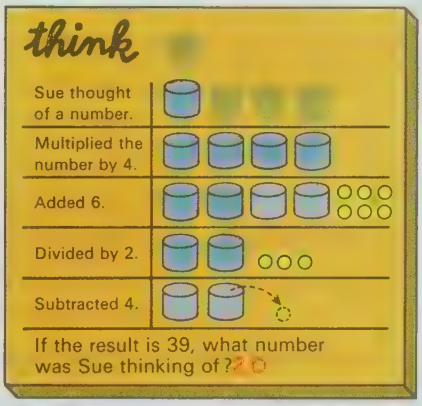
B Three corners of a rectangle are at (2,2), (2,7), and (5,7). Give the co-ordinates of the fourth corner. (5,2)

8. Look at this pyramid.



A On centimetre squared paper, draw a pattern for the pyramid. Be as accurate as you can.

B Cut out the pattern, fold it to form the pyramid. Use tape to fasten the edges together.



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Using the Exercises

Point out to the children the fact that in translations, a point is not relocated or actually moved. Rather a second point is designated as an image of the original point according to a particular motion.

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(See "Using the Exercises," page 353, for teaching suggestions for this page.)

Antique Arithmetic

Arithmetic students were busy scribbling these exercises into their "copybooks" over 100 years ago, in 1856. The exercises first appeared in *Ray's Higher Arithmetic,* by Joseph Ray, M.D. At the time, most North Americans lived on farms and bought their supplies at general stores. Can you do some of them?

Sample solution methods are shown, but others possible.

1. Which is the nearer number to 920736; 1816045 or 25427? *Ans.* Neither. Why? Both are the same "distance" from 920736 on a number line, but in opposite directions.



2. A grocer gave 153 barrels of flour, worth $6 a barrel, for 54 barrels of sugar: what did the sugar cost per barrel? *Ans.* $17.
(6×153)÷54

3. A grocer bought 135 barrels of pork for $2295; he sold 83 barrels at the purchase price and the remainder at an advance of $2 per barrel: how much did he gain? *Ans.* $104.
(135−83)2=104

4. A drover bought 5 horses at $75 each, and 12 at $68 each; he sold them all at $73 each: what did he gain? *Ans.* $50.
(75×17)−[(5×75)+(12×68)]

5. If a pulse beats 28 times in 16 seconds, how many times a minute is that? *Ans.* 105.
(28÷16)×60



6. I can pasture 10 horses or 15 cows on my ground; if I have 9 cows, how many horses can I keep? *Ans.* 4.
15−9=6; 10:15=n:6

7. If 13 men can build a wall in 15 days, in how many days can it be done if 8 men leave? *Ans.* 39.
(13−8):13=15:n

8. If 14 men can perform a job of work in 24 days, in how many days can they perform it with the assistance of 7 more men? *Ans.* 16 days.
(14+7):14=24:n

9. A company of 45 men have provisions for 30 days: how many men must depart, that the provisions may last the remainder 50 days? *Ans.* 18 men.
(50−30):50=n:45

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Mathematical Activities

How to Use the Activity Cards

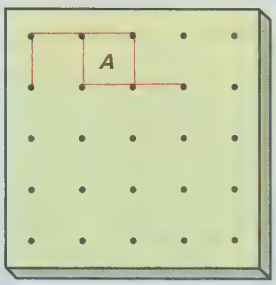
Do you like to explore things for yourself? These Activity Cards will give you some exciting experiences with mathematics. Each card presents a different idea for you to explore. Often you will find that a card will give you ideas for additional activities on your own.



ACTIVITY CARD 1

Use dot paper and try this game with a classmate.

At your turn, you must connect two adjacent dots in the same row or same column. If the connection results in a square, place your initial inside, score one point, and play again. The first player to score 9 points wins.



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Activity Card 1

This activity can stimulate interest and excitement in recreational mathematics. The game is easily played, and requires only paper and pencil and 5-by-5 dot arrays. After playing a few games, children will develop simple game strategies. Children should be encouraged to develop alternative rules of their own for determining the winner. For example, they might agree that a game ends when all possible squares have been formed; the person who completed the most squares would be the winner. Since a total of 16 squares may be formed, however, tie games between two players would be possible.

Activity Card 2

The purpose of this activity is to provide experiences in working with the number of possible ways that given events can occur, and to follow this up by investigating the probability that a given event will occur. As the children answer the last question on the card, you should be sure they observe that they are more likely to land on **A** since there are six possible ways to get to **A** and only four possible paths to get to **B**. An interesting extension of the ideas on this card would be to label the other squares along the diagonal established by squares **A** and **B** and then figure out how many ways there are to get to each of these squares. Of course, the squares on the end of the diagonal have only one possible path that can be used to reach them, because in order to get to the square at the top of the diagonal, one would have to flip four straight heads and to get to the square on the far right of the diagonal, one would have to flip four straight tails. The only other unlabelled square on the diagonal would have four possible ways to get there, just as square B has. The ideas generated in the number of possible paths for this investigation involve the familiar Pascal's triangle.

1					
1	5				
1	4	10			
1	3	6	10		
1	2	3	4	5	
	1	1	1	1	1

Notice along the heavily shaded diagonal that the number of paths is indicated as 1, 4, 6, 4, 1, as we learned through the activity, and if we are to move out to the next row on the diagonal, shaded lightly, we find the number of paths are 1, 5, 10, 10, 5, and 1.

ACTIVITY CARD 2

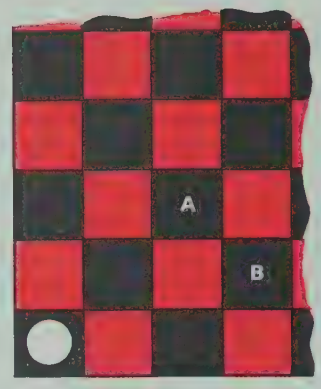
Put a checker in one corner of a checkerboard. To move, flip a penny and

(1) move one space up for a head.

(2) move one space right for a tail.

Use graph paper to show 4 possible paths to **B**.

Can you show all 6 ways to get to **A**? Which square are you more likely to land on, **A** or **B**?



ACTIVITY CARD 3

Can you find all the numbers less than 50 that can be represented by the triangular patterns below?

First	1	Third	6
Second	3	Fourth	10

Can you find a pattern?

Use your pattern to find the thirtieth triangular number.

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Activity Card 3

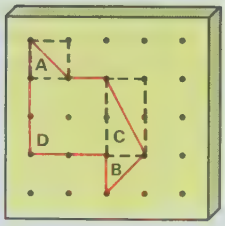
This activity card can be completed by nearly all children, although not all children will use the same method for finding the triangular numbers. The most obvious method is simply to construct the triangular dot pattern and count the number of dots. Other children may see that the number of dots increases in a regular order of 2, 3, 4, 5, ... Still others may see that the number of dots in any triangular pattern is a sum of consecutive whole numbers starting at 1: $1 + 2 + 3 + 4 + 5 \ldots$. The n^{th} triangular number can be found by the formula which follows:

$$n^{th} \text{ triangular number} = \frac{n(n+1)}{2}.$$

Of course, we do not expect very many children to discover this formula, but it is quite possible that one or two children might do so. The triangular numbers less than 50 are 1, 3, 6, 10, 15, 21, 28, 36, 45. The thirtieth triangular number is 465.

ACTIVITY CARD 4

The area of the dotted square is 1,
so the area of triangle **A** is __ ? __. $\frac{1}{2}$
The area of triangle **B** is __ ? __. $\frac{1}{2}$
The area of the dotted rectangle is 2,
so the area of triangle **C** is __ ? __. 1
The area of square **D** is __ ? __. 4
The area of the figure outlined in red is __ ? __. 6



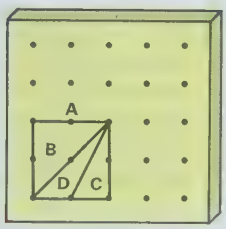
Can you find on the geoboard and draw on dot paper as many as 10 different figures with area 6 ? (Find one such figure that is unusual and compare it with others in your class.)

ACTIVITY CARD 5

What is the area of square **A** ? 4
What is the area of triangle **B** ? 2
What is the area of triangle **C** ? 1

Using the information about the areas of **A**, **B**, and **C**, what is the area of triangle **D** ? 1

There are 8 triangles of different shapes with area 1 that can be formed on the geoboard. How many of them can you find and draw on dot paper ?



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Activity Card 4

In working with this activity card, the child should first discover that the area of a right triangle can be found by finding half the area of an appropriate square or rectangle. The child should also find out that the area of a complicated figure can be found by adding areas of the smaller figures that constitute it. You should give the children considerable freedom in attempting to construct figures of their own that have an area of six. Certainly, the children should come up with a wide variety of figures, including convex figures, concave figures, figures made entirely of squares, and a variety of other kinds of figures composed of squares, rectangles, and triangles. These activities on the geoboard might encourage the children to seek further work with the geoboard. Some references for additional geoboard work follow the suggestions for use of Activity Card 5.

Activity Card 5

This activity card involves having the child find the area of a given figure by a subtractive, rather than an additive, method. For instance, by finding the area of the 2×2 square and subtracting the areas of triangles B and C, the child is left with the area of triangle D. The child may not be able to find all 8 of the triangles with area 1, but should be rewarded for any success he may have. Some children may see that several of the triangles may be found by finding triangles with base 1 and height 2, or triangles with base 2 and height 1.





An interesting extension of the activities on this card would be to have the child find on the 5×5 dot paper or geoboard all possible triangles with areas other than one. For example, you might suggest that the child begin by finding all triangles with areas of $\frac{1}{2}$, and then all triangles with area $2\frac{1}{2}$, and so on up to 8. Of course, such an activity would extend over a considerable length of time, and one certainly would not expect the child to find all such triangles for a given area. (For example, there are 12 possible triangles with area 2!) Some references for additional geoboard activities follow.

Arithmetic Teacher articles (National Council of Teachers of Mathematics, Washington, D.C.)
Geocards, Trivett (Cuisenaire Co., New Rochelle, N.Y.)
Geosquare Activity Cards, Barson; *Geosquare* Teacher's Manual, Bradford and Bartram (Scott Scientific, Fort Collins, Colo.)
Inquiry in Mathematics via the Geo-board, Cohen (Walker, N.Y.)

Activity Card 6

This card is intended to help develop the child's space perception and to strengthen his understandings of concepts of symmetry. By folding and cutting paper, the child should quickly learn that the angle of the cut determines whether the figure will be a square, an octagon, or a four-pointed star. Cuts other than straight cuts will produce other interesting figures. Children should be encouraged to try other types of paper-folding in order to produce other interesting figures. For example, the third fold need not be along the diagonal. The activities that the children participate in beyond those suggested on the card itself can be an important part of the learning experiences suggested by this card.

Activity Card 7

This activity provides a challenging open-ended puzzle which many children will enjoy trying.

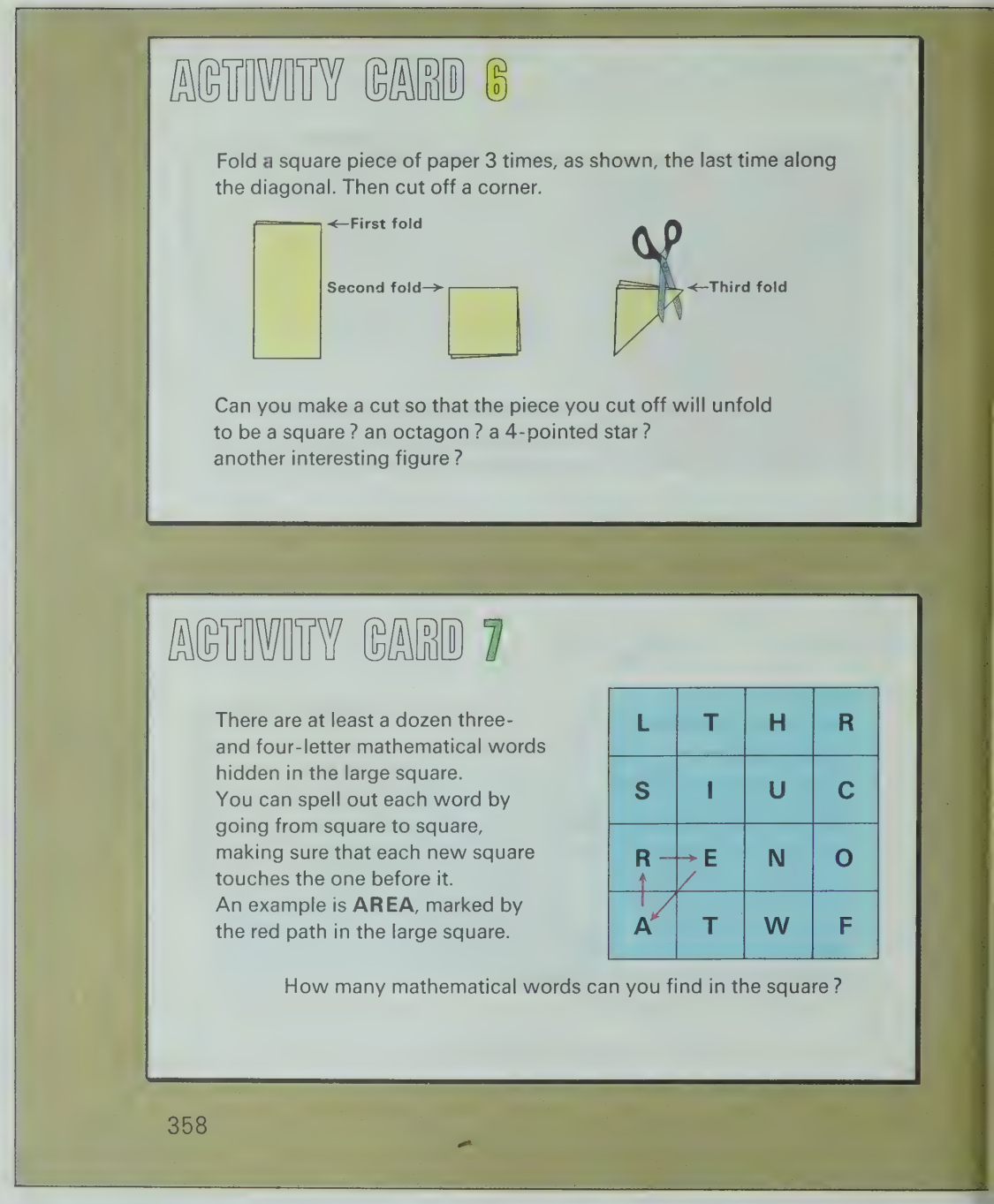
Solutions:

area	four
set	nine
line	ten
cone	net
one	unit
two	rate

Harder or "semi-math" words:

sine	tare
rise	west
list	cut
test	owe

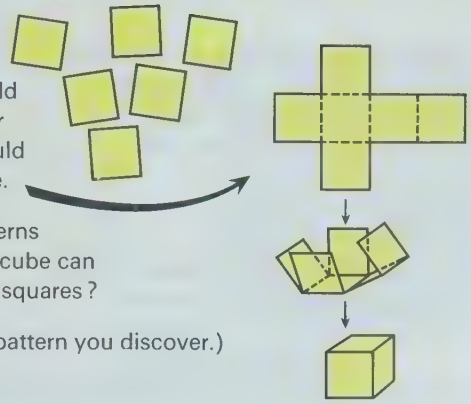
The activity might be extended by having the children find other 3- or 4-letter words which need not be mathematical. As a further extension of the activity, you might encourage a few students to try designing a puzzle like this one for their classmates to try.



ACTIVITY CARD 8

Cut out 6 squares.

Here is one way you could tape the squares together to form a pattern that could be folded to make a cube.



How many different patterns that can be folded into a cube can you make by using the 6 squares?

(Draw a picture of each pattern you discover.)

ACTIVITY CARD 9

One of the numbers below is approximately the number of times an ordinary bicycle wheel goes around in one kilometre. Which do you think it is?



48 480 4800

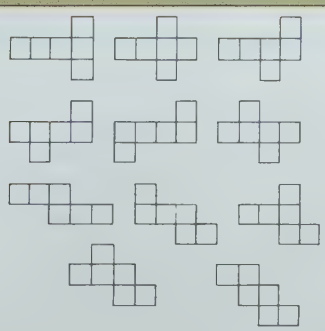
Can you find out how many times your own bicycle wheel goes around in one kilometre?

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Activity Card 8

This card is intended to help the child develop space perception and to broaden his perspective of the cube. Most of the patterns which can be made to fold into a cube involve a row of 4 squares with the 2 other squares on the opposite sides of this row. There are some patterns, however, which can be folded to form a cube which involve more of a stair-step pattern. In all, there are 11 different ways that the 6 squares can be taped together in order that they can be folded to make a cube, as illustrated at the right.



The children should be encouraged to find as many different patterns as they can, but you should by all means be alert to give credit for any small success. This investigation might be extended to further work with putting plane figures together to form other solid figures.

Activity Card 9

While the task to be performed for this card is specific, the activity is open-ended in the sense that the child should be encouraged to use his own methods to attempt to find out how many times the wheel goes around in one kilometre. The guesses or approximations are given as a motivational feature to get the child to want to find out whether or not he made the best guess. Of course, the key to the solution of the problem is for the child to find out the circumference of his own bicycle wheel. The approximations given are for a standard bicycle wheel. The best approximation is that the wheel would go around about 480 times.

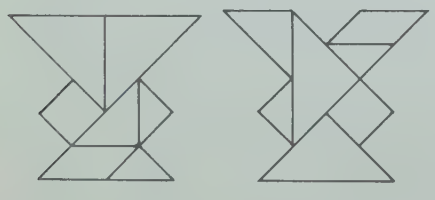
Activity Card 10

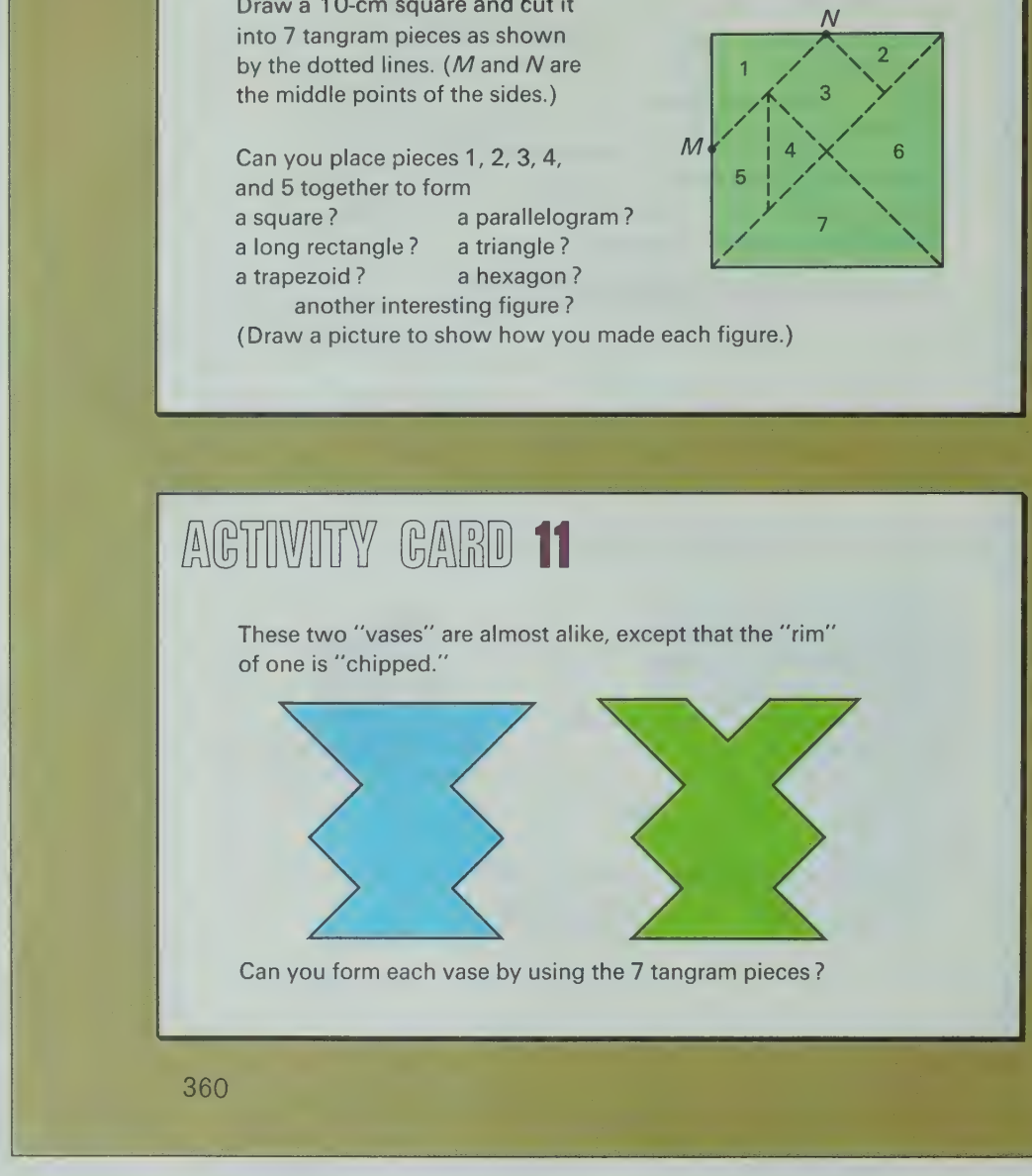
You might want to encourage your children to make a square larger than the one indicated on this activity card. Thus, when they cut out the tangram pieces they will be better able to work with them to form the various figures. To simplify the matter of making the tangram pieces, you might also ditto a number of sheets containing the tangram pattern and simply have the children cut the pieces out of the dittoed sheet. You will want to have the children take care in cutting out the pieces so that the figures they produce will be reasonably accurate. Once the children have accomplished the various figures suggested, encourage them to continue on their own to experiment with finding a variety of other figures.

Activity Card 11

This activity card presents, seemingly, a paradox that utilizes the 7 tangram pieces. The two vases seem to be the same size and shape, except for the chip out of the rim of one. It would seem that a contradiction to the principle of conservation of area has been found. However, it can be proved that the vase with the "chipped rim" is slightly taller and slightly wider at the top—just enough taller and wider to make up for the missing chip. If the side of the square tangram piece is 1 unit long, the height of the solid vase is $2\frac{1}{2} \times \sqrt{2}$, or about 3.53 units. The width of the solid vase at the top is 4 units compared to $3\sqrt{2}$, or 4.23 units, for the chipped vase.

Solution:

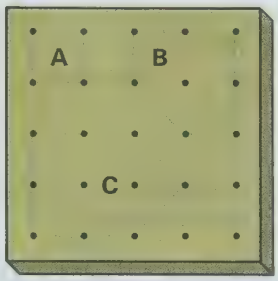




ACTIVITY CARD 12

Copy and fill in the table below.

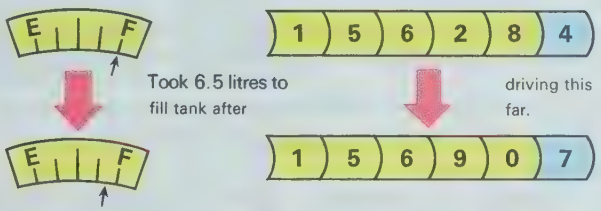
Polygon	A	B	C
Number of nails on the edges	4	7	8
Number of interior nails	0	1	2
Area of polygon	1	$3\frac{1}{2}$	5



Construct several more polygons on dot paper. Make a table like the one above. Can you discover a way to find the areas of polygons on the geoboard if you know the number of edge nails and interior nails?

ACTIVITY CARD 13

Figuring gasoline consumption



Can you find how many kilometres this car can travel per litre of gasoline?

Try this with your family's or someone else's car.

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Activity Card 12

The purpose of this activity card is to encourage discovery. By constructing many different polygons on the geoboard (either concave or convex), the child may discover that there is a relatively simple relation between the number of nails on the edges of the polygon, the number of nails in the interior of the polygon, and the area of the polygon. This relation has been given the name Pic's Theorem. This theorem states that the area of any simple polygon formed on the geoboard is the number of edge nails divided by 2, plus the number of interior nails minus 1.

If n_e = number of edge nails and n_i = number of interior nails, then the area, A, is given by the formula,

$$A = \frac{n_e}{2} + n_i - 1$$

Encourage students to fill out a table like the one illustrated in the text in order to keep a record of their results and as an aid in discovering the theorem.

Activity Card 13

This activity involves the practical problem of calculating how many kilometres a given car can drive on a litre of gasoline. The trick to doing this is to fill the tank and record the distance; then drive for a while and refill the tank, again recording the distance. The figures given on the card show that the car used $6\frac{1}{2}$ litres driving a distance of 62.3 kilometres. Encourage the children not only to solve the problem given on the card, but also to figure this out for either their family car or someone else's car. This can also be an interesting activity for finding the distance for small motor bikes or motor scooters.

Activity Card 14

This activity is designed to extend student work in finding decimal approximation of numbers in fractional form and to motivate student interest in the number π.

It has been proved that π is an irrational number (not a quotient of two integers); thus, its decimal representation is non-repeating and non-terminating. All of the fractions in this activity represent fractional numbers, hence repeating decimals.

Decimals for each fractional number in the activity are as follows. The fraction in part C is the best approximation for π.

A $\frac{22}{7} = 3.\overline{142857}$ (The bar represents the repeating part of the decimal.)

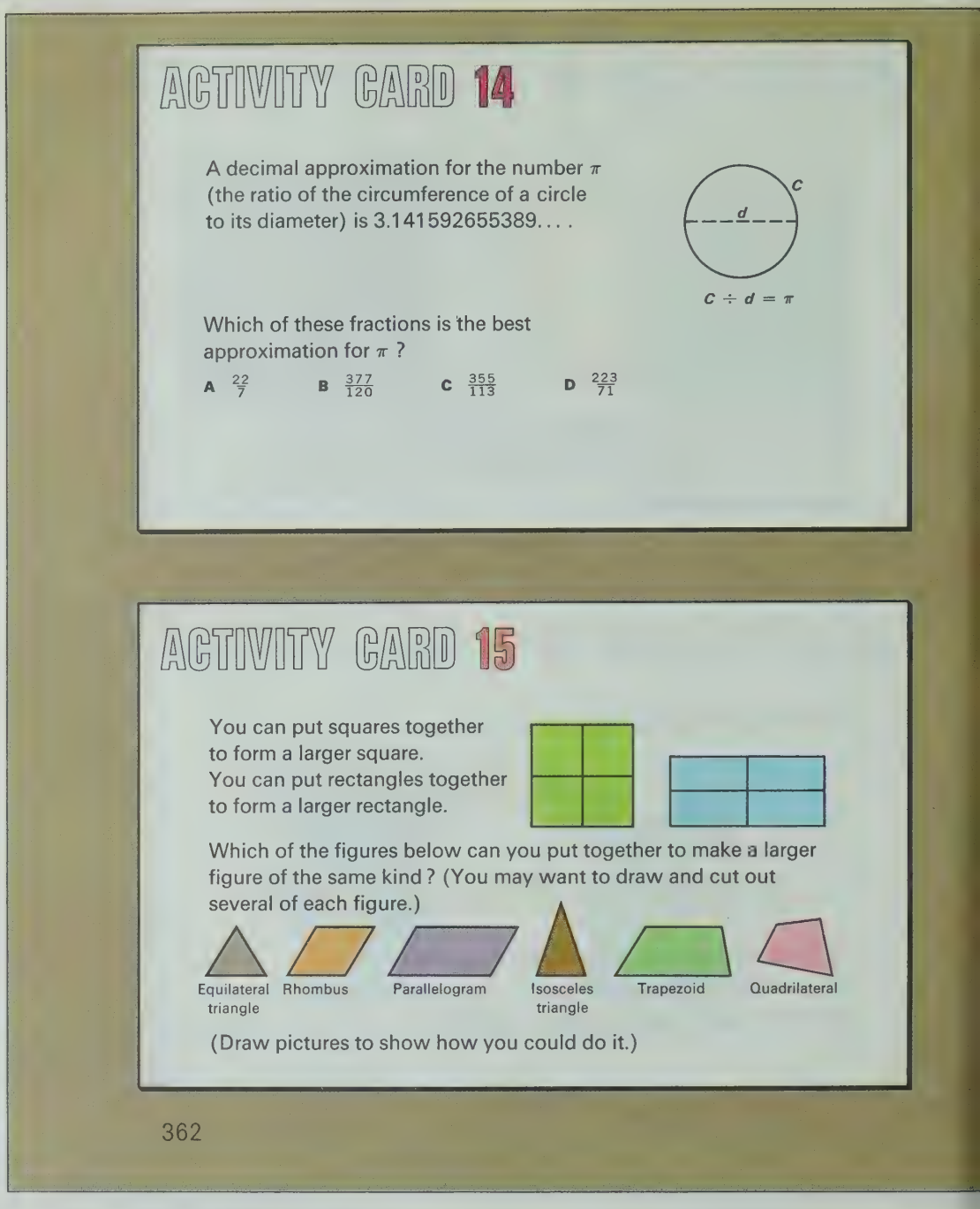
B $\frac{377}{120} = 3.1416666\overline{6}$ (Attributed to Ptolemy, *ca.* 150 A.D.)

C $\frac{355}{113} = 3.1415929203\ldots$ (Attributed to Tsu Ch'ung-chih, 430–501.)

D $\frac{223}{71} = 3.1408456371\ldots$ (Attributed to Archimedes, 287?–212 B.C.)

Activity Card 15

The purpose of this card is to help children investigate some of the tessellation characteristics of various figures. If a certain type of figure can be put with others of the same size to make a larger figure of the same kind, then, clearly, this figure could be used to tile a floor. Children can save work in cutting out these figures of the same size and shape by folding several thicknesses of paper, and then cutting out the figure once. You might encourage the children actually to paste figures together on a display chart in order to give their results. An interesting follow-up to this activity is to have the children invent other figures to see if they can be put together to produce similar figures. For example, they might choose to try this activity with a regular pentagon and find that it is not possible to put regular pentagons together in order to form another pentagon. It is interesting to note that regular hexagons can be put together in order to form other regular hexagons.



ACTIVITY CARD 16

Can you solve these fractional-number problems?

1. Can you find the 3 different missing denominators in the equation $\frac{1}{\square} + \frac{1}{\square} + \frac{1}{\square} = 1$?

2. What are the next four fractional numbers in the sequence $\frac{3}{2}, \frac{7}{5}, \frac{17}{12}, \frac{41}{29}, \frac{\square\square}{\square\square}, \frac{\square\square}{\square\square}, \frac{\square\square}{\square\square}, \frac{\square\square}{\square\square}, \ldots$?

3. What fractional number is $\frac{1}{3}$ of the way between $\frac{3}{7}$ and $\frac{9}{14}$?

ACTIVITY CARD 17

How many of these problems can you solve?

1. Find two whole numbers whose product is 1 000 000 but neither factor has a zero in its representation.

2. Write a numeral of ten digits so that the first digit on the left tells the number of zeros in the numeral, the second digit from the left tells the number of ones, the third digit tells the number of twos and so on until the tenth digit tells the number of nines in the numeral.

3. Place a mathematical symbol between 6 and 7 so that a number between 6 and 7 is represented.

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Activity Card 16

This activity card consists of 3 unusual challenge problems concerning fractional numbers. The first problem can be solved by trial and error.

Solution: $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1$

The second problem requires a pattern discovery by the children. One description of the pattern is as follows:

> The denominator of a given fraction is found by adding the numerator and denominator of the previous fraction; the numerator is found by doubling the previous fraction's denominator and adding its numerator. Squares of the numbers in this sequence are better and better approximations of the number 2.

Solution: $\frac{99}{70}, \frac{239}{169}, \frac{577}{408}, \frac{1393}{985}$

The third problem requires only an understanding of fractional numbers and simple reasoning. The difference between $\frac{9}{14}$ and $\frac{3}{7}$ is $\frac{3}{14}$. $\frac{1}{3}$ of $\frac{3}{14}$ is $\frac{1}{14}$. Adding $\frac{1}{14}$ to $\frac{3}{7}$ gives $\frac{7}{14}$ or $\frac{1}{2}$.

Solution: $\frac{1}{2}$

Activity Card 17

This activity card is intended to provide recreation and motivation for children. The three challenge problems require reasoning which is somewhat unusual.

1. Some children may think this problem is very difficult or even impossible. However, it is easily solved by considering the prime factorization of 1 000 000.

$$\begin{aligned} 1\,000\,000 &= 10 \times 10 \times 10 \times 10 \times 10 \times 10 \\ &= 2 \times 5 \times 2 \times 5 \times 2 \times 5 \times 2 \times 5 \times 2 \times 5 \times 2 \times 5 \end{aligned}$$

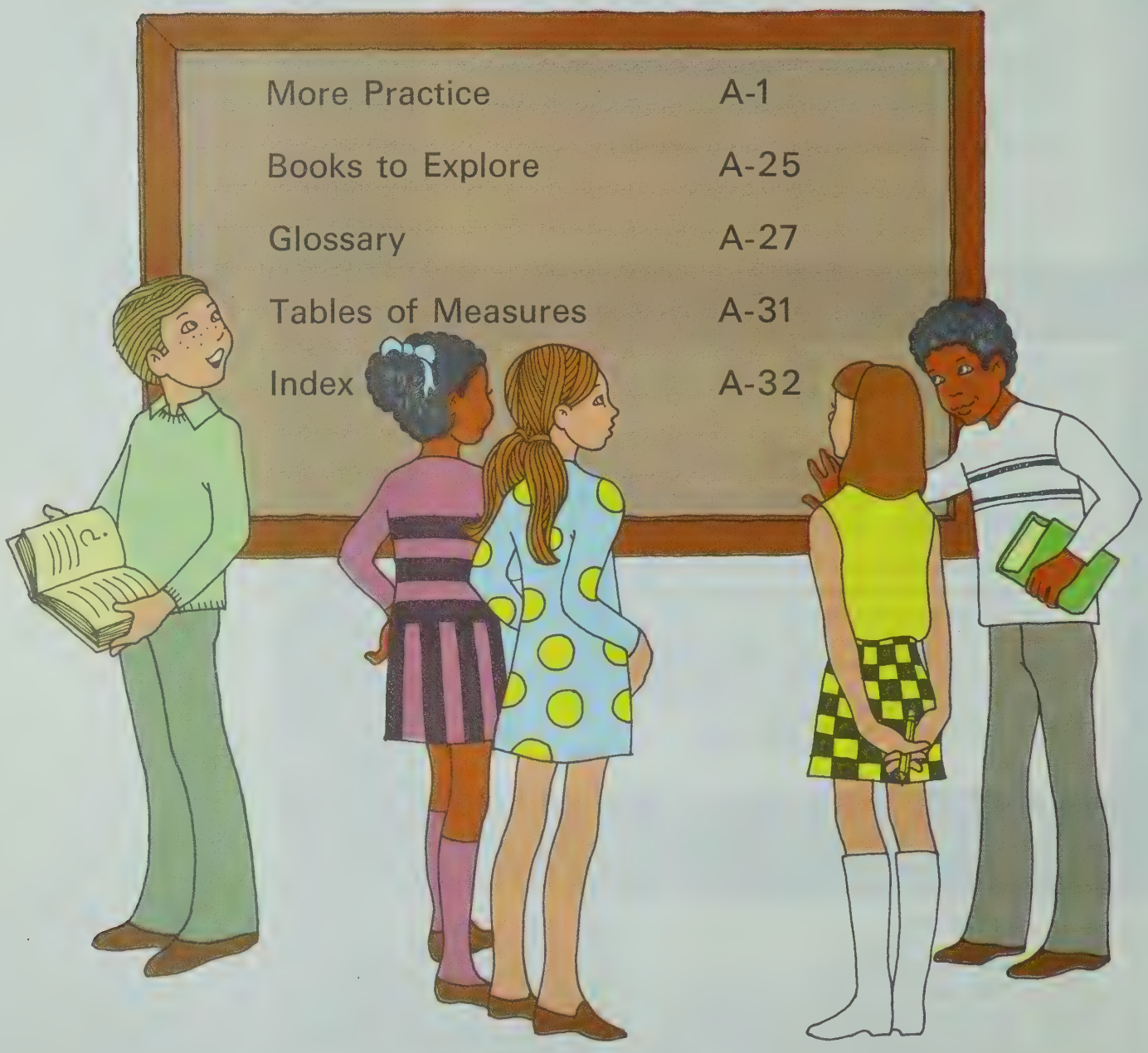
Grouping the factors of 2 together and the factors of 5 together, we have:

$$\begin{aligned} 1\,000\,000 &= (2 \times 2 \times 2 \times 2 \times 2 \times 2) \times (5 \times 5 \times 5 \times 5 \times 5 \times 5) \\ &= 64 \times 15\,625 \end{aligned}$$

2. *Solution:* 6 210 001 000

3. *Solution:* 6.7

Appendix



More Practice	A-1
Books to Explore	A-25
Glossary	A-27
Tables of Measures	A-31
Index	A-32

Set 1 *For use with page 21*

Use expanded notation to represent each number.

1. 37 (3×10)+7
2. 549 (5×100)+(4×10)+9
3. 603 (6×100)+3
4. 4611 (4×1000)+(6×100)+(1×10)+1
5. 7309 (7×1000)+(3×100)+9
6. 9620 (9×1000)+(6×100)+(2×10)
7. 7041 (7×1000)+(4×10)+1
8. 36 482 (3×10 000)+(6×1000)+(4×100)+(8×10)+2
9. 83 722 (8×10 000)+(3×1000)+(7×100)+(2×10)+2
10. 20 631 (2×10 000)+(6×100)+(3×10)+1
11. 761 265 (7×100 000)+(6×10 000)+(1×1000)+(2×100)+(6×10)+5
12. 900 340 (9×100 000)+(3×100)+(4×10)

Give the correct sign (< or >) for each ▨.

13. 261 ▨ 281 <
14. 639 ▨ 631 >
15. 4368 ▨ 4568 <
16. 6721 ▨ 6689 >
17. 12 407 ▨ 13 006 <
18. 84 732 ▨ 84 901 <
19. 23 621 ▨ 22 899 >
20. 32 507 ▨ 31 607 >
21. 53 688 ▨ 53 788 <

Reflected answers, Set 1: 1. (3 × 10) + 7, 13. 261 < 281, 16. 6721 > 6689, 19. 23 621 > 22 899, 10. (2 × 10 000) + (6 × 100) + (3 × 10) + 1, 4. (4 × 1000) + (6 × 100) + (1 × 10) + 1, 7. (7 × 1000) + (4 × 10) + 1

Set 2 *For use with page 27*

Give the number for ***a***. Then give the product for ***b***.

1. $6 \times 10^3 = 6 \times a = b$ 1000; 6000
2. $13 \times 10^2 = 13 \times a = b$ 100; 1300
3. $7 \times 10^4 = 7 \times a = b$ 10 000; 70 000
4. $21 \times 10^5 = 21 \times a = b$ 100 000; 2,100 000
5. $42 \times 10^3 = 42 \times a = b$ 1000; 42,000
6. $35 \times 10^4 = 35 \times a = b$ 10 000; 350 000

Give the number for ***a***. Then give the power of ten for ***b***.

7. $700 = 7 \times a = 7 \times b$ 100; 10^2
8. $4000 = 4 \times a = 4 \times b$ 1000; 10^3
9. $38\,000 = 38 \times a = 38 \times b$ 1000; 10^3
10. $15\,000 = 15 \times a = 15 \times b$ 1000; 10^3
11. $60\,000 = 6 \times a = 6 \times b$ 10 000; 10^4
12. $40\,000 = 4 \times a = 4 \times b$ 10 000; 10^4

Write each of the following as a product of a number between 1 and 10 and a power of 10.

13. 700 7×10^2
14. 3000 3×10^3
15. 7000 7×10^2
16. 80 000 8×10^4
17. 400 000 4×10^5
18. 900 000 9×10^5

Reflected answers, Set 2: 10. 1000; 10^3, 13. 7×10^2, 12. 4×10^2, 1. 1000; 6000, 4. 100 000; 2,100 000, 7. 100; 10^2

Set 3 *For use with page 33*

Find the sums and differences. Use base-six numerals.

1. $2_{(6)} + 4_{(6)} = 10_{(6)}$
2. $4_{(6)} + 3_{(6)} = 11_{(6)}$
3. $3_{(6)} + 5_{(6)} = 12_{(6)}$
4. $10_{(6)} - 5_{(6)} = 1_{(6)}$
5. $10_{(6)} + 4_{(6)} = 14_{(6)}$
6. $15_{(6)} - 5_{(6)} = 10_{(6)}$
7. $13_{(6)} + 5_{(6)} = 22_{(6)}$
8. $11_{(6)} + 4_{(6)} = 15_{(6)}$

Find the products. Use base-six numerals.

9. $3_{(6)} \times 4_{(6)} = 20_{(6)}$
10. $5_{(6)} \times 2_{(6)} = 14_{(6)}$
11. $5_{(6)} \times 5_{(6)} = 41_{(6)}$
12. $3_{(6)} \times 4_{(6)} = 20_{(6)}$
13. $2_{(6)} \times 4_{(6)} = 12_{(6)}$
14. $10_{(6)} \times 2_{(6)} = 20_{(6)}$
15. $13_{(6)} \times 3_{(6)} = 43_{(6)}$
16. $14_{(6)} \times 4_{(6)} = 104_{(6)}$

Reflected answers, Set 3: 1. $10_{(6)}$, 5. $11_{(6)}$, 9. $50_{(6)}$, 13. $14_{(6)}$

Set 4 *For use with page 41*

Find the missing number in the addition and multiplication equations.
Then solve the subtraction and division equations.

1. $a + 4 = 13$, $13 - 4 = a$ — 9
2. $b + 8 = 15$, $15 - 8 = b$ — 7
3. $x + 4 = 12$, $12 - 4 = x$ — 8
4. $w + 8 = 17$, $17 - 8 = w$ — 9
5. $c + 5 = 11$, $11 - 5 = c$ — 6
6. $s + 6 = 14$, $14 - 6 = s$ — 8
7. $r + 9 = 16$, $16 - 9 = r$ — 7
8. $w + 6 = 13$, $13 - 6 = w$ — 7
9. $c \times 7 = 35$, $35 \div 7 = c$ — 5
10. $r \times 3 = 18$, $18 \div 3 = r$ — 6
11. $s \times 7 = 21$, $21 \div 7 = s$ — 3
12. $a \times 8 = 56$, $56 \div 8 = a$ — 7
13. $b \times 6 = 0$, $0 \div 6 = b$ — 0
14. $w \times 5 = 45$, $45 \div 5 = w$ — 9
15. $t \times 7 = 56$, $56 \div 7 = t$ — 8
16. $d \times 6 = 48$, $48 \div 6 = d$ — 8

Solve the equations.

17. $9 + 6 = z$ — 15
18. $r - 9 = 4$ — 13
19. $3 \times s = 24$ — 8
20. $(64 \div 8) \times 8 = a$ — 64
21. $72 \div w = 8$ — 9
22. $6 = 14 - b$ — 8
23. $36 = t \times 6$ — 6
24. $(68 - 38) + 38 = r$ — 68

Reflected answers, Set 4: 1. 9, 2. 7, 3. 8, 4. 9, 5. 6, 6. 8, 7. 7, 8. 7, 9. 5, 10. 6, 11. 3, 12. 7, 13. 0, 14. 9, 15. 8, 16. 8, 17. 15, 18. 13, 19. 8, 20. 64, 21. 9, 22. 8, 23. 6, 24. 68

Set 5 *For use with page 49*

Give the numbers for ***a*** and ***b***.

1. $10 \times 1000 = a$ 10 000
   $10^1 \times 10^3 = 10^b$ 4
2. $100 \times 1000 = a$ 100 000
   $10^2 \times 10^3 = 10^b$ 5
3. $10 \times 10{,}000 = a$ 100 000
   $10^1 \times 10^4 = 10^b$ 5
4. $40 \times 700 = a$ 28 000
   $40 \times 700 = 28 \times 10^b$ 3
5. $500 \times 80 = a$ 40 000
   $500 \times 80 = b \times 10^3$ 40
6. $900 \times 800 = a$ 720 000
   $900 \times 800 = 72 \times 10^b$ 4

Find the products and quotients.

7. 30×60 1800
8. $4000 \div 80$ 50
9. 9×400 3600
10. $1800 \div 20$ 90
11. 4×700 2800
12. $4800 \div 60$ 80
13. 90×80 7200
14. $1200 \div 60$ 20
15. 60×70 4200
16. 50×50 2500
17. 700×9 63 00
18. 80×60 48 00
19. $4900 \div 70$ 70
20. 800×3 2400
21. $3000 \div 50$ 60

Reflected answers, Set 5: 1. 10 000; 4 2. 58 000; 3 3. 1800 4. 8 5. 90 6. 1500 7. 9600 8. 4800

Set 6 *For use with page 53*

Write and solve an equation for each exercise.

1. The sum of 6 and 3 is multiplied by 7. What is the product? $(6+3)\times 7=63$
2. If you divide 42 by 6, then add 2, and multiply by 4, what is the result? $(\frac{42}{6}+2)\times 4=36$
3. Find the product of 5 times the difference of 15 and 8. $5\times(15-8)=35$
4. What is the quotient when you divide the sum of 35 and 5 by 5? $(35+5)\div 5=8$
5. If you multiply 4 and 6, then add 16 to this product, what is the final number? $(4\times 6)+16=40$
6. If 6 is multiplied by 9 and 12 is subtracted from the product, what is the result? $(9\times 6)-12=42$
7. The sum of 6 and 3 is multiplied by 4. What is the result? $(6+3)\times 4=36$
8. Beginning with 36, what is the result if you divide by 9 and then multiply by 8? $(36\div 9)\times 8=32$

Reflected answers, Set 6: 1. $(6+3)\times 7=63$ 2. $(42\div 6)+2)\times 4=40$

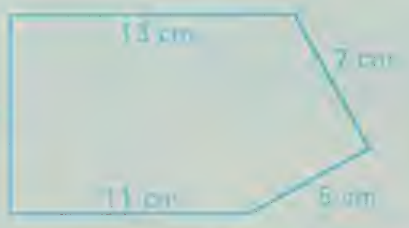
Set 7 *For use with page 69*

Measure each of these segments to the nearest centimetre.

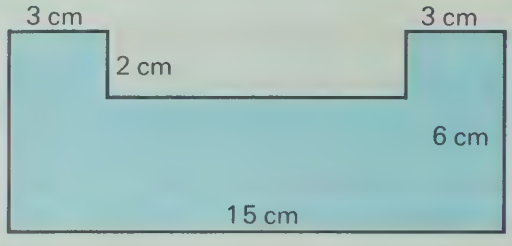
1. 11 cm
2. 6 cm
3. 5 cm
4. 12 cm
5. 5 cm
6. 6 cm

Find the perimeter of each figure below.

7. 46 cm



8. 46 cm



Reflected answers, Set 7: 1. 11 cm 4. 12 cm

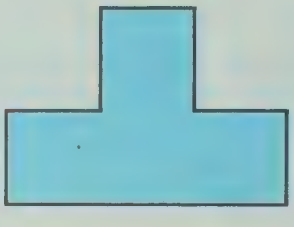
Set 8 *For use with page 75*

Give the number of lines of symmetry for each figure.

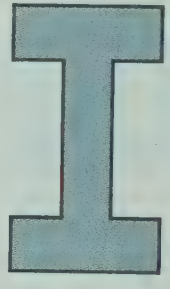
1. 4



2. 1



3. 2



Which figures have exactly 1 line of symmetry? 4
Which figures have no line of symmetry? 6

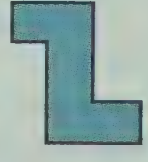
4.



5.



6.



Reflected answers, Set 8: 1. 4 4. 1

Set 9 *For use with page 85*

Find these sums and differences.

1. 42 +38	**2.** 56 +48	**3.** 37 −19	**4.** 66 +78	**5.** 321 +468
6. 640 −558	**7.** 461 −163	**8.** 533 +729	**9.** 520 −483	**10.** 769 +845
11. 825 +466	**12.** 925 −776	**13.** 1376 +4842	**14.** 1761 −1247	**15.** 6538 +5473
16. 5049 −3369	**17.** 4231 −2146	**18.** 2642 −1888	**19.** 7487 +6591	**20.** 8076 +8935
21. 561 48 +361	**22.** 31 257 +3405	**23.** 6483 21 +378	**24.** 3651 4888 +31	**25.** 4863 295 +5906

Reflected answers, Set 9: 1. 80, 2. 104, 3. 18, 4. 144, 5. 789, 6. 82, 7. 298, 8. 1262, 9. 37, 10. 1614, 11. 1291, 12. 149, 13. 6218, 14. 514, 15. 12 011, 16. 1680, 17. 2085, 18. 754, 19. 14 078, 20. 17 011, 21. 970, 22. 3693, 23. 6882, 24. 8570, 25. 11 064

Set 10 *For use with page 93*

Find the products.

1. 42 ×56 2352	**2.** 83 ×36 2988	**3.** 71 ×25 1775	**4.** 17 ×93 1581	**5.** 62 ×34 2108
6. 27 ×95 2565	**7.** 54 ×63 3402	**8.** 74 ×56 4144	**9.** 243 ×37 8991	**10.** 576 ×61 35 136
11. 738 ×42 30 996	**12.** 409 ×73 29 857	**13.** 283 ×96 27 168	**14.** 465 ×64 29 760	**15.** 827 ×36 29 772
16. 543 ×72 39 096	**17.** 615 ×324 199 260	**18.** 753 ×466 350 898	**19.** 358 ×722 258 476	**20.** 408 ×807 329 256
21. 945 ×438 413 910	**22.** 2648 ×321 850 008	**23.** 7407 ×648 4 799 736	**24.** 6532 ×483 3 154 956	**25.** 4309 ×243 1 047 087

Reflected answers, Set 10: 1. 2352, 2. 2988, 3. 1775, 4. 1581, 5. 2108, 6. 2565, 7. 3402, 8. 4144, 9. 8991, 10. 35 136, 11. 30 996, 12. 29 857, 13. 27 168, 14. 29 760, 15. 29 772, 16. 39 096, 17. 199 260, 18. 350 898, 19. 258 476, 20. 329 256, 21. 413 910, 22. 850 008, 23. 4 799 736, 24. 3 154 956, 25. 1 047 087

Set 11 *For use with page 103*

Find the quotients and remainders.

1. 6 R4 27)166	2. 5 R3 51)258	3. 7 R5 34)243	4. 3 R2 69)209	5. 4 R2 56)226
6. 6 R3 42)255	7. 5 R2 21)107	8. 6 R5 64)389	9. 4 R3 73)295	10. 8 R6 32)262
11. 6 R5 75)455	12. 5 R3 48)243	13. 5 R4 81)409	14. 6 R5 58)353	15. 7 R5 25)180
16. 9 R7 67)610	17. 6 R3 19)117	18. 4 R3 92)371	19. 5 R2 31)157	20. 4 R3 88)355
21. 5 R3 95)478	22. 8 R3 46)371	23. 5 R4 70)354	24. 6 R3 83)501	25. 4 R3 28)115

Find the divisor for each exercise.

26. 5 48)240	27. 8 72)576	28. 7 64)448	29. 6 52)312	30. 9 61)549

Reflected answers, Set 11. 1. 6 R4, 2. 5 R3, 3. 7 R5, 4. 3 R2, 5. 4 R2

Set 12 *For use with page 105*

Find the quotients and remainders.

1. 73 R12 34)2494	2. 41 R46 61)2547	3. 56 R15 27)1527	4. 23 R53 87)2054	5. 93 R25 46)4303
6. 25 R11 93)2336	7. 75 R8 41)3083	8. 73 R10 66)4828	9. 66 R21 51)3387	10. 75 R12 22)1662
11. 42 R14 38)1610	12. 61 R17 95)5812	13. 63 R8 39)2465	14. 62 R24 63)3930	15. 56 R15 83)4663
16. 33 R6 24)798	17. 53 R23 48)2567	18. 34 R14 88)3006	19. 73 R8 55)4023	20. 24 R7 69)1663
21. 71 R12 96)6828	22. 48 R8 29)1400	23. 56 R7 32)1799	24. 84 R31 60)5071	25. 42 R31 45)1921
26. 73 R5 84)6137	27. 30 R12 67)2022	28. 71 R6 73)5189	29. 93 R9 57)5310	30. 36 R24 92)3336

2. 83 R52

Reflected answers, Set 12: 1. 73 R12, 2. 41 R46, 3. 56 R15, 4. 23 R53

Set 13 *For use with page 111*

Find the quotients.

1. 328 R 36)11 808	**2.** 506 R 42)21 252	**3.** 641 R 53)33 973	**4.** 434 R 81)35 154
5. 515 R 64)32 960	**6.** 345 R 72)24 840	**7.** 408 R 26)10 608	**8.** 624 R 33)20 592
9. 733 R 19)13 927	**10.** 416 R 84)34 944	**11.** 215 R 93)19 995	**12.** 634 R 46)29 164
13. 326 R 51)16 626	**14.** 453 R 65)29 445	**15.** 872 R 38)33 136	**16.** 536 R 15)8040
17. 412 R 94)38 728	**18.** 315 R 74)23 310	**19.** 514 R 86)44 204	**20.** 723 R 54)39 042
21. 642 R 28)17 976	**22.** 462 R 35)16 170	**23.** 385 R 62)23 870	**24.** 3565 R 15)53 475
25. 815 R 34)27 710	**26.** 509 R 52)26 468	**27.** 436 R 81)35 316	**28.** 941 R 46)43 286

Solve each story problem.

29. There are 12 eggs in one dozen. How many dozen in 60 816 eggs ? 5068

30. Mr. Perez gets 16 km to 1 litre of gas. How many litres of gas is needed to travel 2302 kilometres ? 143.9 Litres

Reflected answers, Set 13: 1. 328, 5. 302, 3. 641, 4. 434, 13. 326, 14. 453, 15. 872, 16. 536, 25. 815, 26. 509, 27. 436, 28. 941

Set 14 *For use with page 113*

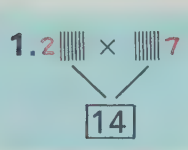
Find the quotients and remainders.

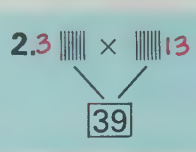
1. 516 R 36)18 576	**2.** 621 R 48)29 808	**3.** 434 R 71)30 814	**4.** 345 R 66)22 770
5. 426 R 82)34 932	**6.** 515 R 53)27 295	**7.** 654 R 8 27)17 666	**8.** 743 R 38)28 234
9. 1346 R 2 62)83 454	**10.** 6212 R 15)93 180	**11.** 4318 R 26 73)315 240	**12.** 5411 R 15 21)113 646
13. 2413 R 91)219 583	**14.** 3402 R 11 56)190 523	**15.** 4064 R 45)182 880	**16.** 6433 R 9 17)109 370
17. 4638 R 28)129 864	**18.** 2651 R 15 84)222 699	**19.** 1843 R 21 63)116 130	**20.** 5064 R 13 32)162 061
21. 7 400)2800	**22.** 9 300)2700	**23.** 4 544)2176	**24.** 7 432)3024

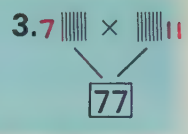
Reflected answers, Set 14: 1. 516, 5. 426, 3. 434, 4. 345, 13. 2413, 14. 3402 R 11, 15. 4064, 16. 6433 R 9, 25. 4638, 26. 2651 R 15, 27. 1843 R 21, 28. 5064 R 13

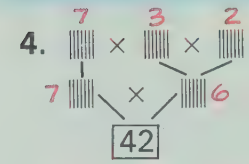
Set 15 *For use with page 127*

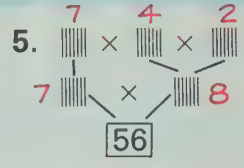
Copy each factor tree and give the missing factors.

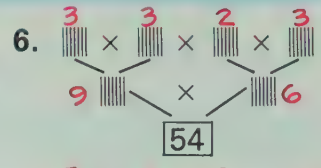
1. 

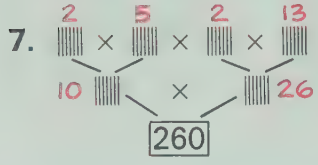
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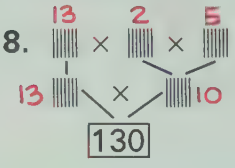
3. 

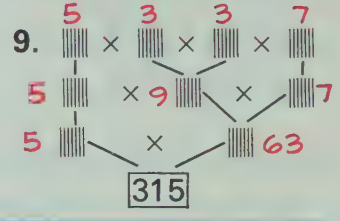
4. 

5. 

6. 

7. 

8. 

9. 

Reflected answers, Set 15: 1. 5, 7, 1 5. 3, 13, 1 3. 7, 11

Set 16 *For use with page 133*

Complete the following.

1. List the factors of 14. 1, 2, 7, 14
2. List the factors of 20. 1, 2, 4, 5, 10, 20
3. List the common factors of 14 and 20. 1, 2
4. What is the greatest common factor of 14 and 20? 2

Use the method above to give the GCF for each pair of numbers.

5. 15, 50 5
6. 12, 36 12
7. 24, 60 12
8. 36, 66 6

Use the prime factorization method to find the GCF for the two numbers.

9. 38, 57 19
10. 26, 52 26
11. 48, 56 8
12. 39, 65 13
13. 49, 63 7
14. 42, 70 14
15. 45, 72 9
16. 36, 160 4

3. 1, 3, 9 4. 2

Reflected answers, Set 16: 1. 1, 2, 5, 7, 14 5. 1, 5, 4, 2, 10, 20

Set 17 *For use with page 135*

Complete the following.

1. List all the multiples (to 54) of 6. 0,6,12,18,24,30,36,42,48,54
2. List all the multiples (to 52) of 4. 0,4,8,12,16,20,24,28,32,36,40,44,48,52
3. List the common multiples (to 48) of 6 and 4. 0,12,24,36,48
4. Give the least common multiple of 6 and 4. 12

Use the method above to find the LCM for each pair of numbers.

5. 3, 7 21 **7.** 12, 14 84 **9.** 4, 24 24 **11.** 28, 40 280
6. 14, 18 126 **8.** 21, 30 210 **10.** 21, 9 63 **12.** 24, 32 96

Use prime factorization to find the LCM for each pair of numbers.

13. 30, 4 60 **14.** 18, 28 252 **15.** 15, 36 180 **16.** 21, 42 42

Find each of the following.

17. The LCM of 3, 9, and 21. 63
18. The LCM of 4, 7, and 12. 84
19. The LCM of 9, 30, and 42. 630
20. The LCM of 7, 15, and 20. 420
21. The LCM of 14, 24, and 33. 1848
22. The LCM of 18, 36, and 42. 252

Reflected answers, Set 17: 1. 6, 12, 18, 24, 30, 36, 42, 48, 54 2. 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52 3. 12, 24, 36, 48 4. 12

Set 18 *For use with page 147*

Tell whether or not these fractions are in lowest terms.

1. $\frac{4}{3}$ yes **3.** $\frac{10}{31}$ yes **5.** $\frac{13}{26}$ no **7.** $\frac{8}{32}$ no **9.** $\frac{16}{32}$ no
2. $\frac{21}{24}$ no **4.** $\frac{11}{31}$ yes **6.** $\frac{4}{21}$ yes **8.** $\frac{15}{40}$ no **10.** $\frac{90}{100}$ no

Give these fractions in lowest terms.

11. $\frac{9}{15}$ $\frac{3}{5}$ **15.** $\frac{20}{25}$ $\frac{4}{5}$ **19.** $\frac{24}{42}$ $\frac{4}{7}$ **23.** $\frac{28}{36}$ $\frac{7}{9}$ **27.** $\frac{42}{54}$ $\frac{7}{9}$
12. $\frac{27}{30}$ $\frac{9}{10}$ **16.** $\frac{14}{36}$ $\frac{7}{18}$ **20.** $\frac{30}{50}$ $\frac{3}{5}$ **24.** $\frac{15}{75}$ $\frac{1}{5}$ **28.** $\frac{32}{48}$ $\frac{2}{3}$
13. $\frac{16}{24}$ $\frac{2}{3}$ **17.** $\frac{8}{18}$ $\frac{4}{9}$ **21.** $\frac{20}{30}$ $\frac{2}{3}$ **25.** $\frac{18}{21}$ $\frac{6}{7}$ **29.** $\frac{72}{81}$ $\frac{8}{9}$
14. $\frac{12}{18}$ $\frac{2}{3}$ **18.** $\frac{6}{32}$ $\frac{3}{16}$ **22.** $\frac{32}{54}$ $\frac{16}{27}$ **26.** $\frac{22}{66}$ $\frac{1}{3}$ **30.** $\frac{75}{100}$ $\frac{3}{4}$

Reflected answers, Set 18: 1. yes 3. yes 5. no 7. no 9. no

Set 19 *For use with page 151*

Give the correct sign (= or ≠) for each ▒.
Use the cross-product to help you find your answer.

1. $\frac{5}{8}$ ▒ $\frac{15}{24}$ =
2. $\frac{7}{10}$ ▒ $\frac{28}{48}$ ≠
3. $\frac{3}{4}$ ▒ $\frac{21}{28}$ =
4. $\frac{6}{9}$ ▒ $\frac{12}{20}$ ≠
5. $\frac{12}{21}$ ▒ $\frac{20}{35}$ =
6. $\frac{15}{18}$ ▒ $\frac{25}{38}$ ≠
7. $\frac{14}{20}$ ▒ $\frac{28}{40}$ =
8. $\frac{6}{9}$ ▒ $\frac{8}{10}$ ≠
9. $\frac{9}{12}$ ▒ $\frac{3}{18}$ ≠
10. $\frac{18}{20}$ ▒ $\frac{27}{30}$ =
11. $\frac{25}{30}$ ▒ $\frac{30}{36}$ =
12. $\frac{12}{18}$ ▒ $\frac{20}{24}$ ≠

Give the missing numerator or denominator.

13. $\frac{4}{5} = \frac{12}{▒}$ 15
14. $\frac{7}{10} = \frac{▒}{30}$ 21
15. $\frac{13}{15} = \frac{▒}{30}$ 26
16. $\frac{12}{20} = \frac{6}{▒}$ 10
17. $\frac{28}{▒} = \frac{21}{3}$ 4
18. $\frac{8}{▒} = \frac{32}{40}$ 10
19. $\frac{12}{5} = \frac{▒}{20}$ 48
20. $\frac{17}{18} = \frac{34}{▒}$ 36
21. $\frac{12}{20} = \frac{▒}{10}$ 6
22. $\frac{30}{▒} = \frac{10}{50}$ 150
23. $\frac{24}{▒} = \frac{12}{30}$ 60
24. $\frac{81}{9} = \frac{▒}{5}$ 45

Give the whole numbers named by these fractions.

25. $\frac{36}{4}$ 9
26. $\frac{56}{8}$ 7
27. $\frac{72}{9}$ 8
28. $\frac{42}{7}$ 6

Reflected answers, Set 19: 1. = ' 4. ≠ ' 7. = ' 10. = ' 13. 15 ' 16. 20 ' 19. 48 ' 22. 150 ' 25. 9 ' 28. 6

Set 20 *For use with page 153*

Give the correct sign (< or >) for each ▒.

1. $\frac{3}{4}$ ▒ $\frac{6}{1}$ <
2. $\frac{4}{5}$ ▒ $\frac{2}{3}$ >
3. $\frac{5}{8}$ ▒ $\frac{3}{16}$ >
4. $\frac{7}{5}$ ▒ $\frac{33}{20}$ <
5. $\frac{15}{4}$ ▒ $\frac{7}{2}$ >
6. $\frac{31}{6}$ ▒ 6 <
7. $\frac{6}{7}$ ▒ $\frac{2}{3}$ >
8. $\frac{7}{9}$ ▒ $\frac{6}{8}$ >

Solve the following short story problems.

9. 57 centimetres. $\frac{2}{3}$ metre. Which is longer? $\frac{2}{3}$ m
10. John works $\frac{3}{5}$ of an hour. Ted works $\frac{5}{6}$ of an hour. Who works longer? Ted
11. Mary answered $\frac{3}{4}$ of the test questions correctly. There were 80 problems. How many did she get wrong? 20
12. 748 metres. $\frac{3}{4}$ kilometre. Which is longer? $\frac{3}{4}$ Km
13. Sam ate $\frac{5}{6}$ of a pie. Mark ate $\frac{7}{8}$ of a pie. Who ate more? Mark
14. Joan bought $\frac{7}{8}$ of a metre of ribbon. Sue bought 88 cm of the same ribbon. Who bought more? Sue

Reflected answers, Set 20: 1. < ' 3. > ' 5. > ' 7. > ' 9. $\frac{2}{3}$ ' 12. $\frac{3}{4}$

Set 21 *For use with page 159*

Make your own lists of equivalent fractions until you find two fractions with the same denominator. Then find the sum or difference.

1. $\frac{3}{8} + \frac{3}{4}$ 9/8
2. $\frac{2}{3} - \frac{1}{6}$ 3/6
3. $\frac{1}{3} + \frac{1}{12}$ 5/12
4. $\frac{2}{5} - \frac{3}{10}$ 1/10
5. $\frac{7}{10} + \frac{3}{8}$ 43/40
6. $\frac{5}{6} - \frac{3}{5}$ 7/30
7. $\frac{3}{4} + \frac{1}{10}$ 17/20
8. $\frac{3}{5} + \frac{1}{20}$ 13/20
9. $\frac{9}{10} - \frac{3}{4}$ 3/20
10. $\frac{53}{100} - \frac{9}{20}$ 8/100
11. $\frac{2}{3} + \frac{7}{10}$ 41/30
12. $\frac{2}{5} + \frac{3}{100}$ 43/100
13. $\frac{1}{6} + \frac{3}{10}$ 14/30
14. $\frac{3}{4} - \frac{3}{8}$ 3/8
15. $\frac{4}{5} - \frac{3}{7}$ 13/35
16. $\frac{1}{6} - \frac{1}{10}$ 2/30
17. $\frac{9}{20} + \frac{11}{25}$ 89/100
18. $\frac{3}{20} + \frac{3}{5}$ 15/20
19. $\frac{2}{3} + \frac{5}{8}$ 31/24
20. $\frac{3}{5} - \frac{2}{7}$ 11/35

Reflected answers, Set 21: 1. 8/9, 2. 3/2, 3. 15/2, 4. 10/1

Set 22 *For use with page 163*

Find the sums and differences. Give your answers in lowest terms.
Mixed numerals may be used instead of improper fractions.

1. $\frac{2}{3} - \frac{1}{5}$ 7/15
2. $\frac{4}{5} + \frac{3}{4}$ 31/20
3. $\frac{5}{8} - \frac{4}{7}$ 3/56
4. $\frac{3}{100} + \frac{1}{10}$ 13/100
5. $\frac{5}{7} + \frac{1}{10}$ 57/70
6. $\frac{3}{4} - \frac{3}{8}$ 3/8
7. $\frac{7}{10} + \frac{5}{6}$ 23/15
8. $\frac{1}{3} - \frac{1}{8}$ 5/24
9. $\frac{5}{8} + \frac{7}{10}$ 13/40
10. $\frac{5}{8} + \frac{3}{4}$ 11/8
11. $\frac{5}{8} - \frac{1}{6}$ 11/24
12. $\frac{5}{7} + \frac{2}{3}$ 29/21
13. $\frac{4}{5} - \frac{1}{3}$ 7/15
14. $\frac{2}{5} + \frac{3}{8}$ 31/40
15. $\frac{2}{5} - \frac{1}{4}$ 3/20
16. $\frac{7}{10} - \frac{4}{15}$ 13/30
17. $\frac{1}{6} + \frac{2}{3} + \frac{1}{4}$ 13/12
18. $\frac{3}{5} + \frac{1}{4} + \frac{3}{10}$ 23/20
19. $\frac{5}{8} + \frac{1}{4} + \frac{1}{2}$ 11/8
20. $\frac{1}{6} + \frac{3}{8} + \frac{1}{4}$ 19/24
21. $\frac{3}{20} + \frac{2}{5} + \frac{7}{10}$ 1/4
22. $\frac{3}{4} + \frac{1}{5} + \frac{1}{2}$ 29/20
23. $\frac{2}{3} + \frac{3}{4} + \frac{5}{8}$ 49/24
24. $\frac{3}{4} + \frac{5}{6} + \frac{1}{2}$ 25/12
25. $\frac{3}{5} + \frac{1}{2} + \frac{1}{6}$ 4/15

Reflected answers, Set 22: 1. 7/15, 2. 50/3, 3. 56/3, 4. 13/100, 5. 50/11, 6. 3/8, 7. 15/23, 8. 24/5, 9. 40/13, 10. 8/11, 11. 24/11, 12. 21/29, 13. 15/7, 14. 40/31, 15. 20/3, 16. 30/13, 17. 12/13, 18. 20/23, 19. 8/11, 20. 24/19, 21. 4/1, 22. 20/29, 23. 24/49, 24. 12/25, 25. 15/4

Set 23 *For use with page 167*

Write an improper fraction for each mixed numeral.

1. $3\frac{5}{8}$ $\frac{29}{8}$
2. $6\frac{1}{3}$ $\frac{19}{3}$
3. $7\frac{3}{5}$ $\frac{38}{5}$
4. $4\frac{3}{4}$ $\frac{19}{4}$
5. $5\frac{2}{3}$ $\frac{17}{3}$
6. $2\frac{3}{10}$ $\frac{23}{10}$
7. $5\frac{5}{6}$ $\frac{35}{6}$
8. $2\frac{1}{10}$ $\frac{21}{10}$
9. $3\frac{1}{5}$ $\frac{16}{5}$
10. $10\frac{3}{5}$ $\frac{53}{5}$

Write a mixed numeral for each improper fraction.

11. $\frac{19}{5}$ $3\frac{4}{5}$
12. $\frac{17}{3}$ $5\frac{2}{3}$
13. $\frac{23}{8}$ $2\frac{7}{8}$
14. $\frac{44}{5}$ $8\frac{4}{5}$
15. $\frac{27}{4}$ $6\frac{3}{4}$
16. $\frac{65}{10}$ $6\frac{1}{2}$
17. $\frac{27}{4}$ $6\frac{3}{4}$
18. $\frac{150}{100}$ $1\frac{1}{2}$
19. $\frac{16}{5}$ $3\frac{1}{5}$
20. $\frac{48}{20}$ $2\frac{2}{5}$

Solve the equations. Give all fractions in lowest terms.

21. $7\frac{4}{3} = 8 + n$ $\frac{1}{3}$
22. $21\frac{12}{10} = 22 + n$ $\frac{1}{5}$
23. $32\frac{13}{9} = n + \frac{4}{9}$ 33
24. $36\frac{13}{4} = 39 + n$ $\frac{1}{4}$
25. $17\frac{10}{8} = n + \frac{1}{4}$ 18
26. $9\frac{7}{5} = 10 + n$ $\frac{2}{5}$
27. $43\frac{10}{7} = 44 + n$ $\frac{3}{7}$
28. $72\frac{13}{6} = 74 + n$ $\frac{1}{6}$
29. $26\frac{11}{4} = n + \frac{3}{4}$ 28

Reflected answers, Set 23: 1. $\frac{85}{8}$, 2. $\frac{61}{3}$, 3. $\frac{28}{5}$, 4. $\frac{61}{4}$, 5. $\frac{71}{2}$, 21. $\frac{1}{3}$, 22. $\frac{1}{5}$, 23. 33

Set 24 *For use with page 169*

Find the sums and differences.

1. $\frac{3}{4} + \frac{7}{12} = 1\frac{1}{3}$
2. $\frac{7}{10} + \frac{2}{5} = 1\frac{1}{10}$
3. $\frac{5}{8} + \frac{5}{6} = 1\frac{11}{24}$
4. $\frac{8}{9} + \frac{1}{6} = 1\frac{1}{18}$
5. $\frac{5}{12} + \frac{7}{8} = 1\frac{7}{24}$
6. $\frac{8}{15} + \frac{9}{10} = 1\frac{13}{30}$
7. $5\frac{3}{4} + 3\frac{7}{8} = 9\frac{5}{8}$
8. $7\frac{3}{10} + 8\frac{4}{5} = 16\frac{1}{10}$
9. $6\frac{1}{2} + 4\frac{1}{6} = 10\frac{2}{3}$
10. $2\frac{2}{3} + 9\frac{3}{5} = 12\frac{4}{15}$
11. $5\frac{1}{4} + 6\frac{1}{12} = 11\frac{1}{3}$
12. $10\frac{2}{5} + 3\frac{1}{2} = 13\frac{9}{10}$
13. $22\frac{5}{8} + 31\frac{1}{6} = 53\frac{19}{24}$
14. $46\frac{1}{3} + 10\frac{1}{5} = 56\frac{8}{15}$
15. $13\frac{2}{7} + 28\frac{5}{6} = 42\frac{5}{42}$
16. $43\frac{3}{4} + 21\frac{9}{10} = 65\frac{13}{20}$
17. $66\frac{1}{3} + 18\frac{3}{10} = 84\frac{19}{30}$
18. $37\frac{3}{4} + 48\frac{7}{10} = 86\frac{9}{20}$
19. $7\frac{1}{6} + 3\frac{3}{10} = 10\frac{7}{15}$
20. $12\frac{1}{2} + 8\frac{3}{4} = 21\frac{1}{4}$
21. $9\frac{1}{3} - 4\frac{2}{5} = 4\frac{14}{15}$
22. $22\frac{3}{50} + 11\frac{1}{20} = 33\frac{11}{100}$
23. $6\frac{1}{4} - 2\frac{1}{3} = 3\frac{11}{12}$
24. $8\frac{3}{7} - 3\frac{1}{6} = 5\frac{11}{42}$

Reflected answers, Set 24: 1. $1\frac{3}{1}$, 2. $1\frac{10}{1}$, 3. $1\frac{24}{11}$, 4. $1\frac{18}{1}$, 5. $1\frac{24}{7}$, 6. $1\frac{30}{13}$

Set 25 *For use with page 175*

Stock Exchange Prices (dollars per share)

	1967	1968	1969	1970
Industrial	52	58	$57\frac{1}{2}$	48
Transportation	$53\frac{1}{2}$	$50\frac{3}{5}$	47	$32\frac{1}{10}$
Utilities	$45\frac{2}{3}$	$44\frac{2}{5}$	$42\frac{4}{5}$	$37\frac{1}{5}$
Finance	$49\frac{4}{5}$	$65\frac{9}{10}$	$70\frac{1}{2}$	$54\frac{3}{5}$

1. In 1968 the average share of transportation stock cost $\$50\frac{3}{5}$. In 1970, the price had fallen to $\$32\frac{1}{10}$ per share. How much more did an average share cost in 1968 than in 1970? $\$18\frac{1}{2}$ more
2. An average share of finance cost $\$49\frac{4}{5}$ in 1967. How much more did an average share cost in 1969? $\$20\frac{7}{10}$ more
3. How much would Mr. Lawson have paid for an average share of transportation and an average share of finance in 1968? $\$116\frac{1}{2}$
4. The price of an average share of utilities steadily decreased from 1967 to 1970. How much more did it cost to buy an average share in 1967 than in 1969? $\$2\frac{13}{15}$ more

Reflected answers, Set 25: 1. $\$18\frac{5}{1}$

Set 26 *For use with page 177*

Give the numerator for ***a*** and the whole number for ***b***.

1. $\frac{42}{4} = \frac{a}{4} + \frac{2}{4} = b + \frac{2}{4}$ 40;10
2. $\frac{68}{5} = \frac{a}{5} + \frac{3}{5} = b + \frac{3}{5}$ 65;13
3. $\frac{124}{7} = \frac{a}{7} + \frac{5}{7} = b + \frac{5}{7}$ 119;17
4. $\frac{37}{3} = \frac{a}{3} + \frac{1}{3} = b + \frac{1}{3}$ 36;12
5. $\frac{51}{2} = \frac{a}{2} + \frac{1}{2} = b + \frac{1}{2}$ 50;25
6. $\frac{93}{8} = \frac{a}{8} + \frac{5}{8} = b + \frac{5}{8}$ 88;11

Give the number for ***q*** and the number for ***r***.
Then give a mixed numeral for ***m***.

7. $\frac{39}{6} = \frac{q \times 6}{6} + \frac{r}{6} = m$ 6;3; $6\frac{1}{2}$
8. $\frac{47}{2} = \frac{q \times 2}{2} + \frac{r}{2} = m$ 23;1; $23\frac{1}{2}$
9. $\frac{57}{4} = \frac{q \times 4}{4} = \frac{r}{4} = m$ 14;1; $14\frac{1}{2}$
10. $\frac{49}{3} = \frac{q \times 3}{3} + \frac{r}{3} = m$ 16;1; $16\frac{1}{3}$
11. $\frac{63}{4} = \frac{q \times 4}{4} + \frac{r}{4} = m$ 15;3; $15\frac{3}{4}$
12. $\frac{72}{5} = \frac{q \times 5}{5} + \frac{r}{5} = m$ 14;2; $14\frac{2}{5}$

Give the mixed numeral for each fraction.

13. $\frac{58}{3}$ $19\frac{1}{3}$
14. $\frac{134}{5}$ $26\frac{4}{5}$
15. $\frac{75}{4}$ $18\frac{3}{4}$
16. $\frac{93}{6}$ $15\frac{1}{2}$
17. $\frac{98}{3}$ $32\frac{2}{3}$
18. $\frac{144}{5}$ $28\frac{4}{5}$
19. $\frac{326}{7}$ $46\frac{4}{7}$
20. $\frac{175}{3}$ $58\frac{1}{3}$
21. $\frac{61}{2}$ $30\frac{1}{2}$
22. $\frac{632}{5}$ $126\frac{2}{5}$

Reflected answers, Set 26: 1. 40; 10 3. 119; 17 5. 50; 25 2. 65; 13 4. 36; 12 6. 88; 11

Set 27 *For use with page 185*

Solve the equations.

1. $7 \times \frac{1}{3} = n$ $\frac{7}{3}$
2. $\frac{1}{7} \times \frac{1}{4} = n$ $\frac{1}{28}$
3. $5 \times \frac{1}{10} = n$ $\frac{5}{10}$
4. $\frac{1}{10} \times \frac{1}{10} = n$ $\frac{1}{100}$
5. $4 \times \frac{1}{5} = n$ $\frac{4}{5}$
6. $6 \times \frac{1}{4} = n$ $\frac{6}{4}$
7. $\frac{1}{3} \times \frac{1}{8} = n$ $\frac{1}{24}$
8. $\frac{1}{10} \times \frac{1}{2} = n$ $\frac{1}{20}$
9. $\frac{1}{6} \times \frac{1}{4} = n$ $\frac{1}{24}$
10. $15 \times \frac{1}{10} = n$ $\frac{15}{10}$
11. $12 \times \frac{1}{5} = n$ $\frac{12}{5}$
12. $\frac{1}{8} \times \frac{1}{4} = n$ $\frac{1}{32}$
13. $\frac{1}{3} \times \frac{1}{9} = n$ $\frac{1}{27}$
14. $\frac{1}{5} \times \frac{1}{4} = n$ $\frac{1}{20}$
15. $\frac{1}{2} \times \frac{1}{6} = n$ $\frac{1}{12}$
16. $11 \times \frac{1}{7} = n$ $\frac{11}{7}$

Write and solve a multiplication equation for each problem.

17. Mary has read 15 books this year. Joe has read only $\frac{1}{5}$ as many books as Mary. How many books has Joe read? $15 \times \frac{1}{5} = 3$
18. John and Steve decided to run from John's house to school. Steve ran $\frac{1}{3}$ of the way to school. John ran only $\frac{1}{2}$ as far as Steve. What part of the way to school did John run? $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

Reflected answers, Set 27: 1. $\frac{3}{7}$, 2. $\frac{2}{7}$, 3. $\frac{5}{10}$, 13. $\frac{5}{27}$, 15. $\frac{1}{2} \times \frac{1}{6} = 3$

Set 28 *For use with page 189*

Give the number for ***n***.

1. $\frac{1}{2} \times \frac{3}{5} = \frac{1}{2} \times n^{3} \times \frac{1}{5}$
2. $\frac{5}{6} \times \frac{1}{8} = n^{5} \times \frac{1}{6} \times \frac{1}{8}$
3. $13 \times \frac{2}{3} = 13 \times n^{2} \times \frac{1}{3}$
4. $\frac{3}{4} \times \frac{1}{6} = n^{3} \times \frac{1}{4} \times \frac{1}{6}$
5. $\frac{7}{9} \times 15 = n^{7} \times \frac{1}{9} \times 15$
6. $\frac{1}{4} \times \frac{3}{8} = \frac{1}{4} \times n^{3} \times \frac{1}{8}$

Give the whole number for ***a***. Give the fraction for ***b***.
Then give the missing product for ***p***.

7. $\frac{3}{4} \times \frac{3}{5} = (3 \times \frac{1}{4}) \times (3 \times \frac{1}{5}) = a \times b = p$ 9; $\frac{1}{20}$; $\frac{9}{20}$
8. $\frac{5}{6} \times \frac{2}{3} = (5 \times \frac{1}{6}) \times (2 \times \frac{1}{3}) = a \times b = p$ 10; $\frac{1}{18}$; $\frac{10}{18}$ or $\frac{5}{9}$

Copy each equation. Give the product for ***n***.

9. $\frac{3}{7} \times \frac{2}{3} = n$ $\frac{6}{21}$ or $\frac{2}{7}$
10. $\frac{2}{5} \times \frac{4}{6} = n$ $\frac{8}{30}$ or $\frac{4}{15}$
11. $\frac{3}{4} \times \frac{6}{5} = n$ $\frac{18}{20}$ or $\frac{9}{10}$
12. $\frac{3}{8} \times \frac{4}{3} = n$ $\frac{12}{24}$ or $\frac{1}{2}$
13. $\frac{7}{10} \times \frac{3}{2} = n$ $\frac{21}{20}$
14. $\frac{2}{5} \times \frac{3}{20} = n$ $\frac{6}{100}$ or $\frac{3}{50}$
15. $\frac{7}{8} \times \frac{3}{10} = n$ $\frac{21}{80}$
16. $\frac{3}{10} \times \frac{5}{2} = n$ $\frac{15}{20}$ or $\frac{3}{4}$
17. $\frac{2}{7} \times \frac{3}{7} = n$ $\frac{6}{49}$
18. $\frac{7}{9} \times \frac{5}{6} = n$ $\frac{35}{54}$
19. $\frac{3}{5} \times \frac{2}{5} = n$ $\frac{6}{25}$
20. $\frac{5}{8} \times \frac{5}{6} = n$ $\frac{25}{48}$

Reflected answers, Set 28: 1. 3, 3. 2, 5. 7, 18. $\frac{35}{54}$

Set 29 *For use with page 193*

Use the distributive principle to find the products.

1. $\frac{3}{4} \times 7\frac{2}{3}$ $5\frac{3}{4}$
2. $8 \times 9\frac{3}{5}$ $76\frac{4}{5}$
3. $6 \times 4\frac{1}{4}$ $25\frac{1}{2}$
4. $\frac{2}{3} \times 6\frac{1}{5}$ $4\frac{2}{15}$
5. $5\frac{3}{5} \times 3$ $16\frac{4}{5}$
6. $4\frac{5}{6} \times 7\frac{2}{3}$ $37\frac{1}{18}$
7. $2\frac{7}{8} \times 8$ 23
8. $3\frac{7}{10} \times 6\frac{3}{5}$ $24\frac{21}{50}$

Replace each mixed numeral with an improper fraction to find these products.

9. $3\frac{3}{4} \times 4\frac{1}{3}$ $16\frac{1}{4}$
10. $\frac{2}{3} \times 5\frac{1}{6}$ $3\frac{4}{9}$
11. $1\frac{1}{8} \times 6\frac{1}{3}$ $7\frac{1}{8}$
12. $2\frac{1}{10} \times 1\frac{2}{5}$ $2\frac{47}{50}$
13. $\frac{5}{2} \times 5\frac{1}{3}$ $13\frac{1}{3}$
14. $7\frac{1}{3} \times 2\frac{1}{5}$ $16\frac{2}{15}$
15. $\frac{7}{8} \times 3\frac{7}{10}$ $3\frac{19}{80}$
16. $\frac{5}{3} \times 1\frac{3}{8}$ $2\frac{7}{24}$
17. $6\frac{3}{5} \times \frac{5}{7}$ $4\frac{5}{7}$
18. $8\frac{1}{4} \times 9\frac{1}{3}$ 77
19. $5\frac{5}{6} \times 1\frac{3}{4}$ $10\frac{5}{24}$
20. $3\frac{5}{9} \times 4\frac{5}{8}$ $16\frac{4}{9}$
21. $\frac{5}{7} \times 8\frac{3}{5}$ $6\frac{1}{7}$
22. $\frac{7}{6} \times 4\frac{3}{8}$ $5\frac{5}{48}$
23. $2\frac{7}{10} \times 4\frac{1}{3}$ $11\frac{7}{10}$
24. $6\frac{1}{6} \times 3\frac{1}{10}$ $19\frac{7}{60}$

Reflected answers, Set 29: 1. $2\frac{1}{4}$, 2. $10\frac{2}{5}$, 3. $52\frac{1}{2}$, 4. $21\frac{1}{5}$, 5. $\frac{1}{10}$, 6. $\frac{1}{18}$, 7. $\frac{1}{8}$, 8. $\frac{1}{24}$, 9. $\frac{1}{4}$, 10. $\frac{4}{9}$, 11. $\frac{1}{8}$, 12. $\frac{47}{50}$, 13. $\frac{1}{3}$, 14. $\frac{2}{15}$, 15. $\frac{19}{80}$, 16. $\frac{7}{24}$, 17. $\frac{5}{7}$, 18. 77, 19. $\frac{5}{24}$, 20. $\frac{4}{9}$, 21. $\frac{1}{7}$, 22. $\frac{5}{48}$, 23. $\frac{7}{10}$, 24. $\frac{7}{60}$

Set 30 *For use with page 197*

Use the shortcut to find these products.

1. $\frac{3}{5} \times \frac{7}{9}$ $\frac{7}{15}$
2. $\frac{4}{7} \times \frac{9}{8}$ $\frac{9}{14}$
3. $\frac{5}{6} \times \frac{9}{20}$ $\frac{3}{8}$
4. $\frac{2}{3} \times \frac{9}{10}$ $\frac{3}{5}$
5. $\frac{5}{12} \times \frac{16}{30}$ $\frac{2}{9}$
6. $\frac{3}{4} \times \frac{12}{18}$ $\frac{1}{2}$
7. $\frac{3}{8} \times \frac{12}{21}$ $\frac{3}{14}$
8. $\frac{2}{9} \times \frac{15}{16}$ $\frac{5}{24}$
9. $\frac{7}{16} \times \frac{32}{42}$ $\frac{1}{3}$
10. $\frac{27}{30} \times \frac{24}{36}$ $\frac{3}{5}$
11. $\frac{14}{22} \times \frac{33}{49}$ $\frac{3}{7}$
12. $\frac{6}{16} \times \frac{24}{33}$ $\frac{3}{11}$
13. $\frac{5}{8} \times \frac{16}{25}$ $\frac{2}{5}$
14. $\frac{16}{7} \times \frac{35}{48}$ $\frac{5}{3}$
15. $\frac{9}{14} \times \frac{32}{45}$ $\frac{16}{35}$
16. $\frac{7}{11} \times \frac{55}{28}$ $\frac{5}{4}$

Complete the function tables.

	Function Rule $\frac{2}{3} \times n$	
	n	$f(n)$
17.	$\frac{9}{16}$	$\frac{3}{8}$
18.	$\frac{21}{32}$	$\frac{7}{16}$
19.	$2\frac{3}{4}$	$1\frac{5}{6}$

	Function Rule $n \times \frac{3}{5}$	
	n	$f(n)$
20.	$\frac{20}{21}$	$\frac{4}{7}$
21.	$5\frac{1}{3}$	$3\frac{1}{5}$
22.	$\frac{35}{39}$	$\frac{7}{13}$

	Function Rule $2\frac{1}{4} \times n$	
	n	$f(n)$
23.	$\frac{32}{27}$	$2\frac{2}{3}$
24.	$3\frac{1}{3}$	$7\frac{1}{2}$
25.	$4\frac{2}{5}$	$9\frac{9}{10}$

Reflected answers, Set 30: 1. $\frac{15}{7}$, 2. $\frac{8}{9}$, 3. $\frac{8}{3}$, 4. $\frac{5}{3}$, 5. $\frac{9}{2}$, 6. $\frac{1}{2}$, 7. $\frac{14}{3}$, 8. $\frac{24}{5}$, 9. $\frac{3}{1}$, 10. $\frac{5}{3}$, 11. $\frac{7}{3}$, 12. $\frac{11}{3}$, 13. $\frac{5}{2}$, 14. $\frac{5}{3}$, 15. $\frac{35}{16}$, 16. $\frac{5}{4}$, 17. $\frac{3}{8}$, 18. $\frac{7}{16}$, 19. $\frac{5}{6}$, 20. $\frac{4}{7}$, 21. $\frac{1}{35}$, 22. $\frac{7}{13}$, 23. $2\frac{2}{3}$, 24. $7\frac{1}{2}$, 25. $9\frac{9}{10}$

Set 31 *For use with page 205*

Solve the equations.

1. $7 \div \frac{3}{4} = n \times \frac{4}{3}$ 7
2. $11 \div \frac{7}{8} = n \times \frac{8}{7}$ 11
3. $13 \div \frac{11}{13} = n \times \frac{13}{11}$ 13
4. $\frac{2}{3} \div \frac{7}{8} = n \times \frac{8}{7}$ $\frac{2}{3}$
5. $\frac{11}{12} \div \frac{4}{7} = n \times \frac{7}{4}$ $\frac{11}{12}$
6. $\frac{5}{9} \div \frac{4}{5} = \frac{5}{9} \times n$ $\frac{5}{4}$
7. $\frac{6}{13} \div \frac{3}{8} = \frac{6}{13} \times n$ $\frac{8}{3}$
8. $\frac{4}{11} \div \frac{6}{13} = \frac{4}{11} \times n$ $\frac{13}{6}$
9. $\frac{12}{13} \div \frac{14}{33} = n \times \frac{33}{14}$ $\frac{12}{13}$

Find the quotients.

10. $\frac{7}{9} \div \frac{2}{3}$ $1\frac{1}{6}$
11. $\frac{5}{8} \div \frac{5}{16}$ 2
12. $\frac{3}{4} \div \frac{7}{24}$ $2\frac{4}{7}$
13. $\frac{7}{6} \div \frac{14}{15}$ $1\frac{1}{4}$
14. $\frac{3}{7} \div \frac{5}{28}$ $2\frac{2}{5}$
15. $7 \div \frac{2}{3}$ $10\frac{1}{2}$
16. $\frac{20}{27} \div \frac{5}{9}$ $1\frac{1}{3}$
17. $\frac{3}{10} \div \frac{4}{15}$ $1\frac{1}{8}$
18. $\frac{9}{8} \div 3$ $\frac{3}{8}$
19. $\frac{15}{18} \div \frac{5}{9}$ $1\frac{1}{2}$
20. $\frac{21}{32} \div \frac{7}{8}$ $\frac{3}{4}$
21. $\frac{18}{7} \div \frac{9}{14}$ 4
22. $\frac{5}{21} \div \frac{6}{27}$ $1\frac{1}{14}$
23. $\frac{9}{16} \div \frac{15}{24}$ $\frac{9}{10}$
24. $\frac{3}{10} \div \frac{12}{25}$ $\frac{5}{8}$
25. $\frac{11}{12} \div \frac{33}{42}$ $1\frac{1}{6}$

25. $5\frac{1}{4}$ 13. $1\frac{4}{7}$

Reflected answers, Set 31: 1. 1 7. 5. 11 3. 13 10. $1\frac{1}{6}$ 11. 5

Set 32 *For use with page 207*

Find the quotients.

1. $2\frac{3}{4} \div \frac{2}{5}$ $6\frac{7}{8}$
2. $3\frac{1}{6} \div 1\frac{1}{2}$ $2\frac{1}{9}$
3. $\frac{7}{8} \div 4\frac{1}{4}$ $\frac{7}{34}$
4. $7\frac{1}{2} \div \frac{5}{6}$ 9
5. $1\frac{3}{5} \div \frac{3}{10}$ $5\frac{1}{3}$
6. $3\frac{1}{7} \div 1\frac{1}{3}$ $2\frac{5}{14}$
7. $4\frac{1}{2} \div 1\frac{7}{8}$ $2\frac{2}{5}$
8. $2\frac{1}{6} \div 3\frac{2}{3}$ $\frac{13}{22}$
9. $4\frac{1}{5} \div 3\frac{1}{3}$ $1\frac{13}{50}$
10. $2\frac{5}{9} \div 3\frac{1}{3}$ $\frac{23}{30}$
11. $3\frac{3}{5} \div 2\frac{1}{10}$ $1\frac{5}{7}$
12. $4\frac{1}{3} \div 3\frac{2}{9}$ $1\frac{10}{29}$

Simplify each expression.

13. $\dfrac{\frac{4}{5}}{\frac{7}{8}}$ $\frac{32}{35}$
14. $\dfrac{\frac{2}{3}}{\frac{3}{10}}$ $2\frac{2}{9}$
15. $\dfrac{\frac{2}{7}}{\frac{9}{8}}$ $\frac{16}{63}$
16. $\dfrac{\frac{5}{7}}{\frac{14}{20}}$ $1\frac{1}{49}$

Find the quotients. Check your work.

17. $49 \div 6$ $8\frac{1}{6}$
18. $37 \div 4$ $9\frac{1}{4}$
19. $123 \div 4$ $30\frac{3}{4}$
20. $239 \div 7$ $34\frac{1}{7}$
21. $371 \div 5$ $74\frac{1}{5}$
22. $184 \div 6$ $30\frac{2}{3}$
23. $431 \div 7$ $61\frac{4}{7}$
24. $165 \div 9$ $18\frac{1}{3}$
25. $2314 \div 6$ $385\frac{2}{3}$
26. $849 \div 7$ $121\frac{2}{7}$
27. $3642 \div 9$ $404\frac{2}{3}$
28. $4733 \div 4$ $1183\frac{1}{4}$

Reflected answers, Set 32: 1. $6\frac{7}{8}$ 5. 3 3. $\frac{7}{34}$ 4. 9

Set 33 *For use with page 211*

1. Mike kept a record of his last three math test scores. They were 78, 83, and 86. What was his average for the three tests? $82\frac{1}{3}$

2. Mr. Haley bought $183\frac{1}{3}$ kilograms of wheat and $465\frac{3}{5}$ kilograms of rye. How many kilograms of grain did he buy all together? $648\frac{14}{15}$

3. Mike weighs $45\frac{2}{3}$ kg, John weighs $66\frac{1}{4}$ kg, and Tom weighs $73\frac{1}{2}$ kg. How much do the three boys weigh together? $185\frac{5}{12}$

4. The temperatures in Vancouver for the last 4 days have been 22°C, 21°C, 25°C, and 23°C. What is the average temperature for the last four days? $22\frac{3}{4}$

5. Sam's beagle weighs $12\frac{3}{4}$ kg. Bill has a collie that weighs $22\frac{2}{5}$ kg. How much more does Bill's collie weigh than Sam's beagle? $9\frac{13}{20}$

Reflected answers, Set 33: 1. $85\frac{3}{1}$, 5. $9\frac{13}{20}$

Set 34 *For use with page 231*

Find the sums and differences.

1. 0.36 + 0.48 = 0.84	2. 3.72 + 0.59 = 4.31	3. 0.73 − 0.62 = 0.11	4. 4.51 − 1.81 = 2.70	5. 2.88 + 4.76 = 7.64
6. 32.7 + 4.89 = 37.59	7. 21.06 − 13.95 = 7.11	8. 5.631 + 4.782 = 10.413	9. 12.347 + 13.89 = 26.237	10. 15.04 − 7.65 = 7.39
11. 403.6 + 702.9 = 1106.5	12. 0.032 − 0.018 = 0.014	13. 46.28 + 39.57 = 85.85	14. 54.07 − 25.95 = 28.12	15. 66.9 + 72.43 = 139.33
16. 72.36 − 7.88 = 64.48	17. 82.031 + 46.799 = 128.830	18. 326.7 − 211.9 = 114.8	19. 52.6 − 14.91 = 37.69	20. 93.66 + 43.77 = 137.43
21. 5.461 + 7.957 = 13.418	22. 6.042 − 3.789 = 2.253	23. 354.22 + 467.95 = 822.17	24. 264.32 − 95.71 = 168.61	25. 48.72 − 3.98 = 44.74

Reflected answers, Set 34: 1. 0.84, 5. 7.64, 3. 0.11, 4. 2.70, 2. 4.31

Set 35 *For use with page 235*

Round each decimal to the nearest hundredth.

1. 0.476 0.48 **2.** 0.506 0.51 **3.** 0.876 0.88 **4.** 0.732 0.73 **5.** 0.322 0.32

Round each decimal to the nearest thousandth.

6. 0.62532 0.625 **7.** 0.53291 0.533 **8.** 0.88385 0.884 **9.** 0.73884 0.739 **10.** 0.48741 0.487

Round each decimal to the nearest ten thousandth.

11. 0.56675 0.5668 **12.** 0.40316 0.4032 **13.** 0.95437 0.9544 **14.** 0.21382 0.2138 **15.** 0.32609 0.3261

Round each decimal to the nearest hundred thousandth.

16. 0.013711 0.01371 **17.** 0.723488 0.72349 **18.** 0.503267 0.50327 **19.** 0.365229 0.36523 **20.** 0.630105 0.63011

Round each decimal to the nearest millionth.

21. 0.3651322 0.365132 **22.** 0.7040321 0.704032 **23.** 0.4379264 0.437926 **24.** 0.9846278 0.984628 **25.** 0.6437885 0.643789

Reflected answers, Set 35: 1. 0.848, 2. 0.525, 3. 0.88, 4. 0.573, 5. 0.35

Set 36 *For use with page 237*

Find the products.

1. 10 × 0.585.8 **2.** 100 × 0.3615 36.15 **3.** 100 × 8.211 821.1 **4.** 10 × 0.473 4.73 **5.** 10 × 3.76 37.6 **6.** 100 × 21.53 2153 **7.** 100 × 32.762 3276.2 **8.** 100 × 0.0541 5.41

9. 100 × 43.7 4370 **10.** 10 × 0.087 0.87 **11.** 10 × 3.2 32 **12.** 100 × 56.03 5603 **13.** 100 × 50.736 5073.6 **14.** 10 × 463.2 4632 **15.** 100 × 0.0064 0.64 **16.** 10 × 3.05 30.5

17. 10 × 0.003.03 **18.** 10 × 0.4 4 **19.** 100 × 0.32 32 **20.** 100 × 4.006 400.6 **21.** 100 × 6.02 602 **22.** 100 × 0.0501 5.01 **23.** 10 × 3.01 30.1 **24.** 1000 × 0.0652 65.2

25. 1000 × 0.0003 0.3 **26.** 10 × 345.62 3456.2 **27.** 100 × 426.703 42670.3 **28.** 1000 × 0.05612 56.12 **29.** 1000 × 0.7 700 **30.** 100 × 0.4 40 **31.** 1000 × 2.006 2006 **32.** 1000 × 0.07 70

Reflected answers, Set 36: 1. 2.8, 8. 0.0541, 15. 0.64, 24. 65.2, 32. 70

Set 37 *For use with page 239*

Find the products.

1. 0.6×0.5 0.3
2. 0.7×0.8 0.56
3. 0.4×0.72 0.288
4. 0.65×0.3 0.195
5. 0.42×0.8 0.336
6. 0.32×0.04 0.0128
7. 0.76×0.21 0.1596
8. 0.05×0.45 0.0225
9. 73×0.9 65.7
10. 0.82×0.06 0.0492
11. 0.7×0.93 0.651
12. 0.21×0.56 0.1176
13. 0.33×0.61 0.2013
14. 0.24×0.7 0.168
15. 0.31×0.48 0.1488
16. 0.74×0.22 0.1628
17. 24.2×0.7 16.94
18. 0.031×0.6 0.0186
19. 0.42×3.61 1.5162
20. 1.45×0.32 0.464

21. 4.9×3.2 15.68
22. 6.8×0.54 3.672
23. 27.2×0.5 13.6
24. 3.84×0.7 2.688
25. 74.2×0.61 45.262
26. 2.43×0.711 1.72773
27. 61.42×8 491.36
28. 4.26×0.21 0.8946

Reflected answers, Set 37: 1. 0.3 2. 0.5376 3. 0.288 4. 0.195 5. 0.336 6. 0.0128 7. 0.1596 8. 0.0225 9. 65.7 10. 0.0492 11. 0.651 12. 0.1176 13. 0.2013 14. 0.168 15. 0.1488 16. 0.1628 17. 16.94 18. 0.0186 19. 1.5162 20. 0.464 21. 15.68 22. 3.672 23. 13.6 24. 2.688 25. 45.262 26. 1.72773 27. 491.36 28. 0.8946

Set 38 *For use with page 243*

Estimate each product. Then find the product and compare it with your estimate.

1. 34.3×6 205.8
2. 48.6×5 243
3. 6.53×71 463.63
4. 0.427×4 1.708
5. 0.596×7 4.172
6. 7.64×3 22.92
7. 2.384×5 11.92
8. 32.2×7 225.4
9. 24.7×3 74.1
10. 0.643×6 3.858
11. 8.26×9 74.34
12. 83.5×4 334

Find the quotients. Check your work.

13. $5\overline{)160.5}$ 32.1
14. $7\overline{)32.41}$ 4.63
15. $4\overline{)3.060}$ 0.765
16. $2\overline{)73.4}$ 36.7
17. $9\overline{)42.408}$ 4.712
18. $12\overline{)57.6}$
19. $10\overline{)3.260}$ 0.326
20. $6\overline{)249.6}$ 41.6
21. $8\overline{)449.68}$ 56.21
22. $30\overline{)16.20}$ 0.54
23. $5\overline{)222.55}$ 44.51
24. $40\overline{)14.440}$ 0.361
25. $20\overline{)96.20}$ 4.81
26. $70\overline{)2275.0}$ 32.5
27. $80\overline{)450.40}$ 5.63
28. $60\overline{)1698.0}$ 28.3

Reflected answers, Set 38: 1. 205.8 2. 243 3. 463.63 4. 1.708 5. 4.172 6. 22.92 7. 11.92 8. 225.4 9. 74.1 10. 3.858 11. 74.34 12. 334 13. 32.1 14. 4.63 15. 0.765 16. 36.7 17. 4.712 18. 0.54 19. 0.326 20. 41.6 21. 56.21 22. 32.5 23. 44.51 24. 0.361 25. 4.81 26. 70 27. 5.63 28. 28.3

Set 39 *For use with page 245*

Find the quotients. Check your work.

1. $0.7\overline{)22.4}$ 32	2. $0.5\overline{)3.355}$ 6.71	3. $0.02\overline{)0.068}$ 3.4	4. $3.4\overline{)27.2}$ 8
5. $0.21\overline{)1.0962}$ 5.22	6. $6.3\overline{)3.276}$ 0.52	7. $0.33\overline{)1.518}$ 4.6	8. $0.62\overline{)0.1426}$ 0.23
9. $5.7\overline{)410.4}$ 72	10. $4.8\overline{)1.488}$ 0.31	11. $2.5\overline{)162.5}$ 65	12. $0.71\overline{)1.136}$ 1.6
13. $0.06\overline{)49.32}$ 822	14. $0.36\overline{)0.0180}$ 0.05	15. $8.2\overline{)0.574}$ 0.07	16. $0.61\overline{)0.854}$ 1.4
17. $3.5\overline{)213.5}$ 61	18. $0.003\overline{)0.375}$ 125	19. $9.4\overline{)432.4}$ 46	20. $0.26\overline{)5.486}$ 21.1
21. $0.52\overline{)16.692}$ 32.1	22. $7.3\overline{)350.4}$ 48	23. $4.4\overline{)360.8}$ 82	24. $0.38\overline{)119.32}$ 314
25. $8.6\overline{)19.584}$ 2.27	26. $4.22\overline{)25.32}$ 6	27. $7.63\overline{)46.543}$ 6.1	28. $50.4\overline{)70.46}$ 1.4

Reflected answers, Set 39: 1. 35, 5. 0.512, 3. 3.4, 4. 8

Set 40 *For use with page 249*

Write a decimal for each fraction. (The denominators are factors of 10 or 100.)

1. $\frac{1}{4}$ 0.25	2. $\frac{9}{20}$ 0.45	3. $\frac{3}{10}$ 0.3	4. $\frac{2}{5}$ 0.4	5. $\frac{17}{25}$ 0.68
6. $\frac{21}{50}$ 0.42	7. $\frac{7}{10}$ 0.7	8. $\frac{23}{25}$ 0.92	9. $\frac{13}{20}$ 0.65	10. $\frac{37}{50}$ 0.74

Use division to find decimals for these fractions.

11. $\frac{7}{25}$ 0.28	12. $\frac{1}{5}$ 0.2	13. $\frac{6}{15}$ 0.4	14. $\frac{7}{20}$ 0.35	15. $\frac{18}{30}$ 0.6
16. $\frac{9}{50}$ 0.18	17. $\frac{3}{25}$ 0.12	18. $\frac{6}{50}$ 0.12	19. $\frac{9}{25}$ 0.36	20. $\frac{1}{20}$ 0.05

Find the repeating decimal for each of these.

21. $\frac{5}{12}$ 0.41666...	22. $\frac{7}{18}$ 0.3888...	23. $\frac{4}{6}$ 0.666...	24. $\frac{4}{9}$ 0.444...	25. $\frac{1}{12}$ 0.08333...
26. $\frac{3}{11}$ 0.272727...	27. $\frac{5}{18}$ 0.2777...	28. $\frac{7}{11}$ 0.636363...	29. $\frac{7}{12}$ 0.58333...	30. $\frac{2}{6}$ 0.333...

Reflected answers, Set 40: 1. 0.25, 5. 0.28, 3. 0.3, 4. 0.4, 2. 0.45

Set 41 *For use with page 251*

Give each quotient rounded to the nearest hundredth.

1. $6\overline{)2.088}$ 0.35
2. $4\overline{)2.93}$ 0.73
3. $7\overline{)4.836}$ 0.69
4. $5\overline{)3.6421}$ 0.73
5. $1.6\overline{)0.825}$ 0.52
6. $2.4\overline{)0.4732}$ 0.20
7. $3.6\overline{)4.271}$ 1.19
8. $4.3\overline{)0.6125}$ 0.14
9. $0.26\overline{)3.436}$ 13.22
10. $0.78\overline{)5.9821}$ 7.67
11. $5.2\overline{)3.7076}$ 0.71
12. $0.34\overline{)7.3478}$ 21.61
13. $0.89\overline{)10.7065}$ 12.03
14. $6.4\overline{)4.609}$ 0.72
15. $2.1\overline{)1.654}$ 0.79
16. $0.46\overline{)10.8149}$ 23.51

Give each quotient expressed as a mixed-decimal numeral in hundredths.

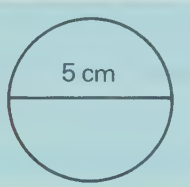
17. $7\overline{)1.72}$ $0.24\frac{4}{7}$
18. $4\overline{)1.47}$ $0.36\frac{3}{4}$
19. $2\overline{)12.71}$ $6.35\frac{1}{2}$
20. $6\overline{)0.73}$ $0.12\frac{1}{6}$
21. $9\overline{)2.36}$ $0.26\frac{2}{9}$
22. $3\overline{)2.56}$ $0.85\frac{1}{3}$
23. $8\overline{)3.72}$ $0.46\frac{1}{2}$
24. $5\overline{)3.68}$ $0.73\frac{3}{5}$
25. $0.4\overline{)0.217}$ $0.54\frac{1}{4}$
26. $0.02\overline{)0.4273}$ $21.36\frac{1}{2}$
27. $0.3\overline{)4.955}$ $16.51\frac{2}{3}$
28. $0.6\overline{)1.047}$ $1.74\frac{1}{2}$

Reflected answers, Set 41: 1. 0.35 5. 0.13 9. 0.69 13. 0.73

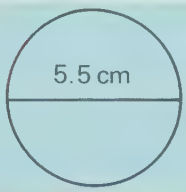
Set 42 *For use with page 263*

Find the circumference of each circle with the given diameter.
$\pi = 3.14$ (to the nearest hundredth)

1. 15.7



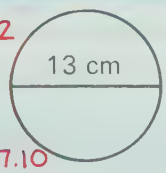
2. 17.27



3. 34.54



4. 40.82



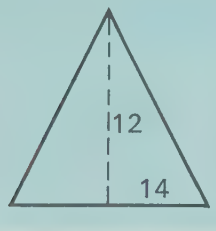
5. $d = 13$ cm 40.82
6. $d = 12.5$ cm 39.25
7. $d = 28.6$ cm 89.80
8. $d = 54.2$ cm 170.19
9. $d = 15$ cm 47.10
10. $d = 23$ cm 72.22
11. $d = 14.6$ cm 45.84
12. $d = 36.4$ cm 114.30
13. $d = 32.2$ cm 101.11
14. $d = 19$ cm 59.66
15. $d = 44.1$ cm 138.47
16. $d = 61$ cm 191.54

Reflected answers, Set 42: 1. 15.7 5. 114.30

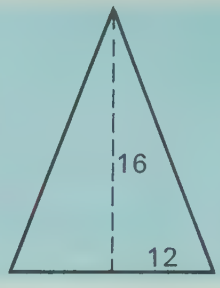
Set 43 *For use with page 267*

Find the area of each triangle.

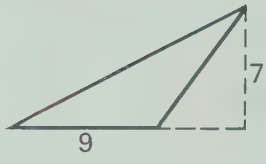
1. 84



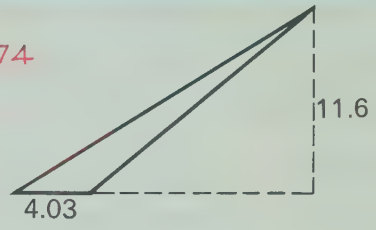
2. 96



3. $31\frac{1}{2}$



4. 23.374



Given the base and height, find the area of each triangle.

5. $b = 16$, $h = 5$ 40
6. $b = 32$, $h = 7$ 112
7. $b = 15$, $h = 42$ 315
8. $b = 9$, $h = 16$ 72
9. $b = 48$, $h = 7$ 168
10. $b = 56$, $h = 11$ 308
11. $b = 10.8$, $h = 12.4$ 66.96
12. $b = 32.6$, $h = 9$ 146.7
13. $b = 14\frac{1}{2}$, $h = 7$ $50\frac{3}{4}$
14. $b = 8$, $h = 10\frac{1}{3}$ $41\frac{1}{3}$
15. $b = 17.5$, $h = 12.6$ 110.25
16. $b = 44.3$, $h = 52.2$ 1156.23

Reflected answers, Set 43: 1. 84 2. 96

Set 44 *For use with page 271*

Find the area of each circle with the radius given. (Use $\pi = 3.14$.)

1. $r = 4$ 50.24
2. $r = 1$ 3.14
3. $r = 8$ 200.96
4. $r = 6$ 113.04
5. $r = 20$ 1256
6. $r = 3.4$ 36.2984
7. $r = 2.6$ 21.2264
8. $r = 9$ 254.34
9. $r = 12.1$ 459.7274
10. $r = 32$ 3215.36
11. $r = 10.4$ 339.6224
12. $r = 42$ 5538.96
13. $r = 5$ 78.5
14. $r = 7.3$ 167.3306
15. $r = 4.6$ 66.4424
16. $r = 12$ 452.16

Reflected answers, Set 44: 1. 50.24 2. 3.14 3. 200.96 4. 113.04

Set 45 *For use with page 273*

Use the volume formula, $V = \ell \times w \times h$, to find the volume of each figure.

1. $\ell = 5$, $w = 6$, $h = 6$ — 180
2. $\ell = 2$, $w = 7$, $h = 9$ — 126
3. $\ell = 5$, $w = 4$, $h = 10$ — 200
4. $\ell = 4$, $w = 4$, $h = 4$ — 64
5. $\ell = 9$, $w = 7$, $h = 12$ — 756
6. $\ell = 4$, $w = 7$, $h = 8$ — 224
7. $\ell = 10$, $w = 12$, $h = 5$ — 600
8. $\ell = 13$, $w = 6$, $h = 5$ — 390
9. $\ell = 2.1$, $w = 3.6$, $h = 4.4$ — 33.264
10. $\ell = 3\frac{1}{2}$, $w = 2\frac{1}{7}$, $h = 1\frac{1}{5}$ — 9
11. $\ell = 5.6$, $w = 2.08$, $h = 1.29$ — 15.02592
12. $\ell = 5\frac{1}{3}$, $w = 2\frac{1}{4}$, $h = 6$ — 72

Reflected answers, Set 45: 1. 180 2. 126 3. 200 4. 64

Set 46 *For use with page 307*

Find the missing numbers in each exercise.

1. $\frac{1}{3} = \frac{4}{\square}$ — 12
2. $\frac{\square}{5} = \frac{16}{20}$ — 4
3. $\frac{9}{8} = \frac{\square}{24}$ — 27
4. $\frac{7}{10} = \frac{21}{\square}$ — 30
5. $\frac{3}{\square} = \frac{12}{16}$ — 4
6. $\frac{\square}{7} = \frac{10}{35}$ — 2
7. $\frac{5}{3} = \frac{\square}{9}$ — 15
8. $\frac{5}{6} = \frac{20}{\square}$ — 24
9. $\frac{4}{9} = \frac{\square}{90}$ — 40
10. $\frac{12}{\square} = \frac{36}{15}$ — 5
11. $\frac{3}{10} = \frac{\square}{40}$ — 12
12. $\frac{7}{2} = \frac{35}{\square}$ — 10

Write a fractional number equation and solve it to answer the following ratio problems.

13. Ratio of tires to cars, 4:1. $\frac{4}{1} = \frac{n}{12}$, $n = 48$
    12 cars. How many tires?
14. Ratio of boys to girls, 7:8. $\frac{7}{8} = \frac{35}{n}$, $n = 40$
    35 boys. How many girls?
15. Ratio of oranges to cents, 5:22. $\frac{5}{22} = \frac{n}{66}$,
    66 cents. How many oranges? $n = 15$
16. Ratio of dogs to cats, 12:17. $\frac{12}{17} = \frac{n}{68}$,
    68 cats. How many dogs? $n = 48$

Reflected answers, Set 46: 1. 12 2. 4 3. 27 4. 30 13. 48 15. 15 14. 40 16. 48

Set 47 *For use with page 317*

1. $\frac{3}{20} =$ 15 %
2. $\frac{4}{5} =$ 80 %
3. $\frac{1}{6} =$ $16\frac{2}{3}$ %
4. $\frac{7}{25} =$ 28 %
5. $\frac{24}{25} =$ 96 %
6. $\frac{9}{25} =$ 36 %
7. $\frac{9}{20} =$ 45 %
8. $\frac{49}{50} =$ 98 %

Solve the following short story problems.

9. 80% of Mr. Jones' orchard: apple trees. 620 trees in all. How many apple trees? 496

10. 760 students at Thomas High School. 45% are girls. How many are girls? 342

11. Book: 575 pages. Read 56%. Read how many pages? 322

12. Paint: 16 litres. Used 75% to paint house. Used how many litres? 12

JJ. 355

Reflected answers, Set 47: J. J2 ' S. 80 ' 3. J0$\frac{3}{5}$ ' ⊅. 58 ' ə. ⊅ə0 '

Set 48 *For use with page 323*

Complete the following table.

Savings	Interest rate (per year)	Amount of interest in one year	Savings plus interest
$34.00	2%	1. $.68	2. $34.68
$125.00	4%	3. $5.00	4. $130.00
$78.00	3%	5. $2.34	6. $ 80.34
$236.00	5%	7. $11.80	8. $247.80
$68.00	4%	9. $ 2.72	10. $ 70.72
$95.00	2%	11. $ 1.90	12. $ 96.90
$430.00	3%	13. $12.90	14. $442.90
$750.00	4%	15. $30.00	16. $780.00
$520.00	5%	17. $26.00	18. $546.00
$1230.00	2%	19. $24.60	20. $1254.60

Reflected answers, Set 48: J. 089 ' S. ⊅3⊅.08

Books to Explore

Adler, Irving. ***The Giant Golden Book of Mathematics.*** New York, Golden Press, 1960. (Available from Whitman Golden Ltd., Cambridge, Ontario.)

Have you ever wondered how a tree grows or why a volcano is shaped as it is or what makes a card trick work? This colorful book answers these and many other questions by exploring the world of mathematics. Here are just a few of the interesting topics:

How to find π by dropping a toothpick 26
Your chance of getting two heads and a tail when you toss three coins 72

Bendick, Jeanne and Levin, Marcia O. ***Take Shapes, Lines and Letters.*** New York, McGraw-Hill Book Company, 1962. (Available from McGraw-Hill Ryerson, Scarborough, Ontario.)

This book teases your imagination with mathematics in art, music, and everyday life; with shapes and curves in nature; and with drawings, graphs, and secret codes. Ideas to explore in this book include:

The golden rectangle in art 44
A strip of paper that has only one side 72
Geometry on a doughnut 74

Boeke, Kees. ***Cosmic View: The Universe in 40 Jumps.*** New York, John Day Company, 1957. (Available from Longman Canada Ltd., Don Mills, Ontario.)

If you have a lively interest in the world in which you live, then take a graphic journey through the universe. Travel from outer space to the nucleus of an atom and take a look at everything between. A series of pictures and drawings begins with a girl shown one-tenth life-size. Each successive picture shows her with other objects in the solar system until the scale is $1:10^{26}$. Similarly, a small portion of her hand is magnified until the scale is $10^{13}:1$ in a picture that compares that part of her hand with the nucleus of a sodium atom.

Gardner, Martin. ***Perplexing Puzzles and Tantalizing Teasers.*** New York, Simon & Schuster, 1969. (Available from Musson Book Company, Don Mills, Ontario.)

This book is a collection of many different kinds of puzzles. None of them is very difficult, but some are tricky. Try your best before you look at the solutions given at the end of the book. Puzzles you may enjoy include:

Fun with palindromes 14
The maze of the Minotaur 20
Find the best words 40
More sneaky arithmetic 49

Other books to look for in your library are listed below.

Diggins, Julia E. ***String, Straightedge, and Shadow.*** New York, Viking Press, Inc., 1965. (Macmillan Company of Canada Ltd., Toronto.)

Based on historical records and legends, this book tells how men made the major discoveries of mathematics by using only three simple tools. It includes an interesting discussion on the origin of the word "mathematician."

Friskey, Margaret. ***ed.*** ***About Measurement.*** Chicago, Children's Press, 1965. (Scholars Choice Limited, Stratford, Ontario.)

Based on "A History of Mathematics," a poster published by the Educational Affairs Department of the Ford Motor Company, this book traces the history of units of length — cubit, foot, inch, fathom, yard, rod, furlong, meter, decimal inch, light-year.

Kaplan, Philip. ***Posers.*** New York, Harper & Row, 1963. (Fitzhenry & Whiteside Ltd., Don Mills, Ontario.)

As you will discover here, posers are not riddles made to deceive, or problems which require mathematical skill, or even puzzles which tease without purpose. They are statements of fact from which a "logical conclusion can be drawn." Anyone with a "reasonably agile mind" should enjoy the eighty posers in this book.

Meadow, Charles. ***The Story of Computers.*** Irvington-on-Hudson, New York, Harvey House, 1970. (Burns & MacEachern Ltd., Don Mills, Ontario.)

Today's computers can tackle problems as intricate as guiding spaceships and predicting the weather. This book tells how they are programmed, or instructed to solve these and other problems.

Rogers, James T. ***Story of Mathematics for Young People.*** New York, Pantheon Books, 1966. (Random House of Canada Limited, Toronto.)

This history explains how man created mathematics to solve his problems. Diagrams and photographs trace the lives of great mathematicians and reveal the part that mathematics plays in all phases of life, from calculating the curves of a suspension bridge to totalling a grocery bill.

Shimek, William. ***Patterns: What Are They ?*** Minneapolis, Minnesota, Lerner Publications, 1969. (J.M. Dent & Sons (Canada) Ltd., Don Mills, Ontario.)

Number patterns have fascinated men for thousands of years. This Math Concept Book tells how mathematics helps record and explore patterns and how such a study brings a deeper understanding of mathematics and even simplifies computation.

Zim, Herbert S. ***Codes and Secret Writing.*** New York, William Morrow and Company, 1948. George J. McLeod Ltd., Toronto.)

Learn to break and make up your own secret codes. This book will show you how.

addend Any one of a set of numbers to be added. In the equation $4 + 5 = 9$, the numbers 4 and 5 are addends.

addition An operation that combines a first number and a second number to give exactly one number. The two numbers are called addends, and the result is called the sum of the addends.

angle Two rays from a single point.



approximation One number is an approximation of another number if the first number is suitably "close" (according to context) to the other number.

area The area of a closed figure or region is the measure of that region as compared to a given selected region called the unit, usually a square region in the case of area.

arithmetic mean (average) The arithmetic mean of a set of numbers is the quotient resulting when the sum of the numbers in the set is divided by the number of addends.

associative (grouping) principle When adding (or multiplying) three numbers, you can change the grouping and the sum (or product) is the same.

Examples: $2 + (8 + 6) = (2 + 8) + 6$
$3 \times (4 \times 2) = (3 \times 4) \times 2$

base of numeration The term "base" refers to the type of grouping involved in the system of numeration. For example, in base 6:
23 means 2 sixes and 3.
321 means 3 thirty-sixes (6×6), 2 sixes, and 1.
In base ten:
23 means 2 tens and 3.
321 means 3 hundreds (10×10), 2 tens, and 1.

base of a polygon Any of the sides of a polygon may be referred to as a base. The term is normally used with reference to the height from a vertex (or parallel side) to the given base. *See* height of a triangle.

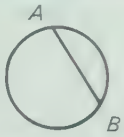
bisect To divide in half or find the midpoint.

centimetre A unit of length. One centimetre is $\frac{1}{100}$ metre.

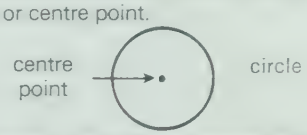
central angle In the figure below, angle BAC illustrates a central angle with respect to a given circle with centre A.



chord A line segment that has its end points on a given circle.



circle The set of all points in a plane which are a specified distance from a given point called the centre or centre point.



circumference The distance around a circle.

clock arithmetic A mathematical system using only the twelve numbers of a clock face. It is also called modular arithmetic or remainder arithmetic.

common factor When a number is a factor of two different numbers, it is said to be a common factor of the two numbers.

common multiple A number is a common multiple of two numbers if it is a multiple of each of the numbers.

commutative (order) principle When adding (or multiplying) two numbers, the order of the addends (or factors) does not affect the sum (or product).

Examples: $4 + 5 = 5 + 4$
$3 \times 6 = 6 \times 3$

composite number Any whole number greater than 1 that is not prime.

cone Generally thought of in terms of a right circular cone, illustrated below.



congruent figures Geometric figures that have the same size and shape.



congruent triangles

coordinate axes Two number lines intersecting at right angles at 0.

coordinates Number pair used in graphing.

cube A rectangular prism (box) such that all faces are squares.

cylinder Generally thought of as a right circular cylinder illustrated below.



decimal Any base-ten numeral that uses place value to represent a number.

degree A unit of angle measure.

denominator The number indicated by the numeral below the line in a fraction symbol.

diagonal A segment joining two nonadjacent vertices of a polygon. In the figure, the diagonal is segment AB.



diameter A chord that passes through the centre point of the circle.



distributive (multiplication-addition) principle This principle is sometimes described in terms of "breaking apart" a number before multiplying.

Example: $6 \times (20 + 4) = (6 \times 20) + (6 \times 4)$

dividend In the problem $33 \div 7$, 33 is called the dividend.

$$\begin{array}{r} 4 \\ 7\overline{)33} \\ \underline{28} \\ 5 \end{array}$$

Example: 33 ← dividend; divisor: 7; 28; 5 → remainder

division An operation related to multiplication as illustrated:

$$3 \times 4 = 12 \begin{cases} \nearrow 12 \div 4 = 3 \\ \searrow 12 \div 3 = 4 \end{cases}$$

divisor In the problem $33 \div 7$, 7 is called the divisor.

empty set A set that has no objects in it.

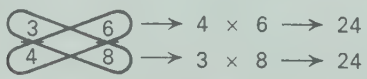
equality (equals; or $=$) A mathematical relation of being exactly the same.

equally likely outcomes Outcomes that have the same chance of occurring.

equation A mathematical sentence involving the use of the equality symbol.

Examples: $5 + 4 = 9$; $7 + \square = 8$; $n + 3 = 7$.

equivalent fractions Two fractions are equivalent when it can be shown that they each can be used to represent the same amount of a given object. Also, two fractions are equivalent if these two products are the same:



equivalent sets Two sets that may be placed in a one-to-one correspondence.

estimate To find an approximation for a given number. (Sometimes a sum, a product, etc.)

even numbers The whole-number multiples of 2 (0, 2, 4, 6, 8, 10, 12, . . .).

exponent In the symbol 10^3, the "3" is the exponent. It indicates that 10 is used as a factor three times. Thus: $10^3 = 10 \times 10 \times 10 = 1000$

$5^4 = 5 \times 5 \times 5 \times 5 = 625$

face The face of a given space figure is any one of the plane geometric figures (regions) making up the space figure. For example, in a cube each of the square regions is a face of the cube.

factor *See* multiplication. The equation $6 \times 7 = 42$ illustrates that both 6 and 7 are factors of 42.

fraction A symbol for a fractional number such as $\frac{2}{3}, \frac{3}{4}, \frac{1}{2}$, and so on.

fractional number The one number we think about for each set of equivalent fractions.

function The set of number pairs (input, output) generated by a function rule applied to a given set of numbers (input numbers).

graph (1) A set of points associated with a given set of numbers or set of number pairs. (2) A picture used to illustrate a given collection of data. The data might be pictured in the form of a bar graph, a circle graph, a line graph, or a pictograph. (3) To draw the graph of.

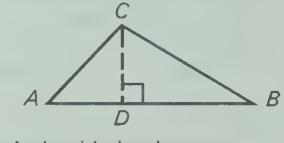
greater than ($>$) One of the two basic inequality relations.

Examples: $8 > 5$, $28 > 25$, $80 > 50$.

greatest common factor The largest, or greatest, number that is a factor of each of two numbers.

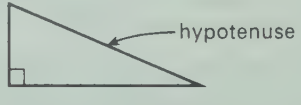
height of a triangle The height of a triangle from any vertex is the perpendicular distance from that vertex to the opposite side (usually called the base).

In the figure, the length of $\overline{CD}$ is the height of the triangle from vertex C to base $\overline{AB}$.



hexagon A six-sided polygon.

hypotenuse The side opposite the right angle in a right triangle.



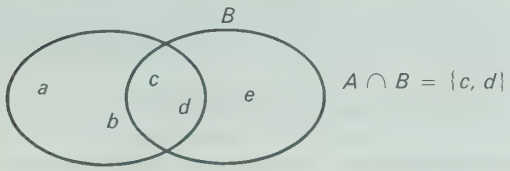
improper fraction A fraction in which the numerator is greater than or equal to the denominator.

Examples: $\frac{8}{5}, \frac{6}{6}, \frac{12}{3}, \frac{7}{7}$

inequality ($<$, $\neq$, $>$) A relation indicating that the two numbers are not the same.

inscribed circle A circle is said to be inscribed in a polygon if the circle lies within the polygon and each side of the polygon is tangent to the circle.

intersection of sets The intersection of two sets is the set of elements common to both of the sets. If A and B are sets, $A \cap B$ denotes the intersection of the sets.

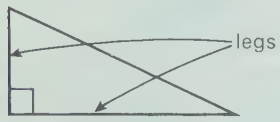


$A \cap B = \{c, d\}$

least common denominator The least common multiple of two denominators. The least common denominator for $\frac{3}{4}$ and $\frac{5}{6}$ is 12.

least common multiple The smallest non-zero number that is a multiple of each of two given numbers. The least common multiple of 4 and 6 is 12.

legs of a right triangle The two sides of a right triangle other than the hypotenuse.



length (1) A number indicating the measure of one line segment with respect to another line segment, called the unit. (2) Sometimes used to denote one dimension (usually the greater) of a rectangle.

less than ($<$) One of the two basic inequality relations. Examples: $5 < 8$, $25 < 28$, $50 < 80$.

lowest terms A fraction is in lowest terms if the numerator and denominator of the fraction have no common factor greater than 1.

metre The basic unit of length in the Metric System.

midpoint A point that divides a line segment into two parts of the same size.

mixed-decimal numeral Numerals such as $0.7\frac{1}{2}$ and $0.33\frac{1}{3}$

mixed numeral A symbol such as $2\frac{1}{2}$ and $5\frac{1}{1}$.

multiple A first number is a multiple of a second number if there is a whole number that multiplies by the second number to give the first number.

Example: 24 is a multiple of 6 since $4 \times 6 = 24$.

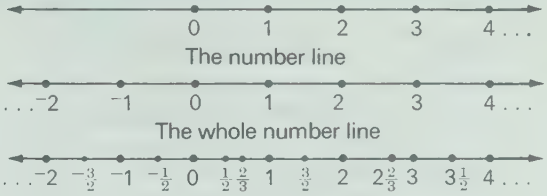
multiplication An operation that combines a first number and a second number to give exactly one number. The two numbers are called factors, and the result is called the product.

negative number A number which will add to a positive number to give a sum of zero.

For example: $5 + {}^{-}5 = 0$

$19 + {}^{-}19 = 0$

number line A line with a subset of its points matched with a subset of the real numbers. We say that the rational number line has "holes" in it because some points are not matched with rational numbers. The real number line is said to be "complete" because each point is matched with some real number.



number pair Any pair of numbers. Each pair of numbers can be matched with a unique point in the co-ordinate plane.

numeral A symbol for a number.

numerator The number indicated by the numeral above the line in a fraction symbol.

odd number Any whole number that is not even.

one principle Any number multiplied by 1 is that same number. Sometimes called the identity principle for multiplication.

one-to-one correspondence A one-to-one correspondence exists between two sets when the elements of one can be matched with the elements of the other in such a way that each element of the first set is matched with exactly one element of the second set and each element of the second set is matched with exactly one element of the first set.

parallel lines Two lines which lie in the same plane and do not intersect.

parallelogram A quadrilateral with its opposite sides parallel.

parentheses A pair of curved symbols ( ), used to indicate grouping or order of performing operations. Examples: $(5 \times 4) - 2 = 18$

$5 \times (4 - 2) = 10$

pentagon A five-sided polygon.

percent (%) Per 100; for each 100; $\frac{1}{100}$.

perimeter The sum of the lengths of the sides of a given polygon.

period In arithmetic, each set of three digits indicated by spaces when writing a numeral is called a period.

Example: 3 4 2 | 6 7 4 | 2 0 8

millions' period | thousands' period | units' period

perpendicular lines Two lines that intersect in right angles are perpendicular to each other.

pi (π) The ratio of the circumference to the diameter of a circle; approximately 3.14.

place value A system used for writing numerals for numbers, using only a definite number of symbols or digits. In the numeral 3257 the 5 stands for 50; in the numeral 36 289 the 6 stands for 6000.

polygon A closed geometric figure made up of line segments. A regular polygon has congruent sides and congruent angles.

prime number A number greater than 1 whose only factors are itself and 1.

probability The probability that an event will occur in a set of equally likely outcomes is the number of ways the event can occur divided by the total number of possible outcomes. For example, the probability that a 3 or a 4 will turn up in a single toss of a die is $\frac{2}{6}$ since there are 2 ways the event can occur and there are 6 possible equally likely outcomes.

product The result of the multiplication operation. In $6 \times 7 = 42$, 42 is the product of 6 and 7.

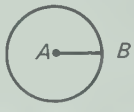
protractor An instrument used for measuring angles.

pyramid A space figure which has a polygonal base and triangular lateral faces.

quadrilateral A four-sided polygon.

quotient The number (other than the remainder) that is the result of the division operation. It may be thought of as a factor in a multiplication equation.

radius (1) Any segment from the centre point to a point on the circle. (2) The distance from the centre point to any point on the circle.



ratio A pair of numbers used in making certain comparisons. The ratio of 3 to 4 is written 3 : 4 or $\frac{3}{4}$.

ray The heavy part of the line shows a ray.



reciprocal Two numbers are reciprocals of one another if their product is 1. Example: $\frac{4}{7}$ and $\frac{7}{4}$ are reciprocals of each other.

rectangle A quadrilateral that has four right angles.

rhombus A parallelogram with 4 congruent sides.

right angle An angle that has the measure of 90 degrees.

right triangle A triangle that has one right angle.

Roman numerals Numerals used by the Romans. Used primarily to record numbers rather than for computing. Examples: IV, IX, XIV.

rotation A motion in which a given figure is turned about a fixed point.

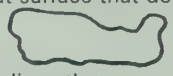
scale drawing A drawing constructed so the ratio of all the dimensions in the drawing to those of the actual object is the same.

segment Two points on a line and all the points on that line that are between the two points.

sequence A collection or set of numbers given in a specific order. Such numbers are commonly given according to some rule or pattern.

set A group or collection of objects.

simple closed curve Can be thought of as a loop of string on a flat surface that does not cross itself.



skew lines Two lines that are not in the same plane.

solution The number or numbers which result from solving an equation or a given problem.

square A quadrilateral that has four right angles and four sides that are the same length.

subtraction An operation related to addition as illustrated:

$$7 + 8 = 15 \begin{cases} \nearrow 15 - 8 = 7 \\ \searrow 15 - 7 = 8 \end{cases}$$

sum The result obtained by adding any set of numbers.

symmetric figure A plane figure which can be divided into two congruent halves by a line.

tangent A line is tangent to a circle if the two figures are in one plane and have exactly one point in common.

tessellation A repeated pattern of regions that can cover a plane.

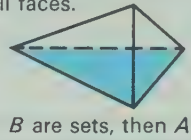


translation A motion in which each point of a figure is moved the same distance and the same direction.

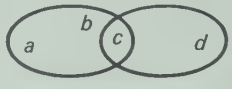
trapezoid A quadrilateral with at least one pair of parallel sides.

triangle A three-sided polygon.

triangular pyramid A 4-sided space figure that has triangular regions for all faces.



union of sets If A and B are sets, then $A \cup B$ (the union of A and B) is the set consisting of all elements that belong to at least one of the two sets.



$A \cup B = \{a, b, c, d\}$

unit An amount or quantity adopted as a standard of measurement.

vertex The point that the two rays of an angle have in common.



volume The measure, obtained using an appropriate unit (usually a cube), of the interior region of a space figure.

whole number Any number in the set $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

zero principle Any number added to zero is that same number. (Also called the identity principle for addition.)

Tables of Measures

LENGTH	
10 millimetres (mm) = 1 centimetre (cm)	1000 millimetres = 1 metre
10 centimetres = 1 decimetre (dm)	100 centimetres = 1 metre
10 decimetres = 1 metre (m)	10 decimetres = 1 metre
1000 metres = 1 kilometre (km)	1 / 1000 kilometres = 1 metre

TIME	
60 seconds (s) = 1 minute (min)	52 weeks = 1 year (yr)
60 minutes = 1 hour (h)	12 months (mo) = 1 year
24 hours = 1 day	365 days = 1 year
7 days = 1 week (wk)	366 days = 1 leap year

CAPACITY
10 millilitres (ml) = 1 centilitre (cl)
10 centilitres = 1 decilitre (dl)
10 decilitres = 1 litre (*l*)
1000 litres = 1 kilolitre (kl)

WEIGHT
1000 grams (g) = 1 kilogram (kg)
1000 kilograms = 1 tonne

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A Text for Teachers

Investigating Mathematics Learning

Phares G. O'Daffer

Robert E. Eicholz

Charles R. Fleenor

Introducing the Metric System

James Sherrill

J. Norman C. Sharp



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INVESTIGATING MATHEMATICS LEARNING

I. Some Thoughts About Learning

Almost everyone has some observations on teaching and learning. A recent quote that has been making the rounds is: "If we tried to teach children to speak, they would never learn." However, in *The Process of Education* (Harvard University Press, 1960), Jerome Bruner observes, "Any subject can be taught effectively in some intellectually honest form to any child at any stage of development." But Linus (of *Peanuts* cartoon fame), considerably less optimistic, laments: "How can I learn 'New Math' with an 'Old Math' mind?"

In a more critical vein, John Holt in *How Children Fail* (Pitman Publishing Co., 1964) asserts: "In our classes, we begin with words, carry on with words, and often fail to get beyond words." He also says, "All too often the mathematics classroom becomes a temple of worship for the right answers, and the way to get ahead is to lay plenty of them on the altar." We know, of course, that many teachers for many years have been doing an excellent job helping elementary school children learn mathematics. Yet, it is worthwhile for us to reevaluate our approaches and, if possible, find even better ways to create situations where children learn more effectively.

The implications of the research of Piaget and others in how children learn mathematics and the observations of countless classroom teachers concerning the directions we should take are well summarized by a familiar Chinese proverb:

I hear and I forget.
I see and I remember.
I do and I understand.

The message of this proverb is that hearing and seeing are not enough: to learn with understanding, the child should experience *active involvement* with mathematical ideas. In order for the child to become actively involved, it has been found that the use of *physical materials* which contain the seeds of the mathematical ideas are valuable and often necessary. Coupled with the idea of active involvement with physical materials is the idea that teachers should encourage *student responsibility* and create conditions in which the student is not always encouraged to rely solely on the teacher but rather to take initiative for figuring out some things for himself.

Z. P. Dienes summarized a multitude of suggestions from researchers and teachers when he said: "It is suggested that we shift the emphasis from teaching to learning, from our experience to the children's, in fact, from our world to their world."

Teachers vary considerably in their views of how best to help children become actively involved with mathematics. While one teacher desires to convert his classroom immediately into a mathematics laboratory, another teacher may prefer a very modest beginning with a limited amount of active student involvement with physical materials inserted into his usual classroom approach. In this text we suggest a number of approaches for modest beginnings and indicate ways in which these approaches might be expanded to provide for a total laboratory approach and a more extensive individualized program.

To introduce one possible approach, let us simulate a teaching strategy by outlining one way to organize a specific lesson. Thus, suppose a teacher wanted to devise a lesson which would help children understand the idea of congruent segments in geometry. First the teacher provides each child with a geoboard and a sheet containing several 5-by-5 arrays of dots. Then he reviews, very briefly, the idea of a segment and the endpoints of a segment. Next, after helping the children see that they can use a rubber-band around two nails to represent a segment on the geoboard, he passes out the investigation suggested in Figure 1.

INVESTIGATION

How many different segments with endpoints just as far apart as these can you find on your geoboard? Record your results on dot paper.

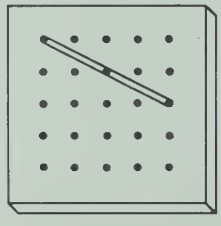


Figure 1

The teacher may choose to have chairs or desks rearranged so that children can communicate with each other as they become involved in the investigation. The teacher will check to be sure that everyone understands the investigation question; then he should encourage the children to find their own way to answer the question and record their findings. (To gain a fuller appreciation of an investigation situation, play the role of the child and complete the investigation yourself.)

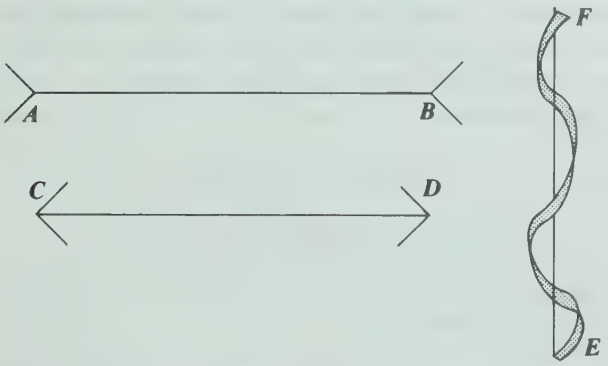
Brief discussions among children or between teacher and children may occur during investigations, but the main discussion is most effective after the investigation has been completed. At this time, the teacher might ask such questions as: "How many different

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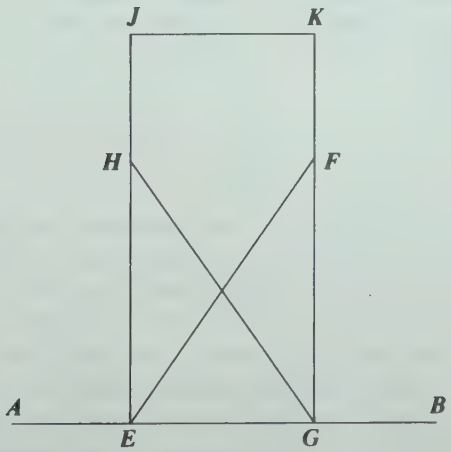
segments did you find?" "How can you be sure that you have found all such segments?" "How could you convince someone that each of your segments has endpoints just as far apart as all the others?" Such questions could then be followed up with a definition of congruent segments: When the endpoints of one segment are just as far apart as the endpoints of another segment, we say the segments are *congruent*. Then ask, "Can you think of some other ways to tell when two segments are congruent?" This question might lead into a discussion of how tracing paper, compasses, or marks on the edge of a piece of paper can be used to determine whether or not two segments are congruent.

After the children have discussed the ideas, the teacher may provide them with some problems which *utilize* these ideas. The child would probably be encouraged to use the ideas for testing congruence of segments that were developed in the discussion. The following are examples of possible exercises.

1. Find 2 segments below that are congruent to each other.



2. Name each pair of congruent segments in this picture.



One way to individualize a lesson is through an *extension* of the exercises. Extending the exercises can provide for remediation, reinforcement, or enrichment. As an extension to individualize this lesson, the teacher might give certain students the follow-up investigation below. (For a fuller appreciation of this lesson, complete the exercises and the investigation yourself.)

INVESTIGATION

Segment AB is not congruent to segment CD.
How many different segments (no two congruent) can you find on your geoboard? Record your results on dot paper.

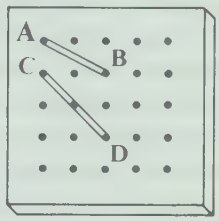


Figure 2

This abbreviated "lesson" provides a preview of one possible technique for encouraging children to become actively involved with physical materials in situations where they take more responsibility in the learning of mathematical ideas.

In the next section of this text, the parts of lessons such as the one described above will be analyzed and discussed. An outline for planning such lessons will be given, and various suggestions for carrying out each part of such a lesson will be proposed.

Since the investigation phase of the lesson provides the encouragement for active involvement by the child and since the kind of investigation used depends upon the type of learning involved, Section III in this text will focus on specific types of learning in elementary school mathematics. For example, the "lesson" described above helped children learn the *concept* of congruent segments; other lessons might be concerned with developing a *skill*, forming a *generalization*, learning a *fact*, or developing an *attitude*. Each type of learning will be analyzed and related to activity-oriented lessons that provide modest beginnings toward an active approach to mathematics learning.

Edith Biggs and James MacLean, in their book *Freedom to Learn* (Addison-Wesley, 1969), state: "A few schools scattered throughout the world are responding with some speed to a message which has been repeated with increasing urgency for some three hundred years. It is a simple message: Schools should be organized, not for teachers to teach, but for children to learn." In the same book, there appears an extensive list of "homemade" materials and devices that can be easily acquired for use in the mathematics classroom. Many materials, from newsprint and drinking straws, to string, popsicle sticks, beans, and homemade geoboards, can be made available to children at minimal cost. Rather than dismiss the possibility of actively involving children with materials in the classroom because no funds are available, a teacher should study this list carefully; he may be amazed by how much can be done with minimum expense.

Teachers sometimes feel that to involve children with physical materials and allow them to communicate with other children in the classroom is to invite

chaos. On the contrary, it has been found that, when children really become involved in using materials to investigate a situation, there may be a bit more low-keyed noise about the room but the usual discipline problems are almost nonexistent. It is helpful if there are tables available in the classroom so that children can work in small groups. If tables are not available, desks could be moved to assist in small-group work. On occasion, an investigation might call for children to leave their desks and to engage in other activity in the room. A simple set of "ground rules" should suffice to make the situation quite manageable.

It is interesting to consider the number of elementary school teachers who prefer to say that they are "helping children learn mathematics" rather than that they are "teaching mathematics." What one says, of course, does not always describe accurately what one does. It does seem important, however, in the light of recent studies and observations about how children learn mathematics, to focus on the child and try to create an environment in which the child has a greater opportunity to make decisions and to become really interested in his study. It is hoped that the following sections of this teachers' text will provide some ideas which may help you improve your ability to "help children learn mathematics."

EXERCISE SET 1

1. What was your reaction to the investigations in this section? **A** Did you become involved in the activity? **B** Were you interested? **C** Did you watch the clock? **D** Did you talk to anyone else while completing the investigation? (If so, was it helpful?) **E** Did the investigation situation help you better understand the idea involved? **F** What other feelings did you have?
2. Which quotation in this section seemed most significant to you? Why?
3. **A** Do you think most teachers teach the same way they were taught as elementary school children? **B** What do they do differently? **C** What are some ways you think our teaching of elementary school mathematics might be improved?
4. Look through the *Investigating School Mathematics* text at your grade level. How do the comments in this first section of the text relate to the approach taken in the child's text?

II. A Plan for a Learning Experience

First consider the practical matter of how the teacher proceeds in the daily task of helping children learn mathematics. A structured outline (inherently flexible) around which daily learning experiences may be planned can be a valuable organizational aid for the teacher and can give him a fresh insight into the role of new approaches to instruction.

Here is the outline that was used in planning the "lesson" in Section I. It has proven to be quite useful, especially for those teachers who have desired to make a beginning toward providing children more opportunities for active involvement with mathematical materials and ideas.

Preparation and Investigation
Discussion
Utilization and Extension

Since this outline offers a variety of possibilities for a teacher to reevaluate his approach to classroom instruction, the following sections provide an examination of its individual elements.

Preparation and Investigation

The investigation phase (often called simply "the investigation") is central to the learning experience. In this phase, the children are encouraged to become actively involved, individually or in groups, in the investigation of a situation that contains the seed for the central idea of the lesson. The investigation should be the "main event" in terms of pupil activity and involvement. The teacher should think of the investigation as a child-centred activity. Completion of the investigation in Figure 3 will help clarify the ideas of investigation.

INVESTIGATION

Can you find an investigation in a text from the *Investigating School Mathematics* series that

(a) uses centimetre strips?
(b) utilizes paper folding?
(c) has a question like "How many can you find?"
(d) involves the geoboard?
(e) encourages children to use graph paper?
(f) asks the children to record their findings?
(g) directs the children to use reference material?



Figure 3

Homemade or commercially produced manipulative materials often provide the stimulus for the situation to be investigated. At other times, even more simple teacher-devised activities provide this stimulus. For example, the suggested investigation in Figure 4 might have been made by a teacher to initiate an investigation in a lesson designed to help children form the generalization, "You can rearrange three addends any way you please, and the sum will always be the same."

Sometimes by asking appropriate questions about a situation of interest to the children the teacher may involve them in an exploration of a central idea to be developed.

Regardless of how an investigation is initiated, a teacher should remember that the investigation situation is specifically designed to encourage children to

INVESTIGATION

Make three slips of paper like these.



Then turn them over and mix them up. Pick any two slips and add the numbers on them. Then add the number on the other slip.

If you do this five times, will you get the same number each time?

Figure 4

take responsibility for the thinking and exploring. Too much "teacher help" can hinder the achievement of these aims.

In an investigation, it is not uncommon to see children deeply involved and assuming full responsibility for completing the task at hand. The teacher, who plays a key role in initiating the investigation, may appear not to be needed as he moves about the room. Occasionally, a brief discussion between teacher and child occurs, but most of the larger-group discussion occurs after the investigation. The investigation itself should embody an attitude toward learning that could be easily stifled by too many words from the teacher. Perhaps, in an investigation, a new adage should replace the old: the teacher, rather than the children, should be "seen but not heard."

The investigation is predicated on the assumption that the best way to minimize the need for words is to substitute an appropriate question for a wordy explanation at a time when a child's interest in a mathematical situation is beginning to ripen.

For example, suppose a certain group of children understand the concept of a triangle and are ready to consider characteristics that distinguish one type of triangle from another. An appropriate question to initiate an investigation might be the one shown in Figure 5. (Try this investigation yourself.)

INVESTIGATION

How many triangles with different shapes can you make that have no nails inside?

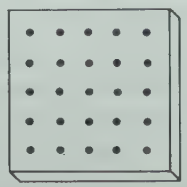


Figure 5

This question is both activity-stimulating and activity-sustaining. It helps involve the child in a search which he will continue with little further motivation. Notice also that the answer is not as important as the experiences the child will have as he responds to the question. Further, the question is sufficiently clear that the child immediately becomes involved with the challenge of the investigation rather than dissipating energy in efforts to understand the question. Another characteristic of this type of question is that it provides for individual differences: when the child is asked "How many can you find?" he can feel successful even if he finds only one. Of course, not all investigations can or should be introduced by this type of question, but it is important for the teacher to recognize that as the children respond to these questions, they will achieve in widely differing ways. In an investigation, the teacher should give recognition for all levels of achievement.

It should be noted that the amounts of time used for the investigations may vary considerably. One investigation may involve a very brief "happening" which sparks a simple idea within the child. Another investigation may utilize a large part of the period of time available for the mathematics lesson and might involve the child in a sustained exploration of a game or a set of manipulative materials.

To set the stage for an investigation of any duration, a preliminary *preparation* phase is sometimes needed. This phase provides for a brief review of key ideas needed for the investigation and for any motivational activity helpful in introducing it. This phase should be kept fairly short and care should be taken to see that this preliminary work does not preempt the central idea or activities involved in the investigation or the work that follows it.

In summary, the investigation phase is the child-involvement phase. It often requires materials, and is usually motivated by a carefully selected question which focusses the student's attention on the central idea of the lesson. Proper consideration of this phase in your lesson planning can be highly rewarding.

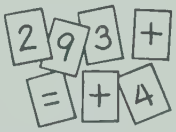
EXERCISE SET 2

1. Find some investigations in the *Investigating School Mathematics* text that contain features not mentioned in Figure 4.
2. Choose a lesson from an *Investigating School Mathematics* text and write a description of the role you think the teacher would play in using the investigation phase of the lesson.
3. Choose an idea to be taught and prepare an investigation situation which has the potential of involving the child in working with this idea.
4. Two investigations follow. Give the central idea of a possible lesson based on the use of each one.

A

INVESTIGATION

Cut out 7 slips of paper. Put one of these numerals or one of these signs on each one.

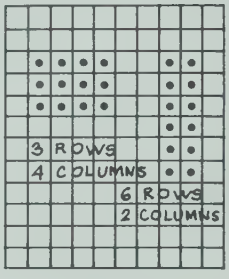


How many different equations with 3 addends can you write with your slips of paper? Record each equation you find.

B

INVESTIGATION

The graph paper shows two different ways to arrange 12 counters in a rectangular array.



How many different ways can you arrange 24 counters in a rectangular array? Record your findings by drawing pictures on graph paper.

5. Here is an interesting investigation you may like to try. Through it, you will be introduced to a basic idea of mathematics. Be sure to record your findings and be ready to discuss them further in the next section.

Copy and continue this array of numbers until you reach 52.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16
17	18	19	20
	. . .		

Then circle all the prime numbers in the array.

Notice that the numbers in the right-hand column can be written as $4 \times$ (a whole number). For example: $8 = 4 \times 2$, $12 = 4 \times 3$, and $20 = 4 \times 5$.

Can you make a statement about prime numbers that is suggested by this activity?

Discussion

Following the investigation, a *discussion* phase allows teacher and children to further share ideas in a discussion of what they found in the investigation. The teacher has an excellent opportunity in this phase to ask questions and to supply examples to help children further develop their understanding of the ideas germinated by the investigation.

Another valuable aspect of the discussion phase is that it provides additional opportunities for children to communicate with other children as a means of shaping their ideas. In a good discussion, it is not unusual for children, having reached an impasse in *their* thinking and communication about an idea, to ask the teacher if he can clarify the point. This is when the teacher as a resource person emerges. At other times, when ideas new to the teacher arise, the teacher participates in the discussion, not as a resource person, but as a fellow-learner. Both of these situations can contribute to a comfortable, meaningful discussion, but its potential benefits may never be realized if the teacher monopolizes the discussion to the extent that the children are denied the opportunity to draw their own inferences and make their own decisions. Since it is the child who is involved in the investigation, the child's ideas about the findings should be of primary importance, and the child should supply as many details leading to the understanding of the idea as possible.

By listening to the child and asking appropriate questions, the teacher can build on the child's initial ideas and help him develop a deeper understanding in preparation for further work. This understanding cannot be developed, however, by always asking questions which require simply that a child remember a fact correctly or perform a practical skill. Nor is it sufficient to ask questions to which a child can respond with a guess of "Yes" or "No." Rather, the questions that should be asked often are those that require a deeper thinking on the part of the child.

For examples of the more effective type of question, consider again the investigation described in Figure 7. This investigation, designed to set the stage for the development of the concepts of isosceles triangle, right triangle, scalene triangle, and equilateral triangle, might be followed by a discussion in which the teacher would ask questions such as the following:

1. Can you choose a pair of triangles you found and describe ways in which one is different from another?
2. In what ways are some triangles you found alike? (Note: Children may respond, "Some have a square corner," "Some have two sides the same," "Some have no sides the same," "Some are large," and so on.)
3. How would you describe a triangle that is different from any of the triangles you formed on the geoboard?

As the teacher asks thought-provoking questions and listens to the children's responses, he will be able to find ways to clarify the basic idea of the lesson and to prepare the children for the independent work which is to follow. It is in the latter stages of the discussion that the teacher may want to explain more carefully, show additional examples, and, in general, lead the child to a deeper mastery of the ideas involved.

EXERCISE SET 3

1. Can you find a question in the "Discussing the Ideas" section of an *Investigating School Mathematics* text which **A** asks the children to recall something previously learned? **B** asks the children to restate or explain an idea in their own words? **C** asks the children to interpret a diagram, picture, or explanation? **D** asks the children to analyze a given situation? **E** asks the children to evaluate a given situation?
2. What do you think about the effectiveness of the investigation described in Figure 5 as a means of meeting the goals indicated?
3. Write five questions you might ask while conducting a discussion in a mathematics lesson of your choice.
4. The following discussion exercises refer to the investigation presented in exercise 5 of Exercise Set 2. **A** What statement did you make about prime numbers? **B** Can you find a prime number that does not appear in the first or the third column? Can you find more than one? **C** $4 \times n$ is an algebraic expression. What algebraic expression can you devise to describe the prime numbers in the third column? in the first column? **D** Of the prime numbers less than 100, which type of prime occurs more often? **E** 113 is a prime number. Which type of prime is it?
5. Investigation questions may be open ("In how many ways can you measure a ball?") or closed ("Can you find the circumference and diameter of this ball?"). Discuss the merits of open and closed questions.

Utilization and Extension

The *utilization* phase allows each child to work on his own and to use the ideas developed in the investigation and discussion phases.

Often children need to practice recalling facts that have been developed or introduced in the lesson. Appropriate exercises requiring written answers are often valuable in providing this practice.

In another lesson, a child may have learned an algorithm or a skill. In order to refine this skill, he may need considerable practice using it. Appropriately designed written exercises which children complete independently can be quite helpful in polishing these skills.

In another lesson, a new idea may have been presented. In order to become more familiar with this idea and to understand how it relates to other ideas, the child may need thought-provoking problems which involve the idea. The *utilization* phase presents an opportunity for the child to solve problems which involve ideas that have been presented previously or to look at an idea that is different but closely related to one he has already encountered.

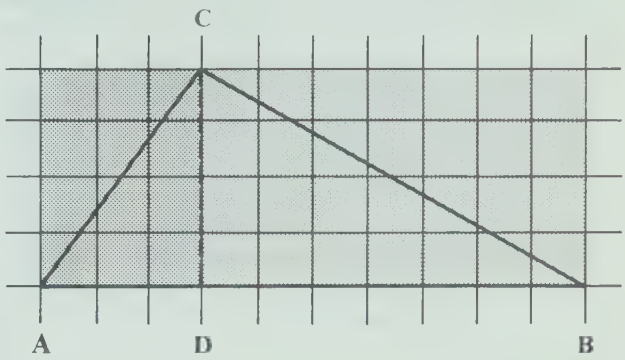
Creative activities for independent work can do much to extend the learnings developed in the investigation and discussion phases. The utilization exercises in examples A and B below are sequenced in such a way that the child has an opportunity to discover a new procedure or new ideas as a result of his work.

EXAMPLE A

Find the differences.

75	75	75	75	75	75	75
-32	-33	-34	-35	-36	-37	-38
43	42	41				

EXAMPLE B



What is the area of the region shaded dark gray?
What is the area of the region shaded light gray?
What is the area of the two regions together?
What is the area of triangle ADC?
What is the area of triangle BDC?
What is the area of triangle ABC?
The area of triangle ABC is what part of the entire shaded region?

The teacher should appreciate the great potential value of discovery-sequenced exercises such as these, and should look for opportunities to make his own exercise sets using such sequences. Another set of utilization exercises might encourage the child to independently delve more deeply into the idea initiated in the investigation. Further activities with mathematical materials often provide opportunities for the child to use and extend the idea of the investigation. Example C provides an opportunity for the child to reinforce his concept of symmetry.

EXAMPLE C

Do this to make symmetrical figures.



Fold a piece of paper.

Make a cut that starts and ends on the fold.

Unfold the piece you cut out. It will be symmetrical.

Make cuts so that the unfolded shape will be:

A a rectangle	**D** a square	**G** a rocket
B a leaf	**E** a house	**H** a hexagon
C a triangle	**F** a pumpkin	**I** a butterfly

Regarding the utilization phase, it should be noted that on occasion it may be more valuable to have pairs or small groups of children work the exercises together.

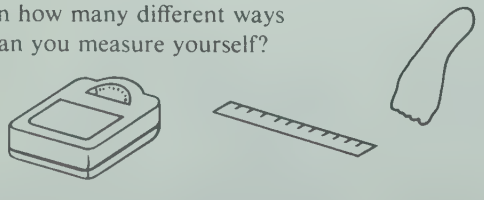
Finally, the *extension* phase provides for use of remedial, maintenance, or enrichment activities to further individualize the learning opportunities. This individualization offers numerous advantages. The slower children can avoid the frustration of having to proceed to new ideas before the previously presented ideas are understood, and the more capable children are spared the tedium of completing long lists of drill problems involving ideas they already understand.

The teacher might look for creative ways to meet individual differences in the ability to learn mathematics. For example, the slower child might profit from additional drill on certain facts and skills. Drill tapes or audio cassettes made by the teacher might provide a novel way to present the necessary practice. Duplicator masters and commercial workbooks are also available to provide extra work for those who need it. For other situations, an appropriate programmed instructional unit might serve the needs of the slower child. Single-concept film loops, which the child can play again and again, often are useful in helping him grasp an important concept. Appropriately conceived tutorial situations, in which classmates who understand the ideas work with children who do not, can be quite effective. Simple investigations utilizing physical objects which clarify more abstract ideas can also provide remedial work for certain children.

The teacher must also be concerned with those children who understand the basic ideas of the lesson and who can quickly work all the utilization exercises provided. These children can often become quite interested in activity cards which contain "open-ended" questions, such as the card shown below. (You are encouraged to try the suggested activity yourself.)

ACTIVITY CARD 10

In how many different ways can you measure yourself?



Make as many different measurements of **you** as you can and make a chart to show the information. Here are just a few suggestions:

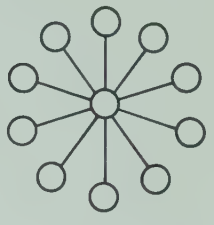
Pulse	Length of step
Height	Number of calories used
Weight	Area of bottom of foot
Arm span	Distance you can jump

Activities such as these give the child an opportunity to make his own decisions about which ideas he uses from the lesson and how he uses them.

Puzzles or riddles can also provide a useful extension of ideas for your children. Consider, for example, those shown in Figure 6.

Think

Draw a figure like this one on your paper. Place the numbers 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 in the circles so that the sum along any line is 21.



Think

I can be found halfway between Twenty-seven and seventeen.

WHO AM I?

27 ? 17

Figure 6

Conceptually fertile games can also provide valuable experiences to supplement the basic lesson. For example, the game "Sleuth" (3M Company) is fun for children and gives them valuable experience in classification and drawing logical inferences.

The methods suggested for extending the ideas for slower children are often suitable for use in certain situations with more capable children. Similarly, the more exciting modes of extension suggested for faster children can often be quite stimulating and valuable if used appropriately for the slower children.

It is to be hoped that the teacher will share a sense of excitement in providing extra stimulation to broaden the mathematical perspective of the children. Perhaps, he will also see that much of the extension activity can truly be fun for children while at the same time inspiring new interest and involvement in mathematical ideas. In using this suggested lesson outline, if the teacher chooses to maximize the investigation phase while deemphasizing the others, it might justly be said he is using the laboratory approach. On the other hand, should he maximize the discussion phase, he may find increased options for a guided discovery approach to mathematics learning. Also, it is possible that maximization of the utilization phase accompanied by appropriate student materials would allow the teacher to embark on a course of individually prescribed instruction.

EXERCISE SET 4

1. Find an example of an exercise set in which a learning sequence occurs in an *Investigating School Mathematics* text.

2. Choose a mathematics topic and write a set of exercises which might lead the student to discovery of a central idea.
3. Can you find a lesson in an *Investigating School Mathematics* text in which the "Using the Ideas" section provides for varying degrees of student ability.
4. Choose a learning experience appropriate for your children and list some possible specific activities for use in the extension phase of this learning experience.
5. Describe your views concerning the role of drill for slow, average, and bright children.
6. Select and play a game that could be used to extend a lesson with children.
7. In Exercise Set 2, you investigated an idea of mathematics. In Exercise Set 3, you had an opportunity to discuss this idea. The exercises below enable you to use the idea you learned, and suggest an extension of the idea.

Complete each exercise.

A List five prime numbers of the "$4n + 1$" type that are greater than 50.

B List five prime numbers of the "$4n - 1$" type that are greater than 50.

C 997 is the largest prime number less than 1000. Is it a "$4n + 1$" or a "$4n - 1$" prime?

D Suppose you used a continuation of the array of numbers shown below and circled all the prime numbers. What does this suggest about another way to classify the primes?

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
...					

III. A Focus on Specific Types of Learning

In considering the more specific aspects of mathematics learning it is helpful to categorize the general types of things children learn. A simplified categorization is given below.

Concepts
Skills
Generalizations
Facts
Attitudes

It is important to recognize that each of these types of learnings has unique characteristics. Because of this, the approaches and children's activities chosen to promote these learnings may often be quite different. In the sections that follow, we will consider each of these types of learning and suggest possible approaches and activities.

Concepts

Suppose that a child is having difficulty and comes to the teacher for assistance. When the teacher asks what the difficulty is, the child points to the multiplication 9×6 and says, "I can't do this because we haven't had it yet." This reflects a common attitude among children who have been in school for a few years. Somehow they learn to feel that they are incapable of figuring out anything new in mathematics. Literally, they can do nothing that they "haven't had yet."

If this child had confidence in his ability to "figure something out" and had a clear understanding of the *concept* of multiplication, he could have found the product by perhaps adding sixes, using sets, or making jumps on the number line. Another child who knew no division "facts" but who had a clear concept of division (as illustrated below) could use his knowledge of multiplication to find any of the basic quotients desired.

$$\begin{array}{lll} \mathbf{P} & \mathbf{F} & \mathbf{F} \\ 72 \div 8 = n & \leftarrow & \text{You find this quotient,} \\ & & \mathbf{F} \quad \mathbf{F} \quad \mathbf{P} \\ \text{when you find this factor.} & \rightarrow & n \times 8 = 72 \end{array}$$

A *concept,* then, may be thought of as an idea which, when properly understood, will help the child to solve problems he "hasn't had yet," to figure something out for himself. As another example, consider the concept of prime number. Once a child understands that a prime number is a whole number with exactly two factors, he has the power, providing he understands how to find the factors of a number, to seek out and list those numbers that are prime. Of course, the task of deciding whether or not a given number is prime may be quite laborious, but understanding the concept does give the child the power to succeed.

To look more carefully at what concepts are and how they are taught, consider a model in which concept learning is relatively easy, namely, that of a set of attribute pieces. Suppose there are pieces of four different shapes (triangles, squares, circles, and rectangles), of three different colors (red, blue, and yellow), and of two different sizes (large and small), as pictured in Figure 7. (In the figures the colors red, blue, and yellow are denoted by the initials R, B, and Y.)

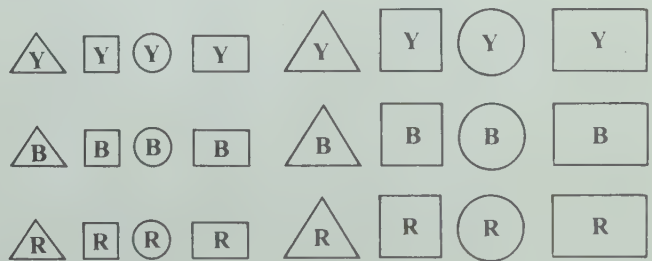


Figure 7

Now consider the Concept Card in Figure 8.

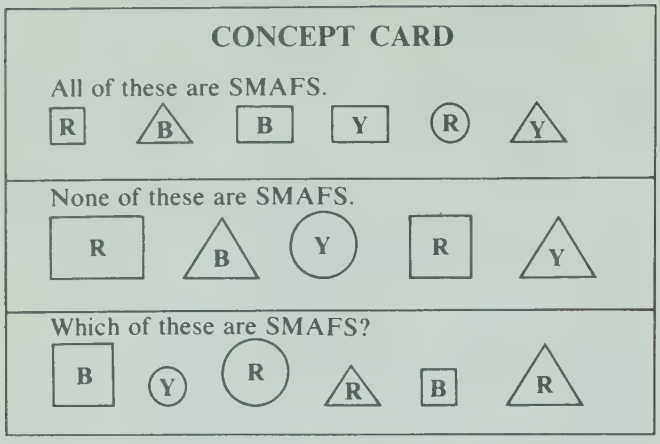


Figure 8

If you study the preceding Concept Card carefully, you will develop the simple concept of a SMAF. Notice that the key means used to teach this concept is by examples, along with *non-examples*. Both examples and non-examples play important roles in teaching many concepts in mathematics. The concept of a triangle may be taught to young children by using the Concept Card in Figure 9.

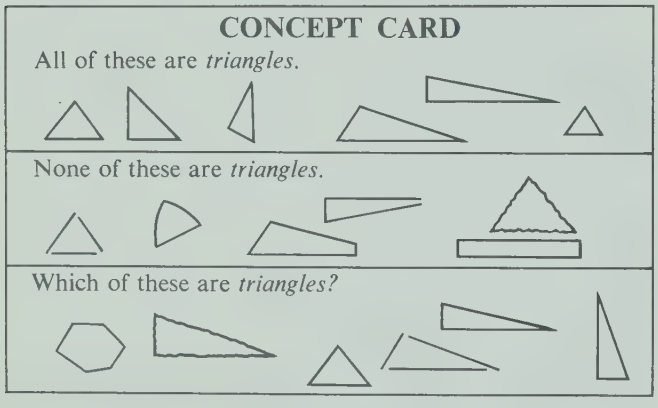


Figure 9

Clearly, the child would need further experiences in order to develop fully the concept of triangle, but the beginnings are embodied in the Concept Card shown in Figure 9.

One of the important ideas to remember when considering concepts is that concepts, unlike some other things that children learn, are developed over a period of time. Simple concepts may be developed very quickly, but other, more complicated concepts must be germinated when the child is very young and broadened through a spiralling return to the concept at various stages throughout the child's development. Many concepts are not fully developed until the child becomes an adult and encounters the idea in a variety of situations. For example, the concept of a fraction or fractional number may be introduced in grade 1 or grade 2, but a full understanding of this concept may not come until many years later. The child may acquire only an embryonic idea of a concept the first time it is presented, so it is important for the teacher to recognize the true nature of concepts and be willing to return often to the idea and carefully nurture its growth within the child. If he does not expect complete mastery after the initial presentation, he will spare himself considerable frustration when he recognizes later that the child needs further development of the basic idea.

Another key feature of concept learning suggested by the experiments of Piaget and supported and extended by the theories developed by Z. P. Dienes concerns the role of physical manipulative materials in young children's concept learning. In general, the implication of these authors' works is that it is through child involvement with physical environment that a firm basis for the development of more abstract concepts is laid. In fact, it is suggested that concept learning is facilitated by exposing children to as many different physical situations which embody the concept as possible.

It should be recognized that there are different levels of concept development and different types of concepts within these levels. For example, in the very earliest stages of mathematical learning, most concept learning involves the *concept of physical objects* such as balls, blocks, and circular or triangular objects. Very soon, the concept of certain *relations between objects* is developed: above, below, taller, shorter, larger, wider, longer, behind, and so on. A subsequent stage involves the concept of a *set of objects* such as a set of golf clubs, a set of dishes, a box of crayons, a set of blocks, a collection of stamps, or the children in a classroom. A slightly higher level of concept learning involves *relations between sets of objects:* equivalent, equal, has more than, has less than, and so on. It is at this stage that the important concept of *number* arises. For, in a sense, the concept of number involves a consideration of a set of equivalent sets. At a higher level of abstraction, the concept of certain *relations between numbers* (is less than, is greater than, is equal to, and so on) is developed. Ascending the ladder of abstraction, another level of development might involve the concept of *sets of numbers,* such as odds, evens, primes, composites, and perfects.

Clearly, the realm of concepts is vast, and the elementary teacher need not concern himself directly with many of the types of higher-level concepts. He must recognize, however, that the beginning stages in the development of many important concepts occur in the elementary school and that, through utilization of a variety of manipulative materials and appropriate strategies, he can do much to help the children learn concepts appropriate for their level.

EXERCISE SET 5

1. Use the attribute pieces shown in Figure 13. Invent a concept, name it, and make an appropriate concept card for it.

2. Choose at least two *Investigating School Mathematics* concepts from the list given below and develop concept cards which illustrate the use of examples and non-examples to teach the concepts you have chosen.
   - **A** quadrilateral
   - **B** simple closed curve
   - **C** odd number
   - **D** greater than (the relation)
   - **E** right triangle
   - **F** is congruent to (the relation)
   - **G** lowest-terms fraction
   - **H** parallelogram
   - **I** diagonal of a polygon
   - **J** parallel lines
   - **K** one half
   - **L** isosceles triangle
   - **M** equivalent sets
   - **N** symmetrical figure
3. Answer the questions on the sets of Creature cards from the set of attribute materials published by the Webster Division of McGraw-Hill Book Company (if available).
4. Choose an unusual concept of your own invention and make a concept card from which a person might discover your concept.
5. The investigation in Figure 1 was used to teach the concept of congruent segments. Make a card to teach this concept using examples and non-examples.
6. Complete "Learning a Concept" on pages I-18 and I-19; then answer the following questions.
   - **A** What are some examples of the concept you learned?
   - **B** Give some characteristics of the concept you learned.
   - **C** What were your feelings about the lesson? How could the lesson be improved to make the learning of the concept easier?

Skills

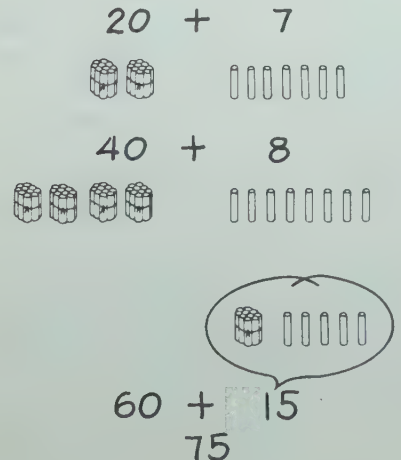
Broadly speaking, there are several types of skills that children develop in the elementary school. Hopefully, many children will develop a skill in estimating distance, weight, capacity, and time. Some teachers may wish to help children develop skill in drawing geometric figures. Some teachers set goals for upper-grade children which include developing skills in reasoning and even in "proof" of simple ideas. In elementary mathematics the most fundamental skill, by far, is that of computation with whole and rational numbers. It is these specific computational skills involving addition, subtraction, multiplication, and division and the processes related to these operations with which we are particularly concerned in the discussion that follows.

Two types of skills, power skills and speed skills, are available for completing each arithmetic process. A *power skill* is any effective way to find an answer. A *speed skill* is the most efficient way to find an answer. A power skill is a process through which a given problem is attacked by means of some technique which, though possibly quite inefficient, can produce a correct solution. This power skill may involve a long, tedious process, one which may be totally unrelated to the most efficient method for arriving at the solution. On the other hand, when a speed skill is employed, the problem is attacked with the most efficient technique available, and the problem is solved relatively quickly, usually in a mechanical fashion.

For example, suppose a child wants to find the sum of 27 and 48. If he simply starts at 48 and counts on 27 more, he is using a power skill. If, however, he finds the answer by using the usual algorithm for addition, then a speed skill is being employed.

Two additional points are worth noting about the previous example. First, in order to utilize the power

POWER SKILL B
– Bundles and Grouping



POWER SKILL C
– Expanded Notation

$$\begin{array}{r} 27 \\ +48 \\ \hline \end{array} \qquad \begin{array}{r} 20 + 7 \\ 40 + 8 \\ \hline 60 + 15 \\ 75 \end{array}$$

POWER SKILL D
– Addition Algorithm with Intermediate Step

$$\begin{array}{r} 27 \\ +48 \\ \hline 15 \\ 60 \\ \hline 75 \end{array}$$

Figure 10

skill, the child needed a clear concept of addition as it relates to counting. Thus, a power skill relies on a previously learned concept. As the child uses the concept in a power-skill situation, he gains new confidence in his ability to do something he "hasn't had yet." Secondly, the teacher should observe the evolution from power to speed. In finding the sum of 27 and 48, the initial power skill involved a basic concept of addition and the counting process. In practice, the child may continue the evolutionary trek from power to speed by next utilizing power skills B, C, and D as shown in figure 10.

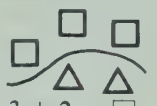
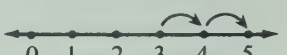
Note that each of these power skills represents a small step toward the ultimate, more efficient speed skill. When considering this process of evolution, it should also be noted that the earlier stages in a power-skill sequence often involve manipulative materials with subsequent power skills exhibiting a transition from the concrete to the more abstract. This physical beginning, which utilizes bundles and grouping, is illustrated as Power Skill B in Figure 10.

The use of power skill is available to all children. The slower child may well attempt the problem by the only means he knows, one which may often be quite laborious. For example, in finding the quotient $5863 \div 72$, the slower child might subtract 1 seventy-two at a time until he has reduced the dividend to some number less than seventy-two. The more able and creative child might tire of this method and attempt to subtract some multiple of seventy-two, such as 10 seventy-twos. Since each child is working on his own for a period of time, the development of power skill is extremely helpful in working with individual differences.

One decision that the teacher must make in relation to each child is the extent to which he should be encouraged to develop an efficient speed skill for a given algorithm. Obviously, skills are important and should be taught in elementary mathematics, yet it is the good judgment of the teacher that plays the crucial role in guiding a given child from power to speed. For certain processes, children should probably never be forced to attain a speed skill, but should be allowed to operate at the power-skill level. Other children should be directed toward the speed skill as quickly as possible in order that they may proceed to more interesting aspects of mathematics. In rare instances, a child might profit from an initial consideration of a speed skill with no previous power-skill development of a given process. The emphasis on the role of conceptual power in the performance of a skill is a key feature of the so-called "new" mathematics. It is quite probable that we cannot predict the future mathematical needs of children in our classes today, but we can help them develop the confidence, even in the area of learning skills, to utilize concepts previously learned to discover some of the basic processes for themselves.

EXERCISE SET 6

1. Write *power* or *speed* depending on the type of skill you think is being employed.

Specific Skill	Example
A Using sets to find sums	 $3 + 2 = \square$
B Using number line to find sums	 0 1 2 3 4 5 $3 + 2 = \square$
C Counting fingers to find sums	 $3 + 2 = \square$
D Memorizing that $3 + 2 = 5$	Think: 3, 2 → 5 $3 + 2 = 5$
E Thinking about "take away" to find differences	 $5 - 2 = \square$
F Using the inverse relation (missing addend) to find differences	Think: $? + 2 = 5$ $5 - 2 = \square$
G Memorizing that $5 - 2 = 3$	Think: 5, 2 → 3 $5 - 2 = 3$
H Using sets to find products	 $3 \times 6 = \square$
I Using the number line to find products	 0 5 10 15 $3 \times 5 = \square$
J Using logic (basic principles) to find products	Since $5 \times 5 = 25$, $6 \times 5 = \square$ or Since $3 \times 5 = 15$, $6 \times 5 = \square$

2. Four different power skills are shown for finding $91 \div 7$. These skills would lead up to finding this quotient by "ordinary short division."

$$7\overline{)91}\quad 13$$

In what order should these be presented?

A
$$\begin{array}{r} 7\overline{)91} \\ \underline{70} \\ 21 \\ \underline{21} \\ 0 \\ \hline 13 \end{array}$$

B Subtract 1 seven at a time.

$$\begin{array}{r} 91 \\ -\ 7 \\ \hline 84 \\ -\ 7 \\ \hline \vdots \end{array}$$

C
$$\begin{array}{r} 7\overline{)91} \\ \underline{70} \\ 21 \\ \underline{21} \end{array}$$

D Group 91 objects into sets of 7.

3. Complete the "Learning a Skill" lesson on pages I-19 and I-20; then do these exercises.
   - **A** Discuss the skill you learned and the way you learned in terms of power skills and speed skills.
   - **B** What part of the lesson helped you evolve a speed skill?
   - **C** What were your feelings about the lesson? How could it be improved?

Generalizations

Imagine that one of your students is engaged in an investigation in which he was asked to cut out a large quadrilateral and draw colored lines connecting the midpoints of each side of the quadrilateral. The question stimulating the investigation was, "Can you make an odd-shaped quadrilateral so that when you connect the midpoints you do not form a parallelogram?" As a result of this investigation and the subsequent discussion of his findings, the child was led to form a generalization: "The segments connecting the midpoints of any quadrilateral form a parallelogram."

In another lesson, a child might be responding to an investigation question which asked: "If you cut off the corners of a triangle and place the tips at the centre of a circle, what part of the circle can you cover? Can you find a triangle for which this is not true?"

As the child completes the investigation and engages in the discussion which follows, he forms this tentative, unproved *generalization:* "If a compass is used to draw arcs on the corners of any triangle and these corners are cut off along the arcs, then these corners will cover exactly one half of a circle drawn with the same compass opening." This tentative generalization, of course, is the forerunner of the familiar generalization that the sum of the degree measures of the three angles of any triangle is 180.

A generalization provides the economy of moving from consideration of isolated, specific cases to a general statement which holds true for a complete set of numbers or geometric figures. For example, the generalizations stated above deal with the set of all quadrilaterals and the set of all triangles. The regular occurrence of the word "any" in the generalization statements implies that the observation is true for *every* such geometric figure.

The key to teaching a generalization effectively is to provide children with appropriately chosen examples (or instances) which lead them to the generalization. An approach often used by teachers to help children learn generalizations is that of *guided discovery.* In this approach the teacher uses carefully sequenced questions and carefully chosen examples to focus the child's thought on the generalization to be discovered.

It is instructive for children in the upper elementary grades to have experiences in forming generalizations which seem obvious from a set of examples, but which, in fact, do not hold true. For example, consider the equations below.

$$\boxed{1} \times \boxed{1} - \boxed{1} + 11 = 11$$
$$\boxed{2} \times \boxed{2} - \boxed{2} + 11 = 13$$
$$\square \times \square - \square + 11 = ?$$

Figure 11

If 1 is written in the box and the operations are performed, the result is 11, which is a prime number. If 2 is written in the box, the result is 13, also a prime number. Upper-grade children are likely to conjecture that the sum is always a prime number. When they try 3, the sum is 17, also a prime. Similarly, the child finds that the numbers 4, 5, 6, 7, 8, 9, and 10, when written in the box produce a prime number. A child accustomed to forming generalizations from even fewer examples than this will likely conclude that this formula will always produce a prime number. It is instructive to note that when the next number, 11, is written in the box, the result is 121, which, being divisible by 11, is not a prime. This example illustrates the important idea that, even though the generalizations the child might make seem quite plausible and are most often true, it is only by means of a mathematical proof of a generalization that one can be completely sure that it is correct. These proofs, of course, are often not accessible to elementary school children. Thus, a healthy attitude might be characterized by references to generalizations which include phrases such as, "appears to be true," "is probably true," or "could most likely be proven."

Often a search for a generalization is initiated by a question such as, "Do you see any patterns?" For example, several simple generalizations might be formulated about the multiplication table in Figure 12. One child might observe that every number on the main diagonal of the table is a square number. Another student might observe that for every number on one side of this main diagonal, such as 10, there is a matching number symmetrically placed on the other side of the main diagonal.

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

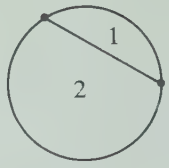
Figure 12

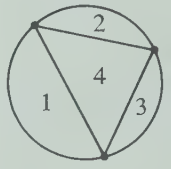
This last generalization is the table counterpart of the commutative principle for multiplication; that is, in the case of 12, $6 \times 2 = 2 \times 6$ or $4 \times 3 = 3 \times 4$. Another generalization that might be reached by careful consideration of the table is that the only primes in the table occur in the one-row or one-column of that table. Still another interesting generalization suggested by the table is that the sum of any number in the two-row and a number below it in the five-row will equal the number below these numbers in the seven-row. Of course, there are many other generalizations ranging from the very simple to the more complex that could be made about this multiplication table.

Perhaps the illustrations above will suggest that the mathematics available to the elementary school child is replete with possibilities for discovery of generalizations. The teacher's task is to create a learning environment in the classroom, not only in terms of physical materials and situations, but in terms of attitude toward learning and toward children, which provides opportunities for discoveries of generalizations and an atmosphere in which it is rewarding to make these discoveries. The teacher should be ever aware of the possibility that the habit of seeking generalizations may well be one of the most valuable things the child learns from his experiences in mathematics.

EXERCISE SET 7

1. Choose a text from the *Investigating School Mathematics* series and list some generalizations which the students who study this text might discover.
2. Investigate the Madison Project shoe boxes and complete the activities for at least two boxes.
3. The illustrations and the table which follow show that if you connect two points on a circle, you divide the interior of the circle into two regions; if you connect three points on a circle, you divide its interior into four regions; if you connect four points on a circle, you divide its interior into eight regions. Note that the points chosen should not be evenly spaced on the circle.







Number of points on a circle	Number of regions formed inside circle
2	2
3	4
4	8
5	
6	

   - **A** Fill in the table to show how many regions are formed if five points on a circle are connected.
   - **B** Form a generalization about the right-hand column of the table.
   - **C** Test your generalization by finding out how many regions are formed inside when six points on a circle are connected.
4. Devise an investigation which might enable a student to discover this generalization: "The sum of the degree measures of the angles of a quadrilateral is 360."
5. Write some questions you would ask and show some examples you would use in guiding a child to discover one of the following generalizations.
   - **A** The commutative principle for multiplication
   - **B** The volume of a "box" is found by multiplying length times width times height.
   - **C** In measuring length, the shorter the unit, the greater the measure.
   - **D** Any angle inscribed in a semicircle is a right angle.
   - **E** Every even number ends in 0, 2, 4, 6, or 8.
6. Complete the "Learning a Generalization" lesson on page I-20; then answer the following questions.
   - **A** What generalization did you learn from the lesson?
   - **B** How many specific examples did you consider before you understood the generalization?
   - **C** In what way did you use the generalization after you discovered it?

Facts

In elementary mathematics, there are certain bits of information that are used so frequently that it is

beneficial for the child to be able to recall them quickly when they are needed. These items are ordinarily called *facts*. There are three main types of facts that are of major concern. The first type of fact is one which evolves from a concept. It might be an example of a specific concept ("Two is a prime number," "25 is a square number," "A parallelogram is a quadrilateral"), or it might be a characteristic of a specific concept, possibly even a part of the definition for the concept ("An isosceles triangle has two congruent sides," "An even number is a number divisible by two," "A pentagon has five sides"). Examples of, or characteristics of, concepts are not always considered as facts; only if such an example or characteristic is deemed important enough to be remembered for immediate recall, is it considered to be a fact and committed to memory.

A second type of fact is a fact derived from a generalization; that is, if a generalization is simple, or deemed important enough to remember for immediate recall, it might often be considered a fact. For example: "The sum of the squares of the lengths of the legs of a right triangle is equal to the square of the length of the hypotenuse of the right triangle"; or "The length of the segment joining the midpoints of two sides of a triangle is one half the length of the third side." Each of these statements might be considered facts since they are sometimes useful for immediate recall. A third type of fact—one that is given a great deal of attention in the elementary school mathematics program—is the type of fact derived from a power skill. For example, the child may have utilized a sequence of power skills for finding sums such as $4 + 3$. He may have used sets of counters, centimetre strips, jumps on the number line, or reasoning from facts such as $3 + 3 = 6$. These power skills, based on certain important concepts, provided the evolutionary progression toward the final speed skill used in finding sums. In this particular case, however, the speed skill used is simply that of memorizing the sum. Whenever the speed-skill stage involves memorization, the particular learning which was classified as a skill or a process during the power-skill stage is reclassified as a fact. The basic addition and multiplication facts fall into this category, and they are given major attention in the elementary school. It is these facts to which primary attention will be given in this section.

A first important point to be made in discussing the teaching of facts is that extensive power-skill work preceding the memorization stage can pay valuable dividends. The broad base of understanding provided by the power-skill work removes the aura of magic from this aspect of mathematics and not only makes the task of memorization of the facts easier, but helps the child view it as a "reasonable thing to do." Figure 13, for example, shows some of the power skills that might be utilized in the initial development of procedures for finding products. Careful development using some or all of these power skills can give the child a basic feeling for a procedure by which products may be found.

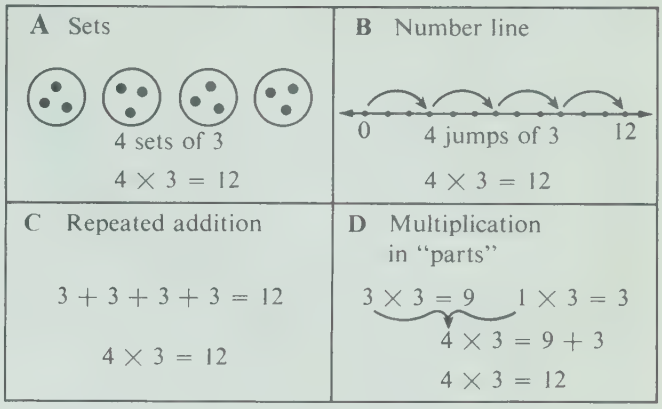


Figure 13

The teacher must use good judgment in deciding when a given child should move from this power-skill stage to memorization of the facts. The appropriate time could vary extensively depending upon the ability and experiences of the child. If the power-skill work is started early in the elementary grades, the child will have ample time to reap the benefits of this basic experience with materials and concepts before the transition to speed skill is made.

When the time has come to memorize the facts, it is important for the child to have a clear idea of the nature of this goal and the reasons it is appropriate. The teacher should even take time to help the child see the very clear difference between "figuring out the fact" and "memorizing the fact." Hopefully, he could help the child develop a feeling for situations in which the facts will be used and in which immediate recall would be quite valuable and time-saving for the child.

After the addition or multiplication facts to be memorized have been placed in perspective, the teacher should seek interesting situations and creative ways in which to practice recalling the facts. For example, the children might make their own flash cards and use a timer to see how long it takes them to give these facts. If desired, two children could work together and see which of them could go through the flash cards most quickly. Another game utilizes a pair of homemade colored dice and an empty multiplication table (Figure 14) for each child. As the game

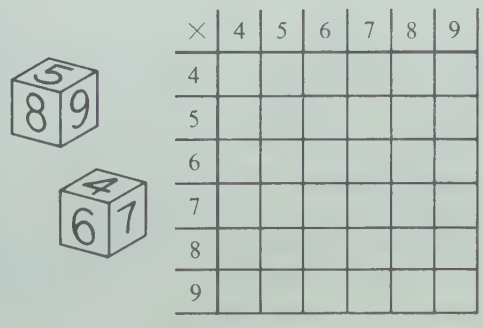


Figure 14

proceeds, a child rolls his dice and writes the product of the numbers on the dice in the appropriate space on his multiplication table. His partner then does the same thing when it is his turn. If a child arrives at an incorrect product or writes the product in the wrong space in the table, he is penalized by missing a turn. The object of the game is to see who can complete the table first. Various modifications of this game are possible, including one in which each child works independently and keeps a tally of the number of times he rolls the dice and also keeps track of the time it takes. The basic objective, of course, is to provide an interesting situation in which the child is motivated to recall multiplication facts rapidly.

Some children may need to spend considerable time in the power-skill stage before they begin to memorize. If there are children who have attempted to memorize the facts and find the job more difficult than anticipated, the teacher may want to consider allowing them to prepare a fact card on which they write the facts that they still do not know. Perhaps it would be realistic and beneficial to let some children use this fact card during the year whenever they desire, thus relieving the tension that could result from difficulties they encounter in memorizing the facts at one specific time. As the school year progresses, the teacher may want to suggest from time to time that a particular child concentrate on one of the troublesome facts and attempt to memorize it so that he can remove it from his fact card. The accomplishment of this goal, of course, would merit recognition and reward. After one fact is removed, the child might start working on removing another fact. The ultimate goal would be to remove all of the facts by the end of the year. Teachers who are interested in helping children learn mathematics in a comfortable way may find that a more realistic, less pressured approach to learning facts may enable the child to find greater enjoyment and success in his mathematical experience.

EXERCISE SET 8

1. Invent a game that could be used to help children practice recalling addition or multiplication facts.
2. Find a commercially produced game that is designed to help children practice recalling facts.
3. Complete the "Learning Some Facts" lesson on pages I-20 and I-21 of this text; then answer the following questions.
   - **A** How many of the facts did you know?
   - **B** What techniques did you use to help you memorize the remaining facts? Did you find this lesson difficult?
   - **C** Can you imagine some of the difficulties your children might have in learning facts?
   - **D** Did you find any mnemonic devices which were helpful in remembering the facts?

ATTITUDES

In his poem "Arithmetic," Carl Sandburg wrote:

Arithmetic is numbers you squeeze from your head
to your hand to your pencil to your paper
till you get the answer.
Arithmetic is where the answer is right
and everything is nice
and you can look out of the window
and see the blue sky—
or the answer is wrong
and you have to start all over and try again
and see how it comes out this time.

. . . . .

Arithmetic is where you have to multiply—
and you carry the multiplication table in your head
and hope you won't lose it.*

The attitude toward mathematics, school, one's ability, and learning in general that one senses on reading this part of Sandburg's poem is surely typical of many children in classrooms today. Perhaps, it was a feeling similar to this that caused Huckleberry Finn to say:

> I had been to school 'most all the time and could spell and read and write just a little and could say the multiplication table up to six times seven is thirty-five, and I don't reckon I could get any further if I was to live forever. I don't take no stock in math, anyway.

There are many different kinds of attitudes exhibited by children who have been exposed to classroom mathematical experiences in different parts of the world. There are, of course, the more general attitudes that a child has toward his teacher, toward his school, toward his fellow students, and toward the process of education. All too often the child's attitude toward education in general is that suggested by Charles Schulz in this *Peanuts* cartoon.



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*From *Complete Poems*, copyright, 1950, by Carl Sandburg. Reprinted by permission of Harcourt Brace Jovanovich, Inc.

Two of the attitudes to be considered here, however, are the child's attitude toward mathematics and the child's attitude toward himself as he relates to mathematics. It has been said that the mathematical experiences of a child before the age of 11, and the responses he has been encouraged to make to those experiences, largely determine his potential mathematical development. If this is so, then a child's attitude toward mathematics and his feelings about how he relates to mathematics are extremely important considerations for the classroom teacher.

A moment's reflection on the number of people who are willing to say that they hate mathematics and on the multitude of others who seem to harbor a fear regarding their inability to cope with ideas of mathematics leads the teacher to realize that he does indeed teach attitudes, whether he tries to or not. Clearly, the teacher who conducts a classroom in which children's achievements are evaluated almost exclusively on the basis of how many right answers they can come up with must surely engender attitudes in children which differ greatly from those engendered by the sensitive teacher who recognizes the child's need to think his own thoughts and to become involved in an exciting exploration of ideas that interest him. Or, consider the difference between the teacher who teaches only speed skills and facts and the teacher who recognizes the central importance of concepts and generalizations, as well as the facts and skills. The child exposed to the first teacher must surely have a feeling toward mathematics, and his ability to interact with it, that is far different from that of the child who learned with the second teacher.

If what happens in the classroom is of such importance in developing attitudes within the child, then the teacher may want to reevaluate his approaches to instruction by reconsidering certain fundamental questions. What subject matter and methods most effectively instill within the child the feeling that mathematics is interesting, fun, and a source of adventure? Will these means provide an opportunity for the child to exercise his freedom of choice and to make decisions about what he does with mathematics? Aldous Huxley said: "A child is a genius until the age of ten." Could it be that our classroom approaches squelch this genius? Can we select mathematical experiences and materials that enable the children to experience success and thus maintain that sense of worthiness and prestige with peers that is of such importance? Can we structure these experiences in such a way that the child maintains within this atmosphere of freedom a sense of security and safety, thus avoiding the fear that can erode his ability to approach mathematical situations with confidence? Can we help children see the usefulness and importance of mathematics without boring them?

Clearly, the questions just raised are difficult to answer and specific techniques for developing healthy attitudes are hard to come by. But even though prescriptions for developing attitudes are scarce, many of the ideas about teaching suggested in earlier sections of this text can provide assistance for the teacher. The investigation, for example, provides the child with an opportunity to make independent decisions and to interact with mathematics and materials and encourages him to take responsibility for his own learning. As difficult as it may seem at times, a child's acceptance of responsibility for his own learning inculcates an attitude that is ultimately invaluable. Also, the manipulative materials or activities that are made available to the child in the investigation situation provide an interaction with the physical world that is often extremely valuable in making mathematics real to a student. Unless a child is ready for more abstract thinking, he cannot be induced to sense the adventure in mathematics without a physical environment to explore. Opportunities for attitude development are implicit not only in the investigation phase of a lesson but in the discussion as well. If a teacher can convince the child that his ideas are important, then the child finds himself in a situation, albeit a mathematical one, in which *he* feels important. His prestige with his peers increases and he feels successful. Exercises in the utilization phase of a lesson that begin simply and gradually increase in difficulty can also help the child feel that he can do mathematics on his own; and, of course, carefully selected extension activities can provide the child with a variety of opportunities to experience the fun of mathematics.

Not only do the phases of the learning experience provide unique opportunities of attitude development, but the particular types of learnings involved within these phases also have their effect. The teaching of concepts and generalizations provides the child with a feeling of power regarding mathematics, for when he experiences the thrill of discovering a concept or a generalization, or when he uses these to solve a problem, he is also developing a useful and wholesome attitude toward mathematics learning. He is developing a habit of reacting to a mathematical situation which will be invaluable when he later encounters mathematical situations possibly undreamed of today. Also, careful teaching of skills and facts can provide the child with that basic sense of security that comes simply from being able to do something or to remember something.



Figure 16

Regarding the child's level of confidence in his ability to cope with mathematical problems, one of the child's paramount needs is to experience success, and as mentioned previously, having entertaining experiences with mathematics might decrease the fear that can erode his confidence. To provide these experiences, the teacher might create in the classroom a "Fun with Mathematics" centre (see Figure 16) that contains mazes, puzzles, design materials, and so on. This centre represents an extra effort to encourage the child to successfully play with mathematics. Some of the materials that might be in such a centre are as follows: the soma cube, the tangram pieces, 2-cm cubes, materials for curve stitching, a kaleidoscope, pattern blocks, Cuisenaire rods, multi-base arithmetic blocks, geoboards, a wide variety of counters, attribute blocks, scales and balances, timers, calendars, measuring tapes and rulers, yarn and string, an assortment of boxes and cans, magazines and catalogues, mirrors, dice, play money, graph paper, assorted plane and solid shapes, abacus, pegboard, compass, mathematical balance, etc.

Perhaps, as you consider the attitudes more carefully and reevaluate the effects of your approaches to instruction, you will find other ways to help children develop a healthy attitude toward mathematics and an enthusiasm for the enjoyment it can offer. Each day as the teacher enters the classroom with plans for a learning experience, he might well ask himself: "What effect will *this* have on the attitudes of the students in my classes?"

EXERCISE SET 9

1. Select a text from the *Investigating School Mathematics* series and find at least five activities which could contribute to the child's development of a positive attitude toward mathematics.
2. Explain how you think some of the other types of learning might also contribute to better child attitude toward learning in general and mathematics specifically.
3. Complete the "Learning an Attitude" lesson on page I-21 of this text; then answer the following questions.
   - **A** Was the lesson fun?
   - **B** How did you feel when you had finished the lesson?
   - **C** Did the lesson change any of your ideas about mathematics?

IV. Some Learning Experiences for the Teacher

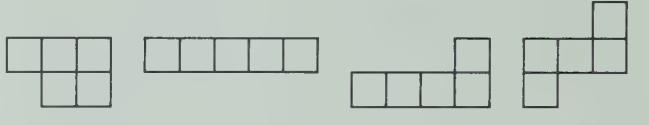
In Section II you were introduced to an outline for a learning experience which involved preparation, investigation, discussion, utilization, and extension. In Section III the types of things children learn—concepts, skills, generalizations, facts, and attitudes—were categorized. In this section, we combine these ideas and use them in presenting five learning experiences designed especially for the teacher. That is, in order to gain a first-hand view of lessons which develop these types

Lesson 1. Learning a Concept

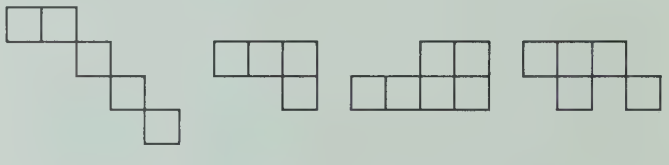
What is a pentominoe?

Investigating the Ideas

Each of these is a **pentominoe.**



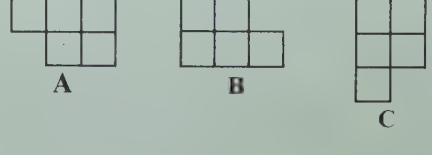
None of these is a **pentominoe.**



How many more pentominoes can you find and show on graph paper?

Discussing the Ideas

1. How many pentominoes did you find?
2. Can you give some characteristics of a pentominoe?
3. How would you "broadly classify" a pentominoe?
4. Can you define a pentominoe?
5. Are the pentominoes in Figures A, B, and C the same?



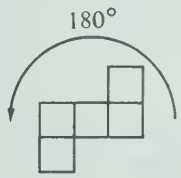
6. How could you convince someone that you have found all possible pentominoes?

of learning, the teacher will have experiences with each of these in the five lessons; and, in order to become more familiar with the suggested structure for a learning experience, each of these five lessons will involve an investigation, a discussion, a utilization, and an extension of the ideas.

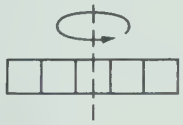
It might be valuable for the teacher, after he has become involved in each of these lessons and has completed the activities, to rethink and discuss his reactions to the various phases of the lesson structure and to the various types of learnings involved. In this way, he might gain a new insight into the way the children in his classes might react to these kinds of situations.

Using the Ideas

1. Which of the pentominoes can be folded to form a box with the "lid missing"?
2. Some pentominoes can be rotated about a point 180° and returned to their starting position. These pentominoes are said to have 180° rotational symmetry. Which pentominoes have 180° rotational symmetry?



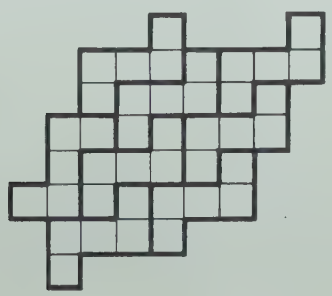
3. Some pentominoes can be flipped about a line and returned to their starting position. Such pentominoes are said to have reflectional symmetry. Which pentominoes have reflectional symmetry?



4. What do you think a hexominoe would be? How many hexominoes can you find?

Extension

Some pentominoes can be used to tessellate (fill without overlapping) the plane, as shown below. Can you find at least two more pentominoes that can be used to tessellate the plane? Show the tessellations on graph paper.

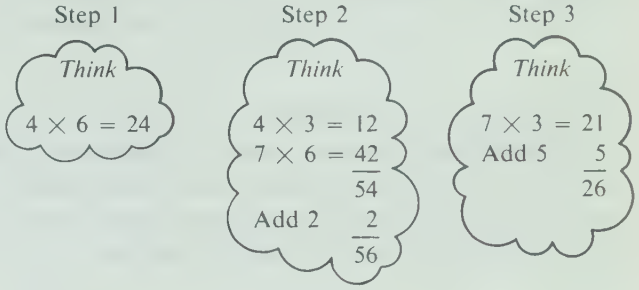


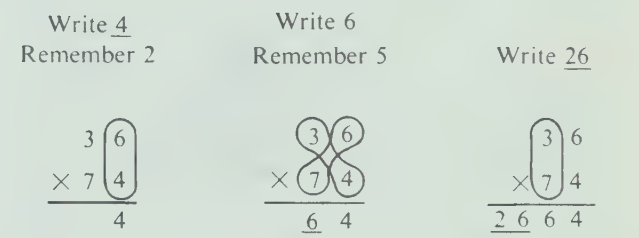
Lesson 2. Learning a Skill

Can you find the product of two 2-digit numbers "in your head"?

Investigating the Ideas

Follow these steps for writing the *answer only* for 74×36.





Can you use this method to write answers only for the products below? Check your answer using the "long" method.

53	37	45	67
×48	×62	×23	×32

Discussing the Ideas

1. Explain this statement: In Step 1 you are finding the number of ones.
2. In Step 2 you are finding the number of ?.
3. The 2 you remembered is really 2 ?.
4. Explain what you are finding in Step 3.

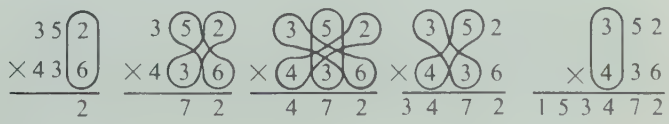
Using the Ideas

Write answers only for each product.

1. 28	**2.** 46	**3.** 37	**4.** 82	**5.** 53
×42	×33	×42	×56	×34
6. 64	**7.** 29	**8.** 48	**9.** 53	**10.** 27
×27	×63	×35	×53	×64

Extension

1. Study the figures below for finding the product of two 3-digit numbers.



2. Use the method shown in exercise 1 to find each product.

125	757	841	525
×365	×426	×215	×525

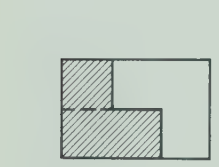
*3. Devise a rule for multiplying two 4-digit numbers.

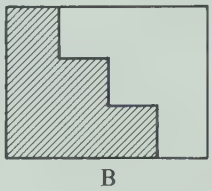
Lesson 3. Learning a Generalization

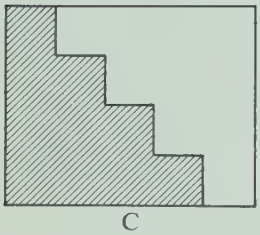
Can you find a pattern?

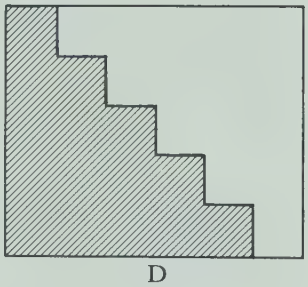
Investigating the Ideas

Use the small square in Figure A as the unit. Can you find the area of each shaded part in two different ways? For each part, write an equation to show that the two ways of calculating the area give the same result.


A

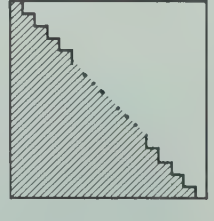

B


C


D

Discussing the Ideas

1. **A** Describe one way you found for finding area in the figures above.
   B Describe another way you found.
   C Did you find any other way?
2. Can you write an equation to show that these two methods give the same area?
3. **A** Suppose there are 50 vertical segments in the "stairsteps" of Figure E. What is the area of the shaded part?
   B Which of the two methods for finding the area would you use?
   C Can you write an equation about this?



4. Can you find the area of the shaded portion of Figure E if there are 100 vertical segments?
5. Can you use what you have learned so far to explain this generalization?

$$1 + 2 + 3 + 4 + 5 + \ldots + n = \frac{n \cdot (n+1)}{2}$$

Using the Ideas

1. Without adding each number, find the sum of the whole numbers through 25.
2. Find the sum of the first 75 whole numbers.
3. Find the sum of the first 200 whole numbers.
4. What is the sum of the first 1000 whole numbers?

Extension

1. What is this sum? $50 + 51 + 52 + 53 + \ldots + 99 + 100$
2. Can you find a short way to find the sum of
   A these even numbers? $0 + 2 + 4 + 6 + 8 + \ldots + 100$
   B these odd numbers? $1 + 3 + 5 + 7 + 9 + \ldots + 99$

*3. Can you state a rule for what you found in exercise 2 by using a variable?

Lesson 4. Learning Some Facts

Can you learn some "new" facts?

Investigating the Ideas

Many rapid "human Calculators" consider these products to be facts.

×	10	11	12	13	14	15
10						
11						
12						
13						
14						
15						

How many of these "facts" can you give without calculating?

(Record the facts you know and shade that portion of the table with a red pencil. Then fill in the remainder of the table by figuring out the remaining facts.)

Discussing the Ideas

1. Which facts in the table need not be memorized provided you know the others and also know the commutative principle? Shade these facts blue.
2. **A** How many facts altogether are in the table?
   B How many facts remain to be memorized?
3. **A** What is the "largest" fact?
   B Which facts are over 200?
   C Which facts are in the 190's?
   D Do you notice other patterns in the table that might help you remember certain facts?

Using the Ideas

1. Give these products as quickly as possible.

A 15×15	**E** 13×13	**I** 11×13
B 15×14	**F** 14×12	**J** 11×14
C 14×14	**G** 11×11	**K** 11×15
D 13×15	**H** 11×12	**L** 12×13

2. Make flash cards for the "facts" in exercise 1 that you do not know. Practice with a friend.
3. In exercise 1, start with part L and, following reverse order, give each of the products as quickly as possible.
*4. Make a large multiplication table with all numbers up to 20. Mark out the "facts" you know. How many of these "facts" are left to memorize?
*5. A person who knew the distributive principle and the facts in the table referred to in exercise 4 looked at the multiplication 143×15 and wrote 2145. How did he do it so quickly?

Extension

Study the facts for these powers of 2.

$$2^2 = 2 \times 2 = 4$$
$$2^3 = 2 \times 2 \times 2 = 8$$
$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$
$$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$$

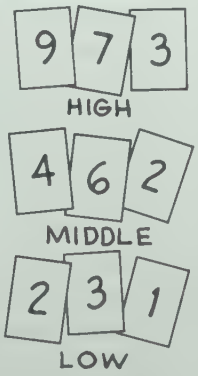
1. Give the next six powers of 2.
*2. Can you find some mnemonic aids to help you memorize the first ten powers of 2?

Lesson 5. Learning an Attitude

Let's try a place-value game.

Investigating the Ideas

Use 3 sets of 9 cards, each with the digits 1 through 9. Shuffle the 27 cards and deal 3 to each player. Each player then forms a 3-digit numeral, places his cards face down in order, and declares (starting to dealer's left and rotating clockwise) whether his number is high, middle, or low. Play the game in groups of three players.



Discussing the Ideas

1. One player arranged his cards like this and declared that he would try for the low hand. What was wrong with his strategy?



2. What is wrong with this arrangement for a middle hand?



3. If you were dealt these cards, would you try for a high or low? Why?



4. Suppose you are last to declare. Everyone else has declared either low or middle. What would you do with these cards?



Using the Ideas

1. Try playing this game with 2 or more other people.
2. Try the game with the rule that you can declare only high or low.
3. Make up rules for a game in which you turn up the cards one at a time starting with the ones' digit card.

Extension

1. Invent a place-value game in which 4 or 5 cards are dealt to each player.
*2. Find or invent another game or activity that strengthens understanding of the concept of place value.

V. Some Thoughts About Evaluation

The strategy of preparation, investigation, discussion, utilization, and extension is a flexible organizational plan that allows each teacher an opportunity to make a modest beginning toward an activity-oriented mathematics program. The lesson categorization of concept, skill, generalization, fact, and attitude provides a framework that allows each teacher an opportunity to apply the teaching strategy to various types of learning situations. Since there are different types of learning, it is reasonable to assume that there should be different types of evaluation used to measure these learnings.

When considering the facts and skills, for example, emphasis should be placed on child accountability. The teacher should determine the learning outcomes, consider performance objectives for these outcomes, and help the child attain these objectives. The evaluation of this attainment is most easily completed by use of fact and skill tests which determine the child's level of achievement. Since the child needs considerable practice in remembering facts and performing skills, the procedure for helping them is reasonably straightforward.

When evaluating concepts, generalizations, and attitudes, however, the desired performance objectives are often quite difficult to verbalize. We have mentioned earlier that concept learning often takes place

over a relatively long time span, that concepts are extended and broadened, and that concepts mature with each subsequent set of related experiences. Clearly, it is difficult to write a performance objective which specifies the exact level of concept maturity appropriate for a given child at a given time. Whenever possible, objectives for simple concepts should be written, and an attempt should be made to write test items which will show whether children understand these concepts. These items should involve requests for children to give examples of concepts, characteristics of a concept, and even, in some cases, a definition of the concept. For more difficult concepts, the evaluation of children's progress might be made through observation and recorded by means of a check-list which specifies certain levels of development for the given concept. The teacher should be alert for situations in which the child actually uses the concept correctly and should recognize also that understandings which are only partially developed indicate positive achievement. The teacher should also search for instances where the child has shown an ability to form concepts, for this is one of the desired learnings.

When evaluating the child's understanding of generalizations, the teacher should specify the simple generalizations which should be learned by all children. Specific performance objectives and the subsequent test items should be written to evaluate these generalizations. Beyond this, the teacher should again evaluate in greater depth through personal observations or interviews with the children. In the area of generalizations, the teacher should be ever aware that a child who is in the habit of looking for patterns or generalizations has learned a great deal. The teacher should also recognize that a child who can form a generalization from a sequence of specific examples has developed an understanding of a process that is extremely important. We would be remiss if we evaluated only the factual part of the learning of generalizations. As noted earlier, however, although these are important goals of mathematics learning, it is very difficult to write performance objectives for these goals. Whenever possible, objectives should be written which go deeper than facts and skills, but in the absence of objectives, the teacher should feel free to use other means of evaluation, including interviews to evaluate student learning.

While attitudes are not easy to measure in a conventional way, it is suggested that teachers frequently observe children and talk to them about their feelings about mathematics. It is important to realize that one's philosophy toward testing can also have a marked influence on the child's attitude toward mathematics. Testing should be reasonable and realistic, and the child should understand its purpose. The spirit of evaluation should be one of helpful assessment, rather than of critical evaluation. If children participate with teachers in understanding (if not in developing) the goals of instruction, the testing procedure can be a positive influence on the child's attitude and ability to improve.

It is hoped that the teacher will constantly take a broad view toward evaluating mathematics learning among his children. In the long run, evaluation of a child's learning should depend upon the interaction of that child and his teacher. For this interaction to be successful, it may be necessary for the teacher to reexamine his own beliefs about how children learn mathematics. As each teacher makes modest beginnings toward an activity-oriented approach to mathematics learning, he might ask himself the following questions:

1. Do I respect each child as an individual with unique interests, abilities, sensitivities, and significant thoughts?
2. Does the learning environment of my classroom provide a natural, free atmosphere in which children can explore, make decisions, be independent, and encounter exciting new experiences?
3. Does the learning experience also include a supportive, non-judgmental atmosphere in which children have enough routine activities to provide a comfortable threshold of security?
4. Is the child's need for earned success recognized in my classroom?
5. Do I recognize and treat mathematics as a dynamic, ever-growing discipline which offers limitless new vistas to be explored and an inexhaustible variety of new problems to be investigated and solved?
6. Do I view mathematics as a subject of beauty and a source of pleasurable fulfillment of intellectual curiosity?
7. Do I appreciate the significance of my role as a fellow-learner rather than merely a source of information?
8. Is my overall attitude toward mathematics one that encourages a basic freedom to learn through use of manipulative materials in an investigative environment, and through free discussion and exchange of ideas?

As a teacher evaluates the children in his class, he should also reevaluate his approach to mathematical learning. The goal of this short text has been to help in that reevaluation by encouraging the teacher to read, study, observe, experience, experiment, and reconsider. If that goal has been achieved, perhaps his resulting basic beliefs about children, mathematics, and evaluation methods will help him create a new climate of interaction that will spark more effective learning experiences in his classroom.

EXERCISE SET 10

1. Give a set of performance objectives for each lesson completed in Section IV.
2. Create an evaluation tool for each set of behavioral objectives given in exercise 1.

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INTRODUCING THE METRIC SYSTEM

Canada is committed to the metric system of measurement. You may be aware of this but may not have a clear idea of exactly what the metric decision means to you as a *teacher*. It is hoped that this section will serve three purposes—

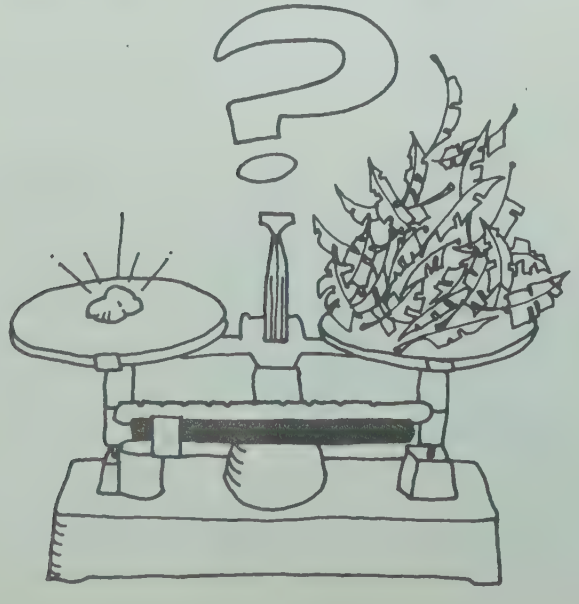
1. give you an idea of how the metric decision will affect you,
2. help you understand the metric system of measurement, and
3. give you some hints for teaching the metric system of measurement to your students.

History and Rationale

The English system of measurement developed from man's need to measure size and distances using units from the most readily available object—himself. He utilized his palm, span, finger, an ell, and a fathom for length; his foot, step, pace, an arrow's flight, and a day's journey for distance; and a handful, shellful, hornful, or gourdful for capacity.

There was little need for standardization until man began to travel and trade with other men. When "standard units" were developed, a new problem arose. Different countries used different definitions for the same unit. The foot was, at first, the length of any man's foot. In some countries, it was the length of the king's foot (since he was the "ruler") and this foot could change as the "rulers" changed. Later an effort was made to standardize some units; for example, England and Scotland decreed the foot to be 12 inches. Unfortunately, England and Scotland didn't use the same definition for the inch.

Today, in the age of technology, one still finds different units in those countries which are not yet metric. Canada and the United States are neighbouring countries, yet they use two different definitions for the gallon. A question at which people in metric countries must laugh is "Which is heavier, a pound of gold or a pound of feathers?" A pound of feathers is heavier since feathers are weighed by the avoirdupois pound (1 avoirdupois pound—7 000 grains) and gold is weighed by the troy pound (1 troy pound—5 760 grains). Which is heavier, an ounce of gold or an ounce of feathers? An ounce of gold is heavier. There are 12 ounces in the troy pound, so one ounce of gold weighs 480 grains; there are 16 ounces in the avoirdupois pound, so an ounce of feathers weighs 437.5 grains.



Out of such confusion there developed a need for a simple, standardized system of measurement. In 1670 Gabriel Mouton, a French abbé, developed a system of measurement organized according to the decimal system of numeration. It took over a hundred years for a system of measurement like the one Mouton put forth to get official sanction. In 1790 the French National Assembly appointed a committee to study the measurement situation and see if a rational system of measurement was possible. In 1795 France adopted a decimal system of measurement, defining the base unit of length to be the *metre* (from the Greek word *metron*, "a measure").

The metric system did not use parts of the human body as units. The metric system did not develop haphazardly adding more and more units as the need arose. The metre was defined as one ten-millionth of the distance from the North Pole to the equator, along the meridian passing near Dunkirk, Paris, and Barcelona. One can see that such a definition would be difficult to replicate in any one country. Also, the length of the metre changes as the position of the North Pole changes; at the time that the metre was defined, scientists were unaware that the position of the North Pole changed.

In 1870, because of the problem of replicating and comparing metric units from country to country, France called a meeting of the metric countries to develop a "unified metric system of measurement". In 1875, the *Treaty of the Metre* was signed to establish the General Conference on Weights and Measures which meets to determine the official definitions for the units used in the metric countries. In 1960 the Conference adopted the Système International des Unités (SI). It is this SI metric system that is most used throughout the world.

A Popular System

The popularity of the metric system stems from two characteristics—the high degree of standardization and its simplicity.

In the entire metric system there are only seven base units! They are **metre** (length), **kilogram** (mass), **second** (time), **ampere** (electric current), **degree kelvin** (thermodynamic temperature), **candela** (luminous intensity), and **mole** (amount of substance).

All units used in the metric system are related to these seven base units. The units you will be most concerned with (because they are the ones used in everyday living) appear in Table 1:

Table 1: Metric Units to be Studied

Quantity	Unit	Symbol
Length	metre	m
Mass	kilogram	kg
Capacity	litre	ℓ*
Temperature	degree Celsius	°C

*As a rule of thumb, the cursive letter (ℓ) is used as a symbol for the litre to avoid confusion with the numeral (1), however, in symbols such as ml (millilitre), kl (kilolitre) the cursive form is not used.

All other units to be discussed can be represented by the product of one of the units and a power of 10. For example, every possible unit of length can be developed by multiplying the number of metres by the appropriate power of 10.

Table 2: Metric Units of Length

Name (Symbol)	Metres
*kilometre (km)	10^3m or 1000 m
hectometre (hm)	10^2m or 100 m
decametre (dam)	10^1m or 10 m
*metre (m)	10^0m or 1 m
decimetre (dm)	10^{-1}m or $\frac{1}{10}$ m
*centimetre (cm)	10^{-2}m or $\frac{1}{100}$ m
*millimetre (mm)	10^{-3}m or $\frac{1}{1000}$ m

*preferred units

To make the system simpler the same prefixes are used with all units. For example, a millimetre (mm) is $\frac{1}{1000}$ of a metre, a millilitre (ml) is $\frac{1}{1000}$ of a litre, a milligram (mg) is $\frac{1}{1000}$ of a gram, etc.

According to the class, you may want to introduce the symbol "m" for metre, "cm" for centimetre, etc. The plurals, metres and centimetres, are also symbolized "m" and "cm", not "ms" or "cms." Remember, these are symbols and not abbreviations and no period is used after a symbol.

Countries which have been completely metric for several years find that some terms such as "decimetre" are not used in everyday living. People will talk of a book being 28 centimetres long rather than 2.8 decimetres long. You may wish to explain the term "decimetre," but it is not necessary.

Most people who feel that the metric system is complex are those who convert back and forth between the metric and English systems of measurement. When teaching the metric system, conversion to the English system is not necessary and should be avoided!

The metre is defined world-wide to be 1 650 763.73 wave lengths in a vacuum of the orange-red line of the spectrum of krypton 86. This is quite a definition! There are two reasons why such a complex definition was adopted –

1. the length never varies and

2. this measurement can be replicated in laboratories throughout the world.

From this brief history of the metric system it is hoped you will take three main thoughts –

1. The metric system resulted from concentrated effort to develop a rational system of measurement. It did not develop haphazardly.

2. The problem of standardization has been solved in the metric system.

3. The metric system is both popular and useful because of its simplicity.

Activities

Experience and activity are key words in the teaching of measurement. Measure things! The success of this material will depend upon the amount of experience each participant has with the activities. The limited number of activities that are presented should stimulate possibilities for many more. Although the content is approached through activities and measuring experiences, there is a need for exercises to further these experiences and to structure metric thinking. Two points should be emphasized –

1. It is *important* that *you* as well as your class do the activities in this section.

2. The activities will be more fun if done in a group situation.

Looking at Table 1 in the *History and Rationale* section, you will notice that you have to be concerned with only four base units. So, let's use the frontal attack, start right in on length, and begin inching our way down the metric road.

Length, Area, and Volume

In the groups where the metric system has been argued for years, there were two camps. One group wanted to use the centimetre, gram, and second for the core of the system and the other the metre, kilogram, and second. The latter group has prevailed.

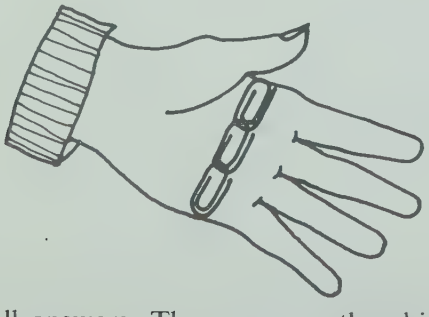
It is strongly urged that first grade teachers **not** start with the metre. It is very difficult for first graders to handle a metre ruler. The same argument may be advanced for the kilogram and litre. Length will be approached as it should be covered with students, i.e., first measure with arbitrary units, then use the centimetre, next use the 10-centimetre (decimetre), and finally the metre. All measurement should be approached as a three step process—

1. Select a unit.
2. Partition the object to be measured into units.
3. Count the number of units used. That number is the measure of the object.

ACTIVITY 1

Measuring objects with an arbitrary unit. Students should do several activities of this type using arbitrary units such as their thumb, a paper clip, pencil, crayon, cutout of their shoe, width of their hand (a unit in the English system used for measuring the height of horses), cubit (another "English" unit, the length of the forearm from the elbow to the tip of the middle finger), or other selected units. For your experience measure the chalk eraser, the width of your hand, the width of this book, and the length of a pencil using a paper clip as the unit.

In the illustration, a "paper clip train" is being used to measure the width of a hand. Follow the three steps mentioned previously in the measurement process.



Record all answers. Then measure the object again using pieces of paper the length of a thumbnail. Repeat the process measuring other objects.

In class emphasize four points—

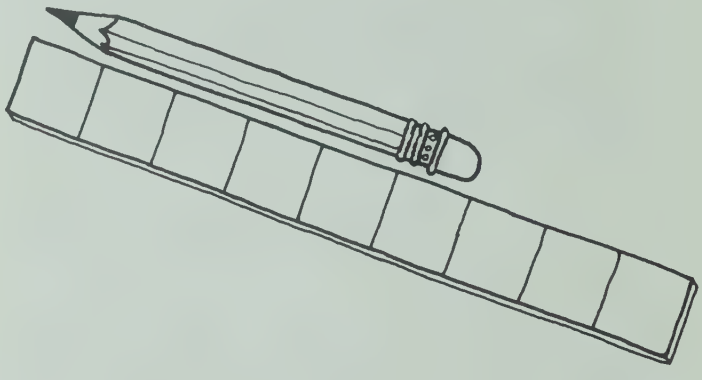
1. The first unit should be lined up with the "starting point" of the object.
2. The units should touch, but not overlap.
3. The "train" should be straight.
4. The units should be "rounded off" to the unit that has its right end nearest to the "finishing point" of the object.

In doing activities where arbitrary units are used, the need for standardized units becomes obvious. Ask several children to measure the same object, each with his own pencil. On the chalkboard, place their statements such as "The table (or whatever object you pick) is 5 pencils wide." "The table is 7 pencils wide." "The table is 8 pencils wide." Children will soon see that when pencils of differing lengths are used, different answers will result.

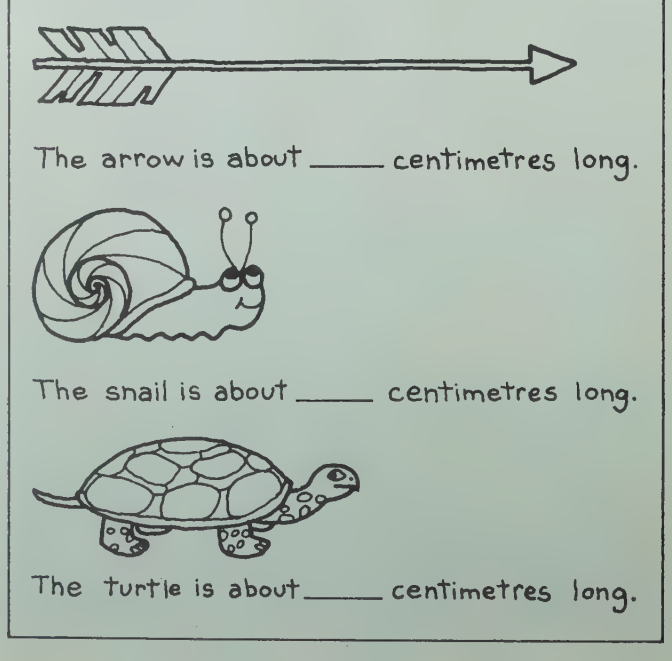
ACTIVITY 2

Developing the concept of a centimetre. Probably the first metric unit the children will make use of is the centimetre. You will need (and each student in the class will need) 9 centimetre strips—9 pieces of paper or cardboard 1 cm by 1 cm square.

The children, especially the younger ones, should have the experience of measuring many objects using centimetre strips. (If at the time you present this activity your students have studied two-digit numbers, have them measure objects longer than 9 cm.)



Using the centimetre strips, measure the length of a paper clip, a piece of chalk, the Cuisenaire 6-rod, the width of a hand, and the width of a thumb to the nearest centimetre. In this initial activity, actually use centimetre strips and not a ruler marked in centimetres. An exercise the children can do at their desks is to measure the pictures of objects drawn on a duplicator master. The pictures can be of predetermined length. Measure the pictures below.



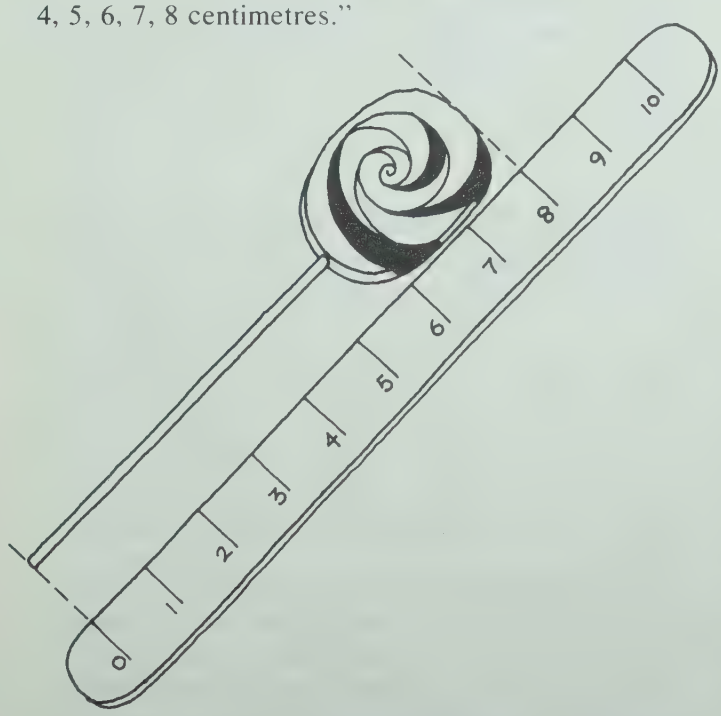
In exercises like these, the length can be controlled. Some answers should require "rounding up," and some "rounding down." The word "about" is important in the sentence since a measurement is an approximation. As the children progress you can have them write not only the number but also the name of the unit.

ACTIVITY 3

Measuring with centimetre rulers. When the children have learned to use the centimetre strips in the measurement process, a ruler marked off in centimetres (not millimetres) should be introduced. It is strongly urged that the child construct his own 10-cm ruler during his first introduction to metric measure. He can do this by constructing a 10-cm train on a 10-cm long piece of paper, pasting the train on the paper, then numbering the cars from 1 to 10. Another approach is to construct a 10-cm ruler in front of the class. Then hand out 10-cm long pieces of paper already marked off in centimetres and have the children number the centimetres from 1 to 10.

The next few activities should involve the measuring of an object with a centimetre train, a 10-cm ruler, and finally with only a 10-cm ruler. When measuring an object with a 10-cm ruler work toward getting your students to "read the ruler" rather than counting the centimetres as they did with the trains.

In the example illustrated the child should learn to round off to the nearest centimetre and then read the ruler, "8 centimetres," instead of counting "1, 2, 3, 4, 5, 6, 7, 8 centimetres."

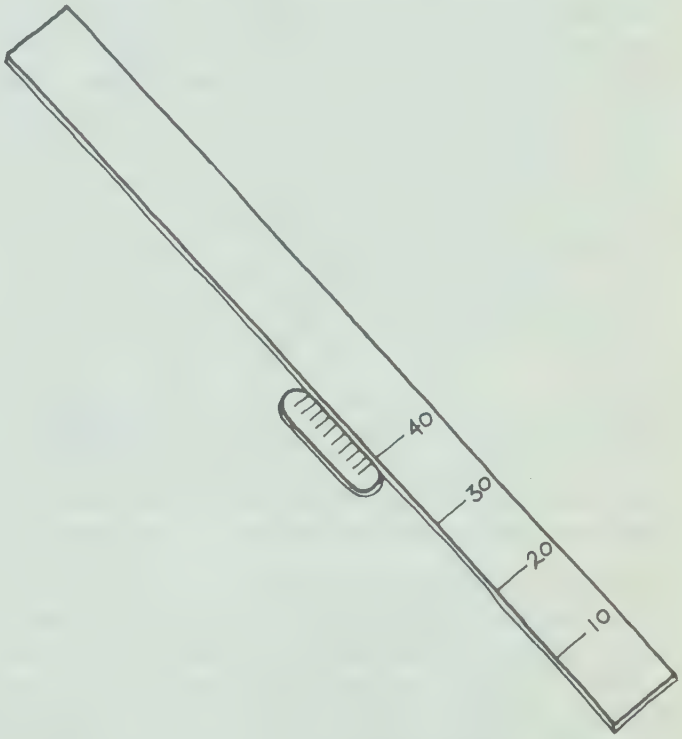


After the children have become skilled in using a 10-cm ruler, they should be given activities requiring them to measure objects which are longer than 10 cm. When working with 5-and-6-year olds, be careful that the measure of the object is not a number the children haven't studied. In the activities concerning measurement it is the process that should be emphasized; the numbers themselves should never be a source of difficulty.

Now, using your 10-cm ruler, measure the length and width of this book and length of your forearm, the length of your foot, and length of your span (what is your span?).

ACTIVITY 4

The metre and notation. Initially, you may want to have your students measure objects with metre-long strips of unmarked cardboard. Then ask them to number the centimetres on the metre strip in groups of 10 using their 10-cm strips. Before proceeding



further, have the class subdivide these cardboard metre rulers into centimetres. It is important that you do the activities with the same type of ruler your students will use. If you have a classroom set of wooden metre rulers, use one of them. Ideally, the rulers used should be marked off in centimetres, but if the ruler is marked off in centimetres (cm) and millimetres (mm) no harm is done. Measure the length, width, and height of your desk rounding off to the nearest metre.

The measurements for a desk, accurate to the nearest metre, might be 2 m long, 1 m wide, and 1 m high. Such measurements would not be helpful. The metre is used for much longer measurements, such as the length and width of the classroom, the playground, the school, the block, etc. To measure the dimensions of objects such as desks, tables, bookshelves, and people, a metre ruler may be used and the results recorded in centimetres. For example, a desk may be 152 cm long, 76 cm wide, and 74 cm high.

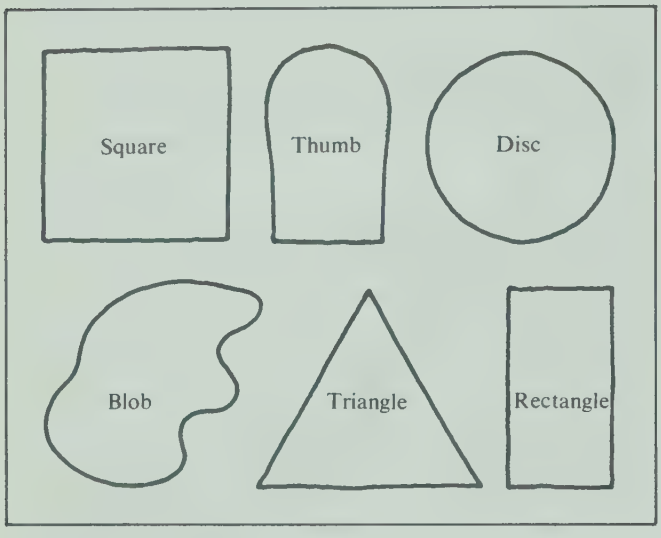
You might say: I am 178 cm tall; what is your height (in centimetres)?

Just as 153 cents is written as \$1.53, 153 centimetres is written as 1.53 metres. This can be interpreted as 1 metre and 53 centimetres which is read as "one point five three" metres. Do not dwell on the mathematical use of the notation—it is not necessary!

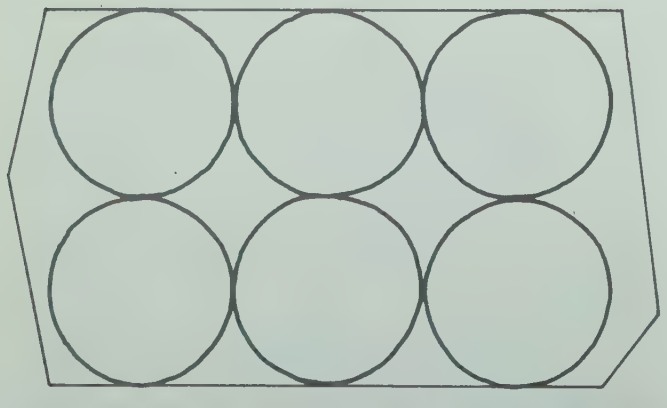
With your class, record the dimensions of your classroom, your desk, their desks, your height, and their heights in terms of centimetres, then in terms of metres using the decimal notation.

ACTIVITY 5

Area using arbitrary units. Here are some examples of area units:



Let the children give names to the units. Then follow the measurement process: select one of these units, match it against the area of some object, and count the number of units used. For example, the irregular figure below has an area of about 6 discs



(if disc is the name given to the unit used). Emphasize that you are trying to "cover" the object. The units should be "even with the edge" of the object, the units should touch, but not overlap, each other. Direct the children's attention to the parts of the object that are not "covered."

Make a cutout of some irregular area such as your thumb and make copies of it out of paper. Use your "thumb" to find the area of the top of a chalk eraser, of the irregular figure measured with the discs, of a cutout of your shoe, and of figure X.

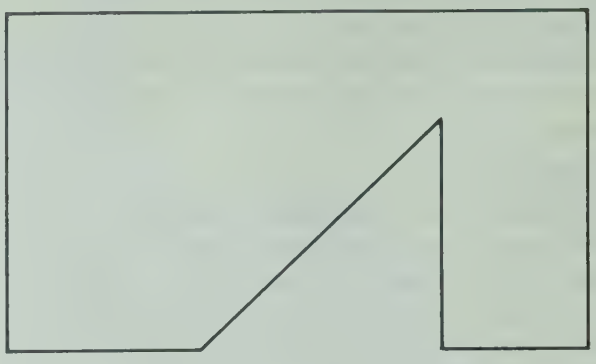


Figure X

Record the answers on the chalkboard in sentence form—

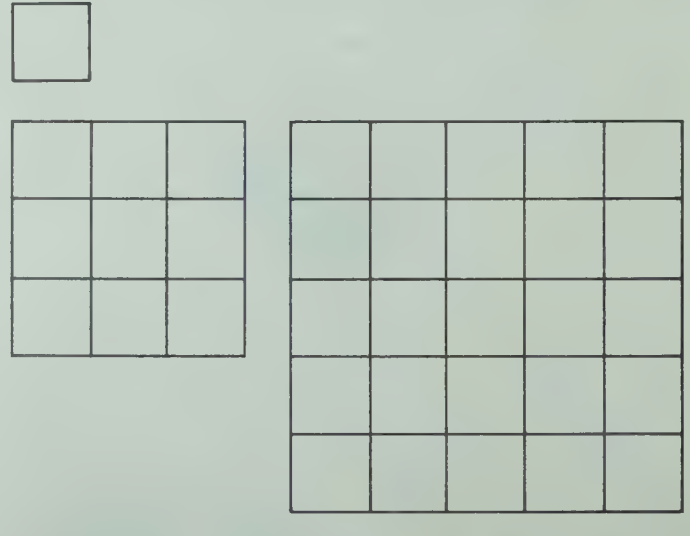
"The figure has an area of about _____ thumbs."

Have your class perform similar activities.

ACTIVITY 6

Area using the centimetre square (cm²). Have the children make centimetre squares (or have them available for use). The children should have the experience of finding the area of many objects.

Make duplicator masters for some areas that the class can measure with their centimetre squares. The figures below are 1 cm², 9 cm², 25 cm², respectively.



You might point out that the square containing the 9 cm² has a side of 3 cm and the square containing the 25 cm² has a side of 5 cm.

Have the children use their centimetre squares to find the *area* of a stamp, a 10-cm ruler, the cutout of their thumb, the irregular figure which had an area of 6 discs, and figure X.

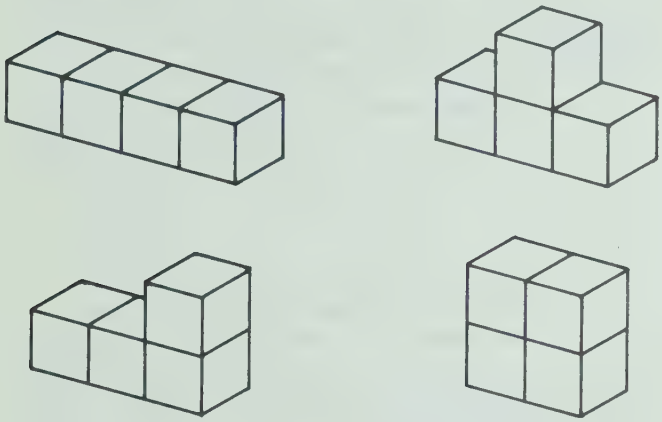
ACTIVITY 7

Volume, using the centimetre cube. In the initial development of the concept of volume, it is important

that children have the opportunity to construct several differently shaped objects each having the same number of volume units.

As with length and area, the study of volume should be introduced with activities making use of arbitrary units of volume, such as blocks, Cuisenaire rods, pencils, erasers, or even marbles.

Use 10 or 12 centimetre cubes in this activity. At first, let the children work on their own, constructing any objects they like. Encourage them to see that an object built of a specific number of cubes has a volume of the same number of cubes regardless of its shape. For example, the illustration shows 4 different constructions, each having a volume of 4 centimetre cubes (4 cm^3).



How many differently shaped objects can be constructed with a volume of 8 centimetre cubes? When those possibilities have been exhausted, try the activity with 10 cubes.

REVIEW: LENGTH, AREA, AND VOLUME

1. Have your class compare the length of their feet, spans, and cubits. Why are these units useless as standard units?
2. Complete these statements.

a.	128 cm = ____ m	**e.**	1.06 m = ____ cm
b.	108 cm = ____ m	**f.**	10.01 m = ____ cm
c.	15 cm = ____ m	**g.**	23.86 m = ____ cm
d.	1010 cm = ____ m	**h.**	0.09 m = ____ cm

3. What would be the length of the sides in a square containing:
   a. 36 cm^2 → ____ cm?
   b. 25 cm^2 → ____ cm?
   c. 4 cm^2 → ____ cm?
   d. 16 cm^2 → ____ cm?
4. How many different-shaped objects can you form with 6 centimetre cubes?

Capacity

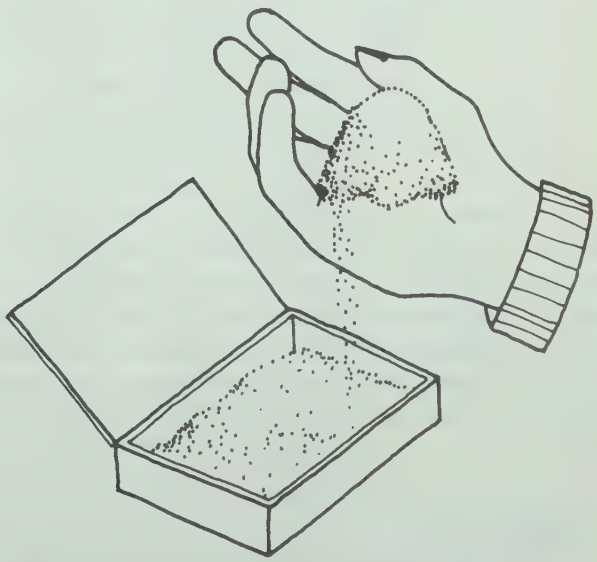
Capacity can be thought of as the amount of material a container will hold. Capacity is usually linked to liquid measure though you may have already had your classes measure capacity by using sand to avoid using liquids.

In the metric system of measurement, volume and capacity are directly related. A container with a volume of 1 cubic centimetre (1 cm^3) will hold 1 millilitre of water. One millilitre (1 ml) is one thousandth of a litre (0.001 ℓ).

The need for fractional names such as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{4}$ etc. will diminish. The parts of the whole which need emphasis are—0.1, 0.2, 0.3, . . . , 0.9. Of course, in measurement, fractions could disappear completely, since $\frac{3}{8}$ of a meter is 0.375 m or 375 mm. However, when working with the litre (the unit of capacity in the metric system) don't worry now about using $\frac{3}{4}\ \ell$, $\frac{2}{3}\ \ell$, etc. if it is the amount you want the children to see or work with. Since the metric system is based on 10 and since 1, 2, 5 and 10 are the only divisors of 10, we will probably talk about halves, fifths, and tenths of metric units. The decimal notation ($\frac{1}{2}$ is 0.5) will prevail eventually, even at the primary level.

ACTIVITY 8

Capacity and arbitrary units. The most obvious capacity units are handfuls. Give each child a container to fill with water or sand or other material you prefer to use. Have the children fill the container



(milk carton, ice cream carton, cigar box, etc.) with "handfuls" of material. Have them record their results on a piece of paper: "My carton holds _____ handfuls of _____." Compare the wide range of results. Re-emphasize the need for a standard unit to measure capacity. If further experience is necessary, you may want to repeat the project with cups brought from home (since there are so many different sized and shaped cups). Try the activity yourself or get several containers such as an ice cream carton, a milk carton, a wastebasket, a big cooking pan, and a litre container.

On a piece of paper write a pair of sentences for each container:

"The (name of container) holds about (guess) litres.

The (name of the container) actually holds (result) litres.

In the first blank "guestimate" the number of litres the container will hold. In the second, write in the results of measuring the object.

Don't forget the three step measuring process—

1. Select the unit—the litre.
2. Match the unit against the object—fill the object using the litre.
3. Count the number of units (litres) used.

When the container is full (it is best to have a "fill line" just below the top of the container) round off to the nearest whole litre according to whether more or less than half of the last litre was used.

ACTIVITY 9

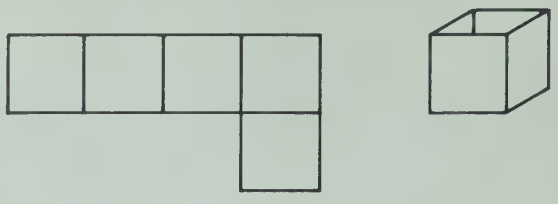
Working with the litre. Get a container that holds a litre of water(and, ideally, has submarkings for each 100 ml). When you are collecting containers for your classroom, try to get as many different shapes as you can. It is important, especially in early experiences, that the children see that litre containers can come in many different shapes. It is the quantity the container will hold, not its shape that determines a capacity of 1 litre.

Once you get a litre container you can make many more. Pour a litre of water into a container and mark a "fill line" for 1 litre on the outside with tape, or, if possible, cut the container so that it holds just 1 litre. Suggested existing containers which can be cut are quart, half-gallon, and gallon milk cartons, round quart, half-gallon, and gallon ice cream cartons. Containers that can be marked might be various shaped pans, cooking bowls, large tin cans, and bottles or jugs. Most activities for introducing the metric units should be accompanied by some estimation exercises. Have the students estimate and record how many litres a container will hold, then measure the container to see about how many litres it does hold. Compare records.

ACTIVITY 10

Introducing the millilitre. The litre is a unit for capacity that is used for milk, gasoline, paint, and other quantities of considerable size. The litre is not used to measure small quantities, such as toothpaste, soda pop, medicines, frozen orange juice, etc. The unit used for the smaller measures is the millilitre (ml). If your school is going to get a set of metric capacity containers, try to get them in these sizes—1 ℓ, 500 ml, 200 ml, 100 ml, 50 ml, 20 ml, and 10 ml. With such a set (whether bought, given, or constructed) one can do all the activities that are necessary.

Construct a container with a volume of 1 cubic centimetre (1 cm^3) to demonstrate the size of the millilitre (ml). Trace the figure below, then cut it out and tape it together along the edges. If you avoid spillage your cube will hold 1 ml of water.



The children need several activities measuring the capacity of objects and recording the results in millilitres. Have them first guess and then measure the capacity of a thimble, a match box, a tablespoon, and a teaspoon. Record the results in sentences like—

"I estimate that the thimble holds about _____ ml.
It actually holds about _____ ml."

Mass

As the metric system becomes the predominant system of measurement you may hear talk about the difference between mass and weight. A lunar example may be the best way to show the difference. Now that we are in the space age, practically everyone knows that a man weighs less on the moon than he does on the earth. For example, a 300-kg man on earth would weigh about 50 kg on the moon, but he would have the same mass on the moon as he does on earth. Weight is dependent upon gravity, mass is not. Begin to stress the use of the correct metric term, mass.

The base unit of mass in the metric system is the kilogram (kg). For example, we say "I have a mass of 78 kg.

ACTIVITY 11

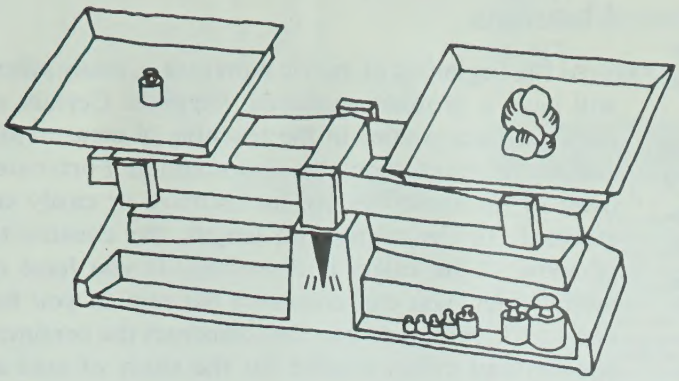
Arbitrary units of mass. To find the mass of an object you will need a balance and some arbitrary units such as paper clips, pencils, Cuisenaire rods, pennies, or other objects. Put a pencil on one side of the beam and then "balance the pencil" with pennies (or multiples of any other small unit). Record the results on paper in a sentence like:

"The pencil has a mass of about _____ pennies."

Repeat the activity with at least three other objects.

ACTIVITY 12

The unit used for small masses is the gram (g). This activity is very similar to the last. You will need gram masses. If you have a classroom set, that's great! If you don't, you can make one.



Put a gram mass on one side of the balance and balance it with a lump of clay or plasticine. Label your clay "1 g." In a similar manner make a set of clay or plasticine "masses" in multiples such as: 5 g, 10 g, 20 g, and 50 g. Use several small objects as test objects (a paper clip, a nickel, a penny, and a pencil). However, before you have the children put one of the test objects on the balance, ask them to estimate its mass in grams. Then find the mass of the object. Record both the guess and the result.

The quarter has a mass of about (guess) grams.

It actually has a mass of (result) grams.

Repeat the activity using other objects. Do you and the class get better at estimating mass?

ACTIVITY 13

Measuring mass using the kilogram. Hopefully, all schools will have metric scales available for finding the mass of children and other large objects using kilograms. For this activity, have each child find his own mass and then make and label a cutout of himself (perhaps using his projected shadow). Have him record his height and mass in metric units on the cutout.

Then you and your class might measure the mass of other objects, such as your own chairs, the textbooks used in the course of one day, litre of water (don't count the container—first find its mass when empty), a dictionary, and even the principal of the school (if he agrees). As mentioned earlier, there is a direct relationship between volume and capacity in the metric system of measurement. In fact, there is a direct relationship between volume, capacity, and mass. A container whose volume is 1 cubic cm (cm^3) holds 1 ml of water and the 1 ml of water has a mass of 1 g. A container whose volume is 1000 cubic cm (or 1 cubic decimetre) holds 1000 ml of water (or 1 litre), and the water has a mass of 1000 g (or 1 kilogram). What did you get for the mass of one litre of water?

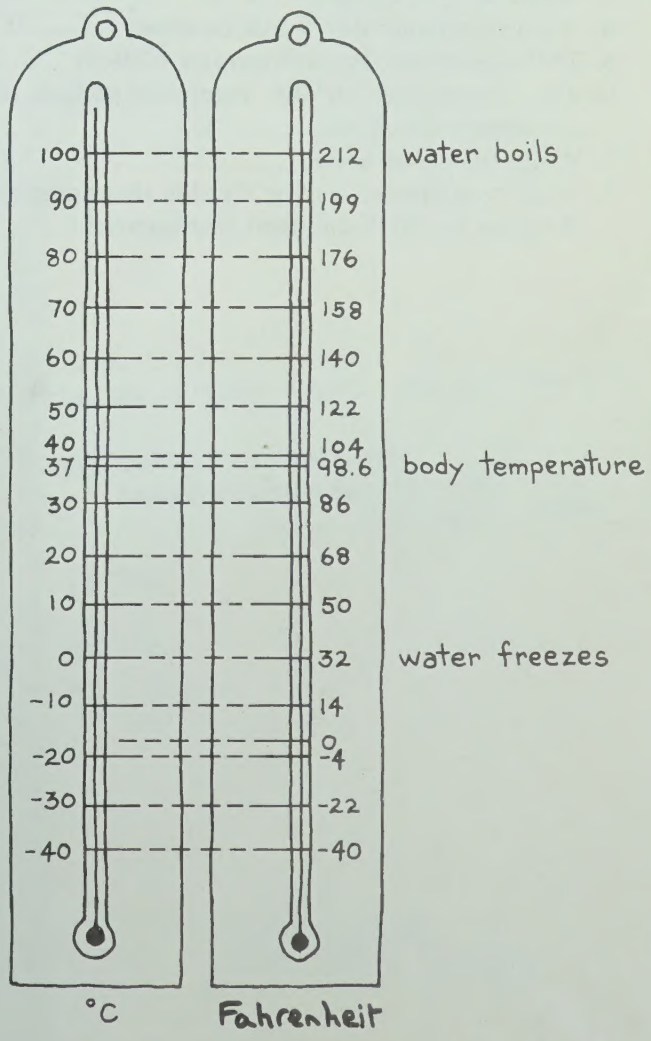
N.B. It is a good idea to label some of the objects in the room as you introduce each metric unit. For example, the aquarium may be 70 cm long, 40 cm wide, 35 cm high; have a water surface area of 2 800 cm^2, volume of 98 000 cm^3; a capacity of 98 ℓ of water and a mass of 12 kg. If the children label the objects as they study particular units, they will begin to think metric.

REVIEW: CAPACITY AND MASS

1. When finding the mass of something using a balance beam, how do you decide which unit to round off to?
2. Fill in the answers:
   - **a.** 28 ml of water has a mass of about _____ grams.
   - **b.** 170 ℓ is _____ ml.
   - **c.** 3.12 kg is _____ g and 438 g or _____ kg.
   - **d.** It would take _____ ml of water to balance 1 kg.
3. **a.** Will a car get a higher or a lower number of miles per litre than miles per gallon? (Is the litre larger or smaller than the gallon?)
   - **b.** Will a car get a higher or a lower number of kilometres per gallon than miles per gallon? (Is the kilometre longer or shorter than the mile?)
   - ★ **c.** Gasoline consumption rates will be given in kilometres per litre. Will a car get a higher or a lower number of kilometres per litre than miles per gallon?

Temperature

This last section covers the introduction of a metric unit, the degree Celsius (°C), for which there is no physical model. On the Celsius scale for temperature, water boils at 100° C and freezes at 0° C. The unit is named after the Swedish scientist, Anders Celsius, who created the centigrade temperature scale. The



Celsius and centigrade scales are the same, but centigrade is no longer the proper term since the centigrade is a unit used to measure angles in the metric system.

The best way to get used to the Celsius temperature scale is to use it! It is almost a necessity that you have a Celsius thermometer. However, if you have a demonstration model of the Fahrenheit thermometer, you can rescale it using the nomograph shown here.

ACTIVITY 14

Graphing temperatures. Be sure to give the children lots of opportunities to read the temperature and record it in degrees Celsius (° C). Perhaps you could institute a morning weather report given by a different child each day to get the class to use Celsius thermometers and to give them a feeling for what the temperature is when expressed in degrees Celsius (° C). The previous day's high and low temperatures (taken from a newspaper account) could be recorded on a wall graph.

REVIEW: TEMPERATURE

1. My body temperature is about _____ °C.
2. Normal room temperature is about _____ °C.
3. Water boils at about _____ °C.
4. A warm summer day would be about _____ °C.
5. The temperature in a refrigerator is about ____ °C.
6. The temperature in the vegetable section of a supermarket is about _____ °C.
7. Water freezes at about _____ °C.
8. What temperature on the Celsius thermometer is the same on the Fahrenheit thermometer? ____ °C

Materia

At the b

will have a problem gathering supplies. Certain materials are necessities in the teaching of measurement and metric measurement is no exception. Fortunately, most of the materials are inexpensive or easily constructed. In the section on length, the construction of some of the rulers is discussed. If you have one metric ruler, you can construct the rest. If you have one metric ruler, you can also construct the centimetre squares and cubes needed for the study of area and volume.

The construction of units of capacity and mass have also been discussed. When it comes to temperature you should have a thermometer available for classroom use. If it is a Fahrenheit thermometer, then you should rescale it to degree Celsius using the nomograph given earlier.

Following is a list of companies and government agencies that are currently producing materials or can give some assistance with this problem of teaching the metric system of measurement.

Addison-Wesley (Canada) Ltd.—Don Mills, Ontario
Buntin Gillies & Co. Ltd.—Ottawa, Ontario
Cameron Products—Bramalea, Ontario
Canadian Metric Association—(P.O. Box 35)—Fonthill, Ontario
Contrasts 20—Calgary, Edmonton, Vancouver, Winnipeg, Regina (Nearest Barber-Ellis Office)
Kruger Pulp and Paper Ltd.—Moncton, Toronto, Hull, Montreal (Nearest Office)
Information Canada (Under Government of Canada) (Nearest Office)
Jack Hood School Supplies Co. Ltd.—Stratford, Ontario
Lufkin Rule Co. of Canada Ltd.—Don Mills, Ontario
Lily Cups Ltd.—Scarborough, Ontario
MacLean-Hunter Learning Materials Co.—Toronto 101, Ontario
Metric-Aids Ltd.—Toronto, Ontario
Moyer-Vico Ltd.—Moncton Weston, Winnipeg, Saskatoon, Edmonton, Vancouver and the Longueuil Co. in Chambly (Nearest Office)
The National Council of Teachers of Mathematics—1906 Association Drive, Reston, Virginia 22091
Sargent-Welch Scientific Co. of Canada Ltd.—Weston, Ontario
Spectrum Education Ltd.—Toronto, Ontario
Spicars International Ltd.—Scarborough, Ontario
Toronto Dominion Bank (Nearest Office)

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